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Production of Massive Fermions at Preheating and Leptogenesis

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Abstract

We present a complete computation of the inflaton decay into very massive fermions during preheating. We show that heavy fermions are produced very efficiently up to masses of order 10^{17}–10^{18} GeV; the accessible mass range is thus even broader than the one for heavy bosons. We apply our findings to the leptogenesis scenario, proposing a new version of it, in which the massive right-handed neutrinos, responsible for the generation of the baryon asymmetry, are produced during preheating. We also discuss other production mechanisms of right-handed neutrinos in the early Universe, identifying the neutrino mass parameters for which the observed baryon asymmetry is reproduced.

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1 Introduction

It is commonly believed that the Universe underwent an early era of cosmological inflation [1]. The flatness and the horizon problems of the standard big bang cosmology are indeed elegantly solved if, during the evolution of the early Universe, the energy density happens to be dominated by the vacuum energy of a scalar field – the inflaton – and comoving scales grow quasi-exponentially.

At the end of inflation the Universe was in a cold, low-entropy state with few degrees of freedom, very much unlike the present hot, high-entropy Universe. At this stage, the Universe does not contain any matter and therefore it looks perfectly baryon symmetric. However, considerations about how the light element abundances were formed when the Universe was about 1 MeV hot lead us to conclude that $n_B/s = (2\text{–}9) \times 10^{-11}$. Here n_B/s is the difference between the number density of baryons and that of antibaryons, normalized to the entropy density of the Universe.

Until now, several mechanisms for the generation of the baryon (B) asymmetry have been proposed [2]. Grand Unified Theories (GUTs) unify the strong with the electroweak forces and predict baryon-number violating interactions at tree level. In these theories, the out-of-equilibrium decay of heavy Higgs particles can indeed explain the observed baryon asymmetry. In the theory of electroweak baryogenesis, baryon number violation takes place at the quantum level, caused by unsuppressed baryon-number violating sphaleron transitions in the hot plasma [3].

Since B and L – where L is the lepton number – are reprocessed by sphaleron transitions, while the anomaly-free linear combination $B - L$ is left unchanged, the baryon asymmetry may be generated from a lepton asymmetry [4]. Indeed, once the lepton number is produced, thermal scatterings redistribute the charges and convert (a fraction of) L into baryon number. In the high-temperature phase of the standard model, the asymmetries of baryon number B and of $B-L$ are therefore proportional [5]:

$$B = a(B - L), \qquad a \equiv \left(\frac{8n_g + 4n_H}{22n_g + 13n_H}\right), \tag{1}$$

where n_H is the number of Higgs doublets and n_g is the number of fermion generations.

In the standard model as well as in its unified extension based on the group $SU(5)$, $B - L$ is conserved and no asymmetry in $B - L$ can be generated. However, adding massive right-handed Majorana neutrinos to the standard model breaks $B - L$ and the primordial lepton asymmetry may be generated by their out-of-equilibrium decay. This simple extension of the standard model can be embedded into GUTs, as in the case of $SO(10)$. Heavy right-handed Majorana neutrinos are the key ingredient to explain the smallness of the light neutrino masses via the see-saw mechanism [6]. The presence of neutrino masses and mixings seems to be the most natural explanation of the recent reports from the Super-Kamiokande [7] and other [8] collaborations indicating the existence of neutrino oscillations. In light of these considerations, the generation of the baryon asymmetry through leptogenesis looks particularly attractive.

The leptogenesis scenario depends crucially on the mechanism that was responsible for populating the early Universe with right-handed neutrinos, and consequently on the thermal history of the Universe, and on the fine details of the reheating process after inflation. One goal of this paper is to discuss several production mechanisms of heavy right-handed neutrinos in the Universe, to compare them and to identify the regions of the appropriate parameter space where the production mechanism is efficient enough to explain the observed baryon asymmetry.

The simplest way to envision the reheating process after inflation is to assume that the comoving energy density in the zero mode of the inflaton decays *perturbatively* into ordinary particles, which then scatter to form a thermal background [9]. It is usually assumed that the decay width of this process is the same as the decay width of a free inflaton field. Of particular interest is a quantity known as the reheat temperature, denoted as T_{RH}. This is calculated by assuming an instantaneous conversion of the energy density in the inflaton field into radiation when the decay width of the inflaton Γ_ϕ is equal to H, the expansion rate of the Universe. This yields

$$T_{RH} \simeq \sqrt{\Gamma_\phi M_{\rm P}}, \tag{2}$$

where $M_{\rm P}$ is the Planck mass. The commonly-accepted assumption is that the heavy right-handed neutrinos with mass M_N were as abundant as photons at very high temperatures. This assumption requires not only that $T_{RH} \gtrsim M_N$, but also that the heavy neutrinos are abundantly produced by thermal scatterings during the reheating stage. This condition, as we will discuss in sect. 3.1, significantly limits the allowed range of neutrino masses compatible with leptogenesis.

There might be one more problem associated with the hypothesis that $T_{RH} \gtrsim M_N$ in the old theory of reheating, and that is the problem of relic gravitinos [10]. If one has to invoke supersymmetry to preserve the flatness of the inflaton potential, it is mandatory to consider the cosmological implications of the gravitino – the spin-3/2 partner of the graviton which appears in the extension of global supersymmetry to supergravity. The slow gravitino decay rate leads to a cosmological problem because the decay products of the gravitino destroy light nuclei by photodissociation and hadronic showers, thus ruining the successful predictions of nucleosynthesis. The requirement that not too many gravitinos are produced after inflation provides an upper bound on the reheating temperature T_{RH} of about 10^8–10^{10} GeV, depending on the value of the gravitino mass [11]. In the following, T_{RH} will be therefore intended as the largest temperature allowed after inflation from considerations of the gravitino problem.

In order to relax the limit on M_N imposed by the gravitino problem, we will consider the possibility that the heavy neutrinos are produced directly through the inflaton decay process. This is kinematically accessible whenever

$$M_N < m_\phi, \tag{3}$$

where m_ϕ is the inflaton mass. In the case of chaotic inflation with quadratic potential, the density and temperature fluctuations observed in the present Universe determine m_ϕ and require M_N to be smaller than about 10^{13} GeV.

The outlook for leptogenesis might be brightened even further with the realization that reheating may differ significantly from the simple picture described above [12, 13, 14, 15, 16]. In the first stage of reheating, called *preheating* [12], nonlinear quantum effects may lead to extremely effective dissipative dynamics and explosive particle production, even when single particle decay is kinematically forbidden. In this picture, particles can be produced in a regime of broad parametric resonance, and it is possible that a significant fraction of the energy stored in the form of coherent inflaton oscillations at the end of inflation is released after only a dozen or so oscillation periods of the inflaton. The preheating stage occurs because, for some parameter ranges, there are new non-perturbative decay channels. In the case of bosonic particle production, coherent oscillations of the inflaton field induce stimulated particle emissions into energy bands with large occupancy numbers [12]. The modes in these bands can be understood as Bose condensates, and they behave like classical waves. A crucial observation for GUT baryogenesis induced by the decay of very heavy Higgs bosons is that particles with masses larger than that of the inflaton may be easily produced during preheating [17].

Indeed, for coupling constants of order unity one would have copious production of boson particles as heavy as 10^{15} GeV, *i.e.* 100 times greater than the inflaton mass. This is a major departure from the old constraint of reheating.

In this paper we wish to provide the first complete calculation of the inflaton decay into heavy fermions during preheating. We will show that fermion production is extremely efficient, in a mass range even broader than the one for heavy bosons: fermions may be generated up to masses of order of 10^{18} GeV. What distinguishes the production of very massive fermions and bosons in an oscillating background is the expression for the total mass. For bosons, the total mass can never vanish and the production reaches the maximum when the amplitude of the inflaton goes through zero. For fermions, the total mass can vanish for particular values of the inflaton field, rendering particle creation much easier.

We will numerically compute the density of the massive fermions produced at the resonance stage. This is the crucial parameter for leptogenesis, when these heavy fermions are identified with the right-handed neutrinos giving rise to the lepton asymmetry. We want to stress that the out-of-equilibrium condition is naturally achieved in this scenario, since the distribution function of the fermionic quanta generated at the resonance is far from a thermal distribution. We will show that the observed baryon asymmetry may be explained by the phenomenon of leptogenesis after preheating, with a reheating temperature compatible with the gravitino problem.

