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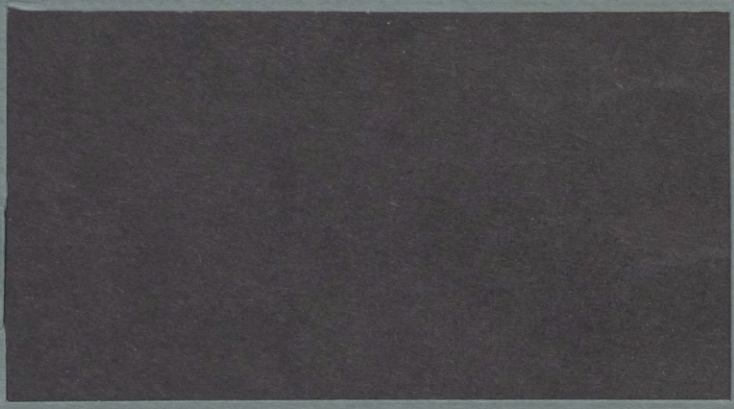
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## THE DISAPPEARANCE OF PRODUCTIVITY CHANGE\*

Robert J. Gordon

Harvard University and the National Bureau of Economic Research

### I. Introduction

Most empirical studies of economic growth attempt to determine the relative importance of increases in inputs and advances in technology in the achievement of growth in per-capita output. This approach is motivated by a desire to explain the sources of that output growth: how much less rapidly would the U. S. economy have expanded in the last 50 years if it had continued to operate with 1918 levels of technology, or if technology had advanced but no net investment in tangible or human capital had occurred? Answers to these questions help us to maximize our future rate of growth by guiding policy makers to an optimal allocation of resources among investment in tangible capital, in education, and in technology-increasing activities, and they help in explaining the reasons for international differences in per-capita income.<sup>1</sup>

Since the mid-1950's a common technique for the separation of

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the respective contributions of input growth and advances in technology has been the calculation of indexes of total factor productivity. Pioneering studies by Solow [24] and others [1][12][19] have suggested capital played only a minor role in per-capita growth, and that most of the long-term increase in U. S. output per capita was due to an increase in the output obtainable per unit of appropriately weighted input. While it was recognized that some of this increase in total factor productivity or "the residual" might have been due to the spread of education, most of it was assumed to have represented technical change. A more refined study in 1962 by Denison [4] reduced the size of the residual by making adjustments for the impact of education on the quality of labor but continued to attribute most of the remaining residual to technological advance. In his latest book [6, p. 334] Denison has confirmed the earlier studies of long-term U. S. growth by showing that only 28 per cent of the 1960 difference in per-capita income levels between the U. S. and Northwest Europe can be explained by differences in capital and in the quality of labor, and that the remainder is primarily due to differences in the "state of knowledge."

But in a recent duet of papers [9][11] Griliches and Jorgenson (G-J) make the startling claim that all previous investigators have committed serious "errors of measurement," resulting in a sizable

exaggeration of the size of the residual. When these errors are eliminated and "if real product and real factor input are accurately accounted for, the observed growth in total factor productivity is negligible" [11, p. 249]. The habits of a decade have led to the association of advances in total factor productivity with technical change, so that G-J appear to be concluding that technical change has been almost non-existent as a source of U. S. growth. Bewildered businessmen and economists, who previously thought that they had been observing rapid advances in managerial techniques and production technology in the postwar United States, may now wonder whether their eyes have been deceiving them. How are they to interpret industry studies (e.g., [10]) which emphasize the importance of technological progress? If we accept G-J's results, are we then forced to conclude that industries enjoying technical change have been atypical and that their achievements have been counterbalanced by technical regress in unstudied industries?

But a closer evaluation, attempted in this paper, suggests that the G-J conclusion is misleading. Increases in total factor productivity appear to be negligible because G-J raise the rate of growth of inputs relative to output, but they ignore the important role of technological change in achieving this rapid growth of inputs. Thus the G-J paper forces us to break our ingrained mental habit of thinking

of technological advance as a number equal to or smaller than the increase in total factor productivity or "residual," and instead, to realize that the contribution of technical change to economic growth may in fact be much larger than the "error-corrected" residual. Unfortunately, G-J repeatedly promote the illusion that their conclusion about total factor productivity change provides information on technological change, contrary to our analysis below. For instance, they argue that "our results suggest that the....advance of knowledge has been substantially overstated, even by Denison" [9, p. 61] and that "Identification of measured growth in total factor productivity with embodied or disembodied technical change provides methods for measuring technical change" [11, p. 249]. Again they imply that calculations of changes in total factor productivity yield information on technical change when they claim their results to "suggest that social rates of return to [expenditures on research and development] are comparable to rates of return on other types of investment" [11, p. 274]. And, most directly, they predict on the basis of their results that "perhaps the day is not far off when economists can remove the intellectual scaffolding of technical change altogether" [9, p. 61].<sup>2</sup>

This paper demonstrates that the measures of total factor productivity provided by G-J tell us nothing about the importance of advances

in technology. In addition, traditional methods of productivity measurement used by Kendrick [12], Solow [24], and others provide no direct evidence on technical change, partly because they ignore cost-increasing advances in knowledge. The Kendrick-Solow methods, however, both algebraically and in computer simulations, appear to give more accurate evidence on the importance of technical change than the "error-free" G-J methods. Therefore the G-J paper is both misleading and irrelevant for the study of economic growth; misleading, since it appears to claim that advances in technology have been unimportant, and irrelevant, since it provides no/<sup>new</sup>information to help us measure the relative contribution of technological advance and other sources of economic growth. In short, G-J have thrown the baby out with the error-ridden bathwater.

## II. Total Factor Productivity and the Social Return to Research

G-J begin by identifying a change in total factor productivity with a shift in the production function, i.e. a "costless" advance in knowledge. They conclude that the 1945-65 increase in total factor productivity has been substantially overstated. Their argument, however, focusses exclusive attention on "costless" advances in knowledge, which Nordhaus has called a "pleasant fiction" [17, p. 3]. Consider in contrast an economy in which knowledge has been advancing steadily,



but only by means of "the employment of scarce resources with alternative uses"--e.g., managers and research workers. If these workers discover new techniques which were previously unknown, a production function relating output to production labor and capital alone may be said to have shifted, even though the fruits of research work have not been "costless." Furthermore, if the research workers are able to appropriate the full social returns of their efforts and if the research portion of labor input is properly weighted to reflect these returns, there will be no apparent increase in indexes of total factor productivity.

Solow [22, pp. 16-28] and Schultz [20, pp. 293-7] have previously argued the advantages of thinking in terms of the rate of return of alternative forms of tangible and intangible capital. But the constancy of total factor productivity in the example of the previous paragraph tells us nothing about the social rate of return of research workers. If the net social rate of return of a research worker is positive, an economy can increase output by reallocating labor from the production to the research sector, even if no increase in total factor productivity occurs because research workers appropriate their full social returns.

Thus the G-J measures of total factor productivity are misleading, both regarding the contribution of technological change to economic growth, and in the implication that social rates of return to expenditures on research are comparable to rates of return on other types of

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Thus the G-J measures of total factor productivity are misleading, both regarding the contribution of technological change to economic growth, and in the implication that social rates of return to expenditures on research are comparable to rates of return on other types of

investment. The rest of the section illustrates this point more precisely with a simple economic model incorporating a distinction between research and non-research workers. It is demonstrated that the approach not only of G-J but also of their predecessors inaccurately measures the true contribution of advances in knowledge to economic growth.

#### A. The Model

In our simple economy there are no "costless" shifts in the production function, yet advances in knowledge play a crucial role in economic growth. All technological change is created by research workers, and the model incorporates both embodied and disembodied research-using technical change. Our aim is to describe the contribution of advances of knowledge to output growth in the hypothetical economy, and then compute how accurately G-J and earlier investigators would measure this contribution. At this initial stage no separate attention is given to education, which will be discussed later in Section III.

In the economy output  $Y(t)$  is produced by effective production-worker man-hours  $L(t)$ , and effective machine-hours  $J(t)$ . In addition there is disembodied technical progress which raises output in response to increases in the accumulated stock of research knowledge  $R(t)$ :

$$(1) \quad Y(t) = F(L(t), J(t), R(t))$$

where (2) 
$$R(t) = \int_0^t S(g) dg$$

and  $S(g)$  is the number of workers engaged in knowledge-increasing activities (including not just conventionally-measured research and development workers but anyone who thinks about or implements new techniques, including managers, foremen, summer interns, and even the share of production worker man-hours spent contributing to suggestion boxes).

The effective machine-hours  $J(t)$  available from a given stock of capital  $K(t)$  may be increased by means of embodied technical progress, which also takes place through increases in the accumulated stock of research knowledge:

$$(3) \quad J(t) = G(K(t), R(t))$$

where (3) is a shorthand representation of the following embodiment process:

$$(3a) \quad J(t) = \int_0^t I(v) \phi(R(v)) dv$$

and

$$(3b) \quad K(t) = \int_0^t I(v) dv$$

$I(v)$  is investment in period  $v$ , and to simplify matters, we assume no depreciation.<sup>3</sup>

The production process in our economy has constant returns to  $L$ ,  $K$ , and  $R$ .<sup>4</sup> The sum of the elasticities of output with respect to

the three inputs is unity:

$$(4) \quad E_{YL} + E_{YK} + E_{YR} = 1$$

where (4a)  $E_{YL} = \frac{F_L L}{Y}$

(4b)  $E_{YK} = \frac{F_J G_K K}{Y}$

(4c)  $E_{YR} = \frac{(F_R + F_J G_R) R}{Y}$

These elasticities, however, do not necessarily describe the distribution of income. In the model, as in most economies, research is not carried on in separate accounting units. Instead, there are only two such units--employees and firms. Firms obtain both production (L) and research man-hours (S) by offering a wage to workers. The market for production workers is perfectly competitive, and thus the observed value share of production workers in the national income  $V_L$  is equal to  $E_{YL}$ . But we make no such restrictive assumption about the market for research workers, and simply write their share as  $V_R$ , allowing their wage to be larger or smaller than their marginal product. The share of all workers ( $M = L + S$ ) is then  $V_M = V_L + V_R = E_{YL} + V_R$ . The share of capital can then be derived from the constant returns assumption (4):

$$(5) \quad V_K = 1 - V_M = E_{YK} + E_{YR} - V_R .$$

If  $E_{YR} > V_R$ , firms are exploiting research workers and pushing the observed rate of return on capital above the marginal product of capital.

Our aim is to isolate in this model the contribution of advances of knowledge to economic growth. To do this, we can compare two economies A and B at time  $t$ , each with production and distribution arrangements described by (1) - (6). The only difference between the two economies is that suddenly at time  $t$  advances in knowledge cease in economy B. Thus, after time  $t$ :

$$\text{ECONOMY A} \quad T_R > 0; G_R > 0$$

$$\text{ECONOMY B} \quad T_R = G_R = 0$$

The next step is to separate observed changes in output in the two economies into portions attributable to changes in the three inputs  $L$ ,  $K$ , and  $R$ . This can be accomplished by differentiating (1) totally with respect to time and converting to elasticities for the two economies. This is straightforward for economy A:

$$(6) \quad \frac{\dot{Y}_A}{Y_A} = E_{YL} \frac{\dot{L}}{L} + E_{YK} \frac{\dot{K}_A}{K_A} + E_{YR} \frac{\dot{R}}{R}$$

No subscript is attached to the symbols  $L$  or  $R$ , since population growth is assumed exogenous and therefore is the same in the two economies. In economy B, all research workers, barren of new ideas, return to production work, where they behave and are paid exactly

like all other production workers:<sup>5</sup>

$$(7) \quad \frac{\dot{Y}_B}{Y_B} = E_{YL} \frac{\dot{M}}{M} + E_{YK} \frac{\dot{K}_B}{K_B} = E_{YL} \frac{\dot{L}}{L} + E_{YK} \frac{\dot{K}_B}{K_B} + V_R^* \frac{\dot{R}}{R}$$

where  $V_R^*$  is the share of research workers when they are paid the same wage as production workers.

The contribution of advances of knowledge ( $\dot{C}/C$ ) is simply the difference between the growth rates of the two economies:

$$\frac{\dot{C}}{C} = \frac{\dot{Y}_A}{Y_A} - \frac{\dot{Y}_B}{Y_B} = (E_{YR} - V_R^*) \frac{\dot{R}}{R} + E_{YK} \left( \frac{\dot{K}_A}{K_A} - \frac{\dot{K}_B}{K_B} \right)$$

This can be simplified if we assume steady-state growth with a proportional saving rate, so that  $\frac{\dot{Y}_A}{Y_A} = \frac{\dot{K}_A}{K_A}$  and  $\frac{\dot{Y}_B}{Y_B} = \frac{\dot{K}_B}{K_B}$ . In that case:

$$(8) \quad \frac{\dot{C}}{C} = \frac{\dot{Y}_A}{Y_A} - \frac{\dot{Y}_B}{Y_B} = \left( \frac{E_{YR} - V_R^*}{1 - E_{YK}} \right) \frac{\dot{R}}{R}$$

The factor inside the parentheses represents the social rate of return to research workers in economy A. When this factor is zero, the contribution of research workers in A just offsets their opportunity cost, the output which they could be producing as production workers. In this case, their research efforts make no extra contribution to economic growth.

