PRODUCTION OF UNIFORM DROPLETS BY LONGITUDINAL VIBRATION OF AUDIO FREQUENCY

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The conditions to produce liquid droplets of uniform size were investigated experimentally. The number of droplets produced and the region which allowed production of uniform-size droplets were related with the operating conditions, design parameters of nozzle and physical properties of liquid.

It was confirmed that the number of liquid droplets produced is equal to the vibration frequency within the uniform region, and the diameter of a liquid droplet which is equivalent to a sphere was calculated by Dabora's equation.

The region producing uniform droplets was not affected by the appropriate amplitude of nozzle vibration, but was affected by nozzle diameter, physical properties of liquid and frequency. Nondimensional empirical equations for their prediction were obtained.

It was found that the range of liquid velocity of the uniform region corresponds to the range of the forced oscillating smooth jet from a vibrating nozzle. Uniform droplets were produced from the forced oscillating smooth jet which is vibrated at an appropriate amplitude and frequency.

Introduction

Application of uniform droplets has been considered useful in simplifying the analysis in studies on mutual interaction, transport phenomena[12] and combustion[11,15] of droplets. Recently, uniform droplets are attracting notice in the field of sprinkling of agricultural pesticides[22] and in pharmaceutical industries[7].

Therefore, many methods of producing uniform droplets have been devised[3,14,21]. The vibration method can produce uniform droplets of desired size and number easily and irrespective of the kind of liquids. Dimmock[5] and Mason *et al.*[10] used a transversely vibrating capillary. Schneider *et al.*[16] produced uniform droplets within the wavelength λ which ranged from $3.5\,d_j$ to $7\,d_j$, using a longitudinally vibrating capillary. Dabora[4] produced uniform droplets using a longitudinally vibrating capillary with the frequency of Rayleigh's maximum instability, $f=u_j/4.508\,d_j$[13]. Araki and Masuda[1,2], using longitudinally and transversely vibrating nozzles, and Nagai *et al.*[23], using a longitudinally vibrating nozzle, investigated the conditions which can produce uniform droplets, i.e. the region of production of uniform droplets, hereafter referred to by the authors as the uniform region. Araki *et al.* experimentally showed for longitudinal vibration that their limits of shorter wavelength, referred to as lower limits of uniform region in this paper, agree with the stable limit of wavelength, $\lambda=\pi d_j$, proposed by Rayleigh[13]. However, Nagai *et al.* reported that uniform droplets were produced with a shorter wavelength than Rayleigh's stable limit.

Regarding the jet diameter d_j, Schneider equated $d_j=d$. Rayleigh and Dabora used d_j but not d. As shown by Kurabayashi[8] experimentally and theoretically, the diameter of a smooth jet column d_j is a function of Weber's number, nozzle diameter d, liquid velocity at the port tip u and axial distance from the port tip, and hence dose not show constant value. Therefore, making use of d_j and u_j to analyse the experimental results is difficult.

In this paper, the authors used inner diameter of capillary d instead of d_j. Those lines in Fig. 5, calculated by the equations of cited investigators, are based of d and u instead of d_j and u_j for the above reason.

Investigations concerning the uniform region have been reported only by Araki *et al.* and Nagai *et al.* However, neither has reported their results regarding the effects of the operating conditions and the physical properties of liquid. This work aims to clarify the relations of prescribed factors to generate uniform droplets, using longitudinally vibrating nozzles.

On the other hand, besides those investigations, many investigations have been reported concerning the stability and flow regimes of a liquid jet issuing

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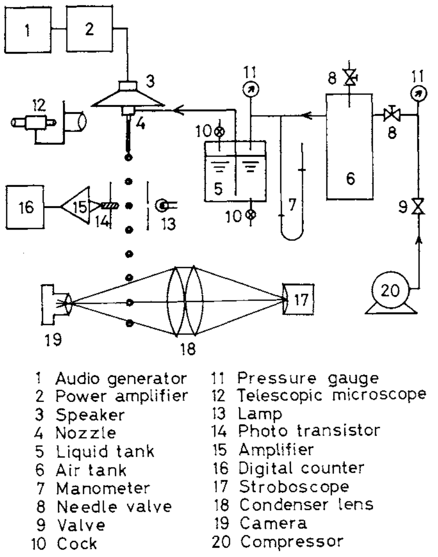