The paper is organized as follows. In sect. 2 we present our calculation, with numerical results as well as some analytical estimates concerning the production of massive fermions during preheating. In sect. 3 we discuss and compare the relevant production mechanisms of right-handed neutrinos in the early Universe, identifying the appropriate neutrino-mass parameters where these mechanisms are the most efficient, as far as the generation of the baryon asymmetry is concerned.

2 Heavy Fermion Production at Preheating

In this section we describe the basic physics underlying the mechanism of heavy fermion generation during the preheating stage, perform the relevant numerical calculations and present some analytical estimates. In the following we will focus on the model of chaotic inflation, with a massive inflaton ϕ having quadratic potential $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$. Here

$m_\phi \sim 10^{13}$ GeV is fixed by the COBE normalization of the cosmic microwave background anisotropy.

We will suppose that the inflaton field is coupled to a very massive Dirac fermion X with bare mass m_X via the Yukawa coupling

$$\mathcal{L}_Y = g\phi \bar{X} X. \tag{4}$$

The total mass of the fermion X is then given by

$$m(t) = m_X + g\phi(t). \tag{5}$$

When we apply our results to the leptogenesis scenario, the fermion X will be identified with the lightest of the right-handed Majorana neutrinos N_1. Although in this section we will restrict our considerations to the case of Dirac particles, the treatment of Majorana fermions is completely analogous. Our final results regarding the abundances of particles (with equal amount of anti-particles being produced) are valid for both Dirac and Majorana fermions.

2.1 The Basic Formalism

We start (see *e.g.* discussion in ref. [18]) by canonically quantizing the action of the massive field X in curved space with Friedmann-Robertson-Walker metric. In the system of coordinates in which the line element is given by $ds^2 = a^2(\eta)(d\eta^2 - d\vec{x}^2)$, where a is the scale factor of the expanding Universe and η is the conformal time defined as $d\eta = dt/a$, the Dirac equation becomes

$$\left(\frac{i}{a}\gamma^\mu \partial_\mu + i\frac{3}{2}H\gamma^0 - m\right) X = 0. \tag{6}$$

Here $H = (a'/a^2)$ is the Hubble rate, the prime denotes derivative with respect to conformal time, and the γ-matrices are defined in *flat* space-time. By defining $\chi = a^{-3/2}X$, eq. (6) can be reduced to the more familiar form

$$(i\gamma^\mu \partial_\mu - a\, m)\, \chi = 0. \tag{7}$$

Since a is a function of η, but not of $\vec{x}$, spatial translations are symmetries of space-time, and we can separate the variables using the decomposition

$$\chi\,(x) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}} \sum_r \left[u_r(k,\eta)a_r(k) + v_r(k,\eta)b_r^\dagger(-k)\right], \tag{8}$$

where the summation is over spin, and $v_r(k) = C\bar{u}_r^T(-k)$. We impose the canonical anticommutation relations on the creation and annihilation operators

$$\left\{a_r(k), a_s^\dagger(k')\right\} = \left\{b_r(k), b_s^\dagger(k')\right\} = \delta_{rs}\delta(\vec{k} - \vec{k}') \tag{9}$$

which, together with the quantization conditions, determine the normalization of the spinors u,

$$u_r^\dagger(k,\eta)u_s(k,\eta) = v_r^\dagger(k,\eta)v_s(k,\eta) = \delta_{rs}, \qquad u_r^\dagger(k,\eta)v_s(k,\eta) = 0. \tag{10}$$

Equations (10) are valid at any conformal time, since they are preserved by the evolution.

In the representation in which $\gamma^0 = \begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix}$ and with the definition $u \equiv \begin{pmatrix} u_+ \\ u_- \end{pmatrix}$, the equation of motion (7) can be written as a set of uncoupled second-order differential equations,

$$\left[\frac{d^2}{d\eta^2} + \omega^2 \pm i(a'm + am')\right] u_\pm(k) = 0 \tag{11}$$

$$\omega^2 = k^2 + m^2a^2. \tag{12}$$

We can now write the Hamiltonian as

$$\begin{aligned} H(\eta) = \int d^3x \chi^\dagger(-i\partial_0)\chi &= \int d^3k \sum_r \Big\{E_k(\eta)\left[a_r^\dagger(k)a_r(k) - b_r(k)b_r^\dagger(k)\right] \\ &+ F_k(\eta)\, b_r(-k)a_r(k) + F_k^*(\eta)\, a_r^\dagger(k)b_r^\dagger(-k)\Big\}. \end{aligned} \tag{13}$$

By using the equations of motion, we find

$$E_k = 2k\ \mathrm{Re}(u_+^*u_-) + am\left(1 - 2u_+^*u_+\right), \tag{14}$$

$$F_k = k\left(u_+^2 - u_-^2\right) + 2am\ u_+u_-, \tag{15}$$

$$E_k^2 + |F_k|^2 = \omega^2. \tag{16}$$

Here we have chosen the momentum k along the third axis, and selected the gamma-matrix representation in which $\gamma^3 = \begin{pmatrix} 0 & 1\!\!1 \\ -1\!\!1 & 0 \end{pmatrix}$.

In order to give a "quasi-particle" interpretation, we diagonalize the Hamiltonian in eq. (13) with a time-dependent Bogolyubov canonical transformation, and define the new creation and annihilation operators

$$\hat{a}(k,\eta) = \alpha(k,\eta)a(k) + \beta(k,\eta)b^\dagger(-k) \tag{17}$$

$$\hat{b}^\dagger(k,\eta) = -\beta^*(k,\eta)a(k) + \alpha^*(k,\eta)b^\dagger(-k). \tag{18}$$

Imposing canonical anticommutation relations on the operators $\hat{a}$ and $\hat{b}$, we find $|\alpha|^2 + |\beta|^2 = 1$. For

$$\frac{\alpha}{\beta} = \frac{E_k + \omega}{F_k^*}, \qquad |\beta|^2 = \frac{|F_k|^2}{2\omega(\omega + E_k)} = \frac{\omega - E_k}{2\omega}, \tag{19}$$

the normal-ordered Hamiltonian in terms of the "quasi-particle" operators is diagonal,

$$H(\eta) = \int d^3k \sum_r \omega(\eta) \left[\hat{a}_r^\dagger(k)\hat{a}_r(k) + \hat{b}_r^\dagger(k)\hat{b}_r(k)\right]. \tag{20}$$

Next, we define a "quasi-particle" vacuum, such that $\hat{a}|0_\eta\rangle = \hat{b}|0_\eta\rangle = 0$. The total number of produced particles up to time η (equal to the number of produced antiparticles) is given by the vacuum expectation value of the particle number operator N,

$$n(\eta) = \langle 0_\eta | N | 0_\eta \rangle = \frac{1}{\pi^2 a^3(\eta)} \int_0^\infty dk\ k^2\ |\beta|^2. \tag{21}$$

The density of produced particles is then computed by integrating the equations of motion (11) with an initial condition at time $\eta = 0$ given by[1]

$$u_\pm(0) = \sqrt{\frac{1}{2}\left(1 \mp \frac{ma}{\omega}\right)}, \qquad u'_\pm(0) = iku_\mp(0) \mp iamu_\pm(0). \tag{22}$$

This boundary condition corresponds to $E_k = \omega$, $F_k = 0$ at $\eta = 0$ or, in other words, to an initial vanishing particle density.

