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Production of intermediate-mass black holes in globular clusters

M. Coleman Miller★ and Douglas P. Hamilton
Department of Astronomy, University of Maryland, College Park, MD 20742-2421, USA

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ABSTRACT

The discovery of numerous non-nuclear X-ray point sources with luminosities $L > 10^{39}\,\mathrm{erg\,s^{-1}}$ in several starburst galaxies has stimulated speculation about their nature and origin. The strong variability seen in several sources points to massive black holes as the central engines. If the flux is isotropic, the luminosities range up to $\approx 10^{41}\,\mathrm{erg\,s^{-1}}$, implying masses of $M \gtrsim 10^3\,\mathrm{M_\odot}$ if the luminosity is sub-Eddington. Here we explore a model for these sources. We suggest that in some tens of per cent of globular clusters a very massive black hole, $M \gtrsim 50\,\mathrm{M_\odot}$, is formed. This black hole sinks in $\lesssim 10^6$ yr to the centre of the cluster, where in the $\sim 10^{10}$ yr lifetime of the cluster it accretes $\sim 10^3\,\mathrm{M_\odot}$, primarily in the form of lighter black holes. Unlike less-massive black holes in binaries, which are flung from clusters by recoil before they can merge gravitationally, a $\gtrsim 50\,\mathrm{M_\odot}$ black hole has enough inertia that it remains bound to the cluster. We suggest that $\sim 10^3\,\mathrm{M_\odot}$ black holes may be common in the centres of dense globular clusters, and may therefore exist in some tens of per cent of current globulars. If the cluster later merges with its host galaxy, accretion from young star clusters in molecular clouds by the black hole can generate luminosity consistent with that observed. We also consider the detectability of massive black holes in globular clusters with gravitational wave detectors, and speculate on future observations that may test our predictions.

Key words: accretion, accretion discs – black hole physics – binaries: close – globular clusters: general – galaxies: starburst.

1 INTRODUCTION

In the last few years, evidence has mounted for a population of spatially unresolved sources in starburst galaxies that have fluxes corresponding to isotropic X-ray luminosities in excess of $\sim 10^{39}\,\mathrm{erg\,s^{-1}}$ (e.g. Fabbiano 1989; Fabbiano, Schweizer & Mackie 1997; Colbert & Mushotzky 1999; Zezas, Georgantopoulos & Ward 1999; Kaaret et al. 2001; Matsumoto et al. 2001; Fabbiano, Zezas & Murray 2001). The variability of these sources on time-scales as short as 10^4 s (Matsumoto et al. 2001) suggests that they are accreting compact objects, probably black holes. If beaming of emission is moderate to negligible, the brightest of these objects (with $L \approx 10^{41}\,\mathrm{erg\,s^{-1}}$) must have masses $M \gtrsim 10^3\,\mathrm{M_\odot}$ so that the luminosity is below the Eddington luminosity $L_\mathrm{E} = 1.3 \times 10^{38} (M/1\,\mathrm{M_\odot})\,\mathrm{erg\,s^{-1}}$. Their non-nuclear locations imply an upper mass limit as low as 10^5–$10^6\,\mathrm{M_\odot}$, otherwise dynamical friction would have caused them to sink to the centre of their host galaxies (Kaaret et al. 2001). The inferred masses are much larger than the ~ 10–$20\,\mathrm{M_\odot}$ masses of black holes usually thought to arise from the evolution of present-day stars (Fryer 1999), yet are much smaller than the $\sim 10^6$–$10^9\,\mathrm{M_\odot}$ masses of black holes that exist in the centres of many galaxies (e.g. Merritt & Ferrarese 2001). The origin of such objects is an intriguing mystery. It has been suggested that the emission is beamed (King et al. 2001), so that the accreting black holes have ordinary stellar masses, or that intermediate-mass black holes formed directly via evolution of Population III stars (Madau & Rees 2001).

Here we propose another scenario, in which a black hole grows significantly by accretion. We show that for black holes with initial mass $M \lesssim 10^3\,\mathrm{M_\odot}$, accretion from the ISM is too slow to increase the mass significantly. Accretion from stars in a dense cluster is more promising (see also Taniguchi et al. 2000), but the cluster must last several billion years for significant growth, which points to globular clusters. However, a $\sim 10\,\mathrm{M_\odot}$ black hole sinks to the cluster core within $\lesssim 10^7$ yr, where it forms a binary or exchanges into one via three-body processes. Further three-body interactions tighten the binary, but each tightening causes the binary to recoil. Before the binary becomes hard enough to merge rapidly via gravitational radiation, the recoil kicks it completely out of the cluster (Sigurdsson & Hernquist 1993). Thus, a light black hole usually cannot increase its mass by a large factor despite being in a high-density stellar environment.

In contrast, as we show in Section 2, a $\gtrsim 50\,\mathrm{M_\odot}$ black hole is so massive that recoil does not eject it from the cluster before the

★E-mail: miller@astro.umd.edu

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binary merges. Hence, the process can be repeated and in a dense cluster a $\gtrsim 50\,\mathrm{M}_\odot$ black hole can increase its mass to $\sim 10^3\,\mathrm{M}_\odot$. This has some similarities to one of the ways in which supermassive black holes may grow in the nuclei of galaxies (see e.g. Frank & Rees 1976; Sigurdsson & Rees 1997).

We suggest that $\sim 10^3\,\mathrm{M}_\odot$ black holes are generated in this fashion in some tens of per cent of clusters. While in the clusters, accretion on to the black hole is usually electromagnetically quiet, as most accretion is of compact objects that are swallowed whole. However, if the cluster merges with the disc of the host galaxy, as may occur for roughly half of globulars (e.g. Gnedin & Ostriker 1997; Gnedin, Lee & Ostriker 1999; Takahashi & Portegies Zwart 1998, 2000), then after the globular has settled into the mean motion of the disc and has been disrupted by galactic tidal fields, the massive black hole is released. If it later enters a molecular cloud, the extra mass of the hole may speed up gravitational contraction and star formation, resulting in the black hole being at the centre of (and accreting from) a cluster of stars. For starburst galaxies, where the number of molecular clouds and volume-averaged density of interstellar gas may be a few times the corresponding values in normal galaxies, several such sources can be detectable at any given time.

In Section 2 we estimate the growth rate of such black holes and the duty cycle of high-luminosity accretion (and hence detectability in globular clusters). In Section 3 we calculate the expected accretion rate once in the disc of the host galaxy. In Section 4 we discuss the implications of this model for the upcoming generation of gravitational wave detectors, and we present our conclusions and future observational tests in Section 5.