Fig. 1 Schematic diagram of the experimental apparatus

Table 1 Physical properties of sample liquid

Sample liquid	Density [g/cm^3]	Surface tension [g/sec^2]	Viscosity [g/cm·sec]
Water	0.999	72.5	1.33×10^{-2}
20 wt% Sugar soln.	1.082	73.3	2.43×10^{-2}
50 wt% Sugar soln.	1.227	76.9	16.3×10^{-2}
60 wt% Sugar soln.	1.283	77.8	50.7×10^{-2}
20 wt% Ethanol soln.	0.971	36.5	2.60×10^{-2}
80 wt% Ethanol soln.	0.846	23.8	2.60×10^{-2}
Ethanol	0.794	21.5	1.49×10^{-2}

from a stationary nozzle into quiescent air[8,13,17–20]. Flow regimes of a liquid jet are classified into *dripping*, *smooth jet*, *wavy jet* and *spray jet*. Among these flow patterns, *smooth jet* is potential flow, and its jet velocity at the tip of a stationary nozzle is constant. However, in this case, the jet velocity observed from stationary coordinates near the tip of the nozzle changes periodically due to the forced longitudinal vibration. Therefore, the authors referred to the jet, which looks like a smooth jet from stationary nozzle though issuing from a vibrating nozzle as a *"forced oscillating smooth jet"*. From the viewpoint of the flow pattern of a liquid jet column, the authors wish to clarify that uniform droplets can be produced from the forced oscillating smooth jet by the appropriate longitudinal vibration.

1. Experimental Apparatus and Method

A schematic diagram of the experimental apparatus is shown in **Fig. 1**. The nozzle is fitted to the center of a speaker, and is vibrated longitudinally by means of an audio generator and a power amplifier, which can control the frequency and the amplitude of sine wave vibration. The forms of generated waves are measured and verified by means of a microphone and a synchroscope when the liquid is issuing from the nozzle. Consequently, it is confirmed that the generated waves are always sine waves.

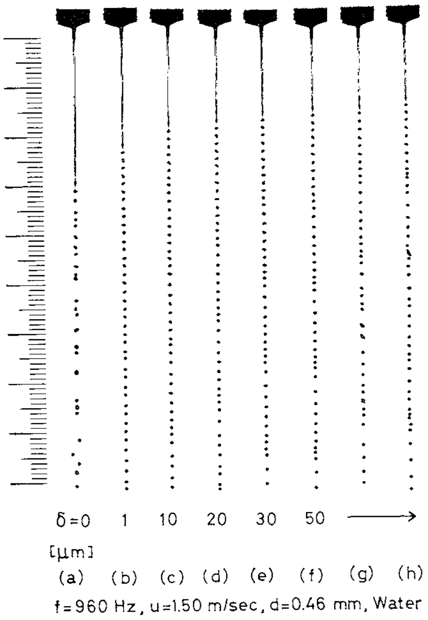

Fig. 2 Disintegration phenomena of liquid jet columns when amplitude is changed under constant vibration frequency and liquid velocity

The liquid is supplied to the nozzle from the liquid tank by air pressure. Disintegration phenomena of the jets are observed by means of a camera and a stroboscope. The amplitude of vibration at the tip of the nozzle is measured by means of a telescopic microscope (100–300 magnifications) and a stroboscope. The number of droplets produced is measured by means of a set of phototransistor and digital counter.