2.2 Numerical Results

The equations of motion (11) describe oscillators with time-varying complex frequency. If m is constant, the time dependence enters only through the scale factor a. The corresponding gravitational creation of heavy fermions was studied in ref. [19]. However, of particular interest is the case in which $m = m(t)$ is a periodic function of time. This is realized when the scalar field ϕ, coupled to X as in eq. (4) is homogeneous and oscillates in time with frequency $V''(\phi)$. It is useful to write the equations of motion in terms of dimensionless variables. We introduce a dimensionless time $\tau \equiv m_\phi \eta$, as well as a dimensionless field $\varphi \equiv \phi/\phi(0)$, so that the scalar field is normalized by the condition $\varphi(0) = 1$. We define $\phi(0)$ as the value of the inflaton field at the moment when its oscillations begin, $\phi(0) \simeq 0.28 M_{\rm P}$ [14]. With these redefinitions, the equation of motion for the background field φ does not contain any parameter, while the fermion

[1] In practice, one has to solve only the evolution equation for u_+, since u_- is determined by the equation $u_- = (amu_+ - iu'_+)/k$.

mass is measured in units of m_ϕ and the strength of the fermion coupling to the external background is determined by the dimensionless combination $g\phi(0)/m_\phi$. For the sake of correspondence with the bosonic case, we introduce the parameter

$$q \equiv g^2\phi^2(0)/4m_\phi^2\,. \tag{23}$$

The production of fermions by an external oscillating background in Minkowski space-time was studied in refs. [20, 21, 22]. The production of massless fermions in a $\lambda\phi^4$ inflaton model, studied in ref. [22], can be reformulated into particle production in an expanding Universe by a simple conformal transformation. Moreover, only the case of moderate q ($q < 100$) has been previously considered. The parametric production of massive fermions in an expanding Universe has never been analyzed before. By analogy with the bosonic case [12, 14, 15], we expect that an efficient production of very massive fermions will require a very large value of q.

The result of our numerical integration is best summarized in fig. 1. For different values of the fermion mass m_X and of the q parameter, fig. 1 shows ρ_X/ρ, the fraction of the inflaton energy density ρ which ends up in the fermionic energy density ρ_X. In our simulation, we have neglected the back reaction of the produced fermions in the evolution of the inflaton field.

Fermion production is efficient up to a time at which ratio ρ_X/ρ freezes out. For fixed q, the larger m_X is, the earlier this freeze-out occurs. In particular, near the cut-off of ρ_X/ρ at large m_X, the production ceases just after a few oscillations of the inflaton field. In fig. 2 we have plotted the final phase-space density of produced fermions in comoving volume for $m_X = 100\ m_\phi$, $q = 10^5$ and $q = 10^8$. For $q = 10^8$ this distribution is reached after twenty inflaton oscillations, while for $q = 10^5$ it is reached just after the first inflaton oscillation. In fig. 1, the curves for $q = 10^7$ and $q = 10^8$ correspond to an evolution up to 20 inflaton oscillations. For $q = 10^8$ and $m_X < 100\ m_\phi$, the freeze-out has not been reached in 20 oscillations and ρ_X/ρ would have grown further, had we integrated for longer times. This explains the slope of the curves in fig. 1 at small m_X for $q > 10^6$. In general, if we integrate up to the freeze-out, we find that the ratio ρ_X/ρ is almost constant with m_X up to a cut-off value $(m_X)_{\rm max}$ much larger than m_ϕ. In this regime, the number density of fermions is inversely proportional to m_X.

The time evolution of the phase-space density of produced fermions in the cases in which more than one inflaton oscillation is required to reach the final distribution

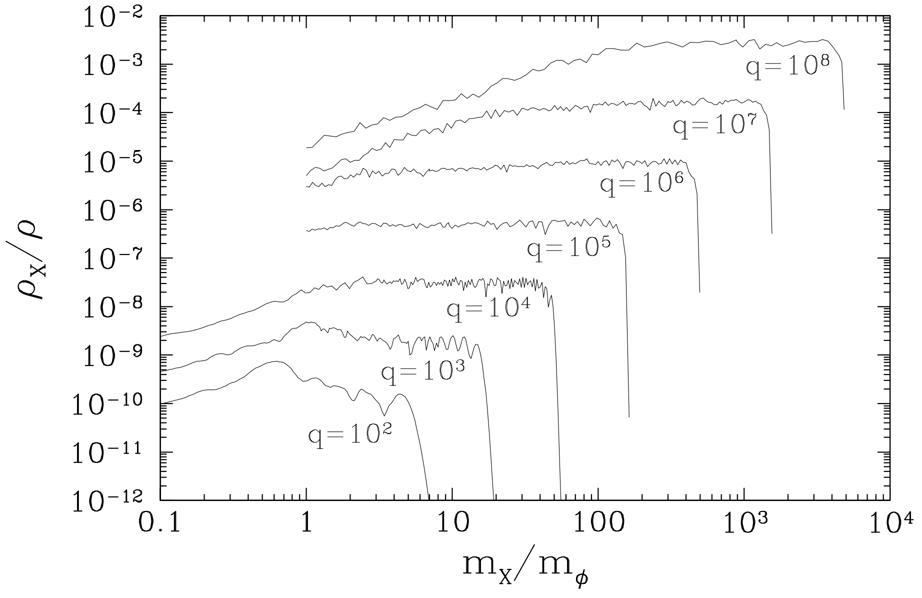

Figure 1: The fraction of the energy density of produced fermions with respect to the total energy density, as a function of the fermion mass m_X in units of the inflaton mass, for various values of q.

is shown in figs. 3 and 4. For the parameters shown in fig. 3, the final distribution was reached after three inflaton oscillations, while in the case of fig. 4 the X-particle production continues up to the twentieth oscillation.

Particle production at large q and large m_X is similar to the "instant preheating" phenomenon discussed in ref. [23]. However, contrary to what is discussed in ref. [23], where the production of heavy fermions was possible only through the coupling of the fermion field to other bosonic degrees of freedom produced by preheating, in our case the instant preheating takes place by coupling the fermion directly to the inflaton field, and therefore this realization is much less model-dependent.

Let us summarize our numerical results in a form suitable for the analytical estimates we will perform in sect. 2.3. Because of the Fermi-statistics, $n(k) \equiv |\beta|^2 \leq 1$. The maximum value of $n(k) \sim 1$ is reached rapidly at some $k = k_{\rm max}$. With time, if particle creation is still efficient, $k_{\rm max}$ grows at each oscillation. We can estimate the number density of the produced fermions as $n = \int d^3k n(k) \propto k_{\rm max}^3$. From our numerical results we observe that, at fixed m_X, the freeze-out value of $k_{\rm max}$ scales as $k_{\rm max} \propto q^\alpha$, with α slightly larger than 1/3; see figs. 1 and 2. On the other hand, $k_{\rm max}$ has to scale as $m_X^{-1/3}$ to explain why ρ_X/ρ is nearly independent of m_X, at constant q. We also

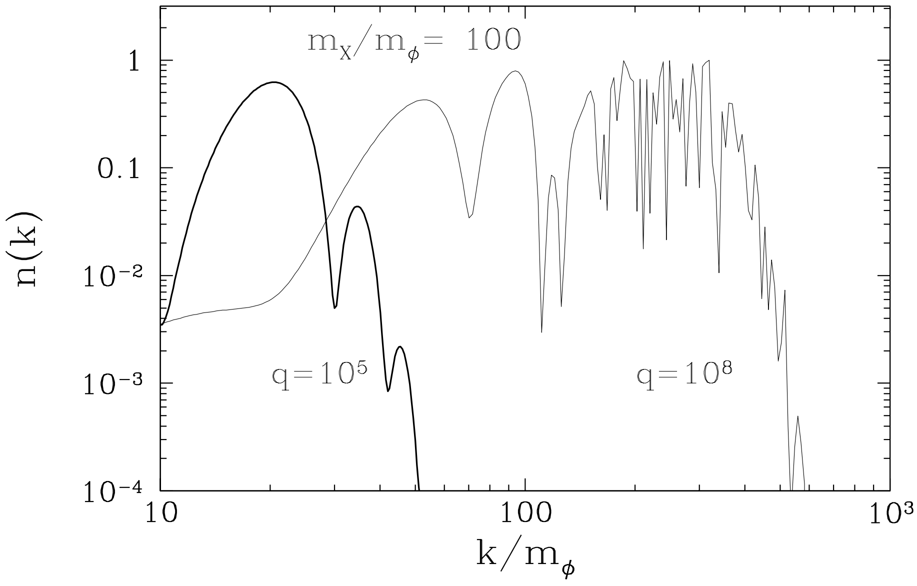

Figure 2: The final phase-space density of produced particles for two values of the parameter q. The X-fermions are taken to be 100 times heavier than the inflaton. At $q = 10^5$ the freeze-out of the particle production was reached after the first inflaton oscillation, while for $q = 10^8$ it required twenty oscillations.