2 FORMATION AND GROWTH OF BLACK HOLES IN GLOBULAR CLUSTERS

2.1 Birth of the initial $\gtrsim 50\,\mathrm{M}_\odot$ black hole

Substantial growth of a black hole by mergers in a globular cluster requires that it have a high enough mass, $M \gtrsim 50\,\mathrm{M}_\odot$, to avoid being kicked out by recoil during three-body interactions (see Section 2.2). How can such black holes form? We identify three possible mechanisms. The first is direct formation from a massive star. A number of recent numerical studies indicate that if a star has a mass $M \gtrsim 30$–$40\,\mathrm{M}_\odot$, when its core collapses then the resulting explosion may not have enough energy to drive off the outer layers of the star (e.g. Fryer 1999). If so, and if the star has low enough angular momentum, then direct fallback from the star may produce a black hole with a mass close to that of the original star (Fryer 1999). For a star with an initial mass of ~ 50–$100\,\mathrm{M}_\odot$, the question is then whether stellar winds and mass ejection during the main sequence and giant phases are enough to reduce the mass well below the $\sim 50\,\mathrm{M}_\odot$ threshold. The amount of mass lost in a wind is challenging to determine, but some recent calculations suggest that, especially for stars in a low-metallicity environment such as a globular cluster, the wind loss is minor; for example, from table 3 of Vink, de Koter & Lamers (2001), the mass loss rate for a 40-$\mathrm{M}_\odot$ star with metallicity $Z = (1/30)Z_\odot$ is less than $10^{-7}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$, implying a total of less than $1\,\mathrm{M}_\odot$ during the lifetime of the star. The number of such stars is difficult to estimate, given the poor statistics. Observations of stars in young clusters imply that for $M > 1\,\mathrm{M}_\odot$ the Salpeter mass function $\mathrm{d}N/\mathrm{d}M \propto M^{-2.35}$ is a decent approximation, with no hint of a breakdown for stars even as massive as $\sim 100\,\mathrm{M}_\odot$ (Massey 1998). This would imply $\sim 10^{-4}$ of stars have $M > 50\,\mathrm{M}_\odot$, and hence that a globular cluster will potentially have tens of black holes of this mass.

The second candidate mechanism is a variant of the first. In a dense stellar environment the most massive stars sink to the centre of a cluster. If this happens more rapidly than the stars evolve off the main sequence (e.g. a few million years for a 30-$\mathrm{M}_\odot$ star), the stars may then undergo frequent collisions and mergers (Portegies Zwart et al. 1999), which produce more massive stars. Recent analyses (Fryer 1999) suggest that when these massive stars undergo core collapse, they may leave 50–$100\,\mathrm{M}_\odot$ black holes, even when mass loss from strong stellar winds is included. In either of these two mechanisms, depending on unknown details of the supernova process, some fraction of black holes formed in binary systems might become unbound from their binaries and even the cluster via recoil during the supernova.

The third possibility is that if hundreds to thousands of $\sim 10\,\mathrm{M}_\odot$ black holes are formed in a cluster, there will be enough interactions that a few will merge with each other, forming a (smaller) set of $\sim 20\,\mathrm{M}_\odot$ black holes, some of which may ultimately be able to reach the $\sim 50\,\mathrm{M}_\odot$ threshold. In this scenario, the overwhelming majority of small black holes are ejected from the cluster; it is only the fortunate minority that can merge. This mechanism has apparently not been explored in detail. We believe that this mechanism is less compelling than the previous two because pairs of 10-$\mathrm{M}_\odot$ black holes are unlikely to form 20-$\mathrm{M}_\odot$ black holes while remaining bound to the cluster (see Section 2.2.2).

2.2 Growth of $\gtrsim 50\,\mathrm{M}_\odot$ black holes in cluster cores

Black holes formed from supernovae are much more massive than the average mass $\langle m \rangle \approx 0.4$–$0.6\,\mathrm{M}_\odot$ of stars in globular clusters, hence they sink rapidly to the centre of the cluster. A black hole of mass M sinks to the centre in a time $\sim \langle m \rangle / M$ times the core half-mass relaxation time t_r. For typical globular clusters, $t_r \sim 10^7$–10^9 yr. Thus, a $\gtrsim 50\,\mathrm{M}_\odot$ black hole will sink to the centre within $\lesssim 10^6$ yr, shorter than the other time-scales of interest. If the black hole is in a binary, the extra mass causes it to sink to the centre slightly faster.

The rate of encounters in the core obviously depends on the number density of field black holes in the vicinity. Numerical simulations of globular clusters (Sigurdsson & Phinney 1995) show that in the cores of clusters in thermal equilibrium with number densities $n \gtrsim 10^5\,\mathrm{pc}^{-3}$ encounters of binaries with single stars are dominated by the most massive field stars, in this case black holes. This is because in thermal equilibrium the massive stars have a low velocity, hence they sink to the centre of the cluster and have a high number density even if their total numbers are comparatively small. For example, in a multimass King model the scaleheight of a stellar species of mass m scales as $(\langle m \rangle / m)^{1/2}$ (see e.g. Sigurdsson & Phinney 1995). Thus, black holes with typical mass $10\,\mathrm{M}_\odot / 0.4\,\mathrm{M}_\odot = 25$ times the average mass of a main sequence star have a scaleheight 1/5 of the scaleheight of the typical star, and hence a number density enhanced by a factor of ~ 100. Therefore, if their number fraction averaged over the cluster is more than ~ 1 per cent, their core number density is comparable to or greater than that of visible stars. In addition, black holes may settle gravitationally with enough rapidity that they are no longer in thermal equilibrium with the lower-mass stars (e.g. Meylan 2000); in this case, their number density in the core and hence all encounter rates are correspondingly increased. For simplicity we assume that the core number density of $\sim 10\,\mathrm{M}_\odot$ black holes equals

that of main sequence stars, but there is considerable uncertainty in this number.

We focus on clusters that are at or near core collapse, and adopt $10^6 n_6\,\mathrm{pc}^{-3} \approx 3\times10^{-50} n_6\,\mathrm{cm}^{-3}$ as a fiducial number density of normal black holes near the massive black hole. Of the 56 Galactic globular clusters studied by Pryor & Meylan 1993 (out of a total of 143; see Djorgovski & Meylan 1993), six had central mass densities greater than $10^6\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$ and 22 had central mass densities greater than $10^5\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$. The average stellar mass is less than $1\,\mathrm{M}_\odot$ (Pryor & Meylan 1993), so the number densities are correspondingly higher. We show in Section 2.2.2 that if $n_6 \gtrsim 0.1$ the cores of clusters are favourable for the growth of black holes to masses $\sim10^3\,\mathrm{M}_\odot$, so this implies that some 40 per cent of Galactic globulars, or $\approx$60, could harbour such black holes. Note that this could be a substantial underestimate; for our purposes the important quantities are the masses and densities of clusters at birth, but to be conservative we are guided by the current properties of Galactic globulars. At formation, the core densities of clusters are likely to have been less than they are now, so interaction rates may have been lower. However, as mentioned by Portegies Zwart & McMillan (2000), zero-age globulars may have had $\sim$3 times the mass and $\sim$ 1/3 of the virial radius of current globulars. Thus, the escape velocity from the clusters was probably greater than it is now, and hence processes leading to the formation of massive black holes are likely to have been significantly more efficient in the past because fewer binaries would have been ejected before merger.