Hypodermic needles are used as nozzles. The tips of the nozzles are squared to the longitudinal direction and are carefully finished with an oilstone to remove excess burrs. City water, sugar aqueous solution of 20, 50 and 60 wt% and ethanol aqueous solution of 20 and 80 wt% are used as the sample liquids. The physical properties of the sample liquids are shown in **Table 1.**

2. Experimental Results and Discussion

2. 1 Observation of disintegration phenomena of liquid jet columns

1) Effect of amplitude of nozzle vibration An example of disintegration phenomena of jets related to amplitude of vibration is shown in **Fig. 2**. In Fig. 2 (a) the jet is a smooth jet, and we can see that a few waves are formed at the tip of the liquid column. Neither the diameters nor the intervals of liquid droplets are uniform. When the nozzle is vibrated

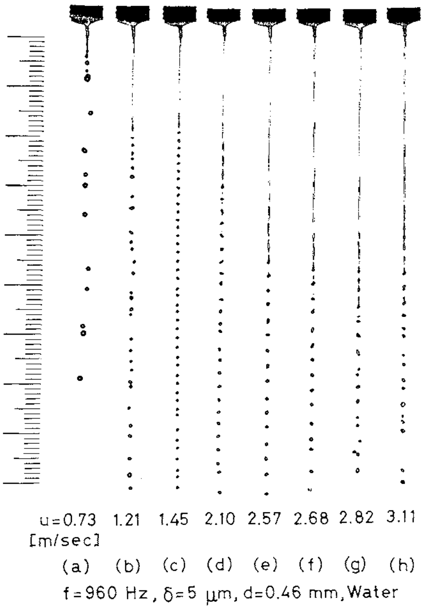

Fig. 3 Disintegration phenomena of liquid jet columns when liquid velocity is changed under constant vibration frequency and amplitude

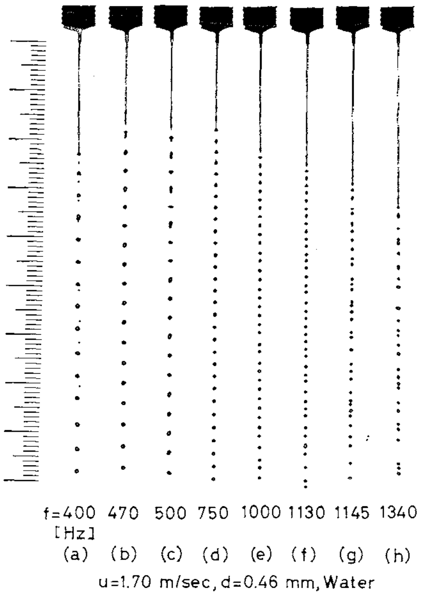

Fig. 4 Disintegration phenomena of liquid jet columns when vibration frequency is changed under constant liquid velocity

with small amplitude, as seen in Fig. 2 (b), the length of liquid column becomes shorter and uniform droplets are produced. As seen in Figs. 2 (c), (d), (e), (f), the length of liquid column becomes much shorter with increasing amplitude of vibration, while both the diameters and the intervals of liquid droplets are uniform. It is found that the liquid column of forced oscillating smooth jet is shorter than that of the smooth jet. (Fig. 2 (a) is a smooth jet and (b)–(f) are forced oscillating smooth jets.) When the nozzle is vibrated at more than 50 μm, as is clearly seen in Figs. 2 (g) and (h), uniform droplets cannot be produced further because of the mutual coalescence of droplets. As seen in the figure, it is observed that the jet is vibrating rotationally. It is considered from this observation that in this amplitude the nozzle is vibrated rotationally and longitudinally.

2) Effect of liquid velocity An example of disintegration phenomena of jets related to liquid velocity is shown in **Fig. 3**. As seen in Fig. 3 (a), the flow pattern of the jet is one of dripping. It is observed in Fig. 3 (b) that a liquid column is formed when the liquid velocity is increased, but the diameters of the liquid droplets are not uniform. When the liquid velocity is increased, as seen in Fig. 3 (c), uniform droplets are produced. As seen in Figs. 3 (d), (e) and (f), the length of liquid column becomes greater with increasing liquid velocity, and by observation with the stroboscope any satellite droplet coalesces with a main droplet as soon as it is produced and consequently uniform droplets are produced. When the liquid velocity is further increased, as seen in Figs. 3 (g) and (h), droplets of various sizes are produced because satellite droplets do not coalesce with main droplets.

