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# 1 Programmable linear quantum networks with a multimode fibre

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## Abstract

Reconfigurable quantum circuits are fundamental building blocks for the implementation of scalable quantum technologies. Their implementation has been pursued in linear optics through the engineering of sophisticated interferometers [1–3]. While such optical networks have been successful in demonstrating the control of small-scale quantum circuits, scaling up to larger dimension poses significant challenges [4, 5]. Here, we demonstrate a potentially scalable route towards reconfigurable optical networks based on the use of a multimode fibre and advanced wavefront-shaping techniques. We program networks involving spatial and polarisation modes of the fibre and experimentally validate the accuracy and robustness of our approach using two-photon quantum states. In particular, we illustrate the reconfigurability of our platform by emulating a tunable coherent absorption experiment [6]. By demonstrating reliable reprogrammable linear transformations, with the prospect to scale, our results highlight the potential of complex media driven by wavefront shaping for quantum information processing.

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16 Linear optical networks are prominent candidates for practical quantum computing [1].  
17 The efficient implementation of quantum information processing tasks requires high di-  
18 mensionality, dense network connectivity and the possibility to actively reconfigure the  
19 network. Currently, bulk and integrated linear optics are the most popular platforms to  
20 implement such networks. The design of the latter is based on a cascade of beamsplitters  
21 and phase-shifters connected by single-mode waveguides [2–5]. However, the scalability of  
22 such architecture is significantly limited by the fabrication process. Alternatively, integrated  
23 multimode waveguides [7–10] and metasurfaces [11] provided new routes towards robust im-  
24 plementation of larger quantum optical circuits, with the strong disadvantages of not being  
25 reprogrammable after fabrication. Coupling spatial modes with other degrees of freedom,  
26 such as time, frequency and polarisation [12], provides a different route towards encoding  
27 and processing information in higher dimensions [13], but remains an engineering challenge  
28 in integrated optics. To date, the quest for a controllable high-dimensional optical network  
29 offering arbitrary connectivity is ongoing.

30 Complex media, from white paint to multimode fibres, can overcome these bottlenecks  
31 when used in combination with wavefront shaping. Many classical and quantum applica-  
32 tions rely on this approach [14], ranging from spatial mode structuring [15–17] to adaptive  
33 quantum optics [18]. As for linear circuits, programmable beamsplitters have been imple-  
34 mented in opaque scattering media [19–21] and multimode fibres [22] through control of  
35 spatial mode mixing. In this work, we report the implementation of fully programmable  
36 linear optical networks of higher dimensions by harnessing spatial and polarisation mix-  
37 ing processes in a multimode fibre driven by wavefront shaping. We first demonstrate the  
38 reliability and versatility of our approach by controlling two-photon interferences between  
39 multiple ports of various networks with high accuracy. We then emulate a circuit for tun-  
40 able coherent absorption, which highlights the reconfigurable nature of our platform. Our  
41 work demonstrates the viability of coherent manipulation of optically encoded information  
42 via multimode scattering from complex media and wavefront shaping, and its potential for  
43 quantum information processing.

44 The experiment is conceptually illustrated in Fig. 1. The multimode fibre (MMF) is a  
45 graded-index fibre supporting  $\sim 400$  propagation modes at  $\lambda = 810$  nm. Complex spatial  
46 and polarisation mixing occurring in the fibre is the key ingredient that enables the design of  
47 a reconfigurable linear transformation  $\mathcal{L}$ . Indeed, measuring the transmission matrix (TM)

48 of the MMF reveals its highly isotropic connectivity across spatial and polarisation modes  
 49 (cf. Supplementary Information (SI) Section 1-2). We exploit the connectivity together with  
 50 the near-unitarity of the MMF to program linear optical transformations  $\mathcal{L}_i$  (cf. Methods for  
 51 details) in a four-dimensional Hilbert space defined across spatial and polarisation degrees  
 52 of freedom, labelled H1, V1, H2, V2.