While in the high stellar density environment of the core, the black hole undergoes a variety of interactions that increase its mass via merger or accretion. If there is a sufficient fraction of binaries in the core (quantified in Section 2.2.1), the massive black hole exchanges into a binary and ultimately merges. If the number of binaries is low, the black hole can capture a field star through tidal interactions (for a main sequence star) or by gravitational radiation (for a compact object). Three-body interactions strongly favour the resulting binary being composed of the two most massive stars of the three that interact. The most massive field stars could be $\sim10\,\mathrm{M}_\odot$ main sequence stars within the first $\sim10^8$ yr of the cluster or, later on, could be $\sim10\,\mathrm{M}_\odot$ black holes (if these are abundant), neutron stars or white dwarfs. Further interactions tend to harden the binary, but they also tend to eject the third, field star. If instead accretion is dominated by captures of field compact objects by emission of gravitational waves, the compact objects are accreted without ejection. We now quantify these scenarios, starting with the demarcation between binary rich and binary poor.

2.2.1 *Division between binary-rich and binary-poor cluster cores*

Which occurs more often, direct capture or binary exchange? A hard binary is defined to have an orbital binding energy greater than the kinetic energy of a typical field star. Assuming equipartition (mv^2 equal to a constant for all species) and a velocity dispersion of $10^6 v_{\mathrm{ms},6}\,\mathrm{cm\,s}^{-1}$ for 0.4-$\mathrm{M}_\odot$ field stars, this implies an orbital separation of $\sim10^{15}$ cm for black hole binaries of total mass $\sim10\,\mathrm{M}_\odot$. Thus, if a massive black hole encounters such a binary with a periastron r_p less than 10^{15} cm, there is likely to be a strong interaction with an exchange. This is $\approx10^4$ times the $\sim10^{11}$ cm periastron necessary for direct capture (see Section 2.2.3); because in the strong focusing limit the cross-section scales linearly with the periastron, $\sigma_\mathrm{coll} \approx \pi r_\mathrm{p}(2GM/v_\infty^2)$ (where v_∞ is the velocity of the field star at infinity), this implies that if the binary fraction is much larger than $\sim10^{-4}$ in the core, a $\sim50\,\mathrm{M}_\odot$ black hole will interact with binaries far more frequently than it will capture single stars.

2.2.2 *Binary-rich clusters*

Now consider the case in which core binaries are plentiful by the above definition. This may be true even if the primordial binary fraction is low, due to processes such as three-body binary formation (e.g. Lee 1995), in which three mutually hyperbolic point masses interact in such a way as to leave a binary and a single star. Once binaries are present, single-binary interactions dominate over the rare convergence of three single stars; henceforth we focus only on the more common interactions, and by three-body interactions we mean encounters between a single star and a binary. If a hard binary is formed, subsequent close three-body interactions pair the massive black hole with the highest-mass species present in abundance (a consequence of exchange interactions), and tend to harden the binary further (Heggie 1975). However, a binary composed of two 'normal' mass $M \lesssim 10\,\mathrm{M}_\odot$ black holes is ejected from the cluster before its separation becomes small enough that merger by gravitational radiation occurs before the next encounter (Sigurdsson & Hernquist 1993). This may not be true for a more massive black hole.

2.2.2.1 *Ejection or retention of binaries* To estimate the threshold mass, we adopt the treatment of Portegies Zwart & McMillan (2000). We are interested here in binaries with very unequal mass, but we assume for simplicity that the field black hole has a mass comparable to the mass of the lighter hole in the binary. For three equal-mass stars the binding energy is usually increased by $\sim$20 per cent by a strong interaction (e.g. Heggie 1975; Sigurdsson & Phinney 1993), but for high-mass ratio binaries Quinlan (1996) finds that in a given hardening interaction the binding energy of the binary typically increases by a fraction $\sim 0.2(m/M) = 0.04 m_{10} M_{50}^{-1}$, where the mass of the large black hole is $50 M_{50}\,\mathrm{M}_\odot$ and of the smaller black holes is $10 m_{10}\,\mathrm{M}_\odot$. The fraction of the change in binding energy carried away by binary recoil also depends on the mass ratio. For three equal-mass objects, conservation of momentum implies that the kinetic energy of recoil of the binary is approximately a third of the change in binding energy. If the binary is much more massive than the third star, $M \gg m$, then the kinetic energy of the binary is instead roughly m/M times the change in binding energy. Let v_esc be the escape velocity from the core, where typically $v_\mathrm{esc} \approx 50\,\mathrm{km\,s}^{-1}$ for a dense cluster (Webbink 1985). Then the binary will recoil at the cluster escape velocity if the binary binding energy is

$$E_\mathrm{b,min} \approx 5(M/m)^2\left(\frac{1}{2}Mv_\mathrm{esc}^2\right) \approx 2\times10^{50} M_{50}^3 m_{10}^{-2}\,\mathrm{erg}. \quad (1)$$

This needs to be compared with the binding energy such that gravitational radiation causes a merger before the next three-body interaction. The merger time for two stars with total mass M and reduced mass μ in an orbit of semimajor axis a and eccentricity e is (Peter 1964)

$$\tau_\mathrm{merge} \approx 3\times10^8 \mathrm{M}_\odot^3 (\mu M^2)^{-1} (a/\mathrm{R}_\odot)^4 (1-e^2)^{7/2}\,\mathrm{yr}. \quad (2)$$

From numerical simulations of three equal-mass black holes (Portegies Zwart & McMillan 2000), the typical eccentricity distribution of binaries after the interaction of three equal-mass objects is $P(e) = 2e$ (with a slight excess at high eccentricities). With this distribution, the average eccentricity weighted over merger time is $e \approx 0.7$. We note, however, that the strong

dependence of merger time on eccentricity means that deviations from this distribution can have large effects on the rate of growth of large black holes in globular clusters, the detectability of gravitational radiation from these systems, and other important quantities. Quinlan (1996) finds that when perturbations are small (as they are when the mass ratio is high) the eccentricity of a hard binary is increased steadily by the hardening process until gravitational radiation becomes important. If so, merger times are decreased and conditions are more favourable for the growth of $\sim 10^3\,\mathrm{M}_\odot$ black holes in the centres of globular clusters. However, in the presence of finite perturbations the outcome is not as clear. An important future project is therefore a numerical study of the growth of $\gtrsim 50\,\mathrm{M}_\odot$ black holes in globulars, with a full treatment of the evolution of the eccentricity. In the meantime, we assume conservatively that $P(e) = 2e$ regardless of the amount of hardening that has occurred, until the point that gravitational radiation is significant.

