Programmable Antenna Design using Convex Optimization

Javad Lavaei, Aydin Babakhani, Ali Hajimiri and John C. Doyle

***Abstract*— This work presents an application of convex optimization and algebraic geometry in devising secure, power-efficient, beam-steerable, and on-chip transmission systems for wireless networks. First, we introduce a *passively controllable smart (PCS) antenna system* that can be programmed to generate different radiation patterns in far field by adjusting its variable passive controller at every signal transmission. To study the programming capability of a PCS antenna system, we consider a PCS antenna transmitting data in z directions, where some voltages $v_1, v_2, ..., v_z$ are induced in different directions in far field. The objective of this paper is to study the set of all feasible vectors $(v_1, v_2, ..., v_z)$ that can be generated by a passive control of the PCS antenna system. To this end, it is shown that all feasible vectors $(v_1, v_2, ..., v_z)$ form a convex semi-algebraic set parameterized by a linear matrix inequality (LMI). Later on, this LMI condition is further studied and it is proven that the geometry of the set of all feasible voltages $(v_1, v_2, ..., v_z)$ is simply an ellipsoid. This significant result makes it possible to compute the feasibility set online to decide how the PCS antenna must be programmed for either directional or simultaneous data transmission. Unlike the existing smart antennas whose programming leads to an NP-hard problem or are made of many active elements, the PCS antenna proposed in the present work has a low-complex programming capability and consists of only one active element.**

I. INTRODUCTION

Conventional antennas for wireless transmission, e.g. omni-directional antennas, radiate in almost all directions. To avoid co-channel interference and unnecessary power consumption in undesired directions, it is preferred not to deploy conventional wireless transmission systems. A great amount of effort has been made in the past several decades to design smart transmitting/receiving antenna systems, which are able to increase the capacity of wireless networks [1], [2], [3], [4], [5], [6]. Two main types of smart antennas are switched beam and adaptive array. A switched beam smart antenna has several pre-designed fixed beam patterns, whereas an adaptive array smart antenna adaptively steers the beam to any direction of interest while simultaneously nulling interfering signals [7], [8]. Note that an array system comprises multiple active (antenna) elements for varying the relative phases and amplitudes of the respective signals in order to generate a desired radiation pattern. Other types of smart antenna systems employ only one active element surrounded by a number of passive parasitic elements, with the disadvantage that they are either non-programmable or their online programming leads to an NP-hard problem [9], [10], [11].

It is noteworthy that smart antennas with several active elements are easy to program but very hard to manufacture because of not being implementable on a single chip, whereas smart antennas with a single active element and several passive elements are easy to implement but very hard to program. To design a smart antenna with easy implementation and programming concurrently, we recently showed in our work [12] that it is possible to design smart antennas with a single active (radiating) element whose programming is tantamount to solving a linear matrix inequality (LMI) problem. This work was based on the recent papers [10] and [11] that introduced the new notion of *near field direct antenna modulation*.

In the present paper, we build on our recent work [12] and study a powerful type of smart antenna system, referred to as *passively controllable smart (PCS) antenna*. A PCS antenna system is composed of a main transmitting antenna, a number of reflectors (or a patch array), and a variable (tunable) passive controller. Since changing the parameters of the passive controller modifies the radiation pattern generated at the far field, this act is regarded as *programming* of the PCS antenna. To study the programming capabilities of a PCS antenna, z receiving nodes are placed around the PCS antenna, which are all equipped with short dipole antennas for signal reception. It is shown that a pre-determined set of voltages $v_1, v_2, ..., v_z$ can be sent to the receiving nodes via a passive control of the PCS antenna if and only if the vector of voltages $(v_1, v_2, ..., v_z)$ belongs to a convex semi-algebraic region specified by an LMI condition. The geometry of this set is further studied and it is shown that the set of all feasible voltage vectors $(v_1, v_2, ..., v_z)$ forms an ellipsoid. This ellipsoid characterizes both the individual signals that can be transmitted to different antennas and the correlation among these signals. Based on the obtained properties, it is shown how the PCS antenna can be programmed to transmit data to an intended user in such a way that many of the unintended users receive a zero signal (no signal) simultaneously.

The rest of the paper is organized as follows. The problem is introduced in Section II and some preliminaries are provided accordingly. The main results are derived in Section III, whose efficacy is demonstrated in Section IV. Concluding remarks are drawn in Section V.

II. PROBLEM STATEMENT AND PRELIMINARIES

The recent papers [10] and [11] have introduced the new notion of near-field direct antenna modulation to design a

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novel type of antenna system for secure wireless transmission, which is on-chip, small-sized, and low-power consuming (due to using only one active element). The antenna system proposed therein has a main dipole transmitting antenna driven by a voltage source, a number of reflectors and several switches mounted on the reflectors. Since each switch can be turned on or off, there exist different switching strategies. Each switching combination creates a possibly unique near-field boundary condition around the antenna, which results in different radiation patterns at the far field. Therefore, each switching combination could possibly generate a new point in the constellation diagram. Figure 1(a) exemplifies the antenna system suggested in [10], which consists of 4 reflectors and 12 switches (shown by arrows). It can be observed that there exist 2^{12} switching combinations, which create a constellation diagram with numerous points. The antenna system introduced in [10] has not only a modulation capability, but also a direction-dependent transmission ability. Indeed, since the reflectors affect the electromagnetic field around the antenna in a non-uniform way, the constellation diagrams seen in different directions are not necessarily correlated. Figure 1(b) illustrates this property via an antenna system with 4 switches, which makes a 16-QAM constellation diagram in the vertical direction and a totally scrambled one in an undesired direction.

Define a *passively controllable smart (PCS) antenna system* as a system with the following components:

- *A dipole transmitting antenna:* This dipole antenna is the only active element of the PCS antenna system, which is driven by a sinusoidal voltage source.
- *A number of reflectors:* These reflectors surround the dipole antenna to shape the electromagnetic field in the space.
- *A number of controllable ports:* These controllable ports are mounted on the reflectors which should be controlled for every signal transmission to form a desired radiation pattern at the far field.
- *An adjustable passive network (controller):* This passive network consists of resistors, capacitors and inductors, and is connected to the controllable ports of the reflectors to control the antenna system for every signal transmission.

A PCS antenna system resembles the antenna system proposed in [10] and [11], with the main difference that the switches are replaced by controllable ports that must be controlled by passive elements for every signal transmission. For simplicity, the term *PCS antenna system* will be abbreviated as *PCS antenna* throughout the paper. Since a PCS antenna has only one active element and its controller solely includes (variable) passive elements, it can be implemented as a low-power integrated on-chip programmable antenna.

