Profit Maximization for Utility Companies in an Oligopolistic Energy Market with Dynamic Prices

Tiansong Cui, Yanzhi Wang, Hadi Goudarzi, Shahin Nazarian, and Massoud Pedram
University of Southern California
Department of Electrical Engineering
Los Angeles CA USA
{*tcui, yanzhiwa, hgoudarz, snazaria, pedram*}*@usc.edu*

Abstract—**Dynamic pricing and demand response are the key elements of the smart grid technologies. Utility companies can incentivize electricity customers to schedule their power hungry tasks during off-peak times of the day whereas demand response manages customers' electricity consumption in response to supply conditions or market prices. The reaction of consumers to dynamic prices creates a feedback system in the smart grid that motivates the utility companies to model the consumers' behavior in the process of determining the price. Letting the consumers select their provider of choice among multiple utility companies, may be modeled as a non-cooperative game. In this paper, we consider the process of determining dynamic electricity prices for electricity based on a modified Bertrand Competition Model of consumer behavior and in view of competition among multiple non-cooperative utility companies in an oligopolistic energy market. The proposed method maximizes the conservative estimate on the profit for each utility company. Results also demonstrate the effectiveness of the oligopolistic electrical market in decreasing the electricity cost to consumers.**

I. INTRODUCTION

There is no doubt that electrical energy is the lifeline of national economy [1]. Electricity is currently provided through an infrastructure, comprised of utility companies, power plants, and transmission lines, which serve millions of electricity customers [2]. Since electricity is generally hard to store and must be used as it is being generated, matching supply to real-time demand has been the usual practice in power networks [3]. This is a challenging problem because power demand depends on exogenous factors and varies dramatically as a function of time of day and seasonal factors [4]. At the same time, the amount of generation, transmission and distribution capacities that utility companies need to provision depends on peak demand rather than the average, and the huge difference between energy consumption levels at peak usage time and off-peak hours has resulted in not only cost inefficiencies and potential brownouts and blackouts, but also environmental pollution due to over provisioning of the Power Grid and the resulting energy waste [5]. For example, the US national load factor is about 55%, and only 10% of generation plants and 25% of distribution facilities are used less than 400 hours per year, i.e., 5% of the time [1].

To shape the demand to reduce the peak and smooth the variation in consumer power demand, dynamic energy pricing method has been used [1]-[4]. Dynamic changes in energy prices provide an incentive for electricity customers to shift their energy consumption from peak-energy-use hours to off-peak hours, thus lower their monthly electric bill. At the same time, by proper shaping of customer demands, utility companies can reduce their capital expenditure by not having to add new power plants to the Grid in order to meet the customers' peak-hour demands. So, dynamic energy pricing can benefit both the consumer and the producer of electricity in an economical way.

Implementing dynamic energy pricing faces many challenges. The most difficult step is how to predict people's reaction to various dynamic energy prices, which calls for accurate behavioral models and practical algorithms. Previous researches have focused on either profit maximization for utility companies [4] or cost minimization for customers [5] and [8]. But for both utility companies and customers, each of them tends to make its decision based on the reaction of the other. The authors in [11] are among the first who combined models of the two sides and concurrently optimized consumer's electrical energy bill and producer's power generation cost. However, the main model in [11] is based on a centralized monopolistic electrical grid, where a single utility company supplies all the power demands of electricity consumers in a local area, and it is the government that puts restriction on price and regulates the profit of the utility company in this monopolistic energy market. As a decentralized "smart grid" is the major trend of electrical power network architecture in the future [1], competition between different utility companies will be increasingly widespread. Moreover, monopoly tends to cause a lot of deadweight losses (also known as excess burden or allocative inefficiency) and other types of economic inefficiencies. Finally, competition is usually encouraged by governments [12]. Considering this fact, an oligopolistic model of dynamic pricing is presented in this paper. Besides creating a feedback system between electricity consumers and utility companies like in [11], the main contribution of this paper is to introduce competitive mechanisms, where several utility companies concurrently make their decisions about dynamic price in order to maximize their expected profit considering customers' reactions and choices.

