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Production, Hedging, and Speculative Decisions with Options and Futures Markets

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This paper analyzes production, hedging, and speculative decisions when both futures and options can be used in an expected utility model of price and basis uncertainty. When futures and option prices are unbiased optimal hedging requires only futures (options are redundant). Options are used together with futures as speculative tools when market prices are perceived as biased. Straddles are used to speculate on beliefs about price volatility and to hedge the futures position used to speculate on beliefs about the expected value of the futures price. Mean-variance analysis in general is not consistent with expected utility when options are allowed.

Key words: futures markets, hedging, options, price uncertainty, risk.

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PRODUCTION, HEDGING, AND SPECULATIVE DECISIONS WITH OPTIONS AND FUTURES MARKETS

One extension of Sandmo's expected utility model of the competitive firm under price uncertainty considers the use of futures or forward contracts. Danthine, Holthausen, and Feder, Just, and Schimtz show that without basis uncertainty the optimal output level is not affected by price risk; also, with an unbiased futures price the optimal hedging level of the competitive firm is the full hedge, while a biased futures price will result in a partly speculative hedge. Related works include Batlin, who allows for basis risk in the form of imperfect time hedging; Paroush and Wolf, and Antonovitz and Nelson, who consider basis risk with the simultaneous availability of futures and forward contracts; Grant, Honda, Losq, Newbery and Stiglitz, and Rolfo, who allow for production uncertainty; Chavas and Pope, who allow for production uncertainty and hedging costs; and Karp, who considers the problem in a dynamic setting.

This paper provides a further extension of this analysis by allowing options as a means of coping with price risk. With the introduction of commodity options on futures for many commodities in the 1980s, this problem appears relevant to a number of production settings, especially in agriculture. Specifically, this paper considers the simultaneous choice of a production level and of hedging levels of futures and options within the general expected utility model. The model allows for basis uncertainty, but the production process is assumed non-stochastic.

Optimal hedging when options are available is considered by Wolf in a linear mean-variance framework without including production decisions.[1] The mean-variance framework has been employed in a number of risk management studies. Under certain assumptions, this framework is consistent with expected utility maximization (Meyer; Robison and Barry). However, the inclusion of commodity options in a decision maker's portfolio leads to a violation of the two main conditions for a mean-variance representation of expected utility. First, options truncate the probability distribution of price so that the argument of the utility function, profit or wealth, is not normally distributed even if the random price is normal. Second, the use of options generally means that the argument of utility is not monotonic in the random attributes. Thus, relaxing the mean-variance framework appears desirable to analyze options in a hedging problem.

The paper is organized as follows. A model of production and hedging with both futures and put options is formulated. Some general results are derived for the pure hedging case in which producer price expectations agree with those embodied in the market price of futures and options. This is followed by an analysis of how changes in asset prices (or expectations) affect optimal portfolios under CARA. Next, the model is reformulated in terms of futures and straddles. This reformulation highlights the impact that individual beliefs have on speculative decisions, particularly on the use of options, and illustrates the limitations of mean-variance analysis. The concluding section summarizes the main contributions of the paper.

Production and Hedging with Futures and Options

The notation is defined as follow: y is the output quantity produced; x is the futures quantity sold; z is the put option quantity sold; p is the futures price at the end of the period; f is the futures price at the beginning of the period; b is the local cash price (including basis risk) at the end of the period; r is the put option price (premium); k is the strike price; v is the terminal value of a put option; π is the profit at the end of the period; and, $\tilde{}$ denotes a random variable. Because one can construct a synthetic call using futures and puts, attention is restricted to put options only. Also, for simplicity only one available strike price for the option is considered.

The random end-of-period profit of the firm using both futures and put options can then be written as:[2]

$$(1) \qquad \tilde{\pi} = \tilde{b}y - c(y) + (f-\tilde{p})x + (r-\tilde{v})z$$

where c(y) is a strictly convex cost function dual to a concave production function,[3] and $\tilde{v}$ is the terminal value of a put option, defined as:

$$(2.1) \qquad \tilde{v} = 0 \qquad \text{if } p \geq k$$

$$(2.2) \qquad \tilde{v} = k - \tilde{p} \qquad \text{if } p < k$$

where p is a realization of the random variable $\tilde{p}$.

The producer's utility is a strictly concave function defined over profit, that is $u = u(\tilde{\pi})$ where $\tilde{\pi}$ is given by (1). Thus, the individual is risk averse, but no other restrictions are placed on his/her preferences. The problem is to choose (y,x,z) to maximize expected utility, that is:

$$(3) \qquad \max_{y,x,z} \mathcal{L} = E[u(\tilde{\pi})]$$

where E denotes the mathematical expectation operator.