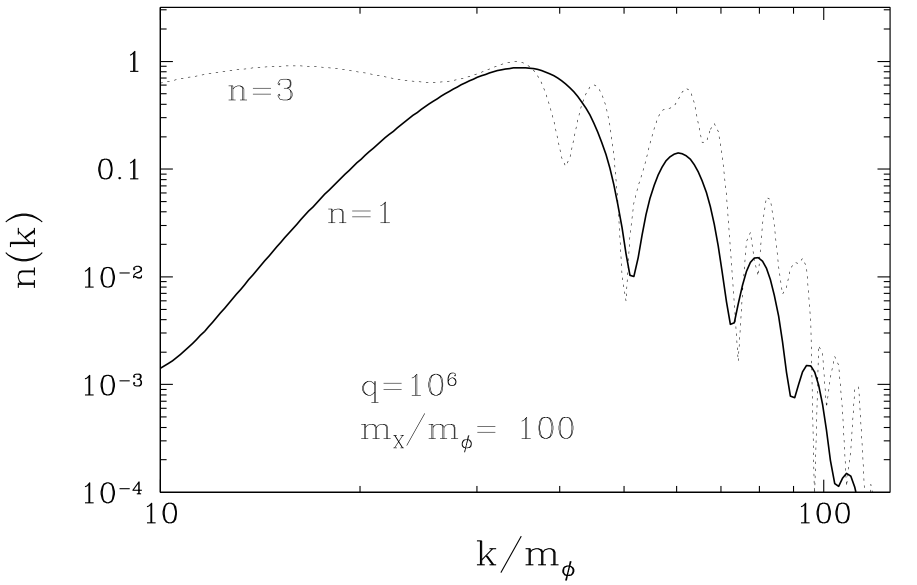

Figure 3: The phase-space density of produced particles after $n = 1$ and $n = 3$ inflaton oscillations for $q = 10^6$ and for X-fermions 100 times heavier than the inflaton. The distribution at $n = 3$ coincides with the final distribution.

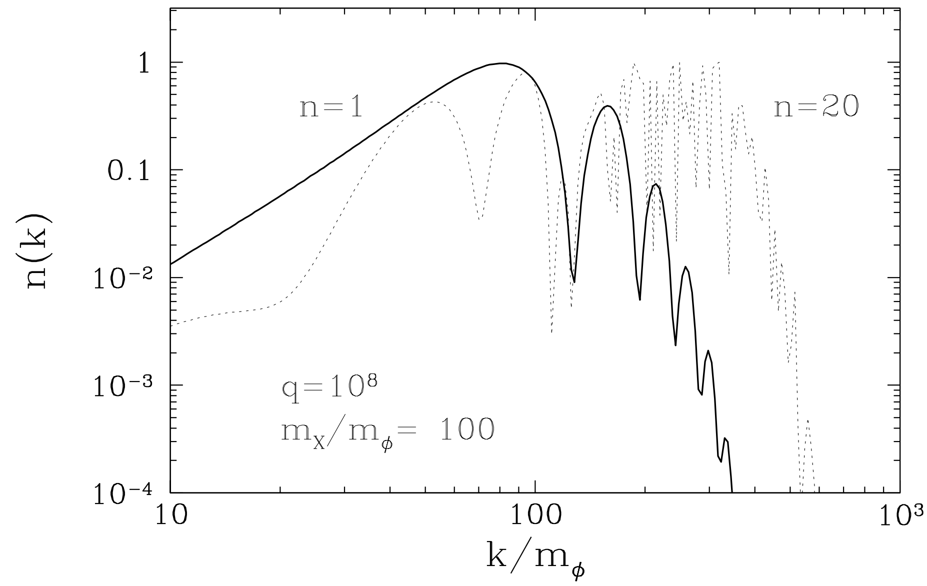

Figure 4: The phase-space density of produced particles after $n = 1$ and $n = 20$ inflaton oscillations for $q = 10^8$ and for X-fermions 100 times heavier than the inflaton. The distribution at $n = 20$ coincides with the final distribution.

find numerically that the cut-off value $(m_X)_{\rm max}$, *i.e.* the maximum mass of produced fermions, is about $(m_X)_{\rm max} \simeq q^{1/2} m_\phi/2$.

2.3 Analytical Estimates

The goal of this section is to provide some analytical estimates that can help us to understand the numerical results presented above.

Compared with the bosonic case, the novel feature of the fermionic production is that much heavier particles can be generated rather efficiently during the oscillations of the inflaton field, see fig. 1. The key difference between the Bose and Fermi cases resides in the expression for the effective particle mass in an external oscillating background. In the Fermi case it is given by eq. (5), $m(t) = m_X + g\phi(t)$, while in the Bose case we have $m^2(t) = m_X^2 + g^2\phi^2(t)$. In both cases, the production of particles with masses larger than the inflaton mass (*i.e.* the frequency of the inflaton oscillations) requires large q. This, in turn, means that $m(t)$ is typically large, and creation of X-particles is impossible at all times, except for very short periods when $m(t)$ fluctuates around its minimum value. In the bosonic case $m(t)$ can never be smaller than m_X. However, in the fermionic case, $m(t)$ can vanish as long as the amplitude of the inflaton field is large enough, $|\phi| > m_X/g$.

In the expanding Universe the amplitude ϕ_0 of the oscillating inflaton field decreases with time. It is reasonable to assume that the sharp cut-off in the particle production at large m_X that we observe in fig. 1 corresponds to a situation in which the condition $m(t) = 0$ cannot be satified even during the first oscillation of the inflaton field. To verify this assumption, let us write $m(t)$, with the help of eq. (23), as

$$m = m_X + 2\sqrt{q} m_\phi \frac{\phi}{\phi(0)} \simeq m_X + \frac{\sqrt{q} m_\phi}{\pi N} \cos(2\pi N) \,. \tag{24}$$

Here we have taken into account that $\phi \propto t^{-1}$, when the energy density of the Universe is dominated by the oscillating inflaton field, and we have denoted with N the number of oscillations of the inflaton field $N = m_\phi t/(2\,\pi)$.

At large m_X, the minimum of m is reached around $N = 1/2$. Therefore, if

$$m_X > m_\phi \frac{2}{\pi} \sqrt{q}, \tag{25}$$

the total mass m never vanishes[2]. The value of $(m_X)_{\rm max}$ given in eq. (25) is already in good agreement with our numerical results. However, eq. (24) cannot be trusted at small N. Actually, our numerical integration shows that the minimum of the inflaton amplitude ϕ_0 during the first oscillation is given by

$$\frac{\phi_0}{\phi(0)} \simeq -0.25. \tag{26}$$

This means that the cut-off value of the mass is at

$$(m_X)_{\rm max} \simeq m_\phi \frac{\sqrt{q}}{2}, \tag{27}$$

in perfect agreement with the results presented in fig. 1.

If $m_X < (m_X)_{\rm max}$, $m(t)$ can vanish more than once during the inflaton oscillations. In particular, in the case in which $m_X \gg m_\phi$, one can suppose that the production of particles continues until the amplitude of the oscillations drops below the critical value

$$(\phi_0)_{\rm crit} = \frac{m_X}{g} = \frac{m_X}{2 m_\phi \sqrt{q}} \phi(0). \tag{28}$$

At later times, when $\phi_0 < (\phi_0)_{\rm crit}$, the ratio ρ_X/ρ is frozen, since particle production has stopped. Let us now estimate analytically the final value of ρ_X/ρ. During the inflaton

[2]We are considering here the case in which $g\phi(0)$ and m_X have the same sign. If these two terms have a relative minus sign, particle production can be extended to even larger values of m_X.

oscillations, the Fermi distribution function is rapidly saturated up to some maximum value of the momentum k, *i.e* $|\beta(k)|^2 \simeq 1$ for $k \lesssim k_{\max}$ and it is zero otherwise. The value of $k_{\max}$ increases at each inflaton oscillation until particle production stops. Therefore, what is relevant for the determination of the final value of ρ_X/ρ is $k_{\max}$ at the freeze-out, *i.e.* when eq. (28) is satisfied and particles are no longer generated. At this moment,

$$\rho_X \simeq \frac{m_X p_{\max}^3}{3\pi^2}\,, \tag{29}$$

where $p_{\max}$ is the maximum *physical* momentum at freeze-out, $p_{\max} = k_{\max}/a$.