Given a natural number z, consider a wireless network with $z+1$ nodes, labeled as $0, 1, 2, ..., z$. Assume that these nodes are geographically distributed so that none of the two nodes in the set $\{1, 2, ..., z\}$ are co-linear with node 0. This assumption is made to ensure an angle diversity

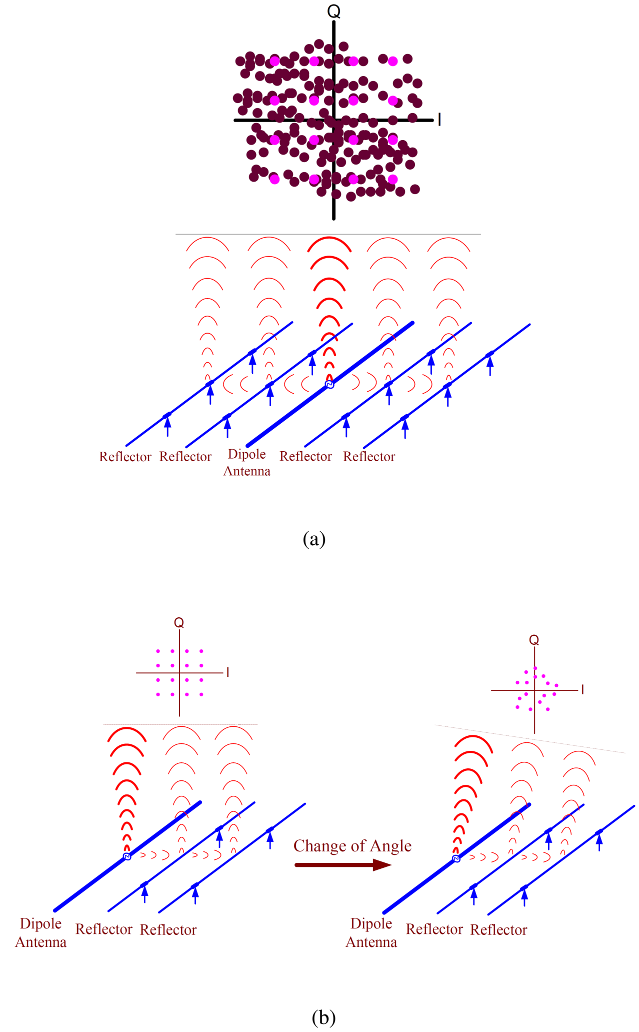


Fig. 1. (a): This figure illustrates the modulation capability of the switch-based antenna system proposed in [10]. (b): This figure illustrates the direction-dependent transmission capability of the switch-based antenna system proposed in [10].

among nodes $1, 2, ..., z$ with respect to node 0. Suppose that node 0 deploys a PCS antenna to transmit data to nodes $1, 2, ..., z$ which are all equipped with regular receiving dipole antennas. Every passive control of the PCS antenna of node 0 induces some voltages on the receiving antennas of nodes $1, 2, ..., z$. Let v_j denote the voltage induced on the antenna of node $j \in \{1, 2, ..., z\}$. The objective of this paper is to study the geometry of the set of all feasible voltage vectors $(v_1, v_2, ..., v_z)$. The geometric shape of this region determines the diversity of a PCS antenna, whose study addresses two important problems:

- *Simultaneous data transmission:* In this case, node 0 wants to send some symbols (voltages) $\alpha_1, \alpha_2, ..., \alpha_z$ to the nodes $1, 2, ..., z$ simultaneously at a single time slot and with using a single frequency.
- *Directional data transmission:* For this application, node 0 intends to transmit data to a node $j \in \{1, 2, ..., z\}$ in such a way that the remaining nodes $1, ..., j-1, j+1, .., z$ receive a zero signal.

Note that both of the above problems reduce to finding the set of all feasible voltage vectors that can be generated by a passive control of the PCS antenna of node 0. Before proceeding with the main results, let some necessary notations and definitions be made in the sequel.

Notation 1: Introduce the following notations:

- i : the imaginary unit;
- $\mathbf{N}$, $\mathbf{R}$ and $\mathbf{C}$: the sets of natural, real and complex numbers, respectively;
- $\mathbf{S}^{k\times k}$: the set of all symmetric matrices in $\mathbf{R}^{k\times k}$ (where $k \in \mathbf{N}$);
- $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$: the operators returning the real and imaginary parts of a complex matrix;
- $*$: the matrix operator taking the conjugate transpose of a complex-valued matrix;
- $\succ$: the matrix inequality sign in the positive definite sense.

Notation 2: For convenience and with a slight abuse of notation, the terms *circle*, *ellipse* and *ellipsoid* in this paper will refer to the interiors of the conventional *circle*, *ellipse* and *ellipsoid*, respectively. For instance, the *unit circle* in this paper refers to the set $\{(x, y) \in \mathbf{R}^2 \mid x^2 + y^2 < 1\}$, as opposed to $\{(x, y) \in \mathbf{R}^2 \mid x^2 + y^2 = 1\}$

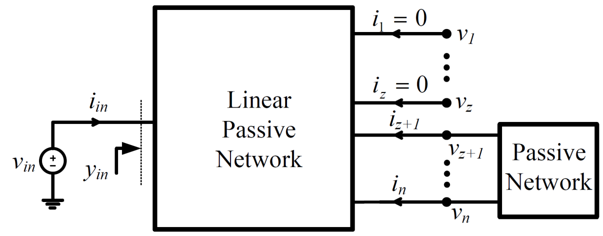
Definition 1: For every real-valued column vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ of the same dimension, define the 2-norm $\|\mathbf{x}_1 + \mathbf{x}_2\mathrm{i}\|$ as $\sqrt{\mathbf{x}_1^*\mathbf{x}_1 + \mathbf{x}_2^*\mathbf{x}_2}$.

Definition 2: Given a scalar $k \in \mathbf{N}$ and a set $\mathcal{H} \subseteq \mathbf{C}^{1\times k}$, define the *real-valued representation of the set* $\mathcal{H}$ as the set of all real vectors in the form of $\begin{bmatrix}\mathrm{Re}\{\boldsymbol{\alpha}\} & \mathrm{Im}\{\boldsymbol{\alpha}\}\end{bmatrix}$ such that $\boldsymbol{\alpha}$ is an element of $\mathcal{H}$. The operator $\mathcal{R}(\cdot)$ will be used henceforth to represent the real-valued representation of a set; for instance, the real-valued representation of $\mathcal{H}$ is denoted by $\mathcal{R}(\mathcal{H})$.