The first order conditions (FOC) require $\mathcal{L}_y = \mathcal{L}_x = \mathcal{L}_z = 0$, where the subscripts to $\mathcal{L}$ denote arguments of partial differentiation, that is:[4]

$$(4.1) \qquad E[\tilde{u}'(\tilde{b}-c')] = 0$$

$$(4.2) \qquad E[\tilde{u}'(f-\tilde{p})] = 0$$

$$(4.3) \qquad E[\tilde{u}'(r-\tilde{v})] = 0$$

where $\tilde{u}'=du/d\tilde{\pi}$, and $c'=dc/dy$.

To characterize the solution of these equations it is necessary to be specific about the relationship between local cash price and futures prices. Following Benninga, Eldor, and Zilcha, and others, the cash price is written as a linear function of the futures price:

$$(5) \qquad \tilde{b} = \alpha + \beta\tilde{p} + \tilde{\theta}$$

where $\tilde{p}$ and $\tilde{\theta}$ are independently distributed and $E(\tilde{\theta})=0$. Because of the definition in (2), equation (5) also uniquely defines the relationship between the terminal value of the put option and the cash price. Using (5), the FOC in (4) can be rewritten as:

$$(6.1) \qquad E[\tilde{u}'(\alpha+\beta f+\tilde{\theta}-c')] = 0$$

$$(6.2) \qquad E[\tilde{u}'(f-\tilde{p})] = 0$$

$$(6.3) \qquad E[\tilde{u}'(r-\tilde{v})] = 0$$

where (6.1) uses (4.2) rewritten as $E[\tilde{u}'(\beta f-\beta\tilde{p})]=0$.

The Case of Unbiased Prices

For any given utility function, the solution of equations (6) will depend crucially on the decision maker's perception of the futures price and option value distribution relative to the prices f and r. To account for this, it is convenient to define the notion of price bias.[5]

Specifically, if the producer's expectation of the end-of-period futures price equals the price of a futures contract, i.e. $\bar{p} = E(\tilde{p}) = f$, then the futures price is unbiased. Similarly, the expected (gross) return from the option position, $\bar{v} = E(\tilde{v})$, is:

$$\bar{v} = \int_0^k (k-p)\ \psi(p)\ dp \tag{7}$$

where $\psi(p)$ is the density function of the distribution of price as perceived by the producer. Following Black and Scholes, general equilibrium option pricing formulae assign a price to the option which, in our framework, is equivalent to the expected returns from the option (i.e. risk attitudes do not matter). Thus, the producer will perceive options to be fairly priced if $r = \bar{v}$, and in this case the option price is unbiased.

To analyze the solution, the strategy is first to consider the benchmark case of unbiased prices. The effects of biased prices are investigated in terms of comparative statics results from this benchmark. For unbiased prices ($\bar{p} = f$ and $\bar{v} = r$), and taking the non-random elements out of the expectation operator, the first order conditions (6) can be rewritten as:

$$\text{Cov}(\tilde{u}', \tilde{\theta})/E\tilde{u}' = c' - \alpha - \beta f \tag{8.1}$$

$$\text{Cov}(\tilde{u}', \tilde{p}) = 0 \tag{8.2}$$

$$\text{Cov}(\tilde{u}', \tilde{v}) = 0 \tag{8.3}$$

where Cov(.,.) denotes the covariance operator.

Consider first the solution for the optimal futures and option positions for any given level of output, which is obtained by solving

(8.2) and (8.3) for x and z conditional on y. Because of the dependence in (5) the random profit can be written as:

$$(9) \qquad \tilde{\pi} = \pi_0 + \tilde{\theta}y + \tilde{p}[\beta y - x] - \tilde{v}z$$

where $\pi_0 = [\alpha y - c(y) + fx + rz]$ is the non-stochastic component of profit. Now consider $x=\beta y$ and $z=0$ as a candidate solution. With these levels of hedging instruments, the only randomness left in $\tilde{\pi}$ and in $\tilde{u}'$ is due to $\tilde{\theta}$. Because $\tilde{\theta}$ and $\tilde{p}$ are by assumption independently distributed, then $Cov(\tilde{u}',\tilde{p})=0$ and $Cov(\tilde{u}',\tilde{v})=0$ so that $x^*=\beta y$ and $z^*=0$ solve equations (8.2) and (8.3). Because the second order conditions hold globally, this solution solves the expected utility maximization problem.