In this paper we assume that each energy consumer is able to schedule its tasks to minimize the total price he/she pays for each day. Moreover, each energy consumer has the ability to freely select any of the existing utility companies without any cost. Based on this model, we propose a solution for determining the hourly price of the electric energy in each utility company to maximize its total profit considering the cost of energy generation and behavior of energy consumers.

The remainder of this paper is organized as follows. In the next section, we present our models for profit maximization problems of both consumers and utility companies in oligopolistic market. Section IV discusses the solution for each model and section V reports the simulation results. The paper is concluded in Section V.

II. Models For Consumers and Utility Companies

As stated above, our ultimate goal is to solve profit maximization problem for each utility company in oligopolistic market. But in the classical economics problems between sellers and buyers, economists always give suggestions to the sellers based on the reaction of the buyers or vice versa because although the government would like to maximize the total social welfare, we still need to consider sellers and buyers as non-cooperative and always making decisions based on their own best solution [12]. This is also the case for energy users and utility companies.

In this section, we start from task scheduling problems in the first model. Under the given daily price function, we decide when to start each task in order to minimize the total electrical energy bill. In the second model, we assume that each of the utility companies offers its corresponding price function and we act as an energy consumer to make decision on which one to choose. Our main contribution is to present the third model for utility companies in an oligopolistic market. Every company has its energy cost function and they need to decide the price distribution in order to maximize its expected total profit. This time we assume that all utility companies will announce their price at the same time in order to comply with fair competition rules. We also assume that consumers are making their own optimal choices on task scheduling and each utility company will find a good solution based on repeatedly simulating the first two models. For each model, an optimal solution is discussed. A unified electricity bill is used in all the models.

A. *Model for Task Scheduling* [11]

Figure 1 shows an example of a task scheduling solution based on the given electricity price function. The height of the task box in this figure signifies the amount of power each household task consumes while running. Under a non-constant price function, consumers tend to assign their tasks to low-price times.

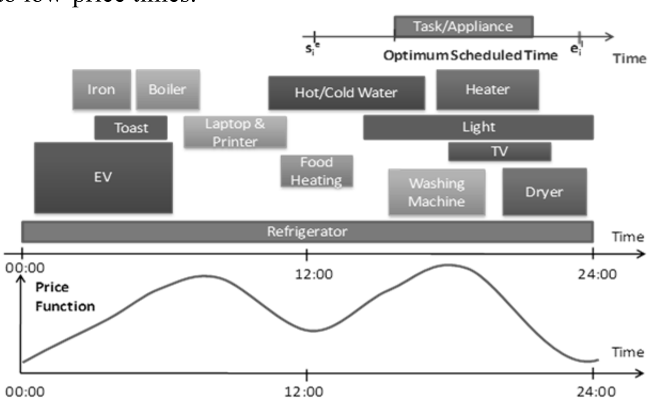


Figure 1. An example of the task scheduling problem.

In this paper, a slotted time model is assumed for all models, i.e., all system cost parameters and constraints as well as scheduling decisions are provided for discrete time intervals of constant length. The scheduling epoch is thus divided into a fixed number of equal-sized time slots (in the experiment, a day is divided into 24 time slots, each with duration of 1 hour). Tasks can be launched only at the beginning of one of these time slots and will be completed at the end of the slots.

We define *Price function, $P[t]$*, as the price of one unit of energy (kWh) at time slot t. In the first model, we assume that $P[t]$ is fixed and pre-announced by the utility company before the start of the day, which means house owners can make their decisions about the whole day but their decisions do not affect the energy price function.

In this model, we also assume that there are a number of tasks in each house that should be executed daily, and each task is independent of other tasks. These tasks are identified by index j. The set of task indexes is denoted by $K=\{1, \ldots, N\}$. For each task j, the earliest start time, $es[j]$, the latest end time, $le[j]$, energy consumption per time slot, $C[j]$, and the duration of task, $Time[j]$, are specified.