III. Passively Controllable Smart Antenna

Let f_0 and v_{in} denote the frequency and magnitude of the sinusoidal voltage driving the dipole transmitting antenna of the PCS antenna system, respectively. Assume that nodes $1, 2, ..., z$ all lie in the far field of node 0, meaning that the distance of each of nodes $1, 2, .., z$ from node 0 is noticeably greater than the wavelength $\frac{3\times10^8}{f_0}$. This assumption normally holds in practice. One can extract the equivalent circuit model of the entire antenna configuration that consists of the transmitting antenna of node 0 and the receiving antennas of nodes $1, 2, ..., z$. This circuit model is given in Figure 2, where

- The block "Linear Passive Network" corresponds to the Y-parameter matrix of the antenna configuration (calculated from scattering parameters), which can be found using an electromagnetic simulation.
- $v_{z+1}, v_{z+2}, ..., v_n$ denote the voltages on the controllable ports of the PCS antenna of node 0 (it is assumed that there are $n - z$ controllable ports).
- The block "Passive Network" represents the adjustable passive controller applied to the controllable ports of the PCS antenna of node 0.



(a)

Fig. 2. Equivalent circuit model of the passively controllable smart antenna system proposed in the present work.

Denote the complex-valued Y-parameter matrix of the antenna configuration at the given frequency f_0 with Y_s. From now on, suppose that the adjustable passive controller of the PCS antenna must be linear and strictly passive because the goal is to implement this controller by an interconnection of a number of variable resistors and possibly some variable capacitors and inductors.

Decompose the complex-valued matrix Y_s in a block form as

$$Y_s = \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix}, \tag{1}$$

where $W_{11} \in \mathbf{C}^{z\times z}$, $W_{22} \in \mathbf{C}^{(n-z)\times(n-z)}$ and $W_{33} \in \mathbf{C}$. Given a complex vector $\boldsymbol{\alpha} \in \mathbf{C}^{1\times z}$, the goal is to investigate whether the PCS antenna of node 0 can be programmed so that it generates the voltage vector $(v_1, v_2, ..., v_z) = \boldsymbol{\alpha}$. We obtained the following result in our recent paper [12].

Theorem 1: Given $\boldsymbol{\alpha} \in \mathbf{C}^{1\times z}$, the PCS antenna of node 0 can be programmed to make the voltages $v_1, v_2, ..., v_z$ satisfy the relation

$$(v_1, v_2, ..., v_z) = \boldsymbol{\alpha} \tag{2}$$

if and only if there exist symmetric matrices $M, N \in \mathbf{R}^{(n-z)\times(n-z)}$ such that

$$\begin{bmatrix} \left(\mathrm{Re}\left\{W_{22} - W_{21}W_{11}^{-1}W_{12}\right\}\right)^{-1} - M & N \\ N & M \end{bmatrix} \succ 0, \tag{3}$$

and

$$\begin{aligned} &-(W_{31}W_{11}^{-1}W_{12} - W_{32})(M + N\mathrm{i})W_{21}W_{11}^{-1} \\ &\qquad\qquad - W_{31}W_{11}^{-1} = \boldsymbol{\alpha}. \end{aligned} \tag{4}$$

Moreover, if there exist such matrices M, N satisfying the above constraints, then one candidate for the admittance of the passive controller at the frequency f_0, denoted by Y_0, is

$$Y_0 = (M + N\mathrm{i})^{-1} - W_{22} + W_{21}W_{11}^{-1}W_{12}. \tag{5}$$

Theorem 1 states that the PCS antenna of node 0 can generate a specific radiation pattern at the far field if and only if a linear matrix inequality (LMI) problem is feasible (see [13] for the definition of LMI). Since the feasibility region of an LMI problem is convex, the set of all possible voltages $(v_1, v_2, ..., v_z)$ has a convex real-valued representation (see

Definition 2). The objective is to further simplify the LMI conditions derived in Theorem 1.

Definition 3: Define $\mathcal{D}$ as the set of all complex-valued z-tuples $(v_1, v_2, ..., v_z)$ that can be generated by the programmable PCS antenna of node 0.

The complex-valued set $\mathcal{D}$ captures the correlation among the individual signals that nodes $1, 2, ..., z$ receive. To characterize this set, two lemmas are required, in addition to Theorem 1, which will be provided in the sequel.

Lemma 1: Given a scalar $m \in \mathbf{N}$ and vectors $\mathbf{x}_1, \mathbf{x}_2 \in \mathbf{R}^{1\times m}$ with the property $\|[\ \mathbf{x}_1 \quad \mathbf{x}_2\ ]\| = 1$, consider the set of all vectors $\boldsymbol{\alpha} \in \mathbf{R}^{1\times 2m}$ for which there exist symmetric matrices $M, N \in \mathbf{R}^{m\times m}$ such that

$$\boldsymbol{\alpha} = \left[\ \mathbf{x}_1 \quad \mathbf{x}_2\ \right] \left[\begin{array}{cc} M & N \\ N & -M \end{array} \right] \tag{6}$$

and

$$\left[\begin{array}{cc} M & N \\ N & -M \end{array} \right] \prec I. \tag{7}$$

This set is identical to the open unit ball $\{\boldsymbol{\gamma} \in \mathbf{R}^{1\times 2m} \mid \|\boldsymbol{\gamma}\| < 1\}$.

Proof: Denote the open unit ball $\{\boldsymbol{\gamma} \in \mathbf{R}^{1\times 2m} \mid \|\boldsymbol{\gamma}\| < 1\}$ with $\mathcal{B}^{2m}$. In order to show that the set of every vector $\boldsymbol{\alpha}$ representable in the form (6) subject to the constraint (7) is equal to $\mathcal{B}^{2m}$, it suffices to prove that every point in this set belongs to $\mathcal{B}^{2m}$ and vice versa. This will be performed in two phases. First, consider an arbitrary vector $\boldsymbol{\alpha} \in \mathbf{R}^{1\times 2m}$ for which there exist symmetric matrices $M, N \in \mathbf{R}^{m\times m}$ such that the relations (6) and (7) both hold. The goal of this step is to prove that $\boldsymbol{\alpha}$ is in the open ball $\mathcal{B}^{2m}$. Notice that since the matrix

$$\left[\begin{array}{cc} M & N \\ N & -M \end{array} \right] \tag{8}$$

is Hamiltonian, its eigenvalues are all symmetric with respect to the imaginary axis in the complex plane. This property, together with the inequality (7), yields that the eigenvalues of this Hamiltonian (and Hermitian) matrix all lie in the interval $(-1, 1)$. As a result,

$$\left[\begin{array}{cc} M & N \\ N & -M \end{array} \right]^2 \prec I. \tag{9}$$