Given this optimal choice of hedging instruments, equation (8.1) will solve for the optimal level of output y^*. Because in this case random profit reduces to $\tilde{\pi} = (\alpha+\beta f+\tilde{\theta})y - c(y)$, the choice of output level reduces to the standard problem of the competitive firm under output price uncertainty, where the random price is $(\alpha+\beta f+\tilde{\theta})$. Using known results of this model, under risk aversion production takes place at a point at which marginal cost is lower than the expected price (with optimal hedging), i.e. $c'(y^*) < \alpha+\beta f$, indicating that a portion of price risk due to the basis cannot be hedged away.

Because there is some residual uncertainty concerning the local cash price, the degree of risk aversion also influences optimal output. Specifically, the output level y^* is inversely related to the degree of risk aversion (Baron). Finally, a <u>ceteris paribus</u> increase in non-diversifiable basis uncertainty (a mean preserving spread of $\tilde{\theta}$) will in general decrease the optimal output level, a sufficient condition being that the Pratt measure of absolute risk aversion is decreasing in profit (Ishii). These conclusions are summarized in the following:

Result 1 - When futures and options prices are unbiased, and cash and futures prices are related as in (5), then: (a) a fraction β of production is hedged in the futures market; (b) options are not used as hedging instruments; (c) the portion of non-diversifiable basis risk affects the production level.

The absence of options from the optimal hedge may seem counter-intuitive and warrants clarification. In the absence of futures and options positions, the risk faced by the producer depends upon the distribution of cash prices which, from (5), is assumed to be a linear function of the end-of-period futures price $\tilde{p}$ plus an orthogonal component $\tilde{\theta}$. Neither the futures contract nor the put option can provide any hedge against the basis risk that is independent of $\tilde{p}$. However, because the remaining hedgeable risk is linear in $\tilde{p}$, it follows that a futures contract (which yields a pay-off that is also linear in $\tilde{p}$) must provide a better hedge than an option contract (the pay-off of which is non-linear in $\tilde{p}$). Thus, in this context the option has no value as a hedging instrument if futures contracts are also present.[6]

The optimal hedge ratio for futures derived above, $x^*/y = \beta$, is the same as that derived under similar conditions for the case of futures only (Benninga, Eldor, and Zilcha; Kahl). This optimal hedge ratio satisfies the condition $\beta = \text{Cov}(\tilde{b},\tilde{p})/\text{Var}(\tilde{p})$ and thus could be estimated by a linear regression of cash on futures prices.

Additional results for the general solution may be obtained under special conditions. First, if there is no orthogonal basis risk ($\tilde{\theta}=0$), then the FOC of equation (8.1) reduces to $c'(y^*) = \alpha+\beta f$. This means that the optimal output level is independent of the distribution of $\tilde{p}$, i.e. production and hedging/speculative decisions are separated. Second, when the basis ($\tilde{b}-\tilde{p}$) and the futures price $\tilde{p}$ are independent (i.e., $\beta=1$), the optimal hedge is the full hedge in the futures market, although in this case production and hedging decisions are not necessarily separated.

A separation result can also occur under our assumptions concerning basis risk if the utility function is of the CARA type. Assume a negative exponential utility function $\tilde{u}=-\exp(-A\tilde{\pi})$, where $A=-u''/u'$ is the (constant) Pratt coefficient of absolute risk aversion, such that $\tilde{u}'= A \exp(-A\tilde{\pi})$. Because $\tilde{p}$ and $\tilde{\theta}$ are independently distributed, it is verified that for CARA equation (6.1) can be written as:

$$(10) \qquad \frac{\int \theta \exp(-A\theta y)\ h(\theta)\ d\theta}{\int \exp(-A\theta y)\ h(\theta)\ d\theta} = c'(y) - \alpha - \beta f$$

where $h(\theta)$ is the density function of $\tilde{\theta}$. Thus, the optimal output level y^* that solves equation (10) will not be affected by parameters of the distribution of $\tilde{p}$. This can be summarized in the following:

Result 2 - If either: (a) there is no basis uncertainty; or, (b) there is a CARA utility function and cash and futures prices are related as in (5); then, there is separation between production and hedging (speculative) decisions.

This separation result means that, when the producer believes that the current futures price is a biased indicator of the end-of-period futures price, it is more efficient for the producer to speculate on this disparity through portfolio decisions than through production decisions. This result with no basis uncertainty is essentially the same as those of Danthine, Holthausen, and Feder, Just, and Schimtz. The separation result with linear basis risk is illustrated by (10). The general case of (8.1) shows that, at the optimal output level, the marginal cost is less than the expected return from an optimally hedged position by an amount which reflects the risk premium of the unhedgeable risk (the left-hand-side of (8.1)). As shown in (10), under CARA this risk premium is independent of that portion of price risk which is orthogonal to $\tilde{\theta}$, leading to separation between production and hedging decisions.