To solve the task assigning problem, two additional definitions are needed: *start time,* $S[j]$, which represents the time slot when a task starts and *task operation matrix,* $M[t][j]$, which represents the operating condition of each task j at time slot t. We set $M[t][j]=1$ when at time slot t, task j is operating. Otherwise $M[t][j]=0$.

Using the above definition, the energy consumer's cost minimization problem can be modeled as follows. Given $P[t]$, $C[j]$, and $Time[j]$, $es[j]$, $le[j]$, we are to assign $S[j]$ for each j. The problem is to minimize the total cost

$$Cost = \sum_{t}\sum_{j} P[t]C[j]M[t][j]$$

subject to:

$$S[j] \geq es[j]$$
$$S[j]+Time[j] < le[j]$$

where $M[t][j]$ can be found by the following method:

```
Initialize M[t][j]=0 for all t and j;
for each j {
    for (t=S[j],t<S[j]+Time[j], t++)
    M[t][j]=1;
}
```

B. Model of Consumer Response to Prices

Energy consumers will be the beneficiaries of competition between utility companies as the competition encourages a larger range of choices and the corresponding lower prices. The authors in [14] applied two kinds of competition models in a conventional electrical energy market, but both of them fail in the future architecture of smart grid. First of all, demand response is a key element of the smart grid technologies, which means the usual practice of power networks is matching supply to demand instead of matching demand to supply. For this reason, the Cournot Model, which is based on competition on the amount of output each industry will produce, is not applicable. On the other hand, the Bertrand Competition Model, which assumes consumers always choose the product with the lowest price, also turns out to be oversimplified, because the introduction of dynamic prices makes it hard for the consumers to determine which utility company really offers a better price, and also the customers may never be totally free to switch from one energy supply to the other [12].

In this paper, we use a modification of the Bertrand Competition Model in the supply selection which yields more realistic results. It is generally agreed by economists (cf. [12]) that each energy consumer has a *threshold cost* (*thre*[*i*]), which represents the expected money they are prepared to pay for their electricity bill. The threshold cost may differ from household to household due to the variation in the energy consumers' income level, total electricity consumption, cultural reasons, and other factors. Utility companies can predict each energy consumer's threshold cost by looking at his previous reactions as well as other statistics. It has been conjectured in [15] that an energy consumer will randomly choose one utility company among those who offer a lower price than his threshold cost. We also assume that the task scheduling model in the previous part is used and the *total electricity bill* (*cost_h*[*c*][*i*]) is calculated after all tasks have been scheduled at proper time slots if utility company c is chosen by a certain energy consumer.

There is another situation in which none of the utility companies offer a price lower than threshold. As electrical energy can be viewed as a kind of life's necessity, each energy consumer must choose an energy supplier. This time, we assume that the energy consumer has already compared all utility company prices and finally chooses the one who offers the lowest price.

C. Model for Price Determination in an Oligoplistic Market

As stated before, sellers will always make their decision based on the reaction of buyers. Knowing the task scheduling and supply selection method of electricity consumers, it is possible for utility companies to maximize their profit by selecting price. As the smart meter will be increasing widespread in the future [1], utility companies will be able to predict each energy consumer's above-mentioned task profiles.

In this model, $P[c][t]$ denotes the price function for each utility company c and at time slot t. The price is the utility company's decision and pre-announced to consumers. In addition to that, each utility company has its *energy cost function* (*cost_e*[*c*][*t*]), which is determined by the type of electricity generation (i.e. steam-power station or solar-energy-power station) as well as weather and seasons.

We define an energy consumer *i* is *satisfied* with a certain utility company *c* if it offers an energy price function that leads to a cost no higher than threshold cost of that energy consumer i.e.,

if (*cost_h*[*c*][*i*] ≤ *thre*[*i*])
  sat[*c*][*i*] = 1;
else
  sat[*c*][*i*] = 0;

Then the utility companies' profit can be classified into two categories: 1) profit from *satisfied* consumers, and, 2) profit from *unsatisfied* consumers. For fair competition, we have assumed that all companies announce their price function at the same time, which means the decision of other utility companies are unknown when one company is deciding its own price function. This makes the profit from unsatisfied consumers unpredictable.