Therefore,

$$\begin{aligned} \boldsymbol{\alpha}\boldsymbol{\alpha}^* &= \left[\ \mathbf{x}_1 \quad \mathbf{x}_2\ \right] \left[\begin{array}{cc} M & N \\ N & -M \end{array} \right]^2 \left[\begin{array}{c} \mathbf{x}_1^* \\ \mathbf{x}_2^* \end{array} \right] \\ &< \left[\ \mathbf{x}_1 \quad \mathbf{x}_2\ \right] \left[\begin{array}{c} \mathbf{x}_1^* \\ \mathbf{x}_2^* \end{array} \right] = 1. \end{aligned} \tag{10}$$

This proves that $\boldsymbol{\alpha}$ belongs to $\mathcal{B}^{2m}$. As the second step of the proof, assume that $\boldsymbol{\beta} \in \mathbf{R}^{1\times 2m}$ is an arbitrary vector in the ball $\mathcal{B}^{2m}$. The objective is to show that there exist two symmetric matrices $M, N \in \mathbf{R}^{m\times m}$ satisfying the relation (7) such that

$$\boldsymbol{\beta} = \left[\ \mathbf{x}_1 \quad \mathbf{x}_2\ \right] \left[\begin{array}{cc} M & N \\ N & -M \end{array} \right]. \tag{11}$$

A constructive proof will be provided here. Decompose the vector $\boldsymbol{\beta}$ as $[\ \boldsymbol{\beta}_1 \quad \boldsymbol{\beta}_2\ ]$, where $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2 \in \mathbf{R}^{1\times m}$. Since the symmetric matrix $\|\boldsymbol{\beta}\|^2\mathbf{x}_1^*\mathbf{x}_1 + \|\boldsymbol{\beta}\|^2\mathbf{x}_2^*\mathbf{x}_2 - \boldsymbol{\beta}_1^*\boldsymbol{\beta}_1 - \boldsymbol{\beta}_2^*\boldsymbol{\beta}_2$ is the sum of four rank-one matrices, it has at most four nonzero eigenvalues. Denote the eigenvalues of this matrix with $\gamma_1, \gamma_2, ..., \gamma_m$, where $\gamma_5 = \cdots = \gamma_m = 0$. Let $\mathbf{q}_j$ represent the unit right eigenvector of the above matrix corresponding to the eigenvalue γ_j, for every $j \in \{1, 2, ..., m\}$. To simplify the proof by avoiding special cases, assume that $m \geq 4$. Define

$$\begin{aligned} \mathbf{p}_1 &:= \frac{1}{2}(-\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{q}_4), \\ \mathbf{p}_2 &:= \frac{1}{2}(+\mathbf{q}_1 - \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{q}_4), \\ \mathbf{p}_3 &:= \frac{1}{2}(+\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q}_3 + \mathbf{q}_4), \\ \mathbf{p}_4 &:= \frac{1}{2}(+\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 - \mathbf{q}_4), \\ \mathbf{p}_j &:= \mathbf{q}_j, \quad \forall j \in \{5, ..., m\}. \end{aligned} \tag{12}$$

It is straightforward to verify that

$$\mathbf{p}_j^*\mathbf{p}_j = 1, \quad \mathbf{p}_j^*\mathbf{p}_k = 0, \quad \forall j, k \in \{1, 2, ..., m\},\ j \neq k.$$

Define the matrix P as $[\ \mathbf{p}_1 \quad \mathbf{p}_2 \quad \cdots \quad \mathbf{p}_m\ ]$. It can be concluded from (13) that $PP^* = I$. Let $\lambda_1, ..., \lambda_m, \bar{\lambda}_1, ..., \lambda_m$ be some scalars given by the equation

$$\left[\begin{array}{c} \lambda_j \\ \bar{\lambda}_j \end{array} \right] = \frac{1}{\|\boldsymbol{\beta}\|} \left[\begin{array}{cc} \mathbf{x}_1\mathbf{p}_j & \mathbf{x}_2\mathbf{p}_j \\ -\mathbf{x}_2\mathbf{p}_j & \mathbf{x}_1\mathbf{p}_j \end{array} \right]^{-1} \left[\begin{array}{c} \boldsymbol{\beta}_1\mathbf{p}_j \\ \boldsymbol{\beta}_2\mathbf{p}_j \end{array} \right], \tag{13}$$

for every $j \in \{1, 2, ..., m\}$. It is desired to show that the relations (7) and (11) are satisfied if M and N are taken as follow:

$$\begin{aligned} M &= \|\boldsymbol{\beta}\|P \times \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_m) \times P^*, \\ N &= \|\boldsymbol{\beta}\|P \times \mathrm{diag}(\bar{\lambda}_1, \bar{\lambda}_2, ..., \bar{\lambda}_m) \times P^*. \end{aligned} \tag{14}$$

For this purpose, it results from the equation (13) that

$$\lambda_j^2 + \bar{\lambda}_j^2 = \frac{\|\boldsymbol{\beta}_1\mathbf{p}_j\|^2 + \|\boldsymbol{\beta}_2\mathbf{p}_j\|^2}{\|\boldsymbol{\beta}\|^2\left(\|\mathbf{x}_1\mathbf{p}_j\|^2 + \|\mathbf{x}_2\mathbf{p}_j\|^2\right)}, \quad \forall j \in \{1, 2, ..., m\}.$$

On the other hand, one can write

$$\mathbf{p}_j^*\left(\|\boldsymbol{\beta}\|^2\mathbf{x}_1^*\mathbf{x}_1 + \|\boldsymbol{\beta}\|^2\mathbf{x}_2^*\mathbf{x}_2 - \boldsymbol{\beta}_1^*\boldsymbol{\beta}_1 - \boldsymbol{\beta}_2^*\boldsymbol{\beta}_2\right)\mathbf{p}_j = 0, \tag{15}$$

for every $j \in \{5, ..., m\}$, due to the equalities $\mathbf{p}_j = \mathbf{q}_j$ and $\gamma_j = 0$. Given an index $j \in \{1, 2, 3, 4\}$, it can be verified that