53 We demonstrate deterministic manipulation of two-photon interference through a de-  
 54 signed optical network  $\mathcal{L}_i$ . First, we generate a two-photon state by spontaneous parametric  
 55 down-conversion (SPDC) process (cf. Methods) and guide it to the experimental platform  
 56 (Fig. 1), in which an optical network  $\mathcal{L}$  is encoded using the spatial light modulators (SLM).  
 57 We implement 4-output  $\times$  2-input optical networks simulating the action of four-dimensional  
 58 Fourier [23] and Sylvester [24] interferometers (cf. SI Section 4 for definitions). These inter-  
 59 ferometers are used for certifying indistinguishability between input photons via verifying  
 60 a suppression criteria [25, 26]. Here, we verify this criteria for a specific two-photon input  
 61 state by measuring the full set of output two-fold coincidence (Fig. 2). Maximum two-photon  
 62 visibility values measured after propagating through the MMF ( $0.96 \pm 0.01$ ) and directly at  
 63 the SPDC source ( $0.95 \pm 0.03$ ) are the same, showing that the platform does not introduce  
 64 significant temporal distinguishability between photon pairs (cf. SI Section 4). The results  
 65 show quantum distinctive features: values of the degree of violation  $\mathcal{D}$ , defined as the prob-  
 66 ability of occupying two-photon states in all suppression configurations [23, 24], are as small  
 67 as  $0.022 \pm 0.009$  (Fourier interferometer, for (1, 3) and (2, 4) input pairs) and  $0.014 \pm 0.008$   
 68 (Sylvester interferometer, for all input pairs).

69 Owing to the high number of propagation modes supported by the MMF, we can manip-  
 70 ulate phase and amplitude of each element in an optical network independently. To demon-  
 71 strate this ability, we implement the non-unitary transformation  $\mathcal{L}_N$ , defined as  $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{\otimes 2}$ ,  
 72 which maps all two-photon interferences into photon anti-coalescences (Fig. 2). The phe-  
 73 nomenon presents a distinct result originating from non-unitarity, which derives from in-  
 74 formation losses stemming from the fact that we do not control all input modes of the  
 75 MMF. The error between the experimentally synthesised transformation and the theoreti-  
 76 cally desired one is quantified by  $\Delta\mathbb{V} = \langle |V_{ij}^{\text{exp}} - V_{ij}^{\text{th}}| \rangle_{ij}$ , where  $V_{ij}^{\text{exp(th)}}$  is the experimental  
 77 (theoretical) visibility over the  $(i, j)$  output ports. We measure  $\Delta\mathbb{V} = 0.05 \pm 0.04$  on average  
 78 over all transformations (cf. SI Section 4), thus demonstrating accurate control over  $4 \times 2$   
 79 linear transformations across spatial-polarisation degrees of freedom.

80 We now illustrate the use of our experimental platform to simulate coherent absorption,  
 81 an intriguing phenomenon in quantum transport [27]. A typical case is the effect of a lossy  
 82 beamsplitter on a two-photon N00N state  $(s|2,0\rangle + e^{2i\phi}|0,2\rangle)/\sqrt{2}$ . This produces a two-  
 83 photon absorption probability that depends on the phase  $\phi$ . The phenomenon has been  
 84 recently demonstrated using a bulk-optics setup with an absorptive graphene layer [27] and  
 85 a plasmonic metamaterial [28, 29].

86 In our work, we use our fibre platform to simulate the coherent absorption experiment  
 87 (Fig. 3a), where the transformation  $\mathcal{L}(\phi, \alpha)$  can be seen as a succession of three linear  
 88 operations: (i) indistinguishable photons are sent onto a beamsplitter to generate a N00N  
 89 state (N=2) with a controllable output phase  $\phi$ ; (ii) the N00N state interacts with a lossy  
 90 phase-tunable beamsplitter (LTBS). The matrix that describes the action of the LTBS is  
 91  $t \begin{pmatrix} 1 & e^{i\alpha} \\ e^{i\alpha} & 1 \end{pmatrix}$  where  $t \leq 0.5$  is the transmission coefficient and  $\alpha$  is a fully tunable phase [27]; (iii)  
 92 the two output ports of the LTBS are distributed into four output ports by two balanced  
 93 beamsplitters in order to measure two-photon survival probability. This overall survival  
 94 probability is defined as a sum of probabilities of detecting two photons in all possible  
 95 output combinations of the LTBS, i.e., both photons on either ports (up or down) or one  
 96 photon at each port.