#### B. The Kendrick-Solow Method

Now let us expose economy A to two pairs of energetic economic

detectives, who can observe only the rates of growth of inputs and outputs and factor shares, but not the underlying structure of production. The first pair to arrive on the scene are Kendrick and Solow (K-S), who propose to measure the contribution of knowledge or the "residual" by the following formula:

$$(9) \quad \frac{\dot{C}_{K-S}}{C_{K-S}} = \frac{\dot{Y}_A}{Y_A} - V_M \frac{\dot{M}}{M} - V_K \frac{\dot{K}_A}{K_A}$$

Since  $V_M \frac{\dot{M}}{M} = V_L \frac{\dot{L}}{L} + V_R \frac{\dot{S}}{S}$ ,  $E_{YL} = V_L$ ,  $E_{YK} + E_{YR} = V_K + V_R$ , and assuming  $\frac{\dot{R}}{R} = \frac{\dot{S}}{S}$ , the K-S residual equals:

$$(10) \quad \frac{\dot{C}_{K-S}}{C_{K-S}} = (E_{YR} - V_R) \left( \frac{\dot{R}}{R} - \frac{\dot{K}_A}{K_A} \right)$$

Does the K-S "residual" correctly identify the contribution of advances of knowledge to the growth of economy A? By subtracting (10) from (8), we can derive an expression for the K-S "error" in measuring the contribution of advances in knowledge:

$$(11) \quad \frac{\dot{C}}{C} - \frac{\dot{C}_{K-S}}{C_{K-S}} = \left( \frac{E_{YK}(E_{YR} - V_R) + V_R - V_R^*}{1 - E_{YK}} \right) \frac{\dot{R}}{R} + (E_{YR} - V_R) \frac{\dot{K}_A}{K_A}$$

There are several possible cases:

1. Research workers are not exploited, and their marginal product is larger than that of production workers,  $E_{YR} = V_R > V_R^*$ . In this case the K-S error becomes:



$$(12) \quad \frac{\dot{C}}{C} - \frac{\dot{C}_{K-S}}{C_{K-S}} = \left( \frac{E_{YR} - V_R^*}{1 - E_{YK}} \right) \frac{\dot{R}}{R}$$

By identifying the contribution of advances of knowledge with increases in total factor productivity, K-S erroneously conclude that there is no residual, even though the true contribution of knowledge from (8)

is  $\left( \frac{E_{YR} - V_R^*}{1 - E_{YK}} \right) \frac{\dot{R}}{R}$ . The mistake stems from counting the salary advantage of research workers over production workers as part of the contribution of labor, rather than as a consequence of the advance of knowledge which causes the salary differentials.<sup>6</sup>

2. Firms exploit research workers by paying them the production worker wage, even though their marginal product exceeds the marginal product of production workers,  $E_{YR} > V_R = V_R^*$ . The K-S error is:

$$(13) \quad \frac{\dot{C}}{C} - \frac{\dot{C}_{K-S}}{C_{K-S}} = (E_{YR} - V_R) \left( \frac{E_{YK}}{1 - E_{YK}} \frac{\dot{R}}{R} + \frac{\dot{K}_A}{K_A} \right)$$

If capital is not growing ( $\dot{K}_A/K_A = 0$ ). K-S err again, although the mistake may not be too serious if  $E_{YK}$  is small.

3. If  $\dot{K}_A/K_A > 0$ , Kendrick-Solow underestimate the contribution of knowledge by erroneously using all of capital's share as a weight on the growth of capital, ignoring the fact that a portion of capital's share really represents the contribution of research. Thus a finding by K-S that their residual (10) is equal to zero should not be accepted as evidence that advances in knowledge have been unimportant

or that the social rate of return to research is zero.

### C. The Griliches-Jorgenson Method

After the team of Kendrick-Solow has issued its report on economy A using formula (9), the team of Griliches-Jorgenson arrives on the scene and discovers "errors of measurement" in the work of Kendrick-Solow. The earlier investigators err in (9) by using the stock of capital K as a measure of capital input, and Griliches-Jorgenson recalculate their procedure "correctly," replacing K in (9) by effective capital J.<sup>7</sup> Thus the Griliches-Jorgenson measure of the contribution of advances of knowledge ( $C_{G-J}$ ) is

$$(14) \quad \frac{\dot{C}_{G-J}}{C_{G-J}} = \frac{\dot{Y}_A}{Y_A} - V_M \frac{\dot{M}}{M} - V_K \frac{\dot{J}_A}{J_A}$$

which can be solved like (9):<sup>8</sup>

$$(15) \quad \frac{\dot{C}_{G-J}}{C_{G-J}} = E_{YL} \frac{\dot{L}}{L} + E_{YR} \frac{\dot{R}}{R} + E_{YK} \frac{\dot{K}_A}{K_A} - V_L \frac{\dot{L}}{L} - V_R \frac{\dot{R}}{R} - V_K (E_{JR} \frac{\dot{R}}{R} + E_{JK} \frac{\dot{K}_A}{K_A})$$

$$= (E_{YR} - V_R - V_K E_{JR}) \left( \frac{\dot{R}}{R} - \frac{\dot{K}_A}{K_A} \right)$$

Again we can compare the G-J estimate (14) with the actual contribution of knowledge to gauge the accuracy of their approach. Subtracting (15) from (10), we can write the G-J error as:

$$(16) \quad \frac{\dot{C}}{C} - \frac{\dot{C}_{G-J}}{C_{G-J}} = \left( \frac{E_{YR} - V_R^*}{1 - E_{YK}} \right) \frac{\dot{R}}{R} - (E_{YR} - V_R - V_K E_{JR}) \left( \frac{\dot{R}}{R} - \frac{\dot{K}_A}{K_A} \right)$$

Again, there are several possible cases:

1. Research workers are not exploited:  $E_{YR} = V_R > V_R^*$ . It appears from (15) that, if  $\dot{R}/R > \dot{K}_A/K_A > 0$ , G-J are further from the truth than K-S, since the former calculate a negative contribution of knowledge in the amount  $-V_K E_{JR} (\dot{R}/R - \dot{K}_A/K_A)$ . This occurs because they double-count the impact of research in making capital more "effective," i.e., in raising the ratio of J to K. In this no-exploitation case, the marginal product of research workers is already fully counted in the research share  $V_R$ , but G-J add to the growth of input an extra quantity reflecting the contribution of research workers in making capital more effective. This is in addition to the basic mistake which G-J make (in common with K-S) in counting the wage differential between research and production workers as part of the contribution of labor. In sum, the G-J error in this case is:

$$(17) \quad \frac{\dot{C}}{C} - \frac{\dot{C}_{G-J}}{C_{G-J}} = \left( \frac{E_{YR} - V_R^*}{1 - E_{YK}} \right) \frac{\dot{R}}{R} - E_{JR} V_K \left( \frac{\dot{R}}{R} - \frac{\dot{K}_A}{K_A} \right)$$

2. When research workers are exploited and paid a wage equal to the marginal product of production workers (so that  $E_R > V_R = V_R^*$ ), G-J still underestimate the contribution of advances of knowledge, even when  $\dot{K}_A/K_A = 0$ . This again occurs because G-J count the contribution of research workers in making capital more effective as an increase in input rather than as a contribution of advances in knowledge.

Their error in the  $\dot{K}_A/K_A = 0$  case is:

$$(18) \quad \frac{\dot{C}}{C} - \frac{\dot{C}_{G-J}}{C_{G-J}} = \left( \frac{E_{YK}(E_{YR} - V_R) + (1 - E_{YK})E_{JR}V_K}{1 - E_{YK}} \right) \frac{\dot{R}}{R}$$

3. And, as was true with K-S, in the exploitation case where  $R/R > \dot{K}_A/K_A > 0$ , G-J underestimate the contribution of knowledge by erroneously applying all of capital's share as a weight on the growth of capital, even though part of capital's share represents the contribution of research rather than capital. In this case their error can be written as equation (16) above.

In short, neither the K-S nor G-J calculations of changes in total factor productivity are reliable indicators of the contribution of advances in knowledge to economic growth. Nor can a finding of negligible growth in total factor productivity be accepted as evidence that the social returns to research activity are similar to returns for those in other kinds of employment. Both methods tend to underestimate the contribution of knowledge by including in the weights on input growth the portion of labor and capital compensation which really represents the return to research workers. And, in addition, the G-J insistence on measuring "effective capital" further understates the contribution of advances in knowledge by ignoring the role of research in raising capital's "effectiveness." (Note that in each of the three cases above, the G-J "error" is larger than that of K-S). It is easy

to conceive of examples, using (14), in which the social return to research workers is strongly positive, yet at the same time G-J could be calculating virtually no growth in total factor productivity.

After a few brief remarks on the G-J treatment of education, we shall use computer simulations of a hypothetical economy to suggest orders of magnitude for the K-S and G-J errors.

### III. The Treatment of Education

To simplify the discussion in the preceding section, the labor force was divided into two homogeneous groups, production and research workers, and no account was taken of possible differences in the quality of labor within these groups. Now, however, we should recognize the role of education in creating quality differences among workers and should consequently examine G-J's method of measuring the contribution of education to economic growth.

Just as embodied research can make some machines more productive than others, so can embodied education make some workers more productive than others. In addition, the efficiency of workers with given education endowments will vary with their "native ability" or intelligence, as well as with their environment, amount of encouragement from parents, and other factors. Thus the input of effective labor  $L(t)$  into the production function (1) is itself a function of

education, ability (where "ability" stands for influences on labor quality other than education), and man-hours:

$$(19) \quad L(t) = \sum_i G(E_i) \sum_j H(A_j) B_{ij}(t)$$

Here total man-hours  $B$  are allocated into groups according to native ability  $A_j$  and educational attainment  $E_i$ .  $B_{ij}$  is the number of man-hours in each education-ability group;  $H(A_j)$  represents the contribution of ability to effective labor input, and  $G(E_i)$  similarly stands for the impact of differing levels of educational attainment on effective labor input. The respective contributions of ability and education to economic growth can be separated as follows. First, we define  $b_{ij}$ , the proportion of man-hours of a given ability group  $j$  in an education group  $i$ :

$$(20) \quad b_{ij} = B_{ij}/B_i$$

and  $e_i$ , the proportion of aggregate man-hours in a group with educational attainment  $i$ :

$$(21) \quad e_i = B_i/B.$$

Substituting (20) and (21) into (19), differentiating with respect to time, and dividing by  $L$ , we obtain:

$$(22) \quad \frac{\dot{L}}{L} = \frac{B}{L} \sum_i G(E_i) (e_i \sum_j \dot{H}(A_j) b_{ij} + e_i \sum_j H(A_j) \dot{b}_{ij}) + \frac{\dot{B}}{B}$$

The average wage within an educational group can be written as

follows (if workers are paid their marginal products):

$$(23) \quad w_i = \frac{\sum_j w_{ij} B_{ij}}{B_i} = \sum_j \frac{\partial L}{\partial B_{ij}} b_{ij} = G(E_i) \sum_j H(A_j) b_{ij}$$

Substituting (23) into (22), and noting that the average wage in the economy is  $\bar{w} = L/B$ , we can write:

$$(24) \quad \frac{\dot{L}}{L} = \sum_i \frac{w_i}{\bar{w}} \left[ e_i + \frac{\sum_j H(A_j) \dot{b}_{ij}}{\sum_j H(A_j) b_{ij}} \right] + \frac{\dot{B}}{B}$$

Here the relative wage  $w_i/\bar{w}$  times the first term inside the brackets represents the contribution to economic growth of the changing educational distribution of the labor force. Even with a stationary population, effective labor input  $L$  will increase as a larger fraction of the labor force enters the educational groups with high relative wages  $w_i/\bar{w}$ . The second term is an adjustment for the changing average ability of each educational group. As a larger and larger fraction of the nation's population attains a twelfth-grade educational level, the average ability of the twelfth-grade group is likely to decline, so the net effect of the second term on economic growth is almost certainly negative.