$$\begin{aligned} &\mathbf{p}_j^*\left(\|\boldsymbol{\beta}\|^2\mathbf{x}_1^*\mathbf{x}_1 + \|\boldsymbol{\beta}\|^2\mathbf{x}_2^*\mathbf{x}_2 - \boldsymbol{\beta}_1^*\boldsymbol{\beta}_1 - \boldsymbol{\beta}_2^*\boldsymbol{\beta}_2\right)\mathbf{p}_j = \\ &= \sum_{k=1}^{4}\gamma_k = \sum_{k=1}^{m}\gamma_k \\ &= \mathrm{trace}\left(\|\boldsymbol{\beta}\|^2\mathbf{x}_1^*\mathbf{x}_1 + \|\boldsymbol{\beta}\|^2\mathbf{x}_2^*\mathbf{x}_2 - \boldsymbol{\beta}_1^*\boldsymbol{\beta}_1 - \boldsymbol{\beta}_2^*\boldsymbol{\beta}_2\right) \\ &= \|\boldsymbol{\beta}\|^2\|\mathbf{x}_1\|^2 + \|\boldsymbol{\beta}\|^2\|\mathbf{x}_2\|^2 - \|\boldsymbol{\beta}_1\|^2 - \|\boldsymbol{\beta}_2\|^2 \\ &= \|\boldsymbol{\beta}\|^2 - \|\boldsymbol{\beta}_1\|^2 - \|\boldsymbol{\beta}_2\|^2 = 0. \end{aligned} \tag{16}$$

Hence, it can be concluded from (15) and (16) that

$$\|\boldsymbol{\beta}\|^2\left(\|\mathbf{x}_1\mathbf{p}_j\|^2 + \|\mathbf{x}_2\mathbf{p}_j\|^2\right) = \|\boldsymbol{\beta}_1\mathbf{p}_j\|^2 + \|\boldsymbol{\beta}_2\mathbf{p}_j\|^2, \tag{17}$$

for every $j \in \{5, ..., m\}$. Combining (15) and (17) yields

$$\lambda_j^2 + \bar{\lambda}_j^2 = 1, \quad \forall j \in \{1, 2, ..., m\}. \tag{18}$$

The above equation, along with the relation PP^*, leads to the fact that the matrices M and N introduced earlier satisfy the equality

$$\begin{bmatrix} M & N \\ N & -M \end{bmatrix}^2 = \|\boldsymbol{\beta}\|^2 I, \tag{19}$$

which implies that the Hamiltonian matrix (8) has m eigenvalues at $\|\boldsymbol{\beta}\|$ and m eigenvalues at $-\|\boldsymbol{\beta}\|$. Since $\|\boldsymbol{\beta}\|$ is strictly less than 1, it can be inferred that the inequality (7) holds for this choice of M and N. Now, it remains to show that the equation (11) is also satisfied. To this end, simplify the equation (13) to obtain

$$\|\boldsymbol{\beta}\|\lambda_j \mathbf{x}_1\mathbf{p}_j + \|\boldsymbol{\beta}\|\bar{\lambda}_j \mathbf{x}_2\mathbf{p}_j = \boldsymbol{\beta}_1\mathbf{p}_j, \ \forall j \in \{1, ..., m\},$$

$$\|\boldsymbol{\beta}\|\bar{\lambda}_j \mathbf{x}_1\mathbf{p}_j - \|\boldsymbol{\beta}\|\lambda_j \mathbf{x}_2\mathbf{p}_j = \boldsymbol{\beta}_2\mathbf{p}_j, \ \forall j \in \{1, ..., m\},$$

or equivalently

$$\mathbf{x}_1 M + \mathbf{x}_2 N = \boldsymbol{\beta}_1, \quad \mathbf{x}_1 N - \mathbf{x}_2 M = \boldsymbol{\beta}_2. \tag{21a}$$

The above equations show the validity of (11), which completes the proof. ∎

Lemma 2: Given scalars $m, k \in \mathbf{N}$, vectors $\mathbf{x}_1, \mathbf{x}_2 \in \mathbf{R}^{1\times m}$ and matrices $G_1, G_2 \in \mathbf{R}^{m\times k}$, consider the set of all complex vectors $\boldsymbol{\alpha} \in \mathbf{C}^{1\times k}$ that can be written as

$$\boldsymbol{\alpha} = (\mathbf{x}_1 + \mathbf{x}_2\mathrm{i})(M + N\mathrm{i})(G_1 + G_2\mathrm{i}) \tag{22}$$

for some symmetric matrices $M, N \in \mathbf{R}^{m\times m}$ with the property

$$\begin{bmatrix} M & N \\ N & -M \end{bmatrix} \prec I. \tag{23}$$

The real-valued representation of this complex set is identical to the ellipsoid

$$\left\{\mathbf{h} \in \mathbf{R}^{1\times 2k} \middle| \mathbf{h}\left(\begin{bmatrix} G_1^* & -G_2^* \\ G_2^* & G_1^* \end{bmatrix}\begin{bmatrix} G_1 & G_2 \\ -G_2 & G_1 \end{bmatrix}\right)^{-1}\mathbf{h}^* < \|\mathbf{x}_1\|^2 + \|\mathbf{x}_2\|^2\right\},$$

provided the matrix $G_1 + G_2\mathrm{i}$ has full column rank over the field of complex numbers.

Proof: Observe that the equation (22) is tantamount to

$$\begin{bmatrix} \mathrm{Re}\{\boldsymbol{\alpha}\} & \mathrm{Im}\{\boldsymbol{\alpha}\} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 & -\mathbf{x}_2 \end{bmatrix}\begin{bmatrix} M & N \\ N & -M \end{bmatrix} \times \begin{bmatrix} G_1 & G_2 \\ -G_2 & G_1 \end{bmatrix}. \tag{24}$$

On the other hand, it follows from Lemma 1 that the set

$$\left\{\begin{bmatrix} \mathbf{x}_1 & -\mathbf{x}_2 \end{bmatrix}\begin{bmatrix} M & N \\ N & -M \end{bmatrix} \middle| M, N \in \mathbf{S}^{m\times m}, \begin{bmatrix} M & N \\ N & -M \end{bmatrix} \prec I\right\}$$

is an open ball centered at the origin with radius $\sqrt{\|\mathbf{x}_1\|^2 + \|\mathbf{x}_2\|^2}$. It can be inferred from this result that the set

$$\left\{\begin{bmatrix} \mathbf{x}_1 & -\mathbf{x}_2 \end{bmatrix}\begin{bmatrix} M & N \\ N & -M \end{bmatrix}\begin{bmatrix} G_1 & G_2 \\ -G_2 & G_1 \end{bmatrix} \middle| M, N \in \mathbf{S}^{m\times m}, \begin{bmatrix} M & N \\ N & -M \end{bmatrix} \prec I\right\}$$

is equal to the ellipsoid given in (24), provided the matrix

$$\begin{bmatrix} G_1^* & -G_2^* \\ G_2^* & G_1^* \end{bmatrix}\begin{bmatrix} G_1 & G_2 \\ -G_2 & G_1 \end{bmatrix} \tag{25}$$

is nonsingular or equivalently $G_1 + G_2\mathrm{i}$ has full column rank. The proof is an immediate consequence of this property and the fact that the equation (22) is the same as (24). ∎