The Role of Expectations

Result 1 above describes the hedging decisions of a producer whose expectations agree with the market forecasts as displayed by the futures price and option premium. When this condition is relaxed, the optimal hedging rule $x^* = \beta y$ and $z^* = 0$ is modified. A convenient way to model the divergence of individual expectations from the market expectations, as aggregated in the futures price f and in the option price r, is to let f and r change while holding the producer's subjective distribution of $\tilde{p}$ unchanged.[7]

Consider first hedging decisions conditional on the output level y. In this case equations (4.2) and (4.3) will solve for x^* and z^*. Totally differentiating these FOC and solving yields:

$$(11) \qquad \begin{bmatrix} dx \\ dz \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -\mathcal{L}_{zz} & \mathcal{L}_{zx} \\ \mathcal{L}_{xz} & -\mathcal{L}_{xx} \end{bmatrix} \begin{bmatrix} \mathcal{L}_{xf} & \mathcal{L}_{xr} \\ \mathcal{L}_{zf} & \mathcal{L}_{zr} \end{bmatrix} \begin{bmatrix} df \\ dr \end{bmatrix}$$

where:

$$\mathcal{L}_{xx} = E[\tilde{u}''(f-\tilde{p})^2]$$

$$\mathcal{L}_{xz} = \mathcal{L}_{zx} = E[\tilde{u}''(r-\tilde{v})(f-\tilde{p})]$$

$$\mathcal{L}_{zz} = E[\tilde{u}''(r-\tilde{v})^2]$$

$$\mathcal{L}_{xf} = E[\tilde{u}''(f-\tilde{p})x] + E\tilde{u}'$$

$$\mathcal{L}_{xr} = E[\tilde{u}''(f-\tilde{p})z]$$

$$\mathcal{L}_{zf} = E[\tilde{u}''(r-\tilde{v})x]$$

$$\mathcal{L}_{zr} = E[\tilde{u}''(r-\tilde{v})z] + E\tilde{u}'$$

$$\Delta = \mathcal{L}_{xx}\,\mathcal{L}_{zz} - (\mathcal{L}_{xz})^2$$

Under CARA, and using the first order conditions (4.2) and (4.3), it is verified that $\mathcal{L}_{xf} = \mathcal{L}_{zr} = E\tilde{u}'$ and $\mathcal{L}_{xr} = \mathcal{L}_{zf} = 0$. Under CARA, therefore, equation (11) reduces to:

$$(12) \qquad \begin{bmatrix} dx \\ dz \end{bmatrix} = \frac{E\tilde{u}'}{\Delta} \begin{bmatrix} -\mathcal{L}_{zz} & \mathcal{L}_{zx} \\ \mathcal{L}_{xz} & -\mathcal{L}_{xx} \end{bmatrix} \begin{bmatrix} df \\ dr \end{bmatrix}$$

From the second order conditions we know that $\mathcal{L}_{xx} < 0$, $\mathcal{L}_{zz} < 0$, and $\Delta > 0$. It can also be proved that under CARA $\mathcal{L}_{zx} = \mathcal{L}_{xz} > 0$.[8] Hence (12) yields:

Result 3 - Under CARA, a _ceteris paribus_ change in the futures price leads to a change in futures and options sold in the same direction, i.e. $\partial x^*/\partial f > 0$ and $\partial z^*/\partial f > 0$. A _ceteris paribus_ change in the option premium leads to a change in futures and options sold in the same direction, i.e. $\partial x^*/\partial r > 0$ and $\partial z^*/\partial r > 0$.

The relative magnitude of the comparative statics effects also provides some insight on how futures and options are used to exploit information on futures and options bias. First, (12) implies that $\partial x^*/\partial r = \partial z^*/\partial f$ because $\mathcal{L}_{xz} = \mathcal{L}_{zx}$. Also, from (12) we obtain:

$$(13) \qquad dz - dx = (E\tilde{u}'/\Delta)\ [(\mathcal{L}_{xz} + \mathcal{L}_{zz})df - (\mathcal{L}_{xx} + \mathcal{L}_{zx})dr]$$

Under CARA $(\mathcal{L}_{xz} + \mathcal{L}_{zz}) > 0$ and $(\mathcal{L}_{xx} + \mathcal{L}_{zx}) < 0$.[9] Thus we can conclude:

> Result 4 - Under CARA, a <u>ceteris paribus</u> change in futures or options price results in a larger change in the option position than in the futures position, with changes in r implying the largest changes. That is: $\partial z^*/\partial r > \partial x^*/\partial r = \partial z^*/\partial f > \partial x^*/\partial f > 0$.