3) Effect of vibration frequency **Figure 4** shows an example of disintegration phenomena of jets related to vibration frequency. As seen in Figs. 4 (a) and (b), the diameters of liquid droplets are not uniform at lower vibration frequencies because of the production of satellite droplets. When the vibration frequency is increased, as shown in Figs. 4 (c), (d) and (e), coalescence of satellite droplet and main droplet was observed with the stroboscope, as soon as the satellite droplet is produced, and consequently uniform droplets are produced. When the vibration frequency is increased, uniform droplets are produced without production of satellite droplets, as shown in Fig. 4 (f). When the vibration frequency is increased still more, as seen in Figs. 4 (g) and (h), the intervals of droplets become irregular, and by observation with the stroboscope the length of liquid column becomes unstable. Consequently, uniform droplets cannot be produced by further increasing the frequency.

According to the above-mentioned observation, the existence of a uniform region is confirmed. The amplitude of nozzle vibration, liquid velocity, vibration frequency, nozzle diameter and physical properties of liquid are considered to be the factors which affect the uniform region.

2. 2 Number and diameter of uniform droplets produced

As shown by Dabora[4], one wave formed at the end tip of the liquid column makes one liquid droplet, (Fig. 3 (e), Fig. 4 (e)), except under satellite-generating conditions at criteria. Thus, using mass conservation, the diameter D of a liquid droplet which is equivalent to a sphere is calculated from Eq. (1).

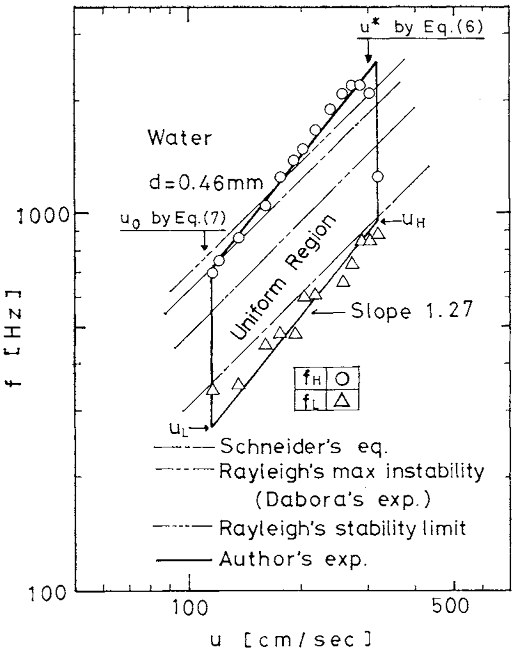

Fig. 5 An example of uniform region and comparison with reported investigations

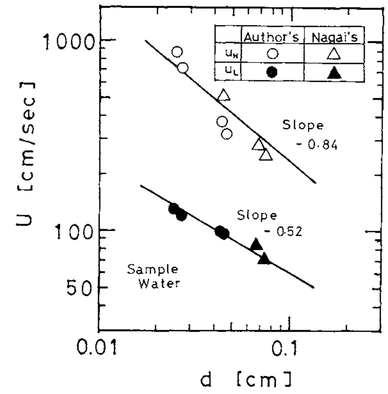

Fig. 6 Effect of nozzle diameter on higher and lower limits of liquid velocity

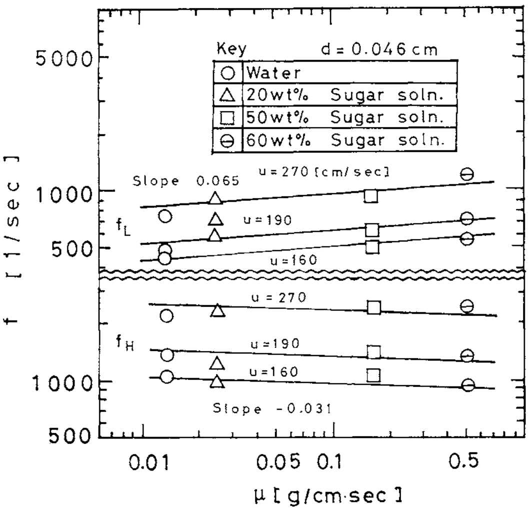

Fig. 7 Effect of liquid velocity on upper and lower limits of vibration frequency