Stated in another way, differences in the relative wages  $w_i/\bar{w}$  used to weight the educational groups occur for reasons other than education. The relative earnings of college-educated workers are high not just because they went to college, but also because of the

relatively high percentage of college graduates "who had obtained high marks in earlier schooling, who had scored well on standardized intelligence tests, who had attended the better schools at lower educational levels, and who also had parents who were themselves well educated and had substantial incomes" [6, p. 83].<sup>9</sup>

Equation (17) above can be compared with G-J's equation (12), which in our notation can be written:

$$(25) \quad \frac{\dot{L}}{L} = \sum \frac{w_i}{w} e_i + \frac{\dot{B}}{B}$$

G-J, therefore, allow for the first term inside the brackets in (17), the positive contribution of education to economic growth, but they make no mention of the second term, the changing ability mix of each educational group. Thus G-J substantially exaggerate the rate of growth of labor input "with errors in the aggregation of labor services eliminated." The order of magnitude of this exaggeration can never be known exactly, although Denison has recently cited several pieces of evidence supporting his original estimate that education is responsible for 60 per cent of observed wage differentials among educational groups, not the 100 per cent assumed by G-J.<sup>10</sup>

#### IV. Advances in Knowledge and Total Factor Productivity in a Hypothetical Economy

Sections II and III demonstrated that the measurement techniques



of Griliches and Jorgenson tend to underestimate the contribution of advances in knowledge to economic growth. But, unfortunately, we can never obtain accurate estimates of the magnitude of their errors, since we can never know how rapidly the U. S. economy would have grown from 1945 to 1965 without any advances in knowledge. As a second-best alternative, it is possible to construct a numerical model of economic growth in a hypothetical economy to reveal the accuracy of the G-J measurement techniques, given the stated assumptions of the numerical model. The model has been designed for computer simulation to facilitate the inclusion of numerous "realistic" assumptions, and so several different experiments can be run to test the sensitivity of the conclusions to alternative parameter values.

#### A. Outline of the Model

1. The Effective Input of Labor. The model is completely production-oriented and has no demand mechanism. Full employment is maintained continuously, since investment is always set equal to saving. There are two production sectors, one producing consumption goods with effective production workers, effective capital, and part of the accumulated stock of knowledge. Effective capital is produced by effective production workers in the investment sector and the rest of the accumulated stock of knowledge. There is no capital input

in the investment goods sector. The allocation of the labor force between research workers and the two groups of production workers is arbitrary; an allocation obtained through the equalization of marginal returns is not desired, since one of the main purposes of the model is to exhibit the effects on growth and productivity measurement of large differences between the marginal returns to research and investment. So the allocation of production workers between the two sectors is governed by a fixed proportional savings rate, and the proportion of the total labor force engaged in research activity is completely exogenous.<sup>11</sup>

The first equation of the model describes the determination of the effective labor force  $M_t$ , given the exogenous supply of "brute-force" or "raw" labor  $B_t$ , the proportion  $e_{it}$  of the labor force in each of  $n$  education-ability classes, and the multiplicative education ( $G_i$ ) and ability ( $H_i$ ) factors which convert the units of raw labor in each class into units of effective labor:

$$(26) \quad M_t = B_t \sum_{i=1}^n e_{it} G_i H_i$$

The multiplicative factors are based on U. S. data on the relative compensation of workers in different educational groups, using Denison's assumption that 60 per cent of compensation differences are due to differences in education and the remainder to differences in ability.<sup>12</sup>

The proportions in the different education-ability groups are based on G-J's figures on the education attainment of the U. S. labor force, and in addition on the assumption that all people moving from one educational level to a higher one have the native ability of an average member of the former class.<sup>13</sup>

As noted above in section 3, Griliches and Jorgenson do not allow for differences in native ability. Their procedure implies that the native ability of persons moving to a higher educational attainment is effortlessly converted to the average native ability of those in the higher educational category.<sup>14</sup> The application by G-J of ability coefficients  $H_i^{G-J}$  which differ from the  $H_i$  values used in (26) requires us to calculate a separate series showing G-J's measure of the labor force:<sup>15</sup>

$$(27) \quad M_t^{G-J} = B_t \sum_{i=1}^n e_{it} G_i H_i^{G-J}$$

Research workers are assumed to constitute a given (and growing) fraction  $u_t$  of "raw" labor, but their share  $S_t/L_t$  of effective labor input is considerably greater than this, since they are assumed to be the most-educated members of the labor force.<sup>16</sup>

$$(28) \quad \frac{S_t}{M_t} = \sum_{i=0}^{n-1} x_{it} e_{n-i,t}^G e_{n-i}^H$$

$$x_{it} = 1; i = 1, \dots, m-1$$

$$x_{it} = \frac{e_{n-i,t} + u_t - \sum_{i=0}^m e_{n-i,t}}{e_{n-i,t}}$$

$$x_{it} = 0; i = m+1, \dots, n-1$$

where  $m$  is the lowest number at which

$$u_t < \sum_{i=0}^m e_{n-i,t}$$

The portion of effective labor input  $M_t$  which is not devoted to research work is available as production labor in the investment ( $L_t^{PI}$ ) and consumption ( $L_t^{PC}$ ) sectors:

$$(29) \quad L_t = L_t^{PI} + L_t^{PC} = M_t - S_t$$

The effective input of research workers ( $S_t$ ) is apportioned arbitrarily to three different research laboratories. One group works on disembodied process improvement in the machinery industry; a second is engaged in product improvement in the machinery industry (performing what we usually mean by "technical change embodied in capital"); and the third group works on disembodied process improvements in the production of consumer goods (improvements in management, organization, etc.). There is no product research in the consumption sector built into the model, reflecting the real-life failure of the national

accounts properly to measure quality change in consumer goods.

In keeping with our deliberately arbitrary allocation of the labor force, we shall assume that the research labor force is evenly divided among the three research laboratories. Effective labor input in each laboratory ( $s_t$ ) is:

$$(30) \quad s_t = S_t/3$$

Technical progress takes place in the model in response to increases in the accumulated stock of the three different types of knowledge produced by the three groups of research workers. The accumulated stock of knowledge in each laboratory ( $R_t$ ) is:

$$(31) \quad R_t = \sum_{g=0}^{t-1} s_g e^{-\lambda(t-g)}$$

where  $\lambda$  represents the obsolescence of ideas.<sup>17</sup> Old ideas lose their usefulness when replaced by newer versions, just as do old machines.

2. Technology in the Consumption Sector. A simple Cobb-Douglas production function is assumed for the consumption goods industry:

$$(32) \quad Q_t = (R_t)^{\alpha_1} (L_t^{PC})^{\alpha_2} J_t^{\alpha_3}$$

Output is a function of the accumulated stock of research knowledge on production processes in the consumption industry  $R_t$ , the effective input of production workers ( $L_t^{PC}$ ), and the effective stock of capital ( $J_t$ ), which is measured not in tons or dollars but in machine

revolutions per unit of time. If there are constant returns to all factors, so that  $\alpha_1 + \alpha_2 + \alpha_3 = 1$ , and if  $\alpha_1 > 0$ , then there are diminishing returns to effective labor and capital alone. A possible rationalization for this assumption is that any economy which grows without process research in the consumption sector becomes disorganized and inefficient.<sup>18</sup> An alternative assumption is increasing returns to the three factors ( $\alpha_1 + \alpha_2 + \alpha_3 > 1$ ) and constant returns to  $L^{PC}$  and  $J$  alone. If the stock of knowledge is growing exponentially, this assumption makes (32) into the traditional constant-returns Cobb-Douglas production function with neutral disembodied technical progress which has been used so often in studies of economic growth. We are not committed to any particular values of the factor elasticities, and below we shall present results for values of  $\alpha_1 + \alpha_2 + \alpha_3$  both equal to and greater than one.

The wage rate for production workers in the consumption industry is competitively determined, for each unit of effective labor receives its marginal product. In the increasing returns case, the wage is assumed to be proportional to the marginal product:

$$(33) \quad w_t = \frac{\partial Q_t}{\partial L_t^{PC}} \left( \frac{1}{\alpha_1 + \alpha_2 + \alpha_3} \right) = \frac{\alpha_2 Q_t}{(\alpha_1 + \alpha_2 + \alpha_3) L_t^{PC}}$$

Research workers, however, are not paid their marginal product but are paid the same wage per unit of effective input as production

workers. Since research workers are all the best-educated members of society, their annual earnings per man will be greater than those of the less-educated production workers. This payment system corresponds to the observable fact in the real world that salaries for research workers are similar to the earnings of employees with similar educational backgrounds.

The compensation of capital  $A_t^K$  is simply the residual product in the consumption sector after all workers have been paid:

$$(34) \quad A_t^K = Q_t - w_t(L_t^{PC} + s_t)$$

3. Technology in the Investment Goods Sector. The conversion of labor into machine revolutions ( $J_t$ ) takes place in two stages. First, production workers in the investment sector join with the accumulated stock of process knowledge to produce structures and equipment ( $I_t$ ):

$$(35) \quad I_t = (R_t)^{\beta_1} (L_t^{PI})^{\beta_2}$$

This production function, like (32), can exhibit either constant or increasing returns in  $R$  and  $L^{PI}$ .  $I_t$  is measured in units of effective labor input.<sup>19</sup>

Although only equipment is used in (32) to produce consumption goods, structures are necessary to house the equipment, in the ratio

$\mu$  units of structures to every  $1-\mu$  units of equipment. Thus, if  $I_t^X$  is the portion of investment output available for expansion after replacement needs have been satisfied,  $K_t^S$  is the accumulated stock of structures,  $\delta$  the depreciation rate of effective machine revolutions, and  $\eta$  the depreciation rate for structures, we have:

$$(36) \quad I_t^X = I_t - \delta J_t - \eta K_t^S$$

$$(37) \quad I_t^E = (1-\mu)I_t^X + \delta J_t$$

$$(38) \quad I_t^S = \mu I_t^X + \eta K_t^S$$

with  $I_t^E$  and  $I_t^S$  as gross investment in equipment and structures, respectively. The machine revolutions  $Z_t$  obtainable from a unit of gross equipment investment  $I_t^E$  do not remain constant over time but are constantly increased through product research in the investment sector. The production function for effective equipment investment is similar to (35):

$$(39) \quad Z_t = (R_t)^{\gamma_1} (I_t^E)^{\gamma_2}$$

If the stock of research knowledge grows exponentially and  $\gamma_2 = 1$ , this equation represents exponential capital-augmenting, capital-embodied technical progress.<sup>20</sup> Only machines improve, however, and structures always remain the same.

Finally, we write two accounting equations which describe the



accumulation of capital:

$$(40) \quad K_t^S = \sum_{g=0}^{t-1} I_t^S e^{-\eta(t-g)}$$

$$(41) \quad J_t = \sum_{g=0}^{t-1} Z_t e^{-\delta(t-g)}$$

Another set of equations is necessary to determine the allocation of the production labor force between the consumption and investment sectors. On the assumption of a constant propensity to save and invest ( $w$ ) out of current-dollar income ( $Y_t^*$ ), current dollar investment ( $p_t^I I_t$ ) can be written:

$$(42) \quad p_t^I I_t = w Y_t^*$$

But current-dollar income and product is:

$$(43) \quad Y_t^* = Q_t + p_t^I I_t$$

where the relative price of investment goods is just the wage bill in the investment sector  $W_t^I$  divided by real investment (in labor units)  $I_t$ :

$$(44) \quad p_t^I = \frac{W_t^I}{I_t} = \frac{w_t(2s_t + L_t^{PI})}{I_t}$$

Note that production labor input in the investment sector is paid the same wage ( $w_t$ ) as in the consumption sector, implying a competitive market for production workers. The three equations (42), (43)

and (44) can be combined with (33) to yield an expression for the input of production labor in the consumption sector.

$$(45) \quad L_t^{PC} = \frac{\alpha_2(1-w)}{w + \alpha_2(1-w)} (M_t - s_t)$$

and then  $L_t^{PI}$  is a residual determined by (29).

A final unknown in the model is the rate of return on the book value of capital ( $r_t$ ), which is:

$$(46) \quad r_t = \frac{A_t^K}{p_t^K K_t}$$

and

$$(47) \quad p_t^K K_t = \sum_{g=0}^{t-1} p_t^I (I_t^E e^{-\delta(t-g)} + I_t^S e^{-\eta(t-g)})$$

## B. Total Factor Productivity and the Contribution of Advances in Knowledge

1. Growth in Economies A and B. Following the scheme laid out in section II above, the contribution of advances in knowledge to economic growth  $\dot{C}/C$  is the difference between the growth rates of two economies, A and B, which are the same in every detail except that research workers are productive in economy A and completely barren of ideas in economy B:

$$(48) \quad \frac{\dot{C}}{C} = \frac{\dot{Y}^A}{Y^A} - \frac{\dot{Y}^B}{Y^B}$$

where  $Y_t^A$  is constant-dollar output of consumption goods plus the

real gross output of capital services in economy A,

$$(49) \quad Y_t^A = Q_t + Z_t + I_t^S$$

and  $Y_t^B$  is a similar expression for economy B. Economy B differs from the model outlined above in that  $\alpha_1 = \beta_1 = \gamma_1 = 0$ , and research workers abandon their desks and drawing boards to return to production work in the same sector (i.e.,  $L_t^{PC}$  in economy B equals  $L_t^{PC} + s_t$  from economy A above and  $L_t^{PI} = L_t - L_t^{PC}$ ). In the cases where there are constant returns to both research and non-research factors, this implies, of course, that the elasticity of  $Q_t$  and  $I_t$  with respect to the remaining non-research inputs is less than one. Diminishing returns would not be implausible in an economy with no advances in technical or managerial knowledge, since capital accumulation would just amount, in Domar's phrase, to "wooden ploughs piled up on top of existing wooden ploughs"[7, p. 712]. The alternative of increasing returns to all inputs with constant returns to non-research inputs will also be included in the experiments.