But the lower bound of profit from satisfied consumers for each company can be estimated if we assume the worst case that those consumers who are satisfied with the targeted company are also satisfied with all other companies, i.e., all other companies offered no higher price than targeted company. In this situation, those satisfied consumers have a probability of 1/*n* to choose the targeted company, where n is the number of total utility companies.

As discussed in oligopolistic model in [12], non-cooperative sellers will come out with a Nash equilibrium, which means economic actors interacting with one another each choose their best strategy given the strategies the others have chosen. Without knowing other companies' price functions, what each utility company can do is to guarantee a maximal conservatively estimated profit regardless of other companies' decisions, which means to adjust its own price function *P*[*c*][*t*] to maximize

$$\frac{1}{n} * \sum_{i} sat[c][i] \left[\sum_{t} (P[c][t] - cost\_e[c][t]) con[c][i][t] \right]$$

where *con*[*c*][*i*][*t*] is the total energy consumption for energy consumer *i* at time *t* if he chooses company *c*, and the task scheduling model in part A is used for each energy consumer, i.e.:

$$con[c][i][t] = \sum_{j} C[i][j] * M[c][t][i][j]$$

III. Profit Optimization Methods

As can be seen from previous section, a feedback system has been created between consumers and utility companies. We first stand on consumers' side to solve the task scheduling problem as discussed in the first model. This is a relatively simple model and we use a greedy algorithm to find the minimal cost: for *Time*[*j*]=*k*, from the earliest start time to the latest possible start time, we calculate all the values of *P*[*t*]+*P*[*t*+1]+….+*P*[*t*+*k*-1] and find the minimal sum. Then we put this task into these timeslots. Repeat the above steps until all the tasks are arranged. It can simply be proven that the proposed greedy algorithm obtains the global optimum solution.

Profit maximization problem considering fixed tasks for energy consumers is a mixed-integer non-linear programming problem. The integer part of the problem comes from the fact that *sat*[*c*][*i*] is 0 or 1 based on the energy prices throughout a day. The complexity of solving this problem is equal to the complexity of selecting the best subset of the energy consumers and finding energy price to satisfy their total cost threshold to maximize the profit of the utility company. By adding the task scheduling model in part A, this problem becomes a multi-level optimization problem and is even harder to find a practical algorithm leadling to the best result. Considering this, we use simulated annealing (SA) to find a nearly-optimal solution for each target utility company *c*. Another reason to use the SA here is that we can easily adjust the run-time and the quality of final result by simply changing some values as well as the running steps [16]. Details of this method are as follows:

1. Set all *P*[*c*][*t*]=*P_initial*, set temperature T=T_{max}.
2. Based on given *P*[*c*][*t*], call the task assignment model, assign all tasks and calculate total energy consumption of each time *con*[*c*][*i*][*t*], also find out whether each energy consumer is satisfied with the certain utility company.
3. Based on the calculated *con*[*c*][*i*][*t*] and *sat*[*c*][*i*], call the utility company model, calculate the expected total *profit* from satisfied consumers.
4. Change the price distribution *P*[*c*][*t*] by randomly pick up one or several successive time slots and increase or decrease them within the constraints, repeat step 2 and 3 and calculate *profit_new*.
5. If *profit_new*>*profit*, accept the new solution, if not accept with a probability *p*=*exp*((*profit* – *profit_new*)/*T*) based on the current temperature *T*.
6. Decrease the temperature by a factor of *a* and repeat from step 2 until *T* reaches a certain value T_{min}. The value of T_{min} is a tradeoff between run-time and the accuracy of final result.

IV. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed solution, cases corresponding to the aforesaid pricing models are examined.

In these simulations, duration of a time slot is set to one hour. For this reason, the minimum duration of a task is also set to one hour, and the durations of tasks are integer multiples of one hour. Moreover, power consumption of the tasks is determined with a granularity of one hour.

The proposed algorithm has been implemented in C++ code and tested for random cases.