The comparative statics of Result 3 and Result 4 should be interpreted with care because they consider the effects of only r or f changing. When the producer differs from the market in terms of his/her perception of the dispersion of price but not in the expected value, only the option price will be perceived as biased, and the comparative statics of a change in r applies. Thus, for example, if one started from the unbiased solution of the basis independence case $x^* = y$ and $z^* = 0$, then an increase in r (i.e. the market is overstating the volatility of $\tilde{p}$ from the individual point of view) would lead to a partly speculative futures open position $x^* > y$ such that more futures are sold. At the same time a position $z^* > 0$ in the option market is also open, such that some put options are also sold. Because the change in option position is larger than the change in the futures position by Result 4, the resulting pay-off of the speculative position resembles a short straddle, a strategy which is deemed useful for speculating on beliefs about the volatility.[10]

Alternatively, changes in the producer's beliefs concerning expected price would bias both futures and option prices. If, at the same time, the producer perceives an offsetting change in the dispersion of price, then only the futures price may be perceived as biased. In this case the comparative statics of a change in f applies. However, this special case may not be very interesting; rather, one may want to know the effects of changing the futures price f when volatility is perceived unchanged. In this case the option premium must be allowed to change when the price of futures changes. A way of doing that would be to let $dr = \delta df$, where the coefficient δ is known as the "delta" of the option (Cox and Rubinstein), and to pursue the comparative statics analysis in terms of total derivatives. An alternative and more fruitful analysis involves reformulating the problem in terms of futures and straddles. This approach is pursued in the next section.

Speculative Hedging with Options

The analysis so far has shown little role for options as a hedging instrument. As long as the exogenous risk is linear in price the futures market provides a perfect hedge. Moreover, the individual speculative decisions depend upon market prices (f and r), and his/her subjective beliefs concerning the "fair" value of these prices. If these prices are perceived as biased, then a speculative position which includes the use of options may occur.

The results concerning speculative decisions are obscured because the futures and put contracts are partial substitutes (a futures equals a long call and a short put). As pointed out before, the bias in the futures price depends on expected price, while the bias in option premiums depends on both expected price and volatility. Thus, changes in price expectations have a direct impact on the expected return (or bias) of both instruments.

The analysis of speculative decisions can be sharpened if the individual speculates using futures contracts and straddles. A long (short) straddle is a combination of a long (short) call and a long (short) put at the same strike price. Because a synthetic call can be constructed using futures and puts, a straddle can also be constructed using futures and puts. Thus, recasting the analysis with this instrument does not entail a change in the choices available to the individual.

The main determinant of the straddle price is the volatility of the end-of-period futures price. From the individual perspective, changes in expectations concerning the futures price affect the bias in the futures but have little effect on the bias in the straddle, while changes in beliefs about the dispersion of price (volatility) affect only the bias in the straddle.

Because basis risk has been investigated earlier, the analysis is simplified by assuming no basis risk ($\tilde{b} = \tilde{p}$) and by concentrating on the speculative decisions given a (fully hedged) output level. To this end, let: $q = (y - x)$ denote the speculative (open) futures market position ($q > 0$ is long); s be the straddle position ($s > 0$ is a long straddle); t be the market price of a long straddle with strike price k (the premium of the put plus the premium of the call); and $\tilde{w}$ be the payoff of the straddle position. The profit defined in terms of open futures and straddle positions then is written as:

(14) $\tilde{\pi} = fy - c(y) + (\tilde{p}-f)q + (\tilde{w}-t)s$

The payoff of the straddle is given by the absolute value function:

(15) $\tilde{w} = | \tilde{p} - k |$

It is convenient to let $\tilde{p} = \bar{p} + \tilde{e}$, where $\tilde{e}$ is a random variable with density function g(e) and satisfying $E(\tilde{e})=0$. Also, assume that the strike price is chosen equal to the expected price, so that $k=\bar{p}$ and $\tilde{w}=|\tilde{e}|$. If one maximizes the expected utility of profit as given in (14) conditional on the output level, the first order conditions are:

(16.1) $E[\tilde{u}'(\bar{p}-f+\tilde{e})] = 0$

(16.2) $E[\tilde{u}'(|\tilde{e}|-t)] = 0$

For the results that follow, assume that the individual perceives the price to be symmetrically distributed, i.e. $g(e) = g(-e)$. Then:

(17) $\int_{e<0} F(e)g(e)de = \int_{e>0} F(-e)g(e)de$

for any function F(e). For notational convenience, define[11]:

(18) $E^{+}[J(e)] = 2 \int_{e>0} J(e)g(e)de$

for any function J(e). Using this symmetry assumption, the FOCs can be expressed in terms of the realizations $e \geq 0$ only. In particular, equations (16) can be rewritten as:

(19.1) $E^{+}[K(\tilde{e})\tilde{e}] + (\bar{p}-f)E^{+}[L(\tilde{e})] = 0$

(19.2) $E^{+}[L(\tilde{e})(\tilde{e}-t)] = 0$

where $K(\tilde{e}) \equiv [u'(\pi(\tilde{e})) - u'(\pi(-\tilde{e}))]$ and $L(\tilde{e}) \equiv [u'(\pi(\tilde{e})) + u'(\pi(-\tilde{e}))]$.[12]

Note that $E^{+}[L(\tilde{e})] > 0$ and, assuming risk aversion ($u'' < 0$), for $e > 0$:

(20) $K(e) \gtrless 0$ as $q \lessgtr 0$

Thus, if the futures price is unbiased ($\bar{p}=f$) then $q^{*} = 0$ for (19.1) to hold. If $\bar{p}>f$ then (19.1) requires $E^{+}[K(\tilde{e})\tilde{e}] < 0$, which implies $q^{*} > 0$ in view of (20). Similarly, if $\bar{p}<f$, then $E^{+}[K(\tilde{e})\tilde{e}] > 0$, which requires $q^{*} < 0$. This can be summarized as:

Result 5 - Under risk aversion and with a symmetric distribution of price, the qualitative optimal speculative futures position depends only upon the bias in the futures price; i.e., $q^{*} \gtrless 0$ as $\bar{p} \gtrless f$. In particular, this speculative futures position is independent of the bias in the straddle price.

The reformulation of the asset mix in terms of futures and straddles allows a clearer representation of how changes in expected price or implied volatility affect optimal speculative decisions. This reformulation also clarifies the comparative statics of Results 3 and 4. For example, a perceived decline in the dispersion of $\tilde{p}$ means that the producer/speculator views both puts and calls as overpriced and hence wishes to sell both, i.e. sell a straddle. If the problem is formulated in terms of futures and puts, then the sale of a (synthetic) straddle can be achieved by selling one futures and two puts. Thus, the use of futures to speculate on the dispersion of price emerging from Result 3 is purely a function of constructing a straddle position.

The first part of this paper showed that options are not useful in hedging exogenous price risk, at least under the assumption of linear price risk. With the reformulation of the model using the straddle, however, we can show that options are desirable instruments to hedge the risk assumed by the agent because of an open speculative position in futures. For example, if the agent has a long futures position, then profit (and utility) will be low when p is low. In this situation, a long straddle that raises income in these states may be desirable. The same argument indicates that a long straddle may be useful when the agent chooses to short the futures contract.

Specifically, the FOC (19.1) determines the sign of q^*, which depends on the perceived bias in the futures price. The sign of s^* then is based on the remaining FOC (19.2). If the straddle is unbiased, then $\bar{w}=t$ where $\bar{w} = E(\tilde{w}) = E[|\tilde{e}|] = E^+[\tilde{e}]$, the last equality following from the symmetry assumption. Thus, the FOC (19.2) can be rewritten as $E^+\{[L(\tilde{e}) - L(\bar{w})](\tilde{e}-\bar{w})\} = 0$, where $L(\bar{w})$ denotes the value of L(e) at $e=\bar{w}$. Now, L(e) is an increasing function of e at s=0 if we assume $u''' > 0$ (as implied by non-increasing absolute risk aversion) because: $dL/de = [u''(\tilde{e}) - u''(-\tilde{e})]q$ at s=0, and $d^2L/de^2 = [u'''(\tilde{e}) + u'''(-\tilde{e})]q^2 > 0$ for $q \neq 0$. Thus, $\text{sgn}[L(\tilde{e})-L(\bar{w})] = \text{sgn}[\tilde{e}-\bar{w}]$, so that the integrand in (19.2) is positive for all $e\neq\bar{w}$ and equal to zero at $e=\bar{w}$. Hence, with an unbiased straddle price at s=0 the FOC in (19.2) is positive. Given the second-order conditions, s^* must also be positive for (19.2) to vanish.

For an unbiased straddle price, the long straddle position will not fully offset the open futures position. To see this note that, from

(19.2), $E^+[L(\tilde{e})(\tilde{e}-t)] = Cov^+[L(\tilde{e}),(\tilde{e}-t)] = 0$ since $E^+(\tilde{e}-t)=0$. From the definition of L(e), $dL/de = u''(\tilde{e})(s+q)+u''(-\tilde{e})(s-q)$. Because L(e) cannot be monotonic in e (if this were the case then $Cov^+[L(\tilde{e}),(\tilde{e}-t)]\neq 0$), it follows that (s+q) and (s-q) must have opposite sign. Thus, $(s+q)(s-q) = s^2 - q^2 < 0$, which implies $|s| < |q|$. For example, if $\bar{p}>f$, then $q^* > s^* > 0$, so that the resulting position is not equivalent to simply being long a call.