In practice, the above model can be written down as a computer program, and given arbitrary values of the parameters  $(\lambda, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \gamma_1, \gamma_2, \delta, \eta, \omega, \mu)$  and initial period values of capital and research stocks, the time path of economies A and B can be traced and the contribution of advances in knowledge C/C can be calculated. The purpose

of the exercise is to compare  $\dot{C}/C$  with the measures of total factor productivity which would be calculated by the rival teams of Kendrick-Solow and Griliches-Jorgenson, if they had access to data on the dependent variables in economy A, but not the parameter values. It is important to evaluate the accuracy of their methods, of course, since we can never learn how a real-world economy B would have behaved without advances of knowledge; hence we cannot calculate  $\dot{C}/C$  for the United States and must rely on some indirect technique.<sup>21</sup>

2. The Measurement of Output. The first difference between Kendrick-Solow (K-S) and Griliches-Jorgenson (G-J) is in the measurement of real output. G-J measure real investment as the real gross output of capital services, so that their output measure  $Y_t^{G-J}$  agrees with (49) above:

$$(50) \quad Y_t^{G-J} = Y_t^A = Q_t + Z_t + I_t^S$$

The K-S measure of output  $Y_t^{K-S}$  differs in two ways, due both to a conceptual difference and to an error in measurement. First, K-S include in output not the gross output of equipment services, but the gross output of equipment in units of base-year cost  $I_t^E$ . And, second, K-S use erroneous structures deflators which are merely averages of input costs and ignore technological advance in the construction part of the investment sector. Since the only input in the sector is labor, the K-S price deflator is the wage  $w_t$ , and

their measure of the real output of structures is  $I_t^{SKS} = I_t^S (p_t^I/w_t)$ .

K-S therefore calculate output as:

$$(51) \quad Y_t^{K-S} = Q_t + I_t^E + I_t^S (p_t^I/w_t)$$

If  $\beta_1$  and  $\gamma_1$  are positive, the growth of  $Z_t$  will be faster than  $I_t^E$ , and  $w_t$  will grow more rapidly than  $p_t^I$ , so that  $Y^{G-J}$  will grow at a faster rate than  $Y^{K-S}$ .

3. The Measurement of Input. G-J make an advance over K-S (as did Denison [4] in 1962) by recognizing that labor is heterogeneous and should be weighted by educational attainment. But, as shown above in equation (27), G-J ignore differences in native ability, with the result that their measure of effective labor input  $M_t^{G-J}$  grows more rapidly than the true measure  $M_t$ , and both grow more rapidly than the homogeneous K-S labor force  $B_t$ .

The differences between G-J and K-S in the measurement of capital parallel those in the measurement of investment. The K-S aggregate capital stock is:

$$(52) \quad K_t^{K-S} = \sum_{g=0}^t I_t^E e^{-\delta(t-g)} + \frac{p_t^I}{w_t} I_t^S e^{-\eta(t-g)}$$

Griliches-Jorgenson, on the other hand, weight together the effective capital input of structures and equipment into a Divisia index:

$$(53) \quad \frac{\dot{K}^{G-J}}{K^{G-J}} = v_t^J \frac{\dot{J}}{J} + v_t^S \frac{\dot{K}^S}{K^S}$$

where the definitions of  $J_t$  and  $K_t^S$  are given above in (40) and (41), and the respective weights are determined by the relative prices of capital services  $c_t^J$  and  $c_t^S$ .<sup>22</sup>

$$(54) \quad v_t^J = \frac{c_t^J J_t}{c_t^J J_t + c_t^S K_t^S}$$

$$(55) \quad v_t^S = 1 - v_t^J$$

$$(56) \quad c_t^J = p_t^I (r_t + \delta_t)$$

$$(57) \quad c_t^S = p_t^I (r_t + \eta_t)$$

Finally, both K-S and G-J calculate the rate of growth of total input and total factor productivity ( $C^{K-S}$  and  $C^{G-J}$ ) by weighting together capital and labor with weights based on share of total compensation:

$$(58) \quad \frac{\dot{C}^{K-S}}{C^{K-S}} = \frac{\dot{Y}^{K-S}}{Y^{K-S}} - v_t^M \frac{\dot{B}}{B} - v_t^K \frac{\dot{K}^{K-S}}{K^{K-S}}$$

$$(59) \quad \frac{\dot{C}^{G-J}}{C^{G-J}} = \frac{\dot{Y}^{G-J}}{Y^{G-J}} - v_t^M \frac{\dot{M}^{G-J}}{M^{G-J}} - v_t^K \frac{\dot{K}^{G-J}}{K^{G-J}}$$

where

$$(60) \quad v_t^M = \frac{w_t^M}{Y_t^*}$$

and

$$(61) \quad v_t^K = \frac{A_t^K}{Y_t^*} = 1 - v_t^M$$

4. The Social Rate of Return to Research and Physical Capital. Griliches and Jorgenson have claimed that their finding of negligible growth in total factor productivity implies that "social rates of return to this type of investment are comparable to rates of return on other types of investment" [11, p. 274]. To evaluate this claim, we can calculate the social rates of return of investment to research and to physical capital in each of our simulations, and observe true differences in rates of return in cases where  $\dot{C}^{G-J}/C^{G-J}$  is very small.

To calculate the one-period rate of return on investment in physical capital, we follow Solow and "sacrifice one unit of consumption at time  $t$  in favor of investment, and then ask what is the largest increment of consumption that can be enjoyed at time  $t+1$  without impairing consumption possibilities in any later period.... This last condition means that the effective stock of capital bequeathed to period  $t+2$  must be no smaller than would have been the case had the extra saving in period  $t$  and the extra consumption in period  $t+1$  not taken place" [22, p. 60]. In practice we begin the calculation by switching one production worker from the consumption to the investment sector. Similarly, the one-period rate of return on investment in research involves the switch of one man from production work in the consumption sector to research work with one third of

the man going to each of the three research laboratories for one time period. We calculate the maximum consumption increment at time period  $t+1$  compared to the original "control solution" on the condition that the effective stock of physical capital and accumulated research bequeathed to period  $t+2$  must not be altered by the experiment. In practice, we must extend our calculations over two time periods, since extra research performed at time  $t$  raises consumption at time  $t+1$  directly through disembodied change in the consumption sector, but also raises consumption indirectly at time  $t+2$  as a consequence of the increased research input in the investment sector at time  $t+1$  and higher resulting quantity of  $J_{t+2}$ . The experimental switch in the allocation of labor lasts only for the one period  $t$ , and in period  $t+1$  the allocation of labor is unchanged from the basic simulation. The labor allocation is affected at time  $t+2$ , however. Since the  $J_{t+3}$  and  $R_{t+3}$  must return to the original values of the control solution and since  $J_{t+2}$  and  $R_{t+2}$  are higher than in the control solution, less investment and research work are necessary in time period  $t+2$  than in the control solution, leaving extra workers for the production labor force in the consumption sector and giving an additional boost to  $Q_{t+2}$ . To preserve symmetry, the rate of return on physical capital, like the rate of return on research, is calculated over two periods.



## V. Simulation Results

Initial experimentation revealed that variations in several structural parameters made little difference in the results, so that arbitrary values were assigned to the three depreciation parameters ( $\lambda=.05$ ,  $\delta=.10$ ,  $\eta=.04$ ) and the structures requirements parameter ( $\mu=.40$ ). In the first part of this section results will be reported for a saving rate ( $w$ ) of .20, but later the effect of alterations in  $w$  will be examined. Growth rates were calculated over fifteen periods.

### A. Embodied and Disembodied Change

1. Constant Returns. Information on the first simulation is presented in Table 1. Technical progress takes place in all three laboratories in economy A; there is disembodied progress in the consumption and investment sectors as well as embodied progress which improves the quality of equipment. There are constant returns to scale in production labor, the stock of knowledge, and effective equipment services in the consumption sector, and to production labor and the stock of knowledge in the equipment sector. The effect of altering this assumption to increasing returns will be examined shortly.<sup>23</sup> The technological parameters are listed in line C of Table 1, and the results are summarized in line D. The rate of

A. Types of Technical Progress: Disembodied in Consumption Sector  
Disembodied in Investment Sector  
Embodied

B. Returns to scale in all factors: Constant

C. Parameter values of the technology:

$$\alpha_1 = .20 \quad \alpha_2 = .60 \quad \alpha_3 = .20 \quad \beta_1 = .20 \quad \beta_2 = .80 \quad \gamma_1 = .20 \quad \gamma_2 = .80$$

D. Summary of results (percentage growth rates):

$$\frac{\dot{Y}_A}{Y_A} = 4.30 \quad \frac{\dot{Y}_B}{Y_B} = 1.44 \quad \frac{\dot{C}}{C} = 2.86 \quad \frac{\dot{C}_{K-S}}{C_{K-S}} = 1.91 \quad \frac{\dot{C}_{G-J}}{C_{G-J}} = .97 \quad \rho_8^R = .365 \quad \rho_8^K = .010$$

E. Components of G-J Correction of K-S:

	Output Aggregation	Price of Structures	Effective Equipment	Education
1. Percentage points subtracted from residual	-.17	.23	.26	.55
2. Per cent of output growth explained by input growth after correction	52.4	58.4	64.8	77.5

F. Explanation of discrepancies between calculations of residual and true contribution of advances in knowledge:

(percentage points)	<u>True</u>	<u>K-S</u>	<u>G-J</u>
1. Calculated contribution	2.86	1.91	.97
2. Sources of Discrepancies:			
a. Growth capital input		.51	.68
b. Price of structures		-.16	
c. Capital share		.61	.84
d. Growth labor input		-.29	.18
e. Labor share		.11	.19
f. Growth of output		.17	
3. True contribution	<u>2.86</u>	<u>2.86</u>	<u>2.86</u>
4. Addendum: Sources of true contribution of advances in knowledge			
a. Direct impact of research	2.61		
b. Indirect impact on capital	.74		
c. Indirect impact on prod'n labor	-.49		

growth of output in economy A ( $Y_A$ ) is 4.30 per cent per year, but only 1.44 per cent in economy B ( $Y_B$ ). The difference between the two rates is the contribution of advances of technology (2.86 per cent per year). In their pioneer calculations of the growth of total factor productivity ("the residual") in economy A, Kendrick and Solow arrive at the figure of 1.91 per cent per year. And shortly thereafter Griliches-Jorgenson announce that the K-S study suffers from "errors in measurement" and that the corrected rate of growth of total factor productivity is really only .97 per cent per year. The social rates of return to investment in research and tangible capital are .3645 and .0095, respectively, so that a considerable increase in the growth rate could be achieved by switching production workers into the research laboratories.<sup>24</sup>

Line E describes the components of the G-J corrections. First, the K-S index of output, aggregated by adding together quantities at constant prices, is replaced by a Divisia index of consumption and investment goods output. There are no corresponding errors of aggregation of labor and capital input, since in (58) and (59) above K-S and G-J both calculate Divisia indexes of total input. After the error in output aggregation is corrected, growth in total inputs explains 52.4 per cent of the growth in total output. Next, the K-S input-cost price index for structures, which does not adjust for

improvements in labor productivity in the construction industry, is replaced by a true price index. With this error corrected, input growth explains 58.4 per cent of output growth. Next, the measurement of the stock of equipment in terms of base-year cost is replaced by a measure of effective equipment services (J). This is equivalent to G-J's replacement of the official producers' durables price index by the price index for consumers' durables, and their adjustment for the secular improvement in the utilization of equipment.<sup>25</sup> This third adjustment on line E also includes a switch to the use of service prices as weights for the aggregation of structures and equipment. After these corrections, input growth explains 64.8 per cent of output growth in economy A. Finally, the K-S measure of man-hour labor input is replaced by G-J's estimate of effective labor input, in which different educational categories of labor are aggregated, using relative wages as weights. With this final correction completed, input growth explains 77.5 per cent of output growth. Notice that the G-J corrections do not lead to a conclusion that the growth in total factor productivity has been zero. This occurs, as we shall see below, only if all advances in technology are embodied in new equipment.

Section F of Table 1 analyzes the sources of discrepancies between the K-S and G-J measures of growth in total factor productivity and

the true contribution of advances in knowledge to the growth of economy A. First, the stock of capital (measured at base-year cost) in economy A grows much faster than in economy B, due to the faster rate of output growth in economy A and the proportional saving assumption (Line F.2.a of Table 1). Thus, even if there had been no embodied technical change, disembodied change would have indirectly caused an increase in the rate of growth of the capital stock, and both the K-S and G-J techniques would exaggerate the growth of capital which would have occurred in the absence of any advances in technology (i.e., in economy B). For this reason alone, calculations of the growth in total factor productivity may be unreliable guides to the importance of advances in technology. The G-J discrepancy in line F.2.a is larger than that of K-S because the G-J effective capital series grows faster than the K-S capital stock series measured at base-year cost. In line F.2.b. the K-S discrepancy is reduced by the use of an erroneous input-cost price index for structures, which reduces the rate of growth of their capital measure.