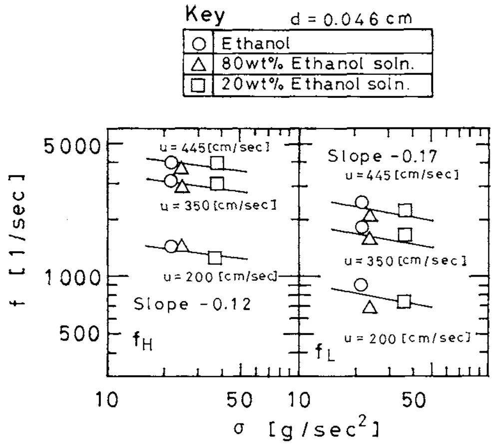

Fig. 8 Effect of liquid surface tension on upper and lower limits of vibration frequency

$$D=(6\cdot Q/\pi\cdot f)^{1/3}=(3\cdot d^2\cdot u/2\cdot f)^{1/3} \qquad (1)$$

Measured diameters agreed excellently with those calculated from Eq. (1).

2. 3 Effects of some factors on the uniform region

1) Effect of amplitude of nozzle vibration The upper and lower limits of liquid velocity at which uniform droplets can be produced are measured when the amplitude of vibration is changed. From observation, it was found that the upper and lower limits of liquid velocity were not affected within the range of $1\leq\delta\leq 50\ \mu$m by the amplitude of vibration. An example of u_H and u_L are shown in **Fig. 5** as the perpendicular sides of a parallelogram which expresses a uniform region.

2) Criteria of uniform region The upper and lower limits of vibration frequency of the uniform region, f_H and f_L, vs. mean jet velocity u are shown in Fig. 5 for the case of water and nozzle diameter 0.46 mm as an example. Uniform droplets can be produced within the enclosed region of a parallelogram surrounded by the lines f_H, f_L, u_H and u_L.

The criteria of f_L are defined, between u_L and u_H, such that when the frequency is below the uniform region satellite droplets begin to be generated, but they coalescence with main droplets, producing uniform droplets. When the vibration frequency is increased between u_L and u_H, the length of the liquid column becomes unstable and generates satellite droplets. f_H is defined as the frequency just before fluctuation of column length occurs, capable of producing uniform droplets without satellite production. From the evidence of the figure, within the range of u_L and u_H, f_H and $f_L \propto u^{1.27}$. When the liquid velocity is increased slightly beyond than that of dripping, the liquid column is formed and uniform droplets can be produced. The liquid velocity of this condition is defined as the lower limit of liquid velocity, u_L, of the uniform region. It is found in Fig. 5 that the range of vibration frequency to produce uniform droplets becomes wider with increasing liquid velocity. As liquid velocity is increased further, not only the tip of the liquid column but all of it vibrates longitudinally and uniform droplets cannot be produced. The liquid velocity just before this condition is defined as the upper limit of liquid velocity, u_H, of the uniform region.

3) Effect of nozzle diameter **Figure 6** shows the effect of nozzle diameter related with u_H and u_L. It is found in the figure that as the nozzle diameter is decreased, u_H and u_L increase and the uniform region becomes wider. From Fig. 6, $u_H \propto d^{-0.84}$ and $u_L \propto d^{-0.52}$. Also, from the experiments, $f_H \propto d^{-0.85}$ and $f_L \propto d^{-0.85}$.

4) Effect of liquid viscosity **Figure 7** shows an example of the effect of liquid viscosity on the uniform region. From the figure, $f_H \propto \mu^{-0.031}$ and $f_L \propto \mu^{0.065}$. It was also observed that u_L is not much affected by viscosity, while u_H became considerably higher when viscosity increased. The relations were $u_H \propto \mu^{0.86}$, $u_L \propto \mu^{0.004}$.

5) Effect of surface tension Examples of the effects of surface tension on the uniform region are shown in **Fig. 8**. From the evidence of the figure, $f_H \propto \sigma^{-0.12}$, $f_L \propto \sigma^{-0.17}$. Also, from the experiments, $u_H \propto \sigma^{0.17}$, $u_L \propto \sigma^{0.32}$.