Define some matrices as follows:

$$\begin{aligned}
K_1 &:= W_{31}W_{11}^{-1}W_{12} - W_{32}, \\
K_2 &:= W_{21}W_{11}^{-1}W_{13} - W_{23}, \\
K_3 &:= W_{31}W_{11}^{-1}W_{13} - W_{33}, \\
K_4 &:= W_{21}W_{11}^{-1}, \\
K_5 &:= W_{31}W_{11}^{-1}, \\
Q &:= \left(\mathrm{Re}\left\{W_{22} - W_{21}W_{11}^{-1}W_{12}\right\}\right)^{-1}, \\
\mathbf{o} &:= -\begin{bmatrix} \mathrm{Re}\left\{\frac{1}{2}K_1QK_4 + K_5\right\} & \mathrm{Im}\left\{\frac{1}{2}K_1QK_4 + K_5\right\} \end{bmatrix}, \\
\Omega &:= \begin{bmatrix} \mathrm{Re}\{K_4^*QK_4\} & \mathrm{Im}\{K_4^*QK_4\} \\ -\mathrm{Im}\{K_4^*QK_4\} & \mathrm{Re}\{K_4^*QK_4\} \end{bmatrix}.
\end{aligned}$$

Since the matrix Q introduced above is positive definite, one can define $Q^{\frac{1}{2}}$ as the unique symmetric positive definite matrix whose square is equal to Q. The next theorem presents one of the main results of this work, which exploits Lemma 2 and Theorem 1 to characterize the feasibility region $\mathcal{D}$.

Theorem 2: If the matrix K_4 has full column rank over the field of complex numbers, then the real-valued representation of the complex set $\mathcal{D}$, i.e. $\mathcal{R}(\mathcal{D})$, is equal to the ellipsoid

$$\left\{\mathbf{h} \in \mathbf{R}^{1\times 2z} \middle| (\mathbf{h} - \mathbf{o})\Omega^{-1}(\mathbf{h} - \mathbf{o})^* < \frac{1}{4}\|K_1Q^{\frac{1}{2}}\|^2\right\}. \tag{26}$$

Proof: It can be concluded from Theorem 1 that a complex vector $\boldsymbol{\alpha}$ belongs to $\mathcal{D}$ if and only if there exist symmetric matrices $M, N \in \mathbf{R}^{(n-z)\times(n-z)}$ such that

$$\begin{bmatrix} Q - M & N \\ N & M \end{bmatrix} \succ 0 \tag{27}$$

and

$$\begin{aligned}\boldsymbol{\alpha} = &-(W_{31}W_{11}^{-1}W_{12} - W_{32})(M + N\mathrm{i})W_{21}W_{11}^{-1} \\ &- W_{31}W_{11}^{-1} = -K_1(M + N\mathrm{i})K_4 - K_5.\end{aligned} \tag{28}$$

The constraint (27) can be re-arranged as

$$\begin{bmatrix} \tilde{M} & \tilde{N} \\ \tilde{N} & -\tilde{M} \end{bmatrix} \prec I, \tag{29}$$

where

$$\tilde{M} := 2Q^{-\frac{1}{2}}MQ^{-\frac{1}{2}} - I, \quad \tilde{N} := 2Q^{-\frac{1}{2}}NQ^{-\frac{1}{2}}. \tag{30}$$

Moreover, the constraint (28) can be expressed in terms of $\tilde{M}$ and $\tilde{N}$ as follows:

$$\begin{aligned}\boldsymbol{\alpha} &= -K_1(M+N\mathrm{i})K_4 - K_5\\ &= -\frac{1}{2}K_1\left(Q^{\frac{1}{2}}\tilde{M}Q^{\frac{1}{2}} + Q + Q^{\frac{1}{2}}\tilde{N}Q^{\frac{1}{2}}\mathrm{i}\right)K_4 - K_5\\ &= -\frac{1}{2}K_1Q^{\frac{1}{2}}(\tilde{M}+\tilde{N}\mathrm{i})Q^{\frac{1}{2}}K_4 - \left(\frac{1}{2}K_1QK_4 + K_5\right).\end{aligned}$$

By applying Lemma 2 to the constraints (29) and (31) and using the relation

$$\begin{bmatrix} \mathrm{Re}\{K_4^*Q^{\frac{1}{2}}\} & \mathrm{Im}\{K_4^*Q^{\frac{1}{2}}\} \\ -\mathrm{Im}\{K_4^*Q^{\frac{1}{2}}\} & \mathrm{Re}\{K_4^*Q^{\frac{1}{2}}\} \end{bmatrix} \times \begin{bmatrix} \mathrm{Re}\{Q^{\frac{1}{2}}K_4\} & \mathrm{Im}\{Q^{\frac{1}{2}}K_4\} \\ -\mathrm{Im}\{Q^{\frac{1}{2}}K_4\} & \mathrm{Re}\{Q^{\frac{1}{2}}K_4\} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\{K_4^*QK_4\} & \mathrm{Im}\{K_4^*QK_4\} \\ -\mathrm{Im}\{K_4^*QK_4\} & \mathrm{Re}\{K_4^*QK_4\} \end{bmatrix},$$

it can be deduced that $\mathcal{R}(\mathcal{D})$ is the same as the ellipsoid given in (26) (note that the matrices $Q^{\frac{1}{2}}K_4$ and K_4 have the same column rank). ■

So far, it is shown that the PCS antenna of node 0 can be programmed to generate a specific voltage vector $(v_1, v_2, ..., v_z)$ if and only if the real-valued representation of this vector belongs to a particular open ellipsoid. This important result completely characterizes the correlation among the voltages received by different nodes of the network. Note that the eigenvalues and eigenvectors of the describing matrix Ω determine the strength of this correlation in diverse directions.