Next, in line F.2.c., both K-S and G-J exaggerate the contribution of capital to economic growth through the use of an oversized weight on capital based on the share of capital compensation in current-dollar output. Since in this model research workers are exploited, part of the reward to capital represents the contribution of research to

output growth. The G-J error is larger despite their use of the same capital share as K-S, because that capital share is applied to a more rapidly growing capital series. Line F.2.d. shows the effect of the failure of K-S to adjust for the contribution of education to economic growth, and the effect of the overcorrection by G-J. The K-S underestimate of the growth rate of labor input reduces the discrepancy between C/C and their calculation of the "residual," so that a correct measure of labor input by K-S would reduce their residual to only 1.57.<sup>26</sup>

As shown in line F.2.e., another discrepancy is due to the use by G-J and K-S of weights which exaggerate the contribution of production workers to output. As we shall see, this discrepancy is eliminated when we assume increasing returns to all factors and raise the elasticity of output with respect to production workers. A final source of discrepancy for K-S is the underestimation of the rate of growth of output, causing an underestimate of the residual relative to the contribution of advances in knowledge.

Line F.4 separates C/C into components showing the routes by which advances in technology affect the growth rate of economy A relative to that of economy B. The direct impact of disembodied technical change is an improvement in the growth rate of 2.61 per cent, of which 1.77 occurs through the growth rate of consumption and .84 through the growth rate of effective investment.<sup>27</sup> The

indirect impact of research through the rate of growth of capital is .74 per cent, of which .28 per cent represents the contribution of embodied technical change to the growth of consumption, and the remainder is due to the over-all impact of faster output growth on capital growth through the proportional saving rate. In fact, the stock of capital measured in base-year cost (i.e., excluding embodiment effects) grows 40 per cent faster in economy A than economy B. The influence of research on the supply of production workers serves to reduce the growth rate. Since the portion of the labor force engaged in research in economy A is steadily rising, the rate of growth of production workers in economy A is slower than in economy B, where all research workers do production work.

In short, the simulation results confirm the analysis of section II above. Both K-S and G-J underestimate the contribution of advances in knowledge to economic growth. The underestimate by G-J is larger, both because they count the effects of embodied technical change as part of the growth of input and because they exaggerate the contribution of education to growth. But even if these two "corrections" in the G-J procedure were to be omitted, the calculated increase in total factor productivity would still be only 1.32 per cent, less than half of the true contribution of advances in knowledge. This, for instance, would be the K-S measure of the residual if K-S (as is

likely) were to agree with G-J on the use of correct structures deflators, on Divisia indexes for output and input aggregation, and on a "correct" adjustment for education. We can call this 1.32 per cent figure the "compromise residual," and it is striking that it explains so little of the true contribution of advances in knowledge.

Other interesting features of the first experiment are not shown in Table 1. Over the fifteen time periods of the simulation, the relative price of investment rises by 55 per cent, due to the more rapid pace of productivity change in the consumption than in the investment sector, combined with the fact that wage rates in the two sectors are the same. The wage rate increases by 64 per cent over this interval, and since the wage rate is used by K-S to measure the price of structures, they overestimate the growth of the latter by 9 per cent. Due to the relatively greater burden of replacement investment in equipment and the rising importance of replacement, the ratio of gross investment in structures to equipment ( $I_t^S/I_t^E$ ) declines over the simulation period from 64 to 53 per cent. The ratio of gross investment in structures to the gross production of equipment services ( $I_t^S/Z_t$ ) declines even more, from 54 per cent to 35 per cent.

Although the assumed saving rate in current prices is 20 per cent, the actual share of gross constant-price investment in output



(when investment is measured at base-year cost) is only 16 per cent in the final period, because the rising relative price of investment goods cuts down on the investment goods which can be purchased with a given sacrifice of consumption goods. But the share of effective investment in output (defined to include consumption plus effective investment) is 23 per cent, due to the contribution of technical change to increasing the equipment services obtainable from a given amount of base-year-cost investment.

2. Increasing Returns. As shown in Table 2, the main points of the first simulation are confirmed if we introduce increasing returns in all factors, which can be accomplished if the technological elasticities of output with respect to production labor and effective capital are raised in proportion by enough to yield constant returns to production labor and effective capital alone. But the magnitudes of the discrepancies between  $\dot{C}/C$  and the two measures of growth in total factor productivity are reduced considerably, enough so that the K-S residual is actually slightly larger than the true contribution of advances in knowledge. The G-J residual is 58 per cent of  $\dot{C}/C$ , as opposed to only 34 per cent in the constant returns case.

The reasons for the main differences between Tables 1 and 2 may be briefly noted. The increase in the  $\alpha_2, \alpha_3, \beta_2,$  and  $\gamma_2$  parameters

A. Types of Technical Progress: Disembodied in Consumption Sector  
Disembodied in Investment Sector  
Embodied

B. Returns to scale in all factors: Increasing

C. Parameter values of the technology:

$$\alpha_1 = .20 \quad \alpha_2 = .75 \quad \alpha_3 = .25 \quad \beta_1 = .20 \quad \beta_2 = 1.00 \quad \gamma_1 = .20 \quad \gamma_2 = 1.00$$

D. Summary of results (percentage growth rates):

$$\frac{\dot{Y}_A}{Y_A} = 5.29 \quad \frac{\dot{Y}_B}{Y_B} = 2.31 \quad \frac{\dot{C}}{C} = 2.98 \quad \frac{\dot{C}_{K-S}}{C_{K-S}} = 3.01 \quad \frac{\dot{C}_{G-J}}{C_{G-J}} = 1.73 \quad \rho_8^R = .351 \quad \rho_8^K = .147$$

E. Components of G-J Correction of K-S:

	Output Aggregation	Price of Structures	Effective Equipment	Education
1. Percentage points subtracted from residual	-.02	.20	.55	.55
2. Per cent of output growth explained by input growth after correction	42.7	46.5	56.9	67.2

F. Explanation of discrepancies between calculations of residual and true contribution of advances in knowledge:

(percentage points)	<u>True</u>	<u>K-S</u>	<u>G-J</u>
1. Calculated contribution	2.98	3.01	1.73
2. Sources of Discrepancies:			
a. Growth capital input		.31	.78
b. Price of structures		-.17	...
c. Capital share		.22	.33
d. Growth labor input		-.37	.22
e. Labor share		-.04	-.08
f. Growth of output		.02	...
3. True contribution	2.98	2.98	2.98
4. Addendum: Sources of true contribution of advances in knowledge			
a. Direct impact of research	2.76		
b. Indirect impact on capital	.90		
c. Indirect impact on prod'n labor	-.68		

raises the growth rate of output in economy A, but economy B, where the output growth rate had been held down by diminishing returns in labor and capital, benefits relatively more; thus the contribution of advances in knowledge, the difference in the growth rates of the two economies, is only slightly larger here than in Table 1. Since the increased growth rate of economy A has been achieved with no increase in the growth rate of labor input and only a moderate increase in the growth rate of capital, both the K-S and G-J "residuals," i.e., output growth minus weighted input growth, are raised considerably. Another result in line D is the reduction in the social rate of return to research (since shifting a unit of labor out of production work now involves more of a sacrifice, given the unchanged elasticity of output with respect to research) and a substantial increase in the social rate of return to tangible capital (which again makes sense, since output is now more responsive to the efforts of production workers in the investment sector).

The difference between the K-S and G-J residuals is a bit larger than before--1.28 percentage points in the increasing returns case as opposed to .94 with constant returns. The G-J correction which converts capital  $\underline{K}$  into effective capital  $\underline{J}$  is more important here, since the increased  $\gamma_2$  coefficient raises the magnitude of the embodiment effect. Another reason for the increase in the difference between

K-S and G-J is the slightly increased share of labor, which raises the importance of the G-J correction for education.

In Section F we notice first that the K-S overstatement of the contribution of capital input is less serious now, mainly because of the faster growth of capital in economy B. In line F.2.c. both overestimates of the capital share are less serious. This occurs because the higher elasticity with respect to labor raises the marginal product of production workers in the consumption sector, hence the wage of research workers relative to the marginal product of research, and thus reduces the degree of exploitation of research workers. The value shares understate the true share of labor, as shown in line F.2.e. An important change in the last section is in line F.4.c., where the negative impact of research on the contribution of production workers is larger, since a larger sacrifice is now involved in switching a worker from production to research employment.

Although the K-S residual overestimates the contribution of knowledge, this is not true after corrections are made for the erroneous price of structures, for the contribution of education, and for errors in output aggregation. This "compromise residual" is 2.48 per cent, or 83 per cent of the true contribution of advances in knowledge. This may be compared to a "compromise residual" in the initial constant-returns trial which is only 46 per cent of the

contribution of advances in knowledge. Thus the degree of returns to scale in the economy is very important in assessing the actual deviation of the "compromise residual" from the true contribution of technical change, but is not decisive in determining the direction of that deviation (unless there are significantly increasing returns in capital and production labor alone).

#### B. Disembodied and Embodied Change Introduced Separately

Tables 3 and 4 present results for the case of technological advance which takes place only in the form of disembodied improvements in the consumption sector. This brings the K-S and G-J residuals much closer together, with the only important differences being due to the G-J corrections for errors in output aggregation and education. K-S would probably agree with these corrections, at least after the educational correction has been adjusted for ability differences, and a "compromise residual" can be calculated. As in Tables 1 and 2, this only explains a fraction of the true contribution of advances in technology--48 per cent in Table 3 and 81 per cent in Table 4. Incidentally, we are reminded in Tables 3 and 4 that there is nothing about the G-J measurement techniques which forces the contribution of advances in knowledge to be zero by definition, as Denison appears to have implied (see footnote 7 above).

A. Types of Technical Progress: Disembodied in Consumption Sector Only

B. Returns to scale in all factors: Constant

C. Parameter values of the technology:

$$\alpha_1 = .20 \quad \alpha_2 = .60 \quad \alpha_3 = .20 \quad \beta_1 = .00 \quad \beta_2 = 1.00 \quad \gamma_1 = .00 \quad \gamma_2 = 1.00$$

D. Summary of results (percentage growth rates):

$$\frac{\dot{Y}_A}{Y_A} = 3.52 \quad \frac{\dot{Y}_B}{Y_B} = 1.76 \quad \frac{\dot{C}}{C} = 1.76 \quad \frac{\dot{C}_{K-S}}{C_{K-S}} = 1.00 \quad \frac{\dot{C}_{G-J}}{C_{G-J}} = .64 \quad \rho_8^R = .574 \quad \rho_8^K = .040$$

E. Components of G-J Correction of K-S:

	Output Aggregation	Price of Structures	Effective Equipment	Education
1. Percentage points subtracted from residual	-.13	...	-.05	.54
2. Per cent of output growth explained by input growth after correction	67.9	67.9	66.5	81.8

F. Explanation of discrepancies between calculations of residual and true contribution of advances in knowledge:

(percentage points)	<u>True</u>	<u>K-S</u>	<u>G-J</u>
1. Calculated contribution	1.76	1.00	.64
2. Sources of Discrepancies:			
a. Growth capital input		.17	.15
b. Price of structures		...	...
c. Capital share		.81	.78
d. Growth labor input		-.34	.21
e. Labor share		-.01	-.02
f. Growth of output		.13	...
3. True contribution	1.76	1.76	1.76
4. Addendum: Sources of true contribution of advances in knowledge			
a. Direct impact of research	1.72		
b. Indirect impact on capital	.17		
c. Indirect impact on prod'n labor	-.13		

A. Types of Technical Progress: Disembodied in Consumption Sector

B. Returns to scale in all factors: Increasing

C. Parameter values of the technology:

$$\alpha_1 = .20 \quad \alpha_2 = .75 \quad \alpha_3 = .25 \quad \beta_1 = .00 \quad \beta_2 = 1.00 \quad \gamma_1 = .00 \quad \gamma_2 = 1.00$$

D. Summary of results (percentage growth rates):

$$\frac{\dot{Y}_A}{Y_A} = 4.37 \quad \frac{\dot{Y}_B}{Y_B} = 2.31 \quad \frac{\dot{C}}{C} = 2.06 \quad \frac{\dot{C}_{K-S}}{C_{K-S}} = 2.02 \quad \frac{\dot{C}_{G-J}}{C_{G-J}} = 1.60 \quad \rho_8^R = .505 \quad \rho_8^K = .080$$

E. Components of G-J Correction of K-S:

	Output Aggregation	Price of Structures	Effective Equipment	Education
1. Percentage points subtracted from residual	.09	...	-.04	.55
2. Per cent of output growth explained by input growth after correction	51.7	51.7	50.8	63.4

F. Explanation of discrepancies between calculations of residual and true contribution of advances in knowledge:

(percentage points)	<u>True</u>	<u>K-S</u>	<u>G-J</u>
1. Calculated contribution	2.06	2.02	1.60
2. Sources of Discrepancies:			
a. Growth capital input		.12	.09
b. Price of structures		...	...
c. Capital share		.24	.23
d. Growth labor input		-.37	.22
e. Labor share		-.04	-.08
f. Growth of output		.09	...
3. True contribution	2.06	2.06	2.06
4. Addendum: Sources of true contribution of advances in knowledge			
a. Direct impact of research	2.06		
b. Indirect impact on capital	.13		
c. Indirect impact on prod'n labor	-.13		

Results in Tables 5 and 6 depict the case of embodied quality improvements in equipment, with no disembodied technical change in either the consumption or investment sectors. The results are qualitatively similar to the initial cases considered in Tables 1 and 2, but the growth rates of all the variables in line D are much smaller, since only one-third as much research is being carried on and its effect is dampened by an elasticity of output with respect to effective capital of only .20. Taking aside the G-J exaggeration of the contribution of education, the G-J residual would be .19 percentage points in the constant returns case and .30 in the increasing returns case. The residual in this embodied-only example would be equal to zero but for a peculiarity of the simulation model--in the simulations it is only effective equipment which directly contributes to output, but the G-J measure of effective capital includes both equipment and slower-growing structures. If the model had been designed so that both structures and equipment contributed directly to output,  $Y_A$  would have grown more slowly and the small remaining G-J residual would have been wiped out.