The typical semimajor axis for a merger time $\tau_{\mathrm{merge}} = 10^6\tau_6$ yr and $M \gg m$ is then

$$a \approx 3\times 10^{11}\tau_6^{1/4}M_{50}^{1/2}m_{10}^{1/4}\ \mathrm{cm}. \tag{3}$$

The binding energy at this separation is

$$E_{\mathrm{bind,merge}} = GMm/a \approx 4\times 10^{50}\tau_6^{-1/4}M_{50}^{1/2}m_{10}^{3/4}\ \mathrm{erg}. \tag{4}$$

Therefore, the ratio of the binding energy necessary to eject the binary from the cluster to the binding energy necessary for a fast merger is

$$x = E_{\mathrm{b,min}}/E_{\mathrm{bind,merge}} \approx 0.5M_{50}^{5/2}m_{10}^{-11/4}\tau_6^{1/4}. \tag{5}$$

If $x > 1$, mergers happen before ejection, so that the hole gains mass and remains near the mass source. If $x < 1$, ejection happens first, so that although the binary might merge in less than a Hubble time it will do so well away from the high stellar density in the cluster, meaning that the black hole mass growth is stopped. This calculation shows that if $M \gtrsim 50\,\mathrm{M}_\odot$, the hole will stay near the centre of the potential and grow rapidly, whereas for much smaller masses the hole will be ejected quickly without additional mass growth. If the eccentricity increases as the orbit shrinks, the critical semimajor axis increases, and hence the ratio x increases. Therefore, eccentricity growth allows lower-mass black holes to grow by mergers without being ejected.

2.2.2.2 *Time-scale for growth by three-body interactions* The interaction cross-section for a massive black hole is dominated by gravitational focusing, so that $\sigma_{\mathrm{coll}} \approx \pi r_{\mathrm{p}}(2GM/v_\infty^2)$. The relevant pericentre distance depends on the type of encounter and the nature of the secondary object. For example, a $10\,\mathrm{M}_\odot$ BH companion with a typical eccentricity $e = 0.7$ and a semimajor axis $\sim 5\times 10^{11}\tau_6^{1/4}$ cm will merge via gravitational radiation within $10^6\tau_6$ yr. A third black hole passing within $\sim$ twice that distance will produce a strong interaction, so $r_{\mathrm{p}} \sim 10^{12}$ cm. In thermal equilibrium the velocity dispersion of stars is inversely proportional to the square root of their mass, so if the average mass of a main sequence star is $0.4\,\mathrm{M}_\odot$ and its velocity is $v_{\mathrm{ms}} = 10^6 v_{\mathrm{ms},6}\,\mathrm{cm\,s^{-1}}$, then black holes have a velocity dispersion $v \approx \frac{1}{5}m_{10}^{-1/2}v_{\mathrm{ms}} = 2\times 10^5 m_{10}^{-1/2}v_{\mathrm{ms},6}\,\mathrm{cm\,s^{-1}}$. Then $\sigma_{\mathrm{coll}} \approx 10^{30}(r_{\mathrm{p}}/10^{12}\,\mathrm{cm})M_{50}m_{10}\,\mathrm{cm}^2$, so the time between interactions is $\tau_{\mathrm{int}} = 1/\langle n\sigma v_\infty\rangle \approx 2\times 10^6 v_{ms,6}n_6^{-1}r_{p,12}^{-1}M_{50}^{-1}m_{10}^{-1/2}$ yr, where $r_{\mathrm{p}} = 10^{12}r_{p,12}$ cm and the average in angle brackets is taken over the velocity distribution of black holes (assumed to be Maxwellian). The binding energy scales with $1/a$, as does the time between interactions. Therefore, the orbit starts shrinking rapidly, but slows down. The total time required to reach $a = 5\times 10^{11}$ cm from an initially much larger semimajor axis is $\sim 5(M/m)\tau_{\mathrm{int}}$ if a fraction $\sim 0.2(m/M)$ of the binding energy is removed in each close encounter. The orbital separation for a fixed merger time-scales as $M_{50}^{1/2}$, so $\tau_{\mathrm{int}} \sim M_{50}^{-3/2}$ and a $\sim 50\,\mathrm{M}_\odot$ black hole will increase its mass by $\sim 10\,\mathrm{M}_\odot$ in a total time $\sim 5\times 10^7 n_6^{-1}M_{50}^{-1/2}m_{10}^{-5/2}$ yr. Integrating, after time t the mass of the black hole is $M_{50} \approx [t/(3\times 10^8 n_6^{-1}m_{10}^{-5/2}\,\mathrm{yr})]^2$. Within a Hubble time there is therefore time for substantial growth in the mass of large black holes in the centres of clusters that are at least moderately dense, with $n_6 \gtrsim 0.1$. If, as argued by Quinlan (1996), three-body effects increase the eccentricity as the binary hardens, the rate of increase of black hole mass is enhanced significantly. Furthermore, if the cluster has a significant number of $\sim 50\,\mathrm{M}_\odot$ black holes, mergers of these with each other might dominate the early growth rate.

2.2.3 *Binary-poor clusters*

Now consider what happens when binary interactions and exchanges can be neglected. The effective capture cross-section of a compact object of mass m by a large black hole of mass $M \gg m$ is (Quinlan & Shapiro 1989)

$$\sigma = 2\pi\left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7}\frac{G^2m^{2/7}M^{12/7}}{c^{10/7}v_\infty^{18/7}}. \tag{6}$$

Here v_∞ is the relative velocity at infinity and c is the speed of light. Numerically, $\sigma \approx 2\times 10^{26}m_{10}^{2/7}M_{50}^{12/7}v_6^{-18/7}\,\mathrm{cm}^2$, where $v_\infty = 10^6 v_6\,\mathrm{cm\,s^{-1}}$. The strong dependence on relative velocity means that for equal number densities the slowest moving population will dominate the encounter rate. In thermal equilibrium, this means the black holes. Averaging the encounter rate $\langle n\sigma v_\infty\rangle$ over the velocity distribution (assumed Maxwellian), this implies $\sigma \approx 2\times 10^{28}m_{10}^{11/7}M_{50}^{12/7}v_{\mathrm{ms},6}^{-18/7}\,\mathrm{cm}^2$ (recall that $v_\infty \propto m^{-1/2}v_{\mathrm{ms}}$). The periastron is only $\sim 10^{11}$ cm for such an encounter, implying a merger time of $\sim 10^4$ yr once the orbit is circularized (see equation 2; circularization also takes place rapidly). Thus, if a compact object is captured in this manner it is swallowed almost immediately, without further interactions.

The encounter rate for a massive black hole is then

$$\nu_{\mathrm{enc}} = \langle n\sigma v_\infty\rangle \approx 2\times 10^{-16}n_6 m_{10}^{11/7}M_{50}^{12/7}v_{\mathrm{ms},6}^{-11/7}\ \mathrm{s}^{-1}. \tag{7}$$

The characteristic time-scale to increase the mass of the black hole by of order itself is then $t_{\mathrm{acc}} \approx (M/m)\nu_{\mathrm{enc}}^{-1}$, or

$$t_{\mathrm{acc}} \approx 2\times 10^9 n_6^{-1}m_{10}^{-18/7}M_{50}^{-5/7}v_{\mathrm{ms},6}^{11/7}\ \mathrm{yr}. \tag{8}$$

Thus, for a dense cluster ($n_6 \sim 1$), or an especially high-mass black hole ($M_{50} \gtrsim 5$), significant growth can occur in a Hubble time even in the absence of binaries in the core. Note that the growth rate accelerates with increasing M, so once growth is significant the massive black hole will consume the remaining field black holes quickly. The ratio of three-body to two-body encounter rates is

$$\frac{\nu_{3b}}{\nu_{2b}} \approx 2\times 10^4 M_{50}^{-5/7}v_{ms,6}^{4/7}. \tag{9}$$

We see that unless there are virtually no binaries in the core into which a massive black hole can exchange, such a black hole will encounter binaries far more often than it encounters single stars. However, note that merger via three-body encounters may require tens to hundreds of interactions (see below), whereas merger via

direct two-body capture is essentially immediate. Moreover, just as three-body rates are modified by gravitational radiation, the direct capture rate may be modified by deflections induced by three-body effects. Numerical simulations are required to determine when the two-body growth rate exceeds the three-body growth rate of a massive black hole in a cluster, and how the two effects couple.