97 As shown in Fig. 3b, the effect of coherent absorption is maximised for  $\alpha = p\pi, p \in \mathbb{Z}$   
 98 (red line). In the case where the relative phase  $\phi = q\pi, q \in \mathbb{Z}$ , which corresponds to having  
 99 a state  $(|2,0\rangle + |0,2\rangle)/\sqrt{2}$  as input, the output state is a superposition of vacuum- and  
 100 two-photon state and the probability of one-photon transmitting to the targeted outputs is  
 101 null. This result hence exhibits the non-linear behaviour of the two-photon absorption in the  
 102 quantum regime. On the other hand, when  $\phi = q\pi + \pi/2$ , thus corresponding to an input  
 103 state  $(|2,0\rangle - |0,2\rangle)/\sqrt{2}$ , only single-photon loss occurs (cf. SI Section 5 for details). Owing  
 104 to our ability of fully control the relative phase  $\alpha$  (Fig. 3c), which is a significant step forward  
 105 with respect to previous experimental arrangements [27–29], we observe a transition of the  
 106 coherent absorption phenomenon from unitary  $\alpha = q\pi + \pi/2$  (blue dots) to the maximal  
 107 coherent absorption situation  $\alpha = \pi$  (red dots).

108 Partial control, which is usually deleterious for a quantum system, here provides the  
 109 ability to coherently control the interaction in a non-unitary way, which can be exploited  
 110 for processing tasks [30]. Note that, as the optical system (SLM and MMF) is nearly loss-  
 111 less, and non-unitarity in our experiment originates from the fact that we control only half

112 of the propagation modes of the MMF in each input port (cf. SI Section 3 for explana-  
113 tion). The unmonitored modes thus embody a sink where information about the desired  
114 optical network leaks, resulting in effective open system dynamics of the latter. The total  
115 energy transmittance  $2|t|^2$  to all targeted outputs of the optical network  $\mathcal{L}_i$  reaches 0.45(0.5)  
116 experimentally (theoretically), which is close to the maximum transmission of the LTBS.

117 The dimensionality of our platform can in principle be scaled up, as the main limit-  
118 ing factor in our experimental implementation is given by the detection architecture. A  
119 significantly larger network can be managed, for instance, by replacing our detection appa-  
120 ratus with an array of correlation detectors [31]. In Fig. 4, we experimentally showcase the  
121 scalability of our platform by designing a larger optical network with 18 targeted outputs  
122 allocated arbitrarily at different positions and taking arbitrary polarisation on the EMCCD  
123 camera. In SI Section 3, we discuss the fidelity, scalability and programmability of this  
124 optical network architecture.

125 We report the use of a multimode fibre to implement fully programmable linear optical  
126 networks across spatial and polarisation degrees of freedom. This platform harnesses the  
127 highly complex coupling between a large number of modes of the MMF, thanks to the ability  
128 to spatially control the input light wavefront. We successfully programmed this platform  
129 to implement circuits able to tackle certification tasks all the way up to the emulation of  
130 coherent absorption. We thus demonstrate the versatility and full reconfigurability of our  
131 approach, including the management of different degrees of freedom of the propagating light.  
132 Complex mixing occurring in an optical mixer, in general, can go beyond path and polar-  
133 isation reported in this work. Spectral, temporal, and spatial (radial and orbital angular  
134 momentum) degrees of freedom can also be manipulated [14, 16]. We anticipate that our  
135 architecture can be applied to those degrees of freedom. We also highlight its scaling po-  
136 tential by demonstrating control over up to 18 output ports, whereas the number of input  
137 ports can also be scaled well beyond 2, provided a multi-photon source is available. Our  
138 architecture provides an efficient and scalable alternative to integrated circuits for linear  
139 quantum networks.