The analysis above has shown that a long straddle provides insurance against adverse outcomes induced by the open futures position. This conclusion is summarized in the following:

> **Result 6** - Under non-increasing absolute risk aversion, and with unbiased straddle price and a symmetric distribution of the futures price, the optimal straddle position will always be long when there is an open speculative futures position (either long or short); also, the straddle position will always be smaller than the open futures position, i.e. $0 < s^* < |q^*|$.

Results 5 and 6 together imply that the speculative position induced by a perceived bias in the futures price will have a nonlinear payoff whose shape is intermediate between that generated by only puts or calls and that generated by futures only. This nonlinear payoff will generate a positively skewed profit distribution even if the price distribution is symmetric. Indeed, the condition of non-increasing risk aversion (or more generally $u''' > 0$) used above is equivalent to assuming a preference for positive skewness (Tsiang).

Note, however, that preference for skewness is not sufficient to justify using only puts or calls to generate a nonlinear payoff. For example, if the agent's expectation of the end-of-period futures price is higher than f, but he/she agrees with the implied price volatility of the market, a viable speculative action would seem to involve buying call options. However, if the agent views the call as underpriced he/she must simultaneously view the put as overpriced. Hence, buying a call and selling a put, i.e. buying futures, is a superior speculative device. Options in a pure sense (straddle) are used only to hedge the risk assumed by this open position.

Comparative statics results concerning the impact of changes in futures and option prices on the optimal portfolio of the individual can be obtained for a CARA utility function. Totally differentiating (16), using the FOC and the constant coefficient of absolute risk aversion $A = -u''/u'$, and solving yields:

$$(21) \qquad \begin{bmatrix} dq \\ ds \end{bmatrix} = \frac{E\tilde{u}'}{\Delta} \begin{bmatrix} \mathcal{L}_{ss} & -\mathcal{L}_{qs} \\ -\mathcal{L}_{qs} & \mathcal{L}_{qq} \end{bmatrix} \begin{bmatrix} df \\ dt \end{bmatrix}$$

where $\mathcal{L}_{ij}$ are the elements of the Hessian matrix for the maximization problem, and hence $\mathcal{L}_{qq} < 0$, $\mathcal{L}_{ss} < 0$, and $\Delta = [\mathcal{L}_{qq}\, \mathcal{L}_{ss} - (\mathcal{L}_{qs})^2] > 0$. Furthermore, with the assumption of symmetry in g(e), $\mathcal{L}_{qs} \gtreqless 0$ as $q \gtreqless 0$.[13] Hence: $\partial q^*/\partial f < 0$, $\partial s^*/\partial t < 0$, and $\partial q^*/\partial t = \partial s^*/\partial f \gtreqless 0$ as $q^* \lesseqgtr 0$. This can be summarized in the following:

Result 7 - Under CARA and with symmetrically distributed prices, a <u>ceteris paribus</u> decrease in the current futures price will lead to an increase in the net long futures position; the net long straddle position will increase (decrease) if the net futures position is long (short). Similarly, a ceteris paribus decrease in the straddle price will increase the net long straddle position; the net long futures position will increase (decrease) if the agent is long (short) in open futures.

In the model reformulated in terms of open futures and straddles, the comparative statics of a change in futures price with the straddle premium held constant illustrate the adjustments when the producer differs from the market in terms of his/her perception of the expected price but not on the price volatility. This allow us to resolve the ambiguity arising from Results 3 and 4. Because of our centering $k=\bar{p}$ and of the assumption of symmetry in the distribution of $\tilde{p}$, a ceteris paribus change in f will not affect the straddle price t (although the price of puts and calls will be affected through their delta). In this light, the own-price effects on the demand for futures of Result 7 are intuitively appealing. For example, starting at the unbiased solution $q^* = s^* = 0$, a decrease in f other things being equal (i.e. an increase in the agent's expected price relative to the market) leads to a net long position.

Similarly, the comparative statics of a change in t with f held constant illustrates the adjustments when the producer differs from the market in terms of his/her perception of the volatility of price but not on the expected price. For example, a ceteris paribus decrease of the straddle price t is equivalent to the producer perceiving an increase in the volatility of the futures price $\tilde{p}$ relative to the market. If one evaluates the comparative statics effects at the unbiased point $q^* = s^* = 0$, the own-price effect of straddle demand implies a movement towards a long straddle, a strategy that is useful when the market is understating the volatility of $\tilde{p}$.