2.3 Mass of central black holes in globular clusters

If the escape velocity from the core is $50\,\mathrm{km\,s^{-1}}$ (Webbink 1985) and the field black hole is ejected with a kinetic energy a factor $\approx 0.2(m/M)$ of the binary binding energy, then field black holes will be ejected when the binary orbital radius is about $10^{13}m_{10}$ cm, independent of the mass of the larger black hole. At a fractional hardening of $0.2(m/M)$ per interaction, the number of interactions required to bring the radius down to $\approx 3\times10^{11}$ cm is

$$\approx \ln[10^{13}m_{10}/(3\times10^{11}M_{50}^{1/2}m_{10}^{1/4})]/\ln(1+0.2m/M)$$

$$\approx 25M_{50}m_{10}^{-1}\ln(30M_{50}^{-1/2}m_{10}^{3/4}) \approx 100M_{50}.$$

Thereafter it merges quickly by gravitational radiation. Thus for each black hole accreted in this manner, $\approx 100M_{50}$ are ejected, so the massive black hole can in principle consume $\sim 0.1-1$ per cent of the smaller black holes that interact with it, ejecting the rest. Here again the precise eccentricity behaviour has a large impact; if high eccentricities are reached in the process of hardening, the binary merges when the semimajor axis is larger, and hence ejects fewer field black holes. The smaller black holes will also interact with and eject each other, further cutting down the mass supply. Portegies Zwart & McMillan (2000) find that interactions among the smaller black holes eject roughly half of them within $\sim$2 Gyr, and about 90 per cent after several billion years. Smaller black holes will also be ejected by the central dominant black hole. If a large black hole grows rapidly, it may be able to accrete a few tenths of a per cent of the initial mass in black holes. How much mass would this be?

If the initial mass function of stars in the cluster is the Salpeter IMF, $\mathrm{d}N/\mathrm{d}M \propto M^{-2.35}$ above $1\,\mathrm{M_\odot}$ (and flatter below one solar mass; see e.g. Meyer et al. 2000), then the mass in stars above some threshold $M_0 > 1\,\mathrm{M_\odot}$ is

$$\int_{M_0}^{\infty} M\frac{\mathrm{d}N}{\mathrm{d}M}\mathrm{d}M \propto M_0^{-0.35}. \tag{10}$$

If black holes form from stars with initial masses $m > 30\,\mathrm{M_\odot}$, this implies that $\sim$20 per cent of the initial mass might have been in stars that form black holes. Wind losses on the main sequence, mass loss during the supernova and ejection of some black holes by supernova recoil reduce this fraction. However, wind losses may be minimal for these low-metallicity stars (Vink et al. 2001), and supernova recoil of black holes is expected to be small compared to that for neutron stars (e.g. Sigurdsson & Hernquist 1993), so mass losses could be unimportant. This is especially true if stars of initial mass $M > 40\,\mathrm{M_\odot}$ produce failed supernova (Fryer 1999) and hence lose little mass. Given the initial total mass in black holes and the fraction lost to ejection, a large central black hole could accrete a fraction $\sim 10^{-3}$ of the initial mass of the cluster. Perhaps coincidentally, this is comparable to the $\sim$0.5 per cent ratio of supermassive black hole mass to bulge mass found for many galaxies (e.g. Merritt & Ferrarese 2001). Then, an initial cluster mass of $\sim 10^6\,\mathrm{M_\odot}$, appropriate for the densest clusters, implies that the black hole could gain a few hundred to a thousand solar masses from just the smaller black holes. If there are multiple $\sim 50\,\mathrm{M_\odot}$ black holes that can merge with each other, or if high eccentricities are reached, this number may become $\sim 10^3-10^4\,\mathrm{M_\odot}$.

Once the small black holes are depleted significantly, the main source of fuel becomes the next most massive stars in the core. If the black holes are consumed before $\gtrsim 1.5\,\mathrm{M_\odot}$ main sequence stars evolve, these will dominate the core. Otherwise, the next in line are neutron stars. These neutron stars have a number density comparable to or smaller than that of black holes (having a larger scaleheight and not being overwhelmingly more numerous) and are also less massive, so the characteristic time for mass gain becomes longer by a large factor, perhaps $\sim$100 or more. Thus, some extra growth is possible, but not factors of many. It is therefore plausible that this mechanism would preferentially generate black holes with masses $M \sim 10^3\,\mathrm{M_\odot}$.

2.4 Detectability of black holes in clusters

Most globular clusters obviously do not have $10^{41}\,\mathrm{erg\,s^{-1}}$ X-ray sources. If a substantial fraction contain high-mass black holes, why are they not emitting profusely? To answer this, consider what happens when stars of various types are accreted. Also, remember that the most massive type of star present in abundance will dominate the encounters. This is because binary exchange interactions tend to yield a binary consisting of the two most massive of the three stars involved, and a large number of such interactions are required to harden the binary sufficiently to produce a merger. If, for example, 100 binary interactions are required before merger, then the encounter rate with black holes must be $\lesssim 1$ per cent of the total for the final binary to not include two black holes. Given that the lower velocities of black holes increase their interaction cross-section, this would require severe depletion of the black holes in the core. If direct capture dominates instead, the higher mass and lower velocities of massive objects give them a far higher cross-section than the less-massive objects. Thus, mergers are primarily with black holes when they are present, then neutron stars and finally white dwarfs and the remaining main sequence stars (which have $M < 0.8\,\mathrm{M_\odot}$ for a typical current cluster age).

A small black hole merging with a massive black hole will generate only gravitational radiation. The same is true with a neutron star. The critical orbital separation r_{tide} between an object of mass M and one with mass $m \ll M$ and radius R, such that the less massive object is destroyed by tidal forces, is $r_{\mathrm{tide}} \approx 2.5(M/m)^{1/3}R$, or about 10^7 cm for $M = 50\,\mathrm{M_\odot}$ and $m = 1.5\,\mathrm{M_\odot}$. Assuming that the massive black hole is not rotating significantly (plausible, since interactions are likely to be uniformly distributed in direction), the radius of the innermost stable circular orbit, inside of which an object will plunge without further loss of angular momentum, is $R_{\mathrm{ISCO}} = 6GM/c^2 = 5\times10^7 M_{50}$ cm. The two radii are equal, $r_{\mathrm{tide}} = R_{\mathrm{ISCO}}$, when $M \approx 5\,\mathrm{M_\odot}$, and since $R_{\mathrm{ISCO}}/r_{\mathrm{tide}} \propto M^{2/3}$ more massive black holes swallow neutron stars even more easily. Therefore, a neutron star is not disrupted before merger.