We estimate $p_{\max}$ as the maximum value of the physical momentum above which the evolution of the vacuum mode function is adiabatic. We choose as a trial mode function the following expression

$$u_+(\eta) = \sqrt{\frac{1}{2}\left(1 - \frac{m_{\rm eff}}{\omega}\right)} \times \exp\left(-i\int^{\eta} \omega \, d\eta'\right), \tag{30}$$

where $m_{\rm eff} = ma$. Notice that this solution corresponds to a vacuum, since it gives $\beta(k) = 0$. This trial function solves to a good approximation the equation of motion if the following condition holds

$$\frac{1}{2}\left(1 + \frac{m_{\rm eff}}{\omega}\right)\left|\frac{m''_{\rm eff}}{\omega} + \frac{1}{2}\left(\frac{m'_{\rm eff}}{\omega}\right)^2\left(1 - \frac{5m_{\rm eff}}{\omega}\right)\right| \ll \omega^2, \tag{31}$$

where we have used the property $\omega' = m_{\rm eff} m'_{\rm eff}/\omega$. As we have explained above, for large q, particle production takes place during short intervals when $m_{\rm eff} \simeq 0$, or $\cos(m_\phi t) \simeq -m_X/g\phi_0$, neglecting the expansion of the Universe during these short periods of particle creation. Under these circumstances, the condition (31) reduces to

$$\left|2pm_X + g^2\phi_0^2 - m_X^2\right| \, m_\phi^2 \ll 4\,p^4\,, \tag{32}$$

where we have used $\omega \simeq k = pa$. As a result, at the end of the particle production epoch, see eq. (28), we get

$$p_{\max}^3 \simeq \frac{1}{2} m_X m_\phi^2. \tag{33}$$

To compare with the numerical results of sect. 2.2, it is convenient to express eq. (33) in terms of the comoving momentum,

$$k_{\max} = \left(\frac{2m_\phi^4}{m_X} q\right)^{1/3}. \tag{34}$$

This reproduces the approximate scaling law $k_{\max} \propto q^{1/3} m_X^{-1/3}$ observed in the numerical results of sect. 2.2. Notice that, at earlier epochs when $g^2\phi_0^2 \gg m_X^2$, the value of $p_{\max}$ is larger, but the corresponding fraction of X-particles produced at this stage becomes subdominant (because it is red-shifted) with respect to the fraction of particles generated right before the freeze-out. In terms of the parameter $q_{\rm eff}(t) \equiv g^2\phi^2(t)/4m_\phi^2$, we obtain that at early times $p_{\max} \propto q_{\rm eff}^{1/4}$, while at the end of the particle production epoch $p_{\max} \propto q_{\rm eff}^{1/6}$.

Our result for $p_{\max}$ amd $(m_X)_{\max}$ and their relative scaling with q for the fermions produced during preheating are different from what argued in ref. [22] and from the behaviour of the same quantities encountered when dealing with boson particle production during preheating. In particular, for bosons, both $p_{\max}$ and $(m_X)_{\max}$ scale like $q_{\rm eff}^{1/4}$ [12, 14].

We can now use eqs. (29) and (33) to find that $\rho_X \simeq m_X^2 m_\phi^2/6\pi^2$. The inflaton energy density at freeze-out is $\rho = m_\phi^2\phi^2/2 = \phi^2(0)m_X^2/8q$, and therefore the energy density fraction is

$$\frac{\rho_X}{\rho} \simeq \frac{4}{3\pi^2}\frac{m_\phi^2}{\phi^2(0)}q = \frac{1}{3\pi^2}\,g^2. \tag{35}$$

This expression describes quite well the behaviour observed in fig. 1 for a large range of q. Equation (35) does not depend upon m_X, it is approximately proportional to q and it gives a reasonable estimate for the overall magnitude. Indeed, for $\phi(0) \simeq 0.28M_{\rm P}$ [14] and $m_\phi/M_{\rm P} \simeq 10^{-6}$, eq. (35) reduces to $\rho_X/\rho \simeq 2\times 10^{-12}q$, in good agreement with our numerical results. At very large values of q the expression (35) underestimates the ratio ρ_X/ρ; for instance at $q = 10^8$, it gives a value of ρ_X/ρ smaller than the numerical result by about one order of magnitude.

In the next section, we will apply our results on fermionic preheating to the case of heavy right-handed neutrinos and leptogenesis. We want to emphasize, however, that the results obtained in this section may be relevant in other cosmological contexts, such as the generation of superheavy dark matter after inflation [24].

3 Leptogenesis

The Lagrangian terms relevant for leptogenesis describe the interactions between the massive right-handed neutrinos N, the lepton doublet ℓ_L, and the Higgs doublet H,

$$\mathcal{L} = -\bar{N} h_\nu H \ell_L - \frac{1}{2} \bar{N}^c M N + \text{h.c.} \tag{36}$$

Here the Yukawa couplings h_ν and the Majorana mass M are 3×3 matrices and generation indices are understood. We choose to work in a field basis in which M is diagonal with real and positive eigenvalues ordered increasingly ($M_1 < M_2 < M_3$). The mass matrix of the light, nearly left-handed, neutrinos is given by

$$m_\nu = -h_\nu^T M^{-1} h_\nu \langle H \rangle^2. \tag{37}$$

The decays of the heavy neutrinos N into leptons and Higgs bosons violate lepton number

$$\begin{aligned} N &\rightarrow \bar{H}\ell, \\ N &\rightarrow H\bar{\ell}. \end{aligned} \tag{38}$$

The interference between the tree-level decay amplitude and the absorptive part of the one-loop diagram can lead to a lepton asymmetry of the right order of magnitude to explain the observed baryon asymmetry, as has been extensively discussed in the literature [4, 25, 26, 27] (for reviews, see ref. [28]). The interference with the one-loop vertex amplitude yields a CP-violating decay asymmetry for N_1 equal to

$$\epsilon_V = \frac{1}{8\pi \left(h_\nu h_\nu^\dagger\right)_{11}} \sum_{j=2,3} \text{Im} \left[\left(h_\nu h_\nu^\dagger\right)_{1j}\right]^2 f\left(\frac{M_j^2}{M_1^2}\right), \tag{39}$$

$$f(x) = \sqrt{x} \left[1 - (1+x) \ln\left(\frac{1+x}{x}\right)\right]. \tag{40}$$

The absorptive part of the one-loop self-energy gives a contribution to the N_1 asymmetry which, in the case of only two-generation mixing, is given by

$$\epsilon_S = \frac{\text{Im}\left[\left(h_\nu h_\nu^\dagger\right)_{1j}\right]^2}{\left(h_\nu h_\nu^\dagger\right)_{11} \left(h_\nu h_\nu^\dagger\right)_{22}} \left[\frac{(M_1^2 - M_2^2) M_1 \Gamma_{N_2}}{(M_1^2 - M_2^2)^2 + M_1^2 \Gamma_{N_2}^2}\right]. \tag{41}$$

Here Γ_{N_i} is the total decay rate of the right-handed neutrino N_i,

$$\Gamma_{N_i} = \frac{\left(h_\nu h_\nu^\dagger\right)_{ii}}{8\pi} M_i. \tag{42}$$

The CP-violating asymmetry ϵ_S is enhanced when the mass difference between two heavy right-handed neutrinos is small, although not smaller than the decay width.

The total CP asymmetry ϵ has an involved dependence on the complete structure of the neutrino matrices h_ν and M. However, let us assume that the lepton asymmetry is generated only at the decay of the lightest right-handed neutrino N_1. This hypothesis is satisfied if the N_1 interactions are in equilibrium at the time of the $N_{2,3}$ decay (erasing any produced asymmetry), or if $N_{2,3}$ are too heavy to be produced after inflation. As will be made clear in the following, this is a very plausible working assumption. In this case, the dynamics of leptogenesis can be described in terms of only 3 parameters [3] : ϵ, M_1, and

$$m_1 \equiv \left(h_\nu h_\nu^\dagger\right)_{11} \langle H\rangle^2/M_1. \tag{43}$$

The parameter m_1, which determines the relevant interactions of N_1, coincides with the light neutrino mass m_{ν_1} only in the limit of small mixing angles, see eq. (37).

Here we will be mostly concerned with the production mechanisms of N_1 in the early cosmology. We will discuss a variety of these mechanisms and identify, in the different ranges of M_1 and m_1, the size of ϵ required to generate the appropriate baryon asymmetry, $n_B/s \sim (2\text{–}9)\times 10^{-11}$. Our results can be used to check if specific particle-physics models for neutrino mass matrices are compatible with the various leptogenesis mechanisms.