If the K-S residual is corrected for the true contribution of education, the price of structures, and errors of output aggregation, we again have the "compromise residual," which is .45 per cent per year in Table 5 and .63 in Table 6, and thus explains 88 and 112



A. Types of Technical Progress: Embodied Only

B. Returns to scale in all factors: Constant

C. Parameter values of the technology:

$$\alpha_1 = .00 \quad \alpha_2 = .80 \quad \alpha_3 = .20 \quad \beta_1 = .00 \quad \beta_2 = 1.00 \quad \gamma_1 = .20 \quad \gamma_2 = .80$$

D. Summary of results (percentage growth rates):

$$\frac{\dot{Y}_A}{Y_A} = 2.46 \quad \frac{\dot{Y}_B}{Y_B} = 1.95 \quad \frac{\dot{C}}{C} = .51 \quad \frac{\dot{C}_{K-S}}{C_{K-S}} = .58 \quad \frac{\dot{C}_{G-J}}{C_{G-J}} = -.03 \quad \rho_8^R = .0995 \quad \rho_8^K = -.0003$$

E. Components of G-J Correction of K-S:

	Output Aggregation	Price of Structures	Effective Equipment	Education
1. Percentage points subtracted from residual	.20	-.03	.23	.62
2. Per cent of output growth explained by input growth after correction	72.4	72.0	76.2	1.013

F. Explanation of discrepancies between calculations of residual and true contribution of advances in knowledge:

(percentage points)	<u>True</u>	<u>K-S</u>	<u>G-J</u>
1. Calculated contribution	.51	.58	-.03
2. Sources of Discrepancies:			
a. Growth capital input		.05	.26
b. Price of structures		.03	...
c. Capital share		.02	.03
d. Growth labor input		-.38	.22
e. Labor share		.02	.03
f. Growth of output		.20	...
3. True contribution	.51	.51	.51
4. Addendum: Sources of true contribution of advances in knowledge			
a. Direct impact of research	.29		
b. Indirect impact on capital	.39		
c. Indirect impact on prod'n labor	-.17		

A. Types of Technical Progress: Embodied Only

B. Returns to scale in all factors: Increasing

C. Parameter values of the technology:

$$\alpha_1 = .00 \quad \alpha_2 = .80 \quad \alpha_3 = .20 \quad \beta_1 = .00 \quad \beta_2 = 1.00 \quad \gamma_1 = .20 \quad \gamma_2 = 1.00$$

D. Summary of results (percentage growth rates):

$$\frac{\dot{Y}_A}{Y_A} = 2.70 \quad \frac{\dot{Y}_B}{Y_B} = 2.14 \quad \frac{\dot{C}}{C} = .56 \quad \frac{\dot{C}_{K-S}}{C_{K-S}} = .80 \quad \frac{\dot{C}_{G-J}}{C_{G-J}} = .07 \quad \rho_8^R = .0852 \quad \rho_8^K = .0536$$

E. Components of G-J Correction of K-S:

	Output Aggregation	Price of Structures	Effective Equipment	Education
1. Percentage points subtracted from residual	-.05	.00	.16	.62
2. Per cent of output growth explained by input growth after correction	66.4	65.7	74.6	97.5

F. Explanation of discrepancies between calculations of residual and true contribution of advances in knowledge:

(percentage points)	<u>True</u>	<u>K-S</u>	<u>G-J</u>
1. Calculated contribution	.56	.80	.07
2. Sources of Discrepancies:			
a. Growth capital input		-.13	.19
b. Price of structures		...	...
c. Capital share		.05	.07
d. Growth labor input		-.38	.23
e. Labor share		.00	.00
f. Growth of output		.23	...
3. True contribution	<u>.56</u>	<u>.56</u>	<u>.56</u>
4. Addendum: Sources of true contribution of advances in knowledge			
a. Direct impact of research	.44		
b. Indirect impact on capital	.33		
c. Indirect impact on prod'n labor	.21		

per cent of the true  $C/C$ . The percentage of explanation is higher in the pure embodied case, since the compromise capital index for economy A does not grow markedly faster than the growth of capital in economy B, as occurs in the presence of disembodied technical change.

### C. Other Examples

Table 7 summarizes the results of the previous tables and several additional trials. Two lines of results are given for each trial, one for constant returns and a second for increasing returns (where in each case the Cobb-Douglas exponents on capital and production labor are raised in proportion by enough to yield constant returns in capital and production labor alone).

Trial 1 duplicates the experiment presented in Tables 1 and 2. Trial 2 is the same with higher research elasticities in the investment sector, which appears further to widen the gap between  $C/C$  and the "residuals." The "compromise residual" explains only 48 and 76 per cent of  $C/C$  in the constant and increasing returns cases, respectively. In general, the higher the coefficients on disembodied change in either sector, the less accurate the "compromise residual." This is confirmed in Trial 3, in which the parameters on capital and disembodied research in the consumption sector are raised, resulting

TABLE 7

## Summary Data for Experiments

Trial	Type of Technical Change			Parameter Values				Returns to Scale		$\frac{Y_A}{Y_A}$	$\frac{C}{C}$	$\frac{C_{K-S}}{C_{K-S}}$	$\frac{C_{G-J}}{C_{G-J}}$	CR	$\rho_R$	$\rho_K$
	DC	DI	E	$\alpha_1$	$\alpha_3$	$\beta_1$	$\gamma_1$	Inc.	Dec.	$\frac{Y_A}{Y_A}$	$\frac{C}{C}$	$\frac{C_{K-S}}{C_{K-S}}$	$\frac{C_{G-J}}{C_{G-J}}$		$\rho_R$	$\rho_K$
1.	x	x	x	.20	.20	.20	.20		x	4.30	2.86	1.91	0.97	1.32	.365	.010
								x		5.29	2.98	3.01	1.73	2.48	.351	.147
2.	x	x	x	.20	.20	.30	.30		x	4.92	3.74	2.15	1.02	1.78	.480	-.044
								x		6.13	3.79	3.33	1.57	2.87	.402	.323
3.	x	x	x	.30	.30	.30	.30		x	6.52	5.45	2.38	0.94	1.80	.921	.065
								x		9.67	6.60	5.46	2.88	4.67	.808	.701
4.	x	x	x	.10	.30	.20	.20		x	4.11	2.36	1.64	0.66	1.28	.267	.085
								x		4.94	2.29	2.33	1.02	1.94	.260	.370
5.	x	x	x	.30	.10	.20	.20		x	4.48	3.42	2.00	1.03	1.65	.464	-.067
								x		5.46	3.52	3.45	2.18	3.02	.360	.127
6.	x			.20	.20	.00	.00		x	3.52	1.76	1.00	0.64	0.85	.574	.040
								x		4.37	2.06	2.02	1.60	1.43	.505	.080
7.		x		.00	.20	.20	.00		x	2.60	0.64	0.79	0.01	0.22	.176	-.033
								x		2.77	0.61	0.75	-.03	0.18	.146	.027
8.		x		.00	.20	.00	.20		x	2.46	0.51	0.58	-.03	0.45	.100	.000
								x		2.70	0.56	0.80	0.07	0.63	.085	.054

TABLE 7 (Continued)

Trial	Type of Technical Change			Parameter Values				Returns to Scale		$\frac{\dot{Y}_A}{Y_A}$	$\frac{\dot{C}}{C}$	$\frac{\dot{C}_{K-S}}{C_{K-S}}$	$\frac{\dot{C}_{G-J}}{C_{G-J}}$	CR	$\rho_8^R$	$\rho_8^K$
	DC	DI	E	$\alpha_1$	$\alpha_3$	$\beta_1$	$\gamma_1$	Inc.	Dec.							
9.			x	.00	.20	.00	.30		x	2.78	1.01	0.78	0.15	0.75	.200	-.017
								x		3.21	1.05	1.08	0.34	1.16	.156	.068
10.		x	x	.00	.20	.20	.20		x	2.58	0.85	0.91	0.04	0.49	.100	-.015
								x		2.87	0.71	0.97	0.01	0.61	.092	.158

New Abbreviations: CR: The "compromise residual."  
 DC: Disembodied technical change in consumption sector.  
 DI: Disembodied technical change in investment sector.  
 E: Embodied technical change.

in a "compromise residual" which only explains 33 and 70 per cent of  $C/C$ . Trial 4 returns to the  $\beta_1$  and  $\gamma_1$  parameters of the initial trial but lowers the disembodied consumption research parameter ( $\alpha_1$ ) and raises the capital parameter ( $\alpha_3$ ). The result is a narrowing of the gap between  $C/C$  and all versions of the residual; the "compromise residual" explains 54 and 85 per cent of  $C/C$ . Trial 5 reverses the change in the  $\alpha_1$  and  $\alpha_3$  parameters, with a slight alteration of the explanation of  $C/C$  by the compromise residual to 48 and 86 per cent. In general, the gap between the social rates of return to investment in research and tangible capital widens in favor of research when  $\alpha_1$  is increased and narrows when  $\alpha_3$  is increased. The increase in  $\beta_1$  and  $\gamma_1$  (Trial 2 compared to Trial 1) appears to raise the gap in the constant returns case and reduce it with increasing returns. Trial 6 corresponds to Tables 3 and 4 above. In Trial 7 there is disembodied technical change only in the investment sector, with results very similar to the embodied-only case in Trial 8, except that the "compromise residual" explains a considerably smaller fraction of  $C/C$ . In Trial 9  $\gamma_1$  in the embodied-only case is raised over its value in Trial 8, with a "compromise" residual which continues to explain most or all of  $C/C$ , depending on the degree of returns to scale. Finally, in Trial 10, there is another variant which differs from the first trial by omitting disembodied change in the

consumption sector, but in which the explanation of C/C by the "compromise residual" is about the same--58 and 86 per cent.

An interesting feature of Table 7 is the existence of several Trials, 7, 8, and 10, in which the Griliches-Jorgenson version of the residual is virtually zero, but in which the social rate of return to research exceeds the social rate of return to investment in tangible capital (an exception is the increasing returns version of Trial 10). As pointed out above in Section II, this contradicts the G-J statement that a small value of their residual implies virtual equality of the two social rates of return.

#### D. Effect of a Higher Saving Rate

Denison has argued on several occasions [4][5] that a substantial boost in the proportion of fixed investment in national income would yield inconsequential increases in the growth rate of output. In his initial study of the U. S., for instance, Denison calculated that "A change of 0.1 points in the growth rate over perhaps 60 years would be achieved by continuing additional net investment equal to... 0.75 per cent [of the national income] if none of the additional investment were devoted to non-farm housing" [4, p.277]. The present simulations lead to smaller effect, as illustrated in Table 8. With the parameter values of the initial trial, as shown on line 1, a five per cent increase in the ratio of gross investment to gross

TABLE 8

Effect of Increase in  
Saving Rate from 20 to 25 Per Cent

<u>Case</u>	<u>Type of Technical Change</u>			<u><math>\alpha_1</math></u>	<u><math>\alpha_3</math></u>	<u><math>\beta_1</math></u>	<u><math>\gamma_1</math></u>	<u>Returns to Scale</u>		<u>Increase in <math>Y_A/Y_A</math></u>	
	<u>DC</u>	<u>DI</u>	<u>E</u>					<u>Inc.</u>	<u>Dec.</u>	<u>Points</u>	<u>%</u>
1.	x	x	x	.20	.20	.20	.20		x	.11	2.6
								x		.22	4.2
2.	x			.20	.20	.00	.00		x	.09	2.6
3.			x	.00	.20	.00	.20		x	.10	4.1



national product (both measured in current prices) yields an increase of only .11 points in the 15-period average annual growth rate in the constant returns case (from 4.30 to 4.41 per cent), and only .22 with increasing returns. Thus a 25 per cent boost in the saving rate produces only a 2.6 (or 4.2) per cent increase in the growth rate.