A white dwarf is disrupted, but the merger is very quick. A white dwarf with typical mass $0.6\,\mathrm{M_\odot}$ has a radius $R \approx 10^9$ cm (e.g. Panei, Althaus & Benvenuto 2000). The tidal disruption radius is then $\approx 10^{10}M_{50}^{1/3}$ cm, well outside the innermost stable circular orbit. The dwarf therefore transfers mass as in a low-mass X-ray binary. However, at such separations the time to merge via gravitational radiation is short: roughly 10 yr for $M_{50} = 1$, scaling as $M^{-2/3}$. Even if this time increases as the white dwarf loses mass,

there is only a short time in which radiation can be detected. In addition, the initial mass transfer rate of $\sim 0.1\,\mathrm{M_\odot\,yr^{-1}}$ is 10^5 times the Eddington rate for a $50\,\mathrm{M_\odot}$ black hole. At such high rates, emission may be in the hypercritical regime in which neutrino emission dominates over photon emission (Houck & Chevalier 1991; Brown, Lee & Bethe 2000). Thus, white dwarfs, if accreted, will have a tiny duty cycle for observable emission, if indeed the emission is observable at all.

A main sequence star of mass $\sim 0.6\,\mathrm{M_\odot}$ has a radius ~ 40 times larger than a white dwarf of the same mass. The separation at disruption is thus 40 times higher, and the gravitational merger time $(40)^4 \approx 10^6$ times greater, than for a white dwarf. Thus, accretion of a main sequence star by a massive black hole will produce copious photon emission. At the Eddington rate a $0.6\,\mathrm{M_\odot}$ star will be consumed by a $\sim 50\,\mathrm{M_\odot}$ black hole in $\sim 2\times 10^6$ yr, smaller than but comparable to the hardening time-scale. If a cluster black hole has exhausted its supply of compact objects with mass greater than the main-sequence turn-off mass, it is therefore possible that it may form a low-mass X-ray binary (LXMB) with a main sequence star.

Accretion from gas in the cluster is unlikely to be detectable. Although much of the primordial gas in globular clusters may be swept out by supernovae, the winds from evolving red giants may be accreted. If we assume that at any given time 1 per cent of the stars are on the red giant branch, even a rich cluster with 10^6 stars will have only 10^4 red giants, which are distributed within a typical half-mass radius of 10 pc. The mass loss rates for low-mass red giants are difficult to determine but seem to be $<10^{-8}\,\mathrm{M_\odot\,yr^{-1}}$ and may be significantly less for the metal-deficient stars in globular clusters (Dupree 1986). The winds come out in a large range of velocities broadly distributed around the escape velocity (Lamers & Cassinelli 1999), which is roughly $50\,\mathrm{km\,s^{-1}}$ at the 1-au radius of a 0.8-$\mathrm{M_\odot}$ red giant. The winds take $3\times 10^{19}\,\mathrm{cm}/5\times 10^6\,\mathrm{cm\,s^{-1}} = 2\times 10^5$ yr to cross the volume, so that on average each star donates $\approx 10^{-3}\,\mathrm{M_\odot}$ in wind material, for a total of $10\,\mathrm{M_\odot}$ in $\frac{4\pi}{3}(10\,\mathrm{pc})^3 \approx 10^{59}\,\mathrm{cm^3}$. The density is then $2\times 10^{-25}\,\mathrm{g\,cm^{-3}}$, consistent with the number density of $\sim 0.1\,\mathrm{cm^{-3}}$ inferred from pulsar dispersion measures in 47 Tuc by Freire et al. (2001). Suppose the central black hole captures the gas via Bondi–Hoyle accretion, at the rate

$$\dot{M} \approx 2\times 10^{14} M_{50}^2 \rho_{-24} [v_6^2 + c_6^2]^{-3/2}\,\mathrm{g\,s^{-1}}, \tag{11}$$

where the hole has a velocity $10^6 v_6\,\mathrm{cm\,s^{-1}}$ relative to the cloud, and at infinity the density and sound speed of the gas are $10^{-24}\rho_{-24}\,\mathrm{g\,cm^{-3}}$ and $10^6 c_6\,\mathrm{cm\,s^{-1}}$, respectively. Then, the accretion rate is $\approx 5\times 10^{13}\,\mathrm{g\,s^{-1}}$ for a $10^3\,\mathrm{M_\odot}$ black hole. This is, however, likely to be an overestimate by several orders of magnitude. The X-ray luminosity generated by accretion ionises and heats its surroundings (e.g. Maloney, Hollenbach & Tielens 1996), so that even well beyond the Bondi–Hoyle capture radius the effective temperature is raised to $T > 10^6$ K by Compton heating. Pressure balance dictates that this heated gas must be less dense than its surroundings by a factor proportional to the temperature, so that the net accretion rate can be far lower than that indicated by a simple Bondi–Hoyle estimate. Accretion from any form of interstellar gas, molecular, atomic or otherwise, is thus unlikely to produce the observed luminosity. It is therefore difficult to rule out the presence of a large black hole based on upper limits to X-ray emission. For example, Grindlay et al. (2001) have recently observed the cluster 47 Tuc with the Chandra X-ray Observatory. Even without including the possible quenching of accretion by preheating, they find that despite the lack of any detectable point source of X-rays within 3 arcsec of the cluster centre, a black hole with a mass of up to $500\,\mathrm{M_\odot}$ could have escaped detection.

A final way in which such a black hole could be detected in a globular would be through its long-range effect on other cluster stars. Current data are ambiguous. For example, Sosin (1997) finds that in the post-core collapse clusters M15 and M30 the radial density cusp of stars is more consistent with the presence of a $\sim 10^3\,\mathrm{M_\odot}$ black hole than with simple equipartition core-collapse models. Similarly, Gebhardt et al. (2000) find evidence for a 2500-$\mathrm{M_\odot}$ black hole in the core of M15. However, there is only marginal consistency with the expected cusp in the velocity dispersion (e.g. Gebhardt et al. 1995). Observations of the pulsars in some clusters may be able to limit the mass of any black holes in their centres (Phinney 1993). Future observations will be required to place more stringent limits on the total black hole mass in various clusters.