3.1 Thermal Production

We start by considering the case in which the right-handed neutrino N_1 reaches thermal equilibrium by scattering with the bath after the inflaton decay. The amount of lepton asymmetry generated by the N decay can then be computed by integrating the appropriate Boltzmann equations [25, 27].

A measure of the efficiency for producing the asymmetry is given by the ratio K of the thermal average of the N_1 decay rate and the Hubble parameter at the temperature $T = M_1$,

$$K \equiv \left.\frac{\Gamma_{N_1}}{2H}\right|_{T=M_1} = \frac{m_1}{2\times 10^{-3}\ \text{eV}}. \tag{44}$$

[3] The other neutrino mass parameters come into play only for very large values of $m_i \equiv \left(h_\nu h_\nu^\dagger\right)_{ii} \langle H\rangle^2/M_i$ $(i = 2, 3)$, when the lepton-number violating interactions mediated by N_2 or N_3 can partially erase the lepton asymmetry.

Here we have expressed the N_1 decay width, see eq. (42), in terms of the parameters m_1 and M_1 as

$$\Gamma_{N_1} = \frac{G_F}{2\sqrt{2}\pi} m_1 M_1^2. \tag{45}$$

For $m_1 \lesssim 2 \times 10^{-3}$ eV, K is less than unity and the decay process is out of equilibrium when N_1 becomes non-relativistic. Under these conditions, the leptogenesis becomes very efficient. Indeed, the produced baryon asymmetry approaches its theoretical maximum value obtained by assuming that each N_1 in thermal equilibrium eventually generates ϵ baryons,

$$\left(\frac{n_B}{s}\right)_{\rm max} = \frac{135\ \zeta(3)\ a}{4\pi^4 g_*}\epsilon = 1 \times 10^{-3}\epsilon. \tag{46}$$

Here a is defined in eq. (1) and g_* counts the number of degrees of freedom (for the standard model particle content, $a = 28/79$ and $g_* = 427/4$).

For very small m_1, $K \ll 1$ and N_1 decouples when it is still relativistic. At temperatures T below M_1, the N_1 contribution to the energy density red-shifts like matter and therefore $\rho_{N_1}/\rho_{total} = (7M_1)/(4g_*T)$. Eventually N_1 matter-dominates the Universe at a temperature

$$T_{dom} = \frac{7\ M_1}{4\ g_*} \simeq 2 \times 10^{-2} M_1. \tag{47}$$

However this can happen only if N_1 does not decay beforehand. Since the decay temperature is

$$T_* = 0.8 g_*^{-1/4}\sqrt{\Gamma_{N_1} M_{\rm P}} = \left(\frac{m_1}{10^{-6}\ {\rm eV}}\right)^{1/2}\left(\frac{M_1}{10^{10}\ {\rm GeV}}\right)\ 3 \times 10^8\ {\rm GeV}, \tag{48}$$

neutrino matter-domination occurs when

$$m_1 < 4 \times 10^{-7}\ {\rm eV}. \tag{49}$$

Under these conditions, the bulk of the energy of the Universe is stored in the non-relativistic N_1. At the time of decay, such energy density is converted into relativistic degrees of freedom whose temperature coincides with T_* given in eq. (48),

$$\rho_{N_1} = M_1 n_{N_1} = \frac{\pi^2}{30} g_* T_*^4. \tag{50}$$

This yields the following baryon asymmetry

$$\frac{n_B}{s} = \epsilon a \frac{n_{N_1}}{s} = \epsilon a \frac{3T_*}{4M_1} = \left(\frac{m_1}{10^{-6}\ {\rm eV}}\right)^{1/2} 8 \times 10^{-3}\epsilon. \tag{51}$$

In order not to reintroduce the cosmological gravitino problem, discussed in the introduction, one has to require that the temperature T_* after the right-handed neutrino decay is less than the maximum value allowed T_{RH}. This implies

$$m_1 < \left(\frac{T_{RH}}{M_1}\right)^2 10^{-3}\ \text{eV}. \tag{52}$$

Therefore, the leptogenesis process is very efficient also when T_{RH} is low enough to suppress gravitino production.

For $K \gg 1$, the departure from thermal equilibrium is reduced, and leptogenesis is less efficient. Larger values of ϵ are now required. However, for $m_1 \sim 10^{-2}$ eV, values of ϵ of about 10^{-5} are sufficient to generate the appropriate baryon asymmetry. Realistic neutrino mass matrices can comfortably reproduce such values of ϵ. Notice that the prediction for the baryon asymmetry depends weakly on M_1, as long as m_1 is not too large [27]. This is because both the production and decay thermal rates of N_1 at $T = M_1$ depend only on m_1, while the M_1 dependence arises from lepton-violating H–ℓ scattering.

Let us now turn to discuss how primordial thermal equilibrium of N_1 can be achieved. The first necessary condition is

$$M_1 < T_{RH}. \tag{53}$$

This can be quite constraining, especially in view of the bound derived from the disruptive gravitino effects on nucleosynthesis discussed mentioned in the introduction. When the inequality (53) is not satisfied, one expects the number density of the N_1-particles generated during the reheating stage to be quite small, making this case marginal, as far as the generation of baryon number is concerned. We would only like to mention here that such a number density depends upon the fine details of the dynamics of the reheating stage itself. In particular, the reheat temperature T_{RH} is not the maximum temperature obtained after inflation; the maximum temperature is, in fact, much larger than T_{RH} [29]. As a result, the abundance of massive particles may be suppressed only by powers of the mass over the temperature, and not exponentially.

A second condition for N_1 thermalization is derived from the requirement that inverse decay or production processes of the kind $\bar{\ell} q_{(3)} \to N_1 t$ (mediated by Higgs-boson exchange) are in thermal equilibrium before N_1 becomes non-relativistic. This implies

$$m_1 \gtrsim 10^{-3}\ \text{eV}. \tag{54}$$

This condition excludes the possibility of the most efficient leptogenesis with $K < 1$. However, even if m_1 is somewhat smaller than the value indicated by eq. (54), a sufficient number of N_1 can be produced. Indeed, for $m_1 \gtrsim 10^{-5}$ eV, values of $\epsilon \gtrsim 10^{-5}$ can give rise to the observed baryon asymmetry. In the case of supersymmetric models, the constraint can be even less stringent [30], and values $\epsilon \gtrsim 10^{-5}$ are sufficient for $m_1 \gtrsim 10^{-6}$ eV.

The constraint on m_1 from thermalization can be evaded if new interactions, different from the ordinary Yukawa forces, bring N_1 in thermal equilibrium at high temperatures. For instance, one could use the extra $U(1)$ gauge interactions included in $SO(10)$ GUTs. These interactions can produce a thermal population of N_1 if, at $T = T_{RH}$,

$$\Gamma(\bar{f}f \to Z' \to NN) = \frac{169\ \alpha_{GUT}^2\ T^5}{3\pi\ M_{Z'}^4} > H. \tag{55}$$

This requires that the mass of the extra gauge boson $M_{Z'}$ should be close to T_{RH} and significantly lower than the GUT scale,

$$M_{Z'} < \left(\frac{T_{RH}}{10^{10}\ \text{GeV}}\right)^{3/4} 4 \times 10^{11}\ \text{GeV}. \tag{56}$$

3.2 Production at Reheating

Since it is very likely that the short period of preheating does not fully extract all of the energy density from the inflaton field, the Universe will enter a long period of matter domination after preheating where the dominant contribution to the energy density of the Universe is provided by the residual small amplitude oscillations of the classical inflaton field and/or by the inflaton quanta produced during the back-reaction processes. This period will end when the age of the Universe becomes of the order of the perturbative lifetime of the inflaton field. At this point the Universe will go through a period of reheating with a reheat temperature T_{RH} given by the perturbative result in eq. (2).

Let us suppose that the inflaton couples to N_1, either directly or through exchange of other particles. In this case, the inflaton decay process can generate a right-handed neutrino primordial population. The condition in eq. (53) is replaced by the weaker constraint

$$M_1 < m_\phi, \tag{57}$$

where m_ϕ is the inflaton mass.

The fate of the right-handed neutrinos produced by the inflaton decay depends on the parameter choice. If $M_1 < T_{RH}$ and $m_1 \gtrsim 10^{-3}$ eV, the Yukawa couplings are strong enough to bring N_1 into thermal equilibrium, and leptogenesis can proceed as in the usual scenario described in sect. 3.1.