In table I, we act as utility companies to decide the price function which maximizes the expected profit. We assume that there are 3 utility companies to serve in total 1000 consumers with 10000 aggregated tasks and each company starts with a relatively high initial price. We use the predicted energy consumer's profile to design the price function. To be realistic, cost functions of those utility companies are different due to the power-station type as well as the technology.

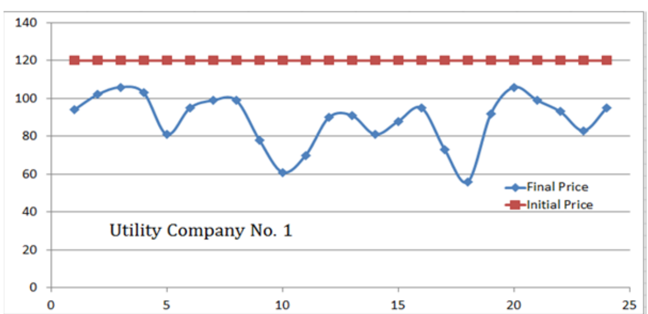
It can be seen from Table I that all those 3 utility companies achieved the expected profit increase by a factor between 2 and 5. This is because they adjusted their price functions to satisfy a certain amount of consumers while maintaining a gap between the energy price and energy cost. Figure 2 shows the change of price function for each utility company.

Table II. Profit Maximization for Utility Companies

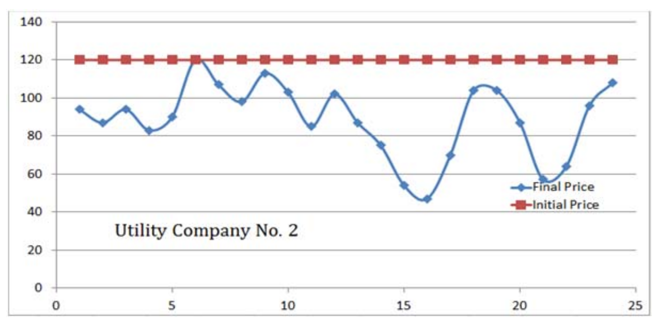
Company	Initial expected profit	Final expected profit	Profit increase factor
1	132520	622960	4.7
2	274284	755646	2.75
3	193419	654224	3.38

In Figure 2, the energy price of each utility company significantly decreases after all of them have tried to maximize their expected profits. Remember that there is no price constraint from government. Instead, it is competition in energy market that has brought price down, which has been proven to be more efficient in promoting overall economic well-being than a centrally planned monopoly market [12].

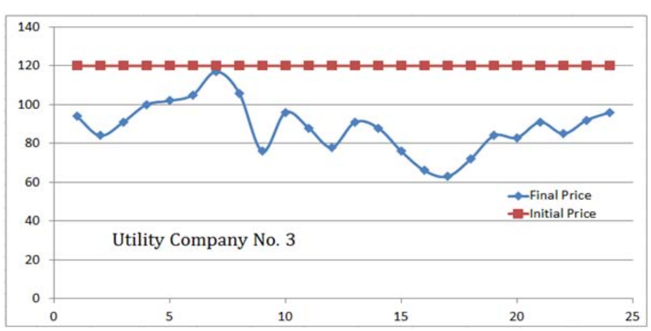
However, all the previous simulations are based on only the profit prediction at utility company's side. In order to verify whether the proposed algorithm really leads to an effective solution, another simulation is presented from the energy consumer's side. In this simulation, we assume the price functions are given from the previous solution and all consumers give their reaction on which utility company to choose based on the rules discussed in section II. After that the real profit of each utility company is calculated and compared with the expected profit, which is shown in Table II.



(a)



(b)



(c)

Figure 2. Initial and final price functions for each utility company

Table II. Comparison of Expected Profit and Real Profit for Utility Companies

Company	Expected profit	Real profit	Profit increase ratio
1	622960	733376	1.18
2	755646	3275627	4.33
3	654224	788670	1.21

It is shown in the above table that all the 3 utility companies received a real profit higher than expected. This is because we were using an underestimating model to calculate the expected profit in the previous step (which results in conservative estimate of final profit). Notice that the second utility company turned out to have a real profit much higher than expected, which comes from the winning of price competition of unsatisfied consumers due to its low energy cost. But this price advantage didn't affect other utility companies' expected profit.