3 CAPTURE OF CLUSTERS BY THEIR HOST GALAXIES AND ACCRETION IN THE GALACTIC DISC

Globular clusters around a galaxy such as our own interact with the galaxy through tidal stripping and other events. Current estimates are that approximately half of the clusters eventually merge with the disc and eventually disperse (e.g. Gnedin & Ostriker 1997; Takahashi & Portegies Zwart 1998, 2000; Gnedin et al. 1999). As the globular penetrates the disc of the galaxy, the damping rate increases, the relative velocity between the globular and the disc is driven toward zero, and globular and massive black hole are driven towards the midplane of the disc. If the central core of the globular survives this process intact, the intermediate mass BH contained inside might be released into the galactic disc with a low velocity ($v < 10\,\mathrm{km\,s^{-1}}$) relative to stars and molecular clouds in the disc. The black hole can then interact with the interstellar medium. An intriguing possible effect of a massive black hole inside a molecular cloud is that it may help precipitate cloud collapse and star formation. Note that in a giant molecular cloud of density $\rho = 10^{-22}\,\mathrm{g\,cm^{-3}}$, a $10^3\,\mathrm{M_\odot}$ black hole contributes most of the mass out to a distance of a few parsecs. The natural gravitational instability of the cloud would then be enhanced by the presence of the hole, and in a few free-fall times $t_{\mathrm{ff}} \sim 1/\sqrt{G\rho} \approx 10^7(\rho/10^{-22}\,\mathrm{g\,cm^{-3}})^{-1/2}$ yr a star cluster could form. This is comparable to the crossing time of a giant molecular cloud by a massive black hole, assuming a relative velocity of a few $\mathrm{km\,s^{-1}}$ (appropriate if the black hole is nearly at rest relative to the local average, since giant molecular clouds tend to have a random bulk velocity of 3–$5\,\mathrm{km\,s^{-1}}$; see Dame, Hartmann & Thaddeus 2001). As in a globular cluster, the most massive stars would sink to the centre, where (as main-sequence stars) they would accrete on to the black hole and generate significant luminosity, in principle up to the Eddington luminosity. This may account for the observation by Matsushita et al. (2000) that the brightest point X-ray source in M82 appears to be in a star cluster within a superbubble.

Note, however, that a black hole with a roughly stellar mass $M \approx 10\,\mathrm{M_\odot}$ cannot easily grow to the required $\sim 10^3\,\mathrm{M_\odot}$ masses in such an environment. This is because the e-folding time for mass increase at even the Eddington rate is 6×10^7 yr, comparable to or longer than the time-scale on which young star clusters dissipate (Portegies Zwart et al. 2001), so that although a black hole may be luminous as it feeds off of massive stars, the amount of mass it can

accumulate is relatively small. These black holes must grow elsewhere.

How likely is it that a $\sim 10^3\,\mathrm{M}_\odot$ black hole is in a dense interstellar cloud at any given time, and what is the resulting prediction for the number of active sources at any given time? Consider first our Galaxy. It currently has $\sim$150 globular clusters, of which $\sim$40 per cent, or 60, have core densities $n_6 > 0.1$ (Pryor & Meylan 1993) and therefore produced massive black holes in their cores according to our model. We assume that a similar number have merged with the Galaxy in its lifetime. Molecular clouds with densities $>10^2\,\mathrm{cm}^{-3}$ occupy a volume fraction $<10^{-2}$. It is therefore not surprising that no massive black holes are currently in dense clouds in our Galaxy.

In contrast, galaxies undergoing active star formation are thought to have surface densities of molecular gas that are higher than ours by a significant factor, perhaps $\sim$10 or so (Taniguchi & Ohyama 1998; Gao et al. 2001). If the molecular clouds in star-forming galaxies are similar to those in our Galaxy, the volume fraction of dense clouds is therefore likely to be up by a similar factor, to $\sim 10^{-1}$. That suggests that such a galaxy will have a few massive black holes currently in star clusters within molecular clouds, emitting $>10^{39}\,\mathrm{erg\,s}^{-1}$. This is consistent with the number observed.

4 DETECTION OF GRAVITATIONAL WAVES FROM A $10^3\,\mathrm{M}_\odot$ BLACK HOLE IN A GLOBULAR CLUSTER

A major prediction of our model is that tens of per cent of globulars have $\sim 10^3\,\mathrm{M}_\odot$ black holes. As discussed in Section 2.4 and Section 3, these holes are usually electromagnetically quiet. However, a massive black hole in a globular cluster around our Galaxy is a potentially strong source of persistent gravitational waves in the 10^{-4}–10^{-2} Hz sensitivity range of the planned Laser Interferometric Space Antenna (LISA). In the final stages of inspiral, some of these sources may be detectable with ground-based detectors including the Laser Interferometry Gravitational Observatory (LIGO) and its follow-up, LIGO II, up to $\sim$1 Gpc away. The strength, detectability, and number of such sources depends on many uncertain parameters, such as the eccentricity distribution and the current number density of lighter black holes in globulars. In this section we make rough estimates of the type and strength of signals that may be found.

The dimensionless gravitational wave strain amplitude measured a distance r from a circular binary of masses M and m with a binary orbital frequency f_{bin} is (Schutz 1997)

$$h = 2^{5/3}(4\pi)^{1/3}\frac{G^{5/3}}{c^4}f_{\mathrm{bin}}^{2/3}Mm(M+m)^{-1/3}\frac{1}{r}. \tag{12}$$

The frequency of gravitational waves from a circular binary is twice the binary frequency, but an eccentric binary will generate gravitational radiation in a range of harmonics. For example, our canonical $e = 0.7$ binary emits $\approx$30 times the power of a circular binary with the same semimajor axis, and has a peak at ten times the binary orbital frequency (Peter 1964). The power emitted at frequency f is proportional to $h^2 f^2$, so the dimensionless amplitude at peak for $e = 0.7$ is by coincidence similar to its value for a circular binary. A black hole binary will emit gravitational waves at all separations, but here we focus on the portion of the inspiral during which gravitational radiation has a significant influence on the orbital evolution. For a inspiral time of $10^7 M_{50}^{-1/2}$ yr and a central density of $10^6 n_6\,\mathrm{pc}^{-3}$, the semimajor axis is $a \approx 4\times 10^{11} n_6^{-1/4} M_{50}^{3/8} m_{10}^{1/4}$ cm, so that the binary orbital frequency is $f_{\mathrm{bin}} = 4\times 10^{-5} n_6^{3/8} M_{50}^{-1/16} m_{10}^{-3/8}$ Hz. Numerically, the gravitational wave amplitude is therefore

$$h \approx 2\times 10^{-21} n_6^{1/4} M_{50}^{5/8} m_{10}^{3/4} (10\,\mathrm{kpc}/r). \tag{13}$$

The sensitivity of LISA in the range 10^{-4}–10^{-3} Hz at a gravitational wave frequency f_{GW} is approximately $h_{\mathrm{min}} = 7\times 10^{-21}(f_{\mathrm{GW}}/10^{-4}\,\mathrm{Hz})^{-2}$ for a signal-to-noise ratio of 5 in an integration time of one year (R. Stebbins, private communication). Thus, for $e = 0.7$ and a peak gravitational wave frequency $f_{\mathrm{GW}} \approx 10 f_{\mathrm{bin}}$, the signal-to-noise ratio for most of the inspiral after the last three-body interaction is

$$S/N = 5h/h_{\mathrm{min}} \approx 25 n_6 M_{50}^{1/2}\frac{10\,\mathrm{kpc}}{r}. \tag{14}$$

If the typical eccentricity is higher than 0.7, the signal-to-noise ratio will also be higher. Therefore, black holes with masses greater than $\sim 100\,\mathrm{M}_\odot$ can be detected with high significance in a year in a dense globular cluster within 10 kpc. Depending on the eccentricity distribution of the binaries and the fraction in a favourable phase of the evolution, several cluster black holes in the Galactic system may be detectable with LISA (see Benacquista 1999, 2001 for an alternate viewpoint on the detectability of gravitational radiation from binaries in globulars).