Let us now assume that the Yukawa couplings are much smaller, and that the right-handed neutrino decay temperature in eq. (48) satisfies $T_* < T_{RH}$, *i.e.* $m_1 < (T_{RH}/M_1)^2 10^{-3}$ eV. After reheating, the N_1 behave like frozen-out, non-thermal, relativistic particles with typical energy $E_{N_1} \simeq m_\phi/2$. The N_1 population will become non-relativistic at a temperature $T_{NR} = T_{RH} M_1/E_{N_1}$. At this moment, the energy of the Universe is shared between the radiation and the N_1 component, with a ratio of the corresponding energy densities which has remained constant between T_{RH} and T_{NR},

$$\left.\frac{\rho_{N_1}}{\rho_R}\right|_{T=T_{NR}} = \left.\frac{\rho_{N_1}}{\rho_R}\right|_{T=T_{RH}} \tag{58}$$

$$\rho_R|_{T=T_{RH}} = \frac{\pi^2}{30} g_* T_{RH}^4 \qquad\qquad \rho_{N_1}|_{T=T_{RH}} = E_{N_1} n_{N_1} = \frac{m_\phi}{2} B_\phi n_\phi. \tag{59}$$

Here n_ϕ is the inflaton number density just before decay, obtained by requiring energy conservation

$$n_\phi|_{T=T_{RH}} = \frac{\pi^2 g_* T_{RH}^4}{30 m_\phi (1 - B_\phi/2)}, \tag{60}$$

and B_ϕ describes the average number of N_1 produced in a ϕ decay. Below T_{NR}, the N_1 density red-shifts like matter and eventually dominates the Universe at a temperature

$$T_{dom} = \frac{B_\phi}{(1 - B_\phi/2)} \left(\frac{M_1}{m_\phi}\right) T_{RH}. \tag{61}$$

Therefore, if

$$m_1 > \left(\frac{B_\phi}{1 - B_\phi/2}\right)^2 \left(\frac{T_{RH}}{10^{10}\ \mathrm{GeV}}\right)^2 \left(\frac{10^{13}\ \mathrm{GeV}}{m_\phi}\right)^2 1 \times 10^{-9}\ \mathrm{eV}, \tag{62}$$

then $T_* > T_{dom}$ and N_1 decays before dominating. In this case, the baryon asymmetry is determined to be

$$\frac{n_B}{s} = \epsilon a \frac{n_{N_1}}{s} = \frac{3\ \epsilon a\ B_\phi\ T_{RH}}{4(1 - B_\phi/2) m_\phi}. \tag{63}$$

If the inequality (62) is not satisfied, N_1 matter-dominates the Universe and we recover the baryon asymmetry result in eq. (51).

A necessary condition to be satisfied is that lepton-number violating interactions mediated by N_i ($i = 1, 2, 3$) exchange are out of equilibrium at the temperature of N_1 decay,

$$\Gamma_{\Delta L} = \frac{4}{\pi^3} G_F^2 m_i^2 T^3 < H \quad \text{at } T = T_*, \tag{64}$$

where T_* is given in eq. (48). This implies

$$m_1 < \left(\frac{10^{12}\ \text{GeV}}{M_1} \right)^{2/5} 0.1\ \text{eV} \quad \text{and} \quad m_1 < \left(\frac{10^{12}\ \text{GeV}}{M_1} \right)^2 \left(\frac{2 \times 10^{-3}\ \text{eV}^2}{\sum_{i=2,3} m_i^2} \right)^2 2\ \text{eV}. \tag{65}$$

Finally, we discuss the case $T_* > T_{RH}$, *i.e.* $m_1 > (T_{RH}/M_1)^2 10^{-3}$ eV, in which N_1 decays immediately after it is produced. In this case, the baryon asymmetry is still given by eq. (63), but the out-of-equilibrium condition of lepton-violating interactions has to be imposed at $T = T_{RH}$. Therefore, eq. (65) is replaced by

$$m_i < \left(\frac{10^{10}\ \text{GeV}}{T_{RH}} \right)^{1/2} 3\ \text{eV}, \qquad i = 1, 2, 3. \tag{66}$$

The combination of the bounds shows that lepton-violating interactions do not give severe constraints on the parameters.

3.3 Production at Preheating

As we have seen in sect. 2, right-handed neutrinos are efficiently produced in a non-thermal state during the preheating stage. In our numerical studies we have tacitly assumed that the superheavy right-handed neutrinos were stable. Of course, the parametric resonance is affected by a nonvanishing decay width of the N_1. However, contrary to what happens for bosons where the presence of a large decay width removes the particles from the resonance bands rendering the preheating less efficient [31], for fermions the presence of a decay width might be even beneficial. Indeed, for stable right-handed neutrinos the distribution function $n(k)$ is rapidly saturated to unity and further particle production is Pauli-blocked. However, if the decay width is large enough, the right-handed neutrinos may be produced at each inflaton oscillation when $m(t) \simeq 0$ and then decay right away. This will give rise to a certain amount of lepton asymmetry at each inflaton oscillation until the condition in eq. (28) is met; the lepton asymmetry would be generated in a cumulative way. Strictly speaking, however, the numerical calculation of fermion preheating presented in sect. 2 applies only to the case in which the right-handed neutrinos have a decay lifetime larger than the typical time-scale of the inflaton oscillation m_ϕ^{-1}

$$\Gamma_{N_1} \lesssim m_\phi \qquad \Rightarrow \quad m_1 < \left(\frac{10^{15}\ \mathrm{GeV}}{M_1}\right)^2 \left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right) 8 \times 10^{-3}\ \mathrm{eV}. \tag{67}$$

The right-handed neutrinos produced during preheating may annihilate into inflaton quanta. This back-reaction will render the final right-handed neutrino abundance smaller and therefore leptogenesis more difficult. Imposing that the back-reaction is negligible requires $\Gamma_A \sim n_{N_1}\sigma_A \lesssim m_\phi$, where $\sigma_A \sim g^4/(4\pi M_1)^2$. Since the energy spectrum of the right-handed neutrinos is dominated by the maximum momentum generated at the last inflaton oscillation, we assume that the number density of right-handed neutrinos is equal to the freeze-out value $n_{N_1} \simeq M_1 m_\phi^2/6\pi^2$, see eq. (35), and we obtain

$$\Gamma_A < m_\phi \qquad \Rightarrow \quad q < 10^{13} \left(\frac{M_1}{10^{15}\ \mathrm{GeV}}\right)^{1/2}. \tag{68}$$

One should also be sure that the number density of the right-handed neutrinos is not depleted by self-annihilations before they decay. This requires

$$\Gamma_A < \Gamma_{N_1} \qquad \Rightarrow \ q < 10^{12} \left(\frac{m_1}{10^{-4}\ \mathrm{eV}}\right)^{1/2} \left(\frac{M_1}{10^{15}\ \mathrm{GeV}}\right)^{3/2}. \tag{69}$$

Suppose now that the right-handed neutrinos N_1 decay before the inflaton energy density is transformed into radiation by perturbative processes. This occurs when $\Gamma_{N_1} > \Gamma_\phi$, which implies

$$m_1 > \left(\frac{T_{RH}}{M_1}\right)^2 1 \times 10^{-3}\ \mathrm{eV}. \tag{70}$$

A crucial point is that, after the generation of non-thermal right-handed neutrinos at the preheating stage, the ratio of the energy densities of N_1 and inflaton quanta remains constant: $\rho_{N_1}/\rho_\phi = (\rho_{N_1}/\rho_\phi)_{ph}$, where $(\rho_{N_1}/\rho_\phi)_{ph}$ is the ratio generated at the preheating stage. Since the energy density of the Universe is dominated by inflaton oscillations, after preheating we obtain

$$\rho_{N_1} = \left(\frac{\rho_{N_1}}{\rho_\phi}\right)_{ph} \frac{3H^2 M_{\mathrm{P}}^2}{8\pi}. \tag{71}$$

Our numerical results discussed in sect. 2 indicate that, for very large q and M_1,

$$\left(\frac{\rho_{N_1}}{\rho_\phi}\right)_{ph} \sim 10^{-11} q \quad \text{for} \quad M_1 \lesssim \frac{q^{1/2}}{2} m_\phi. \tag{72}$$