Runtime of the proposed heuristic for all the 3 utility companies is about 1 minute for 1000 consumers with 10000 aggregated tasks for a machine with a dual core processor with frequency of 2.80 GHz. This run time makes it feasible to utilize our models real-time.

V. CONCLUSION

A model of profit maximization of utility companies in oligopolistic market was presented including its problem formulation and solution. In this model, utility companies are considered as non-cooperative, i.e., always making decisions based on their own best solution. A feedback system is utilized based on consumers' reaction to task scheduling and supply selection. The model was implemented and tested with some arbitrary test schemes. The results showed that all companies achieved significant improvements on their expected profit, and the real-time simulation strengthened the effectiveness of our proposed solution on price function.

REFERENCES

[1] *The Smart Grid*: *An Introduction*. The US Department of Energy, 2008.

[2] S. Caron and G. Kesidis, "Incentive-based Energy Consumption Scheduling Algorithms for the Smart Grid," *Proc. Smart Grid Communications Conf.*, 2010.

[3] L. Chen, S. Low, and J. Doyle, "Two market models for demand response in power networks," *Proc. Smart Grid Communications Conf.*, 2010.

[4] Tariq Samad, "Technology Developments and R&D Challenges for Smart Grid Applications in Homes, Buildings, and Industry, " Presentation slides.

[5] S. Hatami and M. Pedram, "Minimizing the Electricity Bill of Cooperative Users under a Quasi-Dynamic energy pricing Model," *Proc. Smart Grid Communications Conf.*, 2010.

[6] S. Kishore, L. V. Snyder, "Control Mechanisms for Residential Electricity Demand in SmartGrids," *Proc. Smart Grid Communications Conf.*, 2010.

[7] D. O'Neill, M. Levorato, A. Goldsmith, and U. Mitra, "Residential demand response using reinforcement learning," *Proc. Smart Grid Communications Conf.*, 2010.

[8] H. Goudarzi, S. Hatami, and M. Pedram, "Demand-side load scheduling incentivized by dynamic energy prices," *Proc. 2nd Int'l Conf. on Smart Grid Communications,* Oct. 2011.

[9] http://www.powersmartpricing.org/tools

[10] P. Samadi, H. Mohsenian-Rad, R. Schober, V. Wong, and J. Jatskevich, "Optimal real-time pricing algorithm based on utility maximization for smart grid," *Proc. Smart Grid Communications Conf.*, 2010.

[11] T. Cui, H. Goudarzi, S. Hatami, S. Nazarian, and M. Pedram, "Concurrent optimization of consumer's electrical energy bill and producer's power generation cost under a dynamic pricing model," *Proc. of the 3rd IEEE PES Innovative Smart Grid Technologies Conference,* Jan. 2012.

[12] N. Gregory Mankiw, "Principle of Economics," *Dryden Pr*, 1997.

[13] Tikuan Gu, "Thermal Power Station," *China Power Press*, 2008.

[14] J. Valenzuela and M. Mazumdar, "The electricity price duration curve under Bertrand and Cournot models," *Proc. Conf. Probability Methods Appl. Power Syst.*, Ames, IA, Sep. 2004, pp. 38–43.

[15] Y. Narahari, D. Garg, R. Narayanam, H. Prakash, "Game Theoretic Problems in Network Economics and Mechanism Design Solutions," *Springer*, 2009, pp. 21.

[16] S. Kirpatrick, C. D. Gerlatt, and M. P. Vecchi, "Optimization by simulated annealing," *Science*, vol. 220, pp. 671–680, 1983.

[17] Xiaofei Wang, Athanasios V. Vasilakos, Min Chen, Yunhao Liu, Ted Taekyoung Kwon, "A Survey of Green Mobile Networks: Opportunities and Challenges." *MONET* 17(1): 4-20 (2012)