Remark 1: Theorem 2 states that $\mathcal{R}(\mathcal{D})$ is an ellipsoid in the case when the matrix K_4 has full column rank. A question arises as to how the region $\mathcal{R}(\mathcal{D})$ looks like if this condition is violated. To answer this question, notice that a set of feasible voltages $(v_1, v_2, ..., v_z)$ generated by the PCS antenna of node 0 can be represented as

$$\begin{bmatrix} v_1 & v_2 & \cdots & v_z \end{bmatrix} = -K_1(M+N\mathrm{i})K_4 - K_5, \quad (31)$$

for some symmetric matrices M and N (see the equation (28)). The above relation simply implies that if K_4 loses column rank, some of the far-field voltages $v_1, v_2, ..., v_z$ can always be expressed in term of the remaining ones (for every arbitrary matrices M and N). As a result, in the case when K_4 losses column rank, some of the far-field voltages create an ellipsoidal feasibility region and the remaining far-field voltages can be linearly written in terms of these voltages. Note that the matrix K_4 losses column rank if $n-z$ is less than z, which signifies that the number of controllable ports of the antenna determines the maximum number of directions towards which independent data can be sent.

Definition 4: Given $l \in \mathbf{N}$ and distinct indices $j,k \in \{1,2,...,l\}$, define $\mathcal{P}^l_{jk}$ to be the plane consisting of all vectors in $\mathbf{R}^l$ whose elements with the indices in the set $\{1,2,...,l\}\backslash\{j,k\}$ are equal to zero.

Theorem 2 can be used to study whether the PCS antenna of node 0 is capable of transmitting data to an intended node in such a way that unintended nodes all receive a zero signal. This is carried out next.

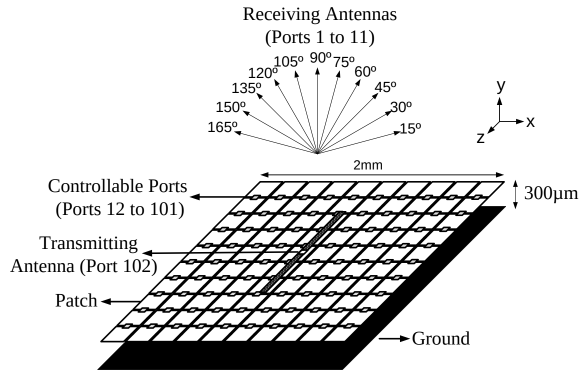


Fig. 3. The PCS antenna system studied in Section IV.

Corollary 1: Let node 0 in the wireless network employ a PCS antenna to generate a radiation pattern at the far field. The following statements hold for every $j \in \{1,2,...,z\}$:

i) The real-valued representation of the set of all possible complex voltages v_j that can be induced on the antenna of node j is an ellipse (circle) obtained by projecting the ellipsoid $\mathcal{R}(\mathcal{D})$ (given in (26)) on the plane $\mathcal{P}^{2z}_{j(z+j)}$.
ii) The real-valued representation of the set of all possible complex voltages v_j that can be induced on the antenna of node j in such a way that other nodes $1,...,j-1,j+1,..,z$ receive a zero signal is an ellipse obtained by intersecting the ellipsoid $\mathcal{R}(\mathcal{D})$ (given in (26)) with the plane $\mathcal{P}^{2z}_{j(z+j)}$.

Proof: Due to the analogy between the two parts of this corollary, only Part (i) will be proved here. Recall from Theorem 2 that a voltage vector $(v_1, ..., v_z)$ can be generated at the far field by a PCS antenna if and only if the vector $(\mathrm{Re}\{v_1\}, ..., \mathrm{Re}\{v_z\}, \mathrm{Im}\{v_1\}, ..., \mathrm{Im}\{v_z\})$ belongs to the ellipsoid $\mathcal{R}(\mathcal{D})$. Since the plane $\mathcal{P}^{2z}_{j(z+j)}$ corresponds to the vector $(\mathrm{Re}\{v_j\}, \mathrm{Im}\{v_j\})$, one can argue that the set of all possible vectors $(\mathrm{Re}\{v_j\}, \mathrm{Im}\{v_j\})$ is equal to the projection of the ellipsoid $\mathcal{R}(\mathcal{D})$ on the plane $\mathcal{P}^{2z}_{j(z+j)}$. The proof of Part (i) is completed by noting that this projection leads to a circle. ■

Given $j \in \{1,2,...,z\}$, the phrase "real-valued representation of the set of all possible complex voltages v_j" in Corollary 1 indeed refers to a constellation digram for v_j. Hence, Corollary 1 characterizes different constellation diagrams that are associated with node j. Although the projection of the ellipsoid $\mathcal{R}(\mathcal{D})$ on the plane $\mathcal{P}^{2z}_{j(z+j)}$ is a non-empty set, the intersection of these ellipsoid and plane might be a null set. As a result, the PCS antenna of node 0 may not be able to transmit data only to node j so that other nodes all receive a zero signal. However, the feasibility region $\mathcal{R}(\mathcal{D})$ can be used to find a maximum number of nodes that can simultaneously receive a zero signal.

A. Online Passive Controller Design

So far, different constellation diagrams associated with every node $j \in \{1,2,...,z\}$ are obtained and shown to be elliptic (see Corollary 1). Assume that based on these of constellation diagrams, node 0 has decided to generate the far-field voltage vector $(v_1, v_2, ..., v_z) = \boldsymbol{\alpha}$ for some $\boldsymbol{\alpha} \in \mathcal{D}$. Note that $\boldsymbol{\alpha}$ can, for instance, be a vector with only one non-zero entry corresponding to a directional data transmission. A question arises as to what passive controller should be applied to the PCS antenna of node 0 to make it generate the voltage vector $(v_1, v_2, ..., v_z) = \boldsymbol{\alpha}$. To address this question, we introduce the following procedure.

Procedure 1:

Step 1: Define a vector $\mathbf{u}$ as

$$\begin{aligned}\mathbf{u} := &\left(\left[\begin{array}{cc} \mathrm{Re}\{\boldsymbol{\alpha}\} & \mathrm{Im}\{\boldsymbol{\alpha}\}\end{array}\right] - \mathbf{o}\right)\\ &\times \left[\begin{array}{cc} \mathrm{Re}\{K_4^* Q K_4\} & \mathrm{Im}\{K_4^* Q K_4\} \\ -\mathrm{Im}\{K_4^* Q K_4\} & \mathrm{Re}\{K_4^* Q K_4\}\end{array}\right]^{-1} \\ &\times \left[\begin{array}{cc} \mathrm{Re}\{K_4^* Q^{\frac{1}{2}}\} & \mathrm{Im}\{K_4^* Q^{\frac{1}{2}}\} \\ -\mathrm{Im}\{K_4^* Q^{\frac{1}{2}}\} & \mathrm{Re}\{K_4^* Q^{\frac{1}{2}}\}\end{array}\right].\end{aligned} \tag{32}$$