Further, the results confirm Denison's argument that the yield of extra saving is little affected by the existence of embodied technical change. For instance, the assumed increase in the saving rate by one-quarter yields a 2.6 per cent (.09 points) increase in the growth rate in the trial which assumes disembodied technical change in the consumption sector, and an increase of 4.1 per cent (.10 points) in the embodied-only trial. In fact, these calculations may overstate the effect of higher saving on the growth rate, since no allowance is made for Denison's point [5, p. 92] that new capital goods are heterogeneous; some new pieces of equipment, which are vastly superior in quality to older vintages, will be installed even at low rates of saving and investment. Marginal increments in the saving rate, however, will be used to purchase lower-priority items which are less superior in quality to earlier vintages and which will thus yield smaller increments in the growth rate than suggested in Table 8.<sup>28</sup>

## VI. Conclusion: Summary and Implications

A. Summary

This paper has demonstrated that total factor productivity or "residual" indexes, whether calculated with or without correction for the "errors" discovered by Griliches and Jorgenson, are not reliable estimates of the contribution of technological change to economic growth. In a theoretical model and in computer simulations, the true contribution of advances in technology is, in most cases, greater than indexes of total factor productivity as calculated by Kendrick-Solow and Griliches-Jorgenson. In all cases the full list of corrections for "errors in measurement" proposed in the G-J papers makes the G-J "residual" smaller than that of K-S and hence a more inaccurate estimate of the contribution of technological advance.

Of the numerous G-J corrections, Kendrick and Solow might agree on the use of accurate price indexes for structures, Divisia indexes for the growth of input and output, and some adjustment of labor input for education--although not as much of an adjustment as made by G-J. An index of total factor productivity adjusted for these corrections can be called a "compromise residual." But Kendrick and Solow would not go beyond this and approve the G-J substitution of measures of effective capital (J) for the base-year-cost stock of capital (K),

since this measure of effective input disguises the role of advances in technology in achieving the increase in J relative to K, i.e., it rules out embodied technical change by definition. And, as we saw above in Tables 5 and 6, the G-J residual (after their erroneous educational adjustment is corrected) is zero in the case of embodied technical change. It is this feature which makes the G-J residual in all of the above simulations further from the true contribution of technological progress than the "compromise residual."

A more novel conclusion is that even the "compromise residual" almost always underestimates the contribution of advances of knowledge to economic growth. The magnitude of this discrepancy varies over a considerable range in the computer simulations, depending on assumptions made in the model regarding the underlying production coefficients and payment arrangements for research workers. The discrepancy depends mainly on:

1. The Degree of Returns to Scale. Simulations with constant returns to scale in production labor, capital, and research, produced larger discrepancies than the assumption of increasing returns in these three factors and constant returns in production labor and capital alone. This is natural, since constant returns to labor and capital is an underlying assumption of the K-S and G-J techniques for calculating indexes of total factor input; if the true elasticity of output with respect to labor and capital is actually less than

one, their weights on the growth of labor and capital (which add to one) are too high, their measures of total factor input grow too rapidly, and their residual is too small. Of course this source of the discrepancy between the "compromise residual" and the true contribution of advances in technology is eliminated or reversed if the true elasticity of output with respect to production labor and capital is sufficiently greater than one. But it seems unlikely that an economy with absolutely no advances in knowledge could avoid diminishing returns to production labor and capital. It could endlessly duplicate plants operating with 1918 or 1818 technology, but how could it overcome problems of transport, organization, and distribution when no one takes time out to think about them? In fact, this is the fate of economy B in Tables 1, 3, and 5 above, in which there is no research, significantly decreasing returns in labor and capital, and a large discrepancy between the "compromise residual" and the true contribution of advances in knowledge.

2. The True Growth Rate of Capital Input. Whether returns to scale are constant or increasing in the three factors, disembodied technical change increases the rate of growth of economy A relative to economy B, and this, due to the proportional saving assumption, raises the growth rate of capital in economy A relative to B, even if there is no embodied technical change. Since the K-S and G-J

indexes of total factor productivity are based on the observed growth of capital in economy A, they overstate the growth of capital which would have occurred without technical change and consequently understate the contribution of advances of technology to economic growth. There is no way this error can be avoided, so that even the "compromise residual" will not accurately identify the contribution of technical change unless in the real world there is no disembodied change at all.

3. Research Compensation in the Capital and Labor Shares. In the computer simulations research workers are paid the same wage as production workers of the same educational attainment, so that they are exploited if the social rate of return to research is positive. For this reason the observed capital share in economy A overstates the elasticity of output with respect to capital, and the K-S and G-J measures of the contribution of the growth of capital to output growth are overstated due to the application of oversized capital shares. This source of error in the approximation of  $C/C$  by the K-S and G-J residuals would not be eliminated if capital were to be paid its marginal product; in this case it would be the oversized value share of labor which would disguise the contribution of research workers, and the contribution of the growth of labor would be overstated. Only with constant returns to scale in production

labor and capital, and increasing returns to all factors, will there be no error when the extra research compensation is included in the shares of the conventional factors.<sup>29</sup>

#### B. Implications

Griliches and Jorgenson claim that "the equality between private and social rates of return is a testable hypothesis within our framework," i.e., that a finding by G-J of no change in total factor productivity would imply that the "contribution of investment to economic growth is....compensated by the private returns to investment"[11,p.274]. Presumably a positive G-J residual would suggest that social returns exceed private returns. Yet in our simulations above there are numerous trials in which the G-J residual is positive, yet private returns exceed the contribution of investment to growth, due to the exploitation of research workers. Without exploitation the social and private returns are equal, but the G-J residuals would be raised due to a smaller weight on the growth of capital input. Thus there is no correspondence between the G-J residual and the difference between the social and private rates of return to investment, since research-using disembodied technical change creates a positive G-J residual without causing the social rate of return to diverge from the private. This point reminds us that previous writers on total

factor productivity, including G-J, have been led to misleading conclusions through excessive concentration on "costless" technical change and insufficient attention to cost-increasing advances in technology.

In the simulations above the G-J residual is much smaller in trials in which all technical change is embodied than those in which part or all of technological advance is disembodied. Does the small G-J residual in [11] for U. S. growth from 1945 to 1965 therefore imply that in reality most U. S. technological advance has been of the embodied type? As yet we do not have sufficient information to answer this question, since the G-J corrections for "errors" in the price of equipment and utilization of capital are notoriously unreliable, as pointed out by Denison [2, pp. 76-78]:

Whether the [equipment] deflator on balance can be assumed to have an upward bias rather than random error depends on the criterion adopted for judging appropriate behavior. I think there is no such presumption if the criterion is the same as for other price indexes, including those for consumers' durables....

Power-driven machinery in manufacturing, to which the [utilization] data refer, is so small a component of total capital input that an increase in the hours it is used would have only a minor effect on the growth rate...[G-J] assume, with no attempt at justification, that the average hours worked by inventories, by structures, and by all producers' durables, including such components as office furniture and restaurant equipment, increased in all industries in proportion to the increase in hours worked by manufacturing machinery driven by electric motors.

There is a noticeable assymetry in the models outlined above in Sections II and IV, since, following the U. S national accounts, investment in tangible capital is considered a part of output and investment in research is excluded from output. If output were redefined to include investment in research and if "investment in scientific research and development could be....cumulated into stocks" (as G-J suggest on p. 275 of [11]) and included in total factor input, changes in total factor productivity would be eliminated in the above models, which do not allow for any shifts in the consumption or investment production functions. But this would be an unrewarding effort for students of economic growth, since such a redefinition would further disguise the true contribution of advances in knowledge to economic growth. No useful information about growth could be gained from such a "broader accounting framework." We could not discover, for instance, how much growth had been due to research, for we would have no way of estimating the contribution of research inputs to growth without assuming in advance that the private earnings of research workers are equal to their social marginal product. Yet the social returns to research are one of the elements which calculations of the "residual" are designed to reveal. This difficulty is in addition to the insuperable problem of measuring the proportion of the labor force which is really engaged in



technology-advancing activities. We would include those formally designated as research and development employees, to be sure, but how many managers would we include, and what fraction of the time of foremen and innovative production workers?

A full evaluation of previous research in the light of our simulation studies is beyond the scope of this paper. But it is interesting to reflect that the work of Denison [4][6], which basically follows the K-S techniques but corrects for the contribution of education to growth, is a close approximation of what we have called the "compromise residual." In most of our simulations the "compromise residual" substantially underestimates the contribution of advances in knowledge to economic growth, suggesting Denison's "state of knowledge" source of U. S. and European growth may be too low.<sup>30</sup>

For potential econometric production function studies, this paper introduces a new note of caution into a file drawer already overstuffed with warnings. Investigators attempting to identify the relative importance of input growth and advances in technology as sources of economic growth should study the results of our simulations, which suggest that the growth in the base-year-cost capital stock which has actually occurred in a technologically advancing economy is larger than that which would have occurred without technical change, so that statistically-estimated "residuals" are likely to

underestimate the true contribution of technical change. Also, it is probable that the observed rate of growth of the labor force is greater than the rate of growth of workers actually engaged in production work. Further, as Nelson has pointed out [15, p.597], a cyclical correlation between investment in research and development and investment in tangible capital will produce estimated capital parameters which are biased upward in regressions which exclude a research variable; this in turn would foil any attempt to estimate the true degree of returns to scale, leading us to be skeptical of G-J's statement that "such production functions provide one means of testing the assumptions of constant returns to scale and equality between price ratios and marginal rates of transformation...."[11, p. 276].

Griliches and Jorgenson claim that their results "suggest a new point of departure for econometric studies of production functions at every level of aggregations" [11, p. 276]. Econometricians should view this advice with caution, for literal interpretation would require the replacement of capital stock data by "surrogate" or "effective" capital series. This would prevent econometric studies from identifying the portion of output growth which is explainable by technological advance, since part of the advance in technology would be disguised in the growth of effective capital. G-J might

counter that econometric production function studies based on "error-corrected" data are at least a guide to disembodied change, but this is true only if we can trust the reliability of the G-J techniques for estimating the ratio of effective capital services to the capital stock--and Denison's remarks suggest that this is very doubtful. (Of course, econometricians should continue to heed Jorgenson's warning that embodied and disembodied technical change cannot in principle be distinguished with standard capital stock data).

Where does research go from here? Since studies of economic growth with existing macro data are suspect for so many reasons, increased resources should be devoted to micro studies of technological improvement at the plant and product level. Nothing in this paper criticizes the laudable earlier attempts of Griliches and others to compute quality-corrected price indexes for machinery and other durables. While we have warned against the use of input indexes computed from such quality-corrected data in studies which attempt to determine the importance of technological advance by residual-type methods, quality-corrected data are clearly desirable for measuring improvements in welfare and the true rate of inflation. And, in the field of human capital, the task of separating the relative contributions of education, ability, experience, and environmental differences to wage differentials has only begun.

## REFERENCES

1. Abramovitz, M., "Resource and Output Trends in the United States Since 1870," American Economic Review, 46, No. 2 (May 1956), 5-23.
2. Denison, E. F., "Discussion" (of [9]), American Economic Review, 56, No. 2 (May 1966), 76-8.
3. \_\_\_\_\_, "Measuring the Contribution of Education (and the Residual) to Economic Growth," in Organisation for Economic Co-operation and Development, The Residual Factor and Economic Growth, Paris, 1965, pp. 13-100.
4. \_\_\_\_\_, The Sources of Economic Growth in the United States and the Alternatives Before Us, Supplementary Paper No. 13, New York, Committee for Economic Development, 1962.
5. \_\_\_\_\_, "The Unimportance of the Embodied Question," American Economic Review, 54, No. 1 (March 1964), 90-4.
6. \_\_\_\_\_, Why Growth Rates Differ: Postwar Experience in Nine Western Countries, Washington, The Brookings Institution, 1967.
7. Domar, E. D., "On the Measurement of Technological Change," Economic Journal, 71, No. 4 (December 1961), 709-29.
8. Fisher, F. M., "Embodied Technical Change and the Existence of an Aggregate Capital Stock," The Review of Economic Studies, 32, No. 4 (October 1965), 263-88.
9. Griliches, Z., and Jorgenson, D., "Sources of Measured Productivity Change: Capital Input," American Economic Review, 56, No. 2 (May 1966), 50-61.
10. Hollander, S., The Sources of Increased Efficiency, Cambridge, The M. I. T. Press, 1965.
11. Jorgenson, D., and Griliches, Z., "The Explanation of Productivity Change," The Review of Economic Studies, 34, No. 3 (July 1967), 249-84.