The above applies to the amplitude of waves emitted during most of the spindown. In addition, some fraction of sources will be in the final stage of their spindown, and hence will emit waves of higher amplitude. The tendency of gravitational radiation to circularize binaries means that in these later stages the binaries are unlikely to have significant eccentricity, and hence almost all of the power will be radiated at twice the binary orbital frequency. For a given pair of masses the amplitude scales as $f_{\mathrm{bin}}^{2/3}$ and the sensitivity threshold of LISA scales as f_{bin}^{-2}, so the signal-to-noise ratio scales as $f_{\mathrm{bin}}^{8/3}$ up to the $\approx 10^{-2}$ Hz sensitivity peak of LISA. The fraction of sources with a given orbital frequency scales as the inspiral time, which is $\sim a^4 \sim f_{\mathrm{bin}}^{-8/3}$. Intermediate-mass black hole binaries may therefore be detectable at large distances; for example, at the 15-Mpc distance of the Virgo cluster, a fraction $\sim 10^{-3}$ of such binaries are likely to be observed, for a total of perhaps tens of sources among the $\approx 10^3$ galaxies in the cluster. Binaries in the last stages of inspiral will of course be visible to even larger distances. For example, from equation (2) a 10-$\mathrm{M}_\odot$ black hole in a circular orbit around a 10^3-$\mathrm{M}_\odot$ black hole at a binary orbital frequency of 5×10^{-3} Hz will spiral in within $\approx$10 yr. The dimensionless strain amplitude at a distance of 10 kpc would be $h = 4\times 10^{-19}$, compared to the LISA $S/N = 5$ sensitivity of $h = 10^{-23}$ at 10^{-2} Hz. Such a signal would therefore yield a 1-yr signal-to-noise ratio of 10 out to 200 Mpc with LISA. Portegies Zwart & McMillan (2000) estimate the number density of globulars in the universe as $8.4\,h^3\,\mathrm{Mpc}^{-3}$; if $\approx$10 per cent of these have $10^3\,\mathrm{M}_\odot$ black holes and core number densities $n_6 \approx 1$, we would expect $\approx$10 such binaries to be detectable at any given time.

The merger itself is at frequencies above the range of LISA. However, if an intermediate-mass black hole in a binary within a few years of merger is found with LISA, the final inspiral could be anticipated and detected with ground-based detectors such as LIGO, because the frequency, phase and location of the source would be known in advance. Unfortunately, the frequency of this final merger [$f_{\mathrm{GW}} \approx 40(M/10^2\,\mathrm{M}_\odot)^{-1}$ Hz at the last stable orbit of a non-rotating black hole] is only marginally observable with

LIGO-I, which has a lower frequency cut-off $\sim$40 Hz (see e.g. Brady & Creighton 2000). However, LIGO-II is expected to be able to detect 10-Hz sources at 99 per cent confidence for a 10-s integration and a dimensionless strain amplitude of $h = 2 \times 10^{-23}$ (scaling from fig. 3 of Brady & Creighton 2000). If black holes at the low end of the masses we consider (100 $M_\odot$ instead of 10^3 $M_\odot$) are present in a significant number of globulars, their final merger is in the LIGO-II sensitivity band. From equation (2), a 10 $M_\odot$ black hole spiraling into a 100 $M_\odot$ black hole spends approximately 10 s at a gravitational wave frequency of 10 Hz or higher. Using equation (12) as a rough guide, this signal would potentially be detectable out to 1 Gpc or more. Thus, advanced ground-based detectors may pick up gravitational waves from a large number of $\sim$100 $M_\odot$ black hole mergers in globular clusters.

5 DISCUSSION AND CONCLUSIONS

We suggest that a population of $\sim 10^3$ $M_\odot$ black holes is generated in globular clusters and then released into the discs of their host galaxies when the clusters are assimilated. Subsequent accretion from young star clusters formed from molecular clouds then produces the luminosity observed from non-nuclear point sources in other galaxies. Such black holes, accreting at less than the Eddington limit, can produce the observed flux without strong beaming.

King et al. (2001) have proposed instead that the black holes are of ordinary stellar mass but are strongly beamed. They argue that larger black holes would have difficulty in producing the required high-energy X-rays if the emission is from blackbody annuli in the disc, because the characteristic blackbody temperature for an Eddington accretor near the inner edge of the disc is $kT \approx 2(M/M_\odot)^{-1/4}$ keV. However, we point out that if a hot Compton corona exists near the black hole, as is usually invoked to explain the highest energy emission, the temperature scales as M/r and is thus independent of mass, so unless the spectrum is clearly of the multicolour blackbody form and has no non-thermal tail the X-ray spectrum does not easily distinguish between beamed and unbeamed emission. For example, we note that the Galactic microquasars GRS 1915+105 and GRO J1655−40 have significantly harder emission than would be expected from a simple multicolour blackbody (Makishima et al. 2000). We also point out that blazars, which are believed to have emission beamed towards us, tend to have relatively flat νF_ν spectra, varying typically by only a factor of $\sim$10 from 10^{10}–10^{18} Hz (up to an X-ray to radio ratio of 10^3 for some *Einstein* Slew Survey blazars; see Fossati et al. 1998). In contrast, the X-ray to radio νF_ν ratio for Galactic microquasars, which are not beamed towards us, is much greater; 10^6 for GRS 1915+105 (Ogley et al. 2000 for radio; Rao et al. 2000 for X-ray) and 10^5 for GRO J1655−40 (Hannikainen et al. 2000 for radio; Zhang et al. 1997 for X-ray). These latter sources are comparable to the brightest X-ray source in M82, which has an X-ray to radio ratio of at least 10^5 (see Matsumoto et al. 2001 and Kronberg et al. 2000 for radio; Kaaret et al. 2001 for X-ray). Although this does not prove that the intermediate-mass black hole candidates are unbeamed, it does argue in that direction.

Additional observations would be very helpful in determining whether beamed or unbeamed models are favoured. Looking at radio spectra of these sources, at high angular resolution, will allow a broader sample to be compared with blazar spectra. Another difference is the expected minimum time-scale of variability. For an unbeamed black hole of e.g. 10^3 $M_\odot$, the minimum time-scale would be the light-crossing time across the diameter of the minimum stable orbit, or $\approx$0.1 s. The time-scale is likely to actually be a few times this; for example, Cyg X-1, with a black hole mass likely to be $\sim$10 $M_\odot$, has a minimum time-scale of significant variability of $\approx$3 ms, three times the light-crossing time (Revnivtsev, Gilfanov & Churazov 2000). Above this time-scale, the fractional variability increases significantly. For a beamed black hole of mass $\sim$10 $M_\odot$, the minimum time-scale is decreased by a factor of $\sim 10^3$ by relativistic effects and the smaller size of the hole itself. A long observation with Chandra in continuous clocking mode, with a time resolution of 3 ms, would help distinguish between beamed and unbeamed models. Such observations are important to determine whether the bright sources are the first representatives of a new class of intermediate-mass black holes.

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