Step 2: By using the procedure presented in the proof of Lemma 1, compute two symmetric matrices $\tilde{M}, \tilde{N} \in \mathbf{R}^{(n-z)\times(n-z)}$ such that

$$\mathbf{u} = \left[\begin{array}{cc} -\frac{1}{2}K_1 Q^{\frac{1}{2}} & \frac{1}{2}K_1 Q^{\frac{1}{2}}\end{array}\right] \left[\begin{array}{cc} \tilde{M} & \tilde{N} \\ \tilde{N} & -\tilde{M}\end{array}\right] \tag{33}$$

and

$$\left[\begin{array}{cc} \tilde{M} & \tilde{N} \\ \tilde{N} & -\tilde{M}\end{array}\right] \prec I. \tag{34}$$

Step 3: One candidate for the admittance of the passive controller at the frequency f_0, denoted by Y_0, is

$$\begin{aligned}Y_0 = &\,2\left(Q^{\frac{1}{2}}\tilde{M}Q^{\frac{1}{2}} + Q + Q^{\frac{1}{2}}\tilde{N}Q^{\frac{1}{2}}\mathrm{i}\right)^{-1} \\ &- W_{22} + W_{21}W_{11}^{-1}W_{12}.\end{aligned} \tag{35}$$

The proofs of Lemma 1, Lemma 2, Theorem 1 and Theorem 2 can all be combined in a clear way to deduce why Procedure 1 described above produces a correct admittance Y_0. After obtaining the matrix Y_0, the next question would be how to design a passive controller (circuit) with the admittance Y_0 at the frequency f_0. This can be accomplished systematically using the existing methods in the literature [15], [16].

IV. Simulation results

Consider the 2mm×2mm PCS antenna system depicted in Figure 3 consisting of a patch array with 90 controllable ports (shown by small squares), which is used for data transmission in the directions $15°, 30°, ..., 150°, 165°$. To study the programming capability of this PCS antenna, a receiving antenna is placed at each of these directions in the far field (at the distance of 20 multiples of the wavelength from the PCS antenna) with the length of 140μm and the fixed terminal impedance 50Ω. Assume that the transmitting dipole antenna of the PCS antenna system is driven by a 300GHz sinusoidal signal with a fixed amplitude of 1 volt. The equivalent circuit model of this antenna configuration is extracted using the electromagnetic software IE3D [17], which consists of 102 ports as follows:

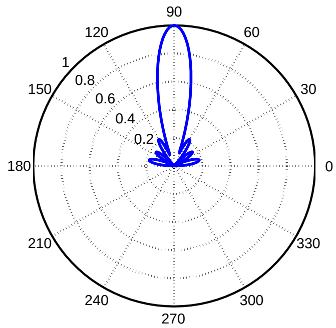
- For every $j \in \{1, 2, ..., 11\}$, port j measures the voltage induced by the PCS antenna on the center of the receiving dipole antenna at the direction $(15j)°$.
- Ports 12 to 101 are the 90 controllable ports on the PCS antenna that are to be controlled by a passive controller for every signal transmission.
- Port 102 lies on the dipole transmitting antenna and is connected to the voltage source.

Every passive control of the PCS antenna generates a set of voltages $(v_1, v_2, ..., v_{11})$ at the far field in the directions $15°, 30°, ..., 150°, 165°$. It follows from Theorem 2 that the set of all feasible voltage vectors $(v_1, v_2, ..., v_{11})$ is simply an ellipsoid given by (26). This nice characterization of the feasibility set enables the PCS antenna to be programmed online for different types of data transmission. Three of such programming problems are tackled in the sequel:

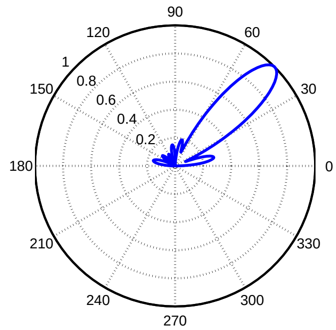
- *Find the maximum power transmitted at the direction* $90°$*:* The ellipsoid can be used to identify the point whose projection on the plane for v_6 yields the farthest point from the origin. This leads to the voltage $v_6 = 0.00303 - 0.002274\mathrm{i}$. The corresponding radiation pattern of the PCS antenna is plotted in Figure 4(a). This figure shows that the antenna has an excellent beamforming capability.
- *Find the maximum power transmitted at the direction* $45°$*:* Similar to the previous case, the point $v_3 = -0.00234 - 0.0030\mathrm{i}$ is obtained with the radiation pattern drawn in Figure 4(b).
- *Find the maximum power transmitted at the direction* $90°$ *subject to the constraint of sending a zero signal at all the directions* $15°, 30°, 45°, 60°, 120°, 135°, 150°, 165°$*:* In this case, the goal is to send no signal in many directions in the course of transmitting data to the vertical direction. It can be concluded from the shape of the ellipsoid that this constrained transmission is possible. The optimal value $v_6 = -0.000866 + 0.000589\mathrm{i}$ is attained and the corresponding radiation pattern is depicted in Figure 4(c).

V. Conclusions

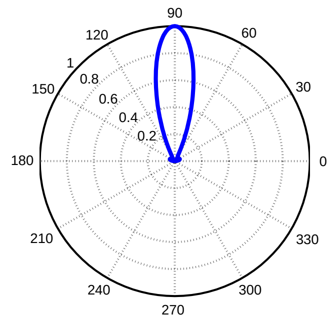
This work proposes a new type of smart antenna system, referred to as *passively controllable smart (PCS) antenna*, which can be used as an efficient transmission device in wireless networks. A PCS antenna system is accompanied by a tunable passive controller whose adjustment at every signal transmission generates a possibly unique radiation pattern. To reduce co-channel interference and optimize the transmitted power, this antenna can be programmed to transmit data in a desired direction in such a way that no signal is transmitted (to the far field) at many pre-specified undesired directions. To study the programming capability of a PCS antenna system, it is crucial to understand how the voltages induced in different directions via a PCS antenna are related to one another. It is shown that the set of all feasible voltage vectors is a convex semi-algebraic region determined by a linear



(a)



(b)



(c)

Fig. 4. (a): The radiation pattern obtained by maximizing the received power at the direction 90°; (b): the radiation pattern obtained by maximizing the received power at the direction 45°; (c): the radiation pattern obtained by maximizing the received power at the direction 90° subject to the constraints $v_1 = v_2 = v_3 = v_4 = v_8 = v_9 = v_{10} = v_{11} = 0$.

matrix inequality. The boundary of this set is further studied and proven to be simply an ellipsoid. This important result implies that every collection of voltages can be transmitted in different directions by a PCS antenna if and only if the corresponding voltage vector belongs to an easy-to-compute ellipsoid.

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