12. Kendrick, J. W., "Productivity Trends, Capital and Labor," Review of Economics and Statistics, 38, No. 3 (August 1956).
13. Mansfield, E., The Economics of Technological Change, New York, W. W. Norton, 1968.
14. \_\_\_\_\_, "Rates of Return from Industrial Research and Development," American Economic Review, 55, No. 2 (May 1965), 310-22.
15. Nelson, R. R., "Aggregate Production Functions and Medium-Range Growth Projections," American Economic Review, 54, No. 5 (September 1964), 575-606.
16. Nordhaus, W. D., "The Optimal Rate and Direction of Technical Change," in Shell, K., ed., Essays on the Theory of Optimal Economic Growth, Cambridge, M. I. T. Press, 1967.
17. \_\_\_\_\_, "A Theory of Endogenous Technological Change," Ph. D. dissertation, M. I. T., 1967, to be published by the M. I. T. Press.
18. Phelps, E. S., "Models of Technical Progress and the Golden Rule of Research," The Review of Economic Studies, 33, No. 2 (April 1966), 133-46.
19. Schmookler, J., "The Changing Efficiency of the American Economy, 1869-1938," Review of Economics and Statistics, 34, No. 3 (August 1952), 214-231.
20. Schultz, T. W., "The Rate of Return in Allocating Investment Resources to Education," The Journal of Human Resources, 2, No. 3 (Summer 1967), 293-309.
21. Schwartzman, D., "Education and the Quality of Labor, 1929-63," American Economic Review, 68, No. 3 (June 1968), 508-13.
22. Solow, R. M., Capital Theory and the Rate of Return, Chicago, Rand-McNally, 1965.
23. \_\_\_\_\_, "Heterogeneous Capital and Smooth Production Functions," Econometrica, 31, No. 4 (October 1963), 623-46.

24. \_\_\_\_\_, "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, 39, No. 3 (August 1957), 312-20.
25. Thurow, L. C., The Economics of Poverty and Discrimination, to be published by The Brookings Institution.
26. U. S. Department of Commerce, Bureau of the Census, The Statistical History of the United States, Stamford, Fairfield Publishers, 1965.
27. Uzawa, H., "Optimal Technical Change in an Aggregative Model of Economic Growth," International Economic Review, 6, No. 1 (January 1965), 18-31.

## FOOTNOTES

1. In this discussion we adopt Mansfield's definition of technical change as the advance of technology, which is "society's pool of knowledge regarding the industrial arts. It consists of knowledge used by industry regarding the principles of physical and social phenomena ..., knowledge regarding the application of these principles to production ..., and knowledge regarding the day-to-day operations of production" [13, p. 10].

2. These statements are contradicted by the remark, "our conclusion is not that advances in knowledge are negligible" [11, p. 274], but are nowhere retracted or rescinded. In fact, the admission that no conclusion is reached regarding technological change is immediately followed by the statement on the comparability of social returns to research and other kinds of investment, which requires the identification of advances in knowledge (i.e., the returns to research) with changes in total factor productivity.

3. Since all capital-embodied technical change is capital-augmenting, it is legitimate to write the capital aggregate  $J(t)$ . See [8]. For a more precise description of the production process in such an economy, see [22, p. 75].

4. Possible justifications for this assumption, which implies diminishing returns to the "conventional" factors  $K$  and  $L$  alone, have

been enumerated by Nordhaus [17, pp. 172-4]. There are no strong reasons to expect that this assumption is more realistic than the alternative of constant returns in L and K, but it simplifies the exposition. The computer simulations below include experiments assuming both diminishing and constant returns to L and K. One reason to expect at least slightly diminishing returns to L and K is that a doubling of L and K probably requires some improvements in managerial techniques and distribution and handling techniques, and if these technological improvements do not occur, Y may not be able to double in size.

5. The expression for economy B is derived as follows. We differentiate (1) with respect to time on the condition that the marginal product of research workers is the same as that of production workers.

$$(a) \quad \dot{Y}_B = F_L \dot{L} + F_J G_{JK} \dot{K}_B + F_L \dot{S}$$

Now we divide both sides by  $Y_B$ , multiply the three right-hand terms by  $\frac{L}{L}$ ,  $\frac{K}{K}$ ,  $\frac{LS}{LS}$ , respectively, and note that, on the assumption of steady-state exponential growth in S,  $\frac{\dot{R}}{R} = \frac{\dot{S}}{S}$ :

$$(b) \quad \frac{\dot{Y}_B}{Y_B} = \frac{F_L}{Y_B} \frac{L}{L} + \frac{F_J G_{JK} K_B}{Y_B K_B} \frac{K_B}{K_B} + \frac{F_L S}{Y_B} \frac{R}{R}$$

6. This point has been made before: "Surely it is a mistake



to measure the contribution of technological change to economic growth after subtracting the higher incomes that R&D scientists and engineers receive" [15, p. 591]. One might add that this point is valid also for a large portion of managerial salaries, for fewer and less well-paid managers would be necessary if there were no technological change.

7. At this point it is important to clarify a disagreement between G-J and Denison [11, p. 254, fn. 1]. Denison claims:

Since advances in knowledge cannot increase national product without raising the marginal product of one or more factors of production, they, of course, disappear as a source of growth if an increase in a factor's marginal product resulting from the advance of knowledge is counted as an increase in the quantity of factor input.

G-J respond that Denison's interpretation implies the measurement of input growth as the sum of the growth of input prices and input quantities, whereas G-J clearly state in their equation (4) that input growth only includes the growth of input quantities.

In fact, Denison is partly right. Consider the model in the text above. On the assumption that factor shares equal factor elasticities, G-J measure the growth of inputs as:

$$(a) \quad \frac{\dot{X}}{X} = E_{YL} \frac{\dot{L}}{L} + E_{YJ} \frac{\dot{J}}{J}$$

That is, they measure the sum of growth in input quantities. But what is  $\dot{J}/J$ ? Substituting from (4) above:

$$(b) \quad \frac{\dot{X}}{X} = E_{YL} \frac{\dot{L}}{L} + E_{YJ} E_{JK} \frac{\dot{K}}{K} + E_{YJ} E_{JR} \frac{\dot{R}}{R}$$

This last term is what Denison means as "an increase in a factor's marginal product resulting from the advance of knowledge," and G-J indeed include it in their measurement of input, as long as the "factor" to which Denison refers is understood to be K, not J. However, Denison is not correct that advances in knowledge "disappear as a source of growth" through this procedure, as can be seen by subtracting the G-J expression for the growth of inputs (b) from (4) above:

$$(c) \quad \begin{aligned} \frac{\dot{P}}{P} &= \frac{\dot{Y}}{Y} - \frac{\dot{X}}{X} = E_{YL} \frac{\dot{L}}{L} + E_{YK} \frac{\dot{K}}{K} + E_{YR} \frac{\dot{R}}{R} \\ &\quad - E_{YL} \frac{\dot{L}}{L} - E_{YJ} E_{JK} \frac{\dot{K}}{K} - E_{YJ} E_{JR} \frac{\dot{R}}{R} \\ &= \frac{F_R}{Y} \frac{\dot{R}}{R} \end{aligned}$$

This last term is the contribution of research-using disembodied advances of knowledge to output growth, which the G-J procedure does allow them to identify. In short, Denison is right to the extent that advances in knowledge operate in the form of embodied technical change, but not to the extent that they are disembodied.

8. This ignores the Griliches-Jorgenson recalculation of output as a weighted average of the output of consumers' goods and "effective" investment goods. As long as the share of capital exceeds the share of investment goods in output, as in the United States, our algebraic

manipulations can safely ignore this adjustment to output. See [11, p. 259, fn. 1]. Also, we ignore the G-J treatment of education, which is discussed below.

9. Note that our discussion above ignores Denison's adjustment for the secular increase in the number of school days per school year, which is explained by him in detail in [3, p. 28] and which has the effect of doubling the contribution of education to growth. This adjustment has been questioned recently by Schwartzman [21].

10. See [3, pp. 86-100][6, p. 84].

11. Thus the model differs considerably, both in form and in purpose, from the models of Phelps [18], Uzawa [27], and Nordhaus [16], which are designed to calculate the optimal allocation of the labor force between research and non-research work. Here we want the allocation to be non-optimal, as it may well be in the real world due to bottlenecks and long gestation periods in the supply of research workers.

12. The figures on relative compensation were obtained from [6, p. 68].

13. Resources to furnish education are not specified in the model, for they are assumed to be provided from outside the private sector. Specific assumptions on our seven education-ability classes are:

Class (i)	Educational Attainment (1)	Native Ability (2)	Education	Ability	Proportion in	
			Coefficient $G_i$ (3)	Coefficient $H_i$ (4)	Class i at $t = 1$ (5)	Class i at $t = 25$ (6)
1	0-4 grade	0-4 grade	.70	.71	.10	.04
2	5-11 grade	0-4 grade	1.00	.71	.00	.06
3	5-11 grade	5-11 grade	1.00	1.00	.63	.34
4	12-15 grade	5-11 grade	1.24	1.00	.00	.29
5	12-15 grade	12-15 grade	1.24	1.13	.22	.14
6	16+ grade	12-15 grade	1.81	1.13	.00	.08
7	16+ grade	16+ grade	1.81	1.30	.05	.05

Data for columns (5) and (6) from [11, p. 279, Table XI, columns 1 and 8].

14. Thus their ability coefficients  $H_i^{G-J}$  for the seven groups would be, respectively, .71, 1.00, 1.00, 1.13, 1.13, 1.30, 1.30.

15. Recently David Schwartzman [21] has claimed that the earnings statistics used by Denison and G-J (and copied for use here) exaggerate the contribution of education, due to the inclusion of agricultural workers, the unemployed, and those not in the labor force. An offsetting bias may be the failure to allow for differences in experience. Since inexperienced young workers are on the average better educated than older, more experienced workers, the figures on relative compensation by educational attainment of all workers may understate the effect of education with experience held constant. For calculations of the contribution of experience, see [25, Chapter 5].

16. The annual rate of growth of  $u_t$  was determined by the

increase in the proportion of professional, technical, and kindred workers (excluding teachers) plus managers, officials, and proprietors (excluding retail trade proprietors) in the U. S. labor force from 1900 to 1950 [26, p. 75]. The 1900 proportion was .054 and had increased by 1950 to .121.

17. (31) is similar to Mansfield's expression for the stock of research in [13, equation (1)]. This expression has the undesirable property that obsolescence will continue even if new research ceases, which is unrealistic, since obsolescence is caused by the appearance of new ideas.

18. (32) implies that with a constant population, a constant share of research workers in the labor force, and no obsolescence of ideas, the rate of increase of technology would approach zero because of decreasing returns ( $\alpha_1 < 1$ ).

19. See [23] for another model in which investment is measured in units of labor input.

20. And we are allowed to write the capital aggregate  $J_t$ , since all capital-embodied technical change is capital-augmenting. See [8].

21. As Nelson [15, pp. 591-2] points out, the returns to education would be lower if there were no technological change, and thus the growth of effective labor input in economy A would probably be greater than in economy B. This point would cause our procedure to

underestimate the contribution of technological advance to the growth of economy A.

22. In practice, year-to-year growth rates are calculated from (53) and averaged over the period of the simulation.

23. The constant returns assumption here, which refers to capital services (J), thus differs somewhat from the constant returns assumption in the theoretical model of Section II above, which referred to the capital stock (K) uncorrected for improvements in equipment quality. The present assumption is more convenient in the two-sector model of the simulations and does not differ for an instantaneous doubling of L, J, and R, since the increased stock of knowledge would have no time to affect the ratio of J to K.

24. In the calculations of social rates of return the initial switch of a production worker from the consumption sector takes place in period 8.

25. G-J attempt a parallel treatment of labor and capital utilization, implying that the secular improvement in equipment utilization has been caused, like any reduction in the unemployment rate, by an improvement in aggregate demand. But in fact, the main cause of the secular improvement in equipment utilization has probably been technical change, e.g., improvements in machine quality which reduce downtime and allow the stretching of maintenance and overhaul intervals.

Otherwise, why wouldn't manufacturing firms in the 1920's have chosen to invest less and utilize their existing capital more?

26. This calculation assumes a value of .00 for line F.2.d. and .16 for line F.2.e.

27. Although the share of effective investment in total real output is only about one-fifth, the direct impact of research is relatively greater than this implies, since there are twice as many research workers in the investment sector as in the consumption sector.

28. Alterations in the saving rate do not yield important changes in our comparisons of  $C/C$  with the G-J, K-S, and "compromise" residuals. Experiments were run with saving rates of .15, .20, and .25. Smaller saving rates cannot in general be used, since simulations with  $\beta_1$  and  $\gamma_1$  positive required a saving rate of at least .15 to pay the salaries of the research workers in the two laboratories in the investment sector. And even then, with  $\omega = .15$ , there was little of the wage bill left over for production workers in the investment sector, resulting in some cases in negative net investment.

29. More precisely, there will be no error when (a) there are constant returns to capital and labor, and (b) the excess of the contribution of research workers over the marginal product of production workers is distributed between capital and labor in proportion

to the contribution of each to output growth.

30. In addition, Schwartzman [21] presents evidence that Denison's educational adjustment is too high, further raising the probable contribution of advances in knowledge.