The cross-price effects can be understood by considering the underlying use of straddles as hedging instruments to offset the futures position. Thus, in the case of a decrease in f the agent will change his/her straddle position to hedge against the net change in futures position. For $q > 0$ this means an increase in s. For $q < 0$ this means that the absolute value of q decreases, so that fewer straddles are needed to hedge the position.

Pitfalls of Mean-Variance Analysis

The introduction pointed out that the truncation and non-linearity introduced by options leads to a violation of commonly used justifications for mean-variance analysis. The results of the preceding section gives us an opportunity to illustrate this point further. Consider a mean-variance formulation of this model. For the profit equation (14) we have:

$$(22.1) \qquad E(\tilde{\pi}) = fy - c(y) + (\bar{p}-f)q + (\bar{w}-t)s$$

$$(22.2) \qquad Var(\tilde{\pi}) = q^2 \, Var(\tilde{e}) + s^2 \, Var(|\tilde{e}|)$$

because $Cov(\tilde{e}, |\tilde{e}|) = 0$ due to the assumed symmetry in g(e).

Maximizing any function that is increasing in $E(\tilde{\pi})$ and decreasing in $Var(\tilde{\pi})$, as given in (22), would imply $s^*=0$ for $\bar{w}=t$. Hence, in the mean-variance framework the optimal straddle position is zero if the straddle price is unbiased, regardless of the optimal position in the futures. This conclusion contrasts sharply with the results of the previous section. Because the distribution of profit cannot be normal in the presence of options, the ultimate rationale for mean-variance analysis must hinge on the undesirable assumption of quadratic utility.[14]

Conclusions

This paper has analyzed the production and hedging decisions of the competitive firm facing both futures and options markets within an expected utility model that allows for basis risk. When futures prices and options premiums are perceived as unbiased, options are redundant hedging instruments. The optimal hedging strategy involves using only futures, and the amount of futures is determined by the covariance of cash and futures prices. However, if futures prices and/or options premiums are perceived as biased, options are typically used along with futures. Thus, in this model options are appealing more as speculative tools to exploit private information on the price distribution, and less so as an alternative hedging instrument.

The qualitative effects of biased prices on the use of options and futures was investigated in terms of comparative statics effects. The nature of the speculative activity brought about by biased prices is clarified if the problem is formulated in terms of open futures and straddle positions. The sign of the open position in the futures depends only on the bias in the futures price, and (long) straddles are used even if the straddle price is unbiased whenever the futures position is open. In this context, options emerge as a useful device to insure against the price risk in an open speculative position.

FOOTNOTES

(lead unnumbered footnote)

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1. Wolf also reports simulation results based on the logarithmic utility function.

2. Input prices and the option premium are implicitly compounded to the end of the period using the (constant) market interest rate, so that all monetary variables in (1) are commensurable.

3. Because choosing a profit maximizing level of inputs is equivalent to choosing a level of output when production is nonstochastic, in the production/hedging problem y is the decision variable of the producer. The effects of exogenous variables on the optimal input levels could be obtained using the (non-stochastic) conditional input demand functions implied by the cost function via the derivative property.

4. Because the utility function is strictly concave, and the cost function is strictly convex in output, the second order conditions are satisfied.

5. This definition of bias illustrates the producer's beliefs about the price distribution, and does not warrant any implication about market efficiency.

6. There are circumstances in which both futures and options may be useful hedging devices. In general, these cases will display a distribution of

profit which is nonlinear in the random price. Examples of these situations may involve a non-linear basis relationship or the presence of production risk which is not orthogonal to price risk. The characterization of these cases is left for further research.

7. This is consistent with the assumptions that f and r are exogenously given to the producer. Alternatively, one could hold the prices f and r constant and investigate how changes in the subjective distribution affect optimal hedging. Because the assumptions do not rule out distributions with more than two moments this alternative may be impractical.

8. Proof of this result is available from the authors upon request.

9. Proof of this result is available from the authors upon request.

10. A short straddle position is obtained by selling one futures and two put options, or equivalently by selling one put and one call options. See Cox and Rubinstein for more details on this and other strategies.

11. The normalization factor 2 in (18) is the reciprocal of the probability of positive realizations of $\tilde{e}$ (which is 0.5 because of the symmetry assumption), so that E^+ is properly interpreted as an expected value.

12. $u'(\pi(\tilde{e}))$ denotes the marginal utility evaluated at the profit level associated with a realization of the random variable $\tilde{e}$, while $u'(\pi(-\tilde{e}))$ denotes the marginal utility evaluated at the profit level associated with the negative of the same realization of the random variable $\tilde{e}$.

13. Proof of this result is available from the authors upon request.

14. Note that this utility function displays increasing absolute risk aversion with $u''' = 0$, which violates the conditions used to derive Result 6 and rules out preference for positive skewness.

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