At $t_{N_1} \sim \Gamma_{N_1}^{-1}$ the right-handed neutrinos decay and the energy density ρ_{N_1} is converted into a thermal bath with temperature

$$\frac{\pi^2}{30} g_* \widetilde{T}^4 = \rho_{N_1}, \tag{73}$$

where ρ_{N_1} is computed at $H = \Gamma_{N_1}$. Using eqs. (71) and (72), we find

$$\widetilde{T} = \left(\frac{q}{10^{10}}\right)^{1/4} \left(\frac{m_1}{10^{-4}\ \mathrm{eV}}\right)^{1/2} \left(\frac{M_1}{10^{15}\ \mathrm{GeV}}\right) 2 \times 10^{14}\ \mathrm{GeV}. \tag{74}$$

Before the inflaton decay, this thermal bath never dominates the energy density of the Universe since $\rho_{N_1} \ll \rho_\phi$ at $H \simeq \Gamma_{N_1}$ and the energy density in the inflaton field ρ_ϕ is red-shifted away more slowly than the radiation. Notice that at this time the asymmetry is still in the form of lepton number since the sphalerons which are responsible for converting the lepton asymmetry into baryon asymmetry are still out-of-equilibrium at $T = \widetilde{T}$. One might be worried that, since $\widetilde{T}$ is usually larger than T_{RH}, too many gravitinos are produced at the stage of thermalization of the decay products of the right-handed neutrino. However, one can estimate the ratio $n_{3/2}/s$ after reheating to be of the order of $10^{-15}(q/10^{10})^{3/2}\,(T_{RH}/10^{10}\ \mathrm{GeV})$, which is quite safe. However, we have to require that the lepton-number violating processes within the thermal bath at temperature $\widetilde{T}$ are out-of-equilibrium in order not to wash out the lepton asymmetry generated by the right-handed neutrino decays. Therefore, we demand that

$$\Gamma_{\Delta L} = \frac{4}{\pi^3} G_F^2 m_i^2 T^3 < H \quad \text{at } T = \widetilde{T}. \tag{75}$$

This implies

$$m_1 \;<\; \left(\frac{10^{10}}{q}\right)^{3/10} \left(\frac{10^{15}\ \mathrm{GeV}}{M_1}\right)^{2/5} 1 \times 10^{-2}\ \mathrm{eV}, \tag{76}$$

$$m_1 \;<\; \left(\frac{10^{10}}{q}\right)^{3/2} \left(\frac{10^{15}\ \mathrm{GeV}}{M_1}\right)^{2} \left(\frac{2 \times 10^{-3}\ \mathrm{eV}^2}{\sum_{i=2,3} m_i^2}\right)^2 6 \times 10^{-5}\ \mathrm{eV}. \tag{77}$$

At $H \lesssim \Gamma_{N_1}$, the ratio n_L/n_ϕ keeps constant until the time $t_\phi \sim \Gamma_\phi^{-1}$ when the inflaton decays and the energy density in the inflaton field $\rho_\phi(t_\phi)$ is transferred to radiation. After reheating we obtain the following lepton asymmetry to entropy density ratio

$$\frac{n_L}{s} = \frac{3 n_L T_{RH}}{4 \rho_\phi(t_\phi)} = \frac{3\,\epsilon\, T_{RH}}{4\, M_1} \left(\frac{\rho_{N_1}}{\rho_\phi}\right)_{ph}. \tag{78}$$

Using eq. (72), the baryon asymmetry can be expressed as

$$\frac{n_B}{s} = \epsilon \left(\frac{T_{RH}}{10^{10}\,\mathrm{GeV}}\right) \left(\frac{10^{15}\ \mathrm{GeV}}{M_1}\right) \left(\frac{q}{10^{10}}\right) 3 \times 10^{-7}. \tag{79}$$

The result in eq. (79) shows that the preheating production mechanism can lead to a successful leptogenesis even for M_1 as large as 10^{15} GeV, if $\epsilon q \sim 10^6$. Values of q as large as 10^{10} correspond to a perturbative coupling between N_1 and the inflaton $g^2 \simeq 0.4$ and are compatible with the constraints in eqs. (68) and (69). Values of ϵ of the order of 10^{-4} are quite large, but can be attained with realistic neutrino mass matrices. For values of M_1 so close to the GUT scale, we expect that all the right-handed neutrino Majorana masses are comparable in size. Moreover, the atmospheric neutrino results, together with the requirement of perturbative Yukawa couplings, indicate that at least one Majorana mass is less than about 8×10^{15} GeV. This situation of comparable Majorana masses and some large Yukawa couplings naturally leads to large values of ϵ. Also, notice that the out-of-equilibrium condition is automatically satisfied in the preheating scenario for all 3 right-handed neutrinos, in a large range of parameters.

Let us finally consider the case in which N_1 decays after the reheating process and the inequality (70) is not satisfied. Again, it is important to establish whether N_1 dominates the Universe before decaying. Since the inflaton energy density is converted into radiation and the N_1 are non-relativistic, the temperature at which ρ_{N_1} dominates is given by

$$T_{dom} = \left(\frac{\rho_{N_1}}{\rho_\phi}\right)_{ph} T_{RH}. \tag{80}$$

Therefore, for $T_* > T_{dom}$, *i.e.* for

$$m_1 > \left(\frac{T_{RH}}{M_1}\right)^2 \left(\frac{q}{10^{10}}\right)^2 10^{-7}\ \mathrm{eV}, \tag{81}$$

the estimate for the baryon asymmetry in eq. (79) is still valid. Otherwise, for $T_* < T_{dom}$, we obtain the result in eq. (51). Notice that, in this case, the constraint $T_* < T_{RH}$ is automatically satisfied.

3.4 Comparison of the Different Production Mechanisms

We want to compare here the different mechanisms discussed in this section for leptogenesis from N_1 decay. We summarize their most important features and estimate the size of the CP-violating parameter ϵ necessary to reproduce the observed baryon asymmetry.

Thermal production. This is the right-handed neutrino production mechanism usually considered in the literature for conventional leptogenesis. In the range 10^{-5} eV $< \tilde{m}_1 < 10^{-2}$ eV (or 10^{-6} eV $< \tilde{m}_1 < 10^{-2}$ eV in the case of the minimal supersymmetric model) and for $M_1 < T_{RH}$, the thermal production of unstable N_1 is efficient, and values of ϵ in the range 10^{-7}–10^{-5} can account for the present baryon asymmetry. The boundaries of the allowed region of neutrino mass parameters are determined as follows. For small values of $\tilde{m}_1$, the N_1 production rate is suppressed and larger values of ϵ are required. For large $\tilde{m}_1$, the Yukawa couplings maintain the relevant processes in thermal equilibrium for longer times, partially erasing the produced asymmetry. The values of M_1 are limited by the reheat temperature after inflation T_{RH}, which in turn is bounded by cosmological gravitino considerations to be below 10^8–10^{10} GeV.

Production at reheating. If N_1 is directly or indirectly coupled to the inflaton, the decay of the small amplitude oscillations of the classical inflaton field at the time of reheating can produce a right-handed neutrino population. This production mechanism enables us to extend the leptogenesis-allowed region to neutrino mass parameters which correspond to non-thermal N_1 populations. In particular, M_1 can be as large as $m_\phi \simeq 10^{13}$ GeV. The observed baryon asymmetry is reproduced for $\epsilon \sim 10^{-6}(10^{10}\ \mathrm{GeV}/T_{RH})(10^{-1}/B_\phi)$, where B_ϕ is the average number of N_1 produced by a single inflaton decay.

Production at preheating. The non-perturbative decay of large inflaton oscillations during the preheating stage can produce a non-thermal population of very massive right-handed neutrinos. In this case, the range of M_1 can be extended to values close to the GUT scale, while $\tilde{m}_1$ is bounded by the condition that lepton-number violating interactions are out of equilibrium after the N_1 decay, see eq. (77). A successful leptogenesis requires $\epsilon \sim 10^{-4}(10^{10}\ \mathrm{GeV}/T_{RH})(M_1/10^{15}\ \mathrm{GeV})(10^{10}/q)$, where q is related to the initial inflaton configuration and is defined in eq. (23).

In conclusion, leptogenesis provides an interesting and simple way to explain the present cosmic baryon asymmetry. The study of the different mechanisms in which it can be realized provides us with precious information on the neutrino mass parameters and the early history of the Universe.

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