2. 4 Region of uniform droplets

The effects of factors on the uniform region were investigated. The amplitude of nozzle vibration does not affect it, but the nozzle diameter and physical properties of the liquid do affect it. Thus the uniform regions defined by f_H, f_L, u_H and u_L are obtained quantitatively, related with the nozzle diameter and physical properties of liquid.

The experimental results are expressed as:

$$f_H \propto u^{1.27} d^{-0.85} \mu^{-0.031} \sigma^{-0.12}$$

$$f_L \propto u^{1.27} d^{-0.85} \mu^{0.065} \sigma^{-0.17}$$

and

$$u_H \propto d^{-0.85} \mu^{0.86} \sigma^{0.17}$$

$$u_L \propto d^{-0.52} \mu^{0.004} \sigma^{0.32}$$

Dimensional analysis is performed for the upper and the lower limits of vibration frequency and the higher and the lower limits of liquid velocity respectively, and the factors and liquid density ρ are arranged in nondimensional terms. The exponents of nondimensional terms and the experimental coefficients are determined by the least square method by means of a digital computer. The following empirical equations are obtained.

Upper limit of vibration frequency within the range of $u_L \leqq u \leqq u_H$ is

$$\frac{f_H \cdot d}{u} = 0.18 \left(\frac{d \cdot u \cdot \rho}{\mu} \right)^{0.031} \left(\frac{d \cdot u^2 \cdot \rho}{\sigma} \right)^{0.12} \tag{2}$$

Lower limit of vibration frequency, also within $u_L \leqq u \leqq u_H$ is

Key	d [mm]	Sample liquid	Investigator
⊖	0.25	Water	Author's
⦶	0.27	Water	
⊗	0.43	Water	
○	0.46	Water	
⏃	0.46	60wt% Sugar soln.	
△	0.46	50wt% Sugar soln.	
⩠	0.46	20wt% Sugar soln.	
□	0.46	80wt% Ethanol soln.	
◫	0.46	20wt% Ethanol soln.	
▲	0.44	Water	Nagai et al.
●	0.732	Water	
■	0.732	40vol% Glycerine soln.	

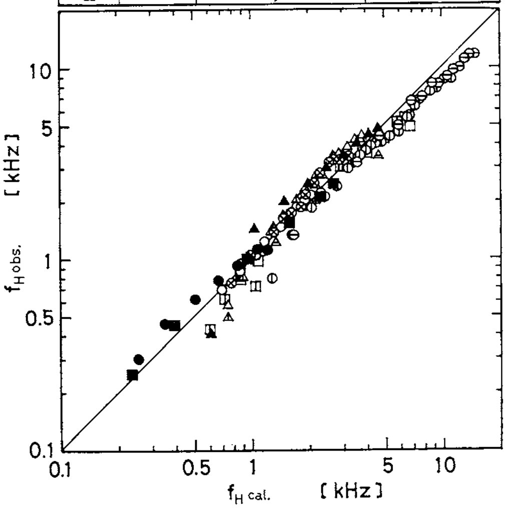

Fig. 9 An example of comparison of calculated and observed values regarding upper limits of vibration frequency

$$\frac{f_L \cdot d}{u} = 0.11 \left(\frac{d \cdot u \cdot \rho}{\mu} \right)^{-0.066} \left(\frac{d \cdot u^2 \cdot \rho}{\sigma} \right)^{0.17} \tag{3}$$

Experimental range: $1.3 \times 10 < (d \cdot u \cdot \rho/\mu) < 2.1 \times 10^3$
$5.0 < (d \cdot u^2 \cdot \rho/\sigma) < 8.1 \times 10^2$

and $d = 0.25$–0.46 mm, $\mu = 1.33$–50.7×10^{-2} g/cm·sec, $\sigma = 23.8$–77.8 g/sec^2, $\rho = 0.846$–1.283 g/cm^3.

Upper limits of liquid velocity:

$$\frac{d \cdot u_H \cdot \rho}{\mu} = 300 \left(\frac{\mu}{\sqrt{d \cdot \rho \cdot \sigma}} \right)^{-0.33} \tag{4}$$

Lower limit of liquid velocity:

$$\frac{d \cdot u_L \cdot \rho}{\mu} = 3.0 \left(\frac{\mu}{\sqrt{d \cdot \rho \cdot \sigma}} \right)^{-0.96} \tag{5}$$

Applicable range: $4.4 \times 10^{-3} < (\mu/\sqrt{d \cdot \rho \cdot \sigma}) < 2.7 \times 10^{-2}$.

Experimental range is the same as above.

As an example of the comparison of observed values with calculated values, **Fig. 9** shows the comparison for the upper limits of vibration frequency. As is evident from the figure, both the authors' data and the Nagai and others' data agreed with those calculated from Eq. (2) excellently. The values calculated from Eqs. (3), (4) and (5) also agreed with observed values.

On the other hand, flow regimes of a liquid jet have been classified and referred to as dripping, smooth jet, transitional jet, wavy jet and spray jet by Tanasawa[19]. The transitional velocity u^* from smooth jet to transitional jet is reported as Eq. (6) by Middleman *et al.*[6]. And the transitional velocity u_0 from dripping to smooth jet is reported as Eq. (7) by

Schneider *et al.*[9].

Middleman *et al.*[6]: $$\frac{d\cdot u^{*}\cdot\rho}{\mu}=325\left(\frac{\mu}{\sqrt{d\cdot\rho\cdot\sigma}}\right)^{-0.28} \tag{6}$$

Schneider *et al.*[9]: $$\frac{d\cdot u_0^2\cdot\rho}{\sigma}=8.0 \tag{7}$$

When we compare u_H by Eq. (4) with u^* by Eq. (6), and u_L by Eq. (5) with u_0 by Eq. (7), we notice that both equations show near values of u. However, there is also a little difference between them. As the liquid velocity at the tip of the nozzle is changed periodically due to the forced longitudinal vibration, the *forced oscillating smooth jet* differs from the *smooth jet*, which has only self-induced oscillation from the stationary nozzle. Therefore it is referred to as the forced oscillating smooth jet. It is considered that the difference between Eq. (4) and Eq. (6), and between Eq. (5) and Eq. (7) is owing to the difference between the forced oscillating smooth jet and the smooth jet. Thus it is concluded that the range of liquid velocity of the uniform region corresponds to the range of formation of the forced oscillating smooth jet, and uniform droplets can be produced by giving the appropriate vibration to the forced oscillating smooth jet.

Conclusions

1) Within the region of uniform droplets production, the number of liquid droplets produced per unit time is equal to the vibration frequency, and the diameter of a liquid droplet which is equivalent to a sphere is calculated from Eq. (1).

2) The region of uniform droplets production is not affected by the amplitude of nozzle vibration, but is affected by nozzle diameter, frequency of vibration and physical properties of the liquid within the limits of liquid velocity.

3) The region of uniform droplets production can be predicted by Eqs. (2), (3), (4) and (5).

4) The range of liquid velocity of the uniform region corresponds to the range of the forced oscillating smooth jet. The uniform droplets can be produced from the forced oscillating smooth jet which is vibrated with appropriate amplitude and frequency.

Nomenclature

D = diameter of liquid droplet equivalent to sphere [cm]
d = inner diameter of nozzle [cm]
d_j = diameter of liquid jet [cm]
f = frequency of vibration [Hz]
N = number rate of produced droplets [1/sec]
Q = volumetric flow rate of liquid jet [cm^3/sec]
u = average liquid velocity in nozzle [cm/sec]
u_j = velocity of liquid jet [cm/sec]
u_0 = transitional velocity from dripping to a smooth jet [cm/sec]
u^* = transitional velocity from a smooth jet to a transitional jet [cm/sec]

δ = amplitude of nozzle vibration [cm]
μ = viscosity of liquid [g/cm·sec]
ρ = density of liquid [g/cm^3]
σ = surface tension [g/sec^2]

⟨Subscripts⟩
cal. = calculation
H = upper limit
L = lower limit
obs. = observation

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