## 140 METHODS

### 141 Two-photon source

142 The frequency-degenerate photon pairs are produced from a type-II polarisation-separable  
143 collinear spontaneous parametric down-conversion (SPDC) source (Fig. 1a), using a 10-mm  
144 periodically poled potassium titanyl phosphate crystal (ppKTP) pumped by a single-mode  
145 continuous-wave laser in a single spatial mode configuration. The photon pairs transmit  
146 through a spectral filter ( $\lambda = 810 \pm 5$  nm) and are separated by a polarising beamsplitter.  
147 The indistinguishability of photon pairs is controlled by a temporal delay  $\delta$ . The photon  
148 pairs are then prepared in the same horizontal polarisation, and collected with polarisation-  
149 maintaining single-mode fibres, which are then connected to the MMF platform. A coinci-  
150 dence window is set at 2.5 ns for all experiments. All coincidence counts are corrected for  
151 accidental coincidence counts.

### 152 Network programming

153 After the TM acquisition using a phase-shifting holographic technique with a co-  
154 propagating reference [32] (cf. SI Section 1), a given linear transformation  $\mathcal{L}_i$  (network) is  
155 programmed. The input electric fields  $\tilde{E}_{\text{in}}^{(j)}$  and the corresponding SLM phase pattern for  
156 each  $j$ -th input port is calculated by solving an inverse scattering problem  $\tilde{E}_{\text{in}}^{(j)} = \mathbb{T}^{(j)\dagger} \mathcal{L}_i^{(j)}$ ,  
157 where  $\mathbb{T}^{(j)}$  is the sub-part of the measured TM linking the relevant input modes for each  
158  $j$ -th input port to the targeted output modes. Imperfections in generating the input electric  
159 fields  $\tilde{E}_{\text{in}}$  with the spatial light modulator (SLM) lead to errors in the coefficients of the  
160 linear transformation  $\mathcal{L}_i$ . In the case of our first experiment (the control of two-photon  
161 interference), we additionally performed an amplitude correction when a new  $\mathcal{L}_i$  is pro-  
162 grammed by adjusting on the amplitudes of the co-propagating reference fields. This was  
163 done by means of minimising the mean squared error between implemented amplitudes and  
164 desired ones. For the experiment on the control of the coherent absorption, we compensated  
165 the amplitude variations using the normalised second-order correlation function  $g^{(2)}$ .

166 **DATA AVAILABILITY STATEMENT**

167 The data that support the plots within this paper and other findings of this study are  
168 available from the corresponding author upon reasonable request.

169 **CODE AVAILABILITY STATEMENT**

170 The code for data analysis and simulation that support the plots within this paper and  
171 other findings of this study are available from the corresponding author upon reasonable  
172 request.

173 **COMPETING FINANCIAL INTERESTS**

174 The authors declare no competing financial interests.

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## 252 STATEMENT OF AUTHOR CONTRIBUTION

253 SL, TJ, HD carried out the experiment and the analysis of the data, SL LI performed  
254 numerical simulations and LI, AF, MP provided a theoretical analysis of the results. SL  
255 proposed the coherent absorption experiment. SG proposed the original idea and supervised  
256 the project. All authors discussed the implementation, the experimental data and the results.  
257 All authors contributed to writing the paper.

## 258 FIGURE CAPTIONS

FIG. 1. Multimode-fibre based programmable linear-optical network (a) Conceptual schematics of the apparatus. Photon pairs produced by spontaneous parametric down-conversion (SPDC) are injected into a multimode fibre (MMF) along orthogonal polarisation using spatial light modulators (SLMs). We use commercial MMF (Thorlabs, GIF50C) as a tool to achieve mode mixing. The transmission matrix (TM) is measured across spatial and polarisation modes of the MMF (cf. Section I). The wavefront corresponding to a desired linear transformation  $\mathcal{L}_i$  is calculated and displayed on the SLMs (cf. Methods). Output ports of interest are selected by two single-mode fibre-based polarisation beamsplitters (fPBS) mounted on translation stages. These correspond to two spatial modes and two polarisations labelled as (H1, V1, H2, V2). Light is detected by avalanche photodiodes (APDs) connected to a coincidence electronics. The output plane of the MMF is imaged onto an electron multiplying charge-coupled device (EMCCD) camera along both polarisations (H and V). (b) An arbitrary  $4 \times 2$  linear network  $\mathcal{L}_i$  is implemented by shaping the spatial phases of each input port  $H_{in}$  and  $V_{in}$ . For each input, the predicted output fields after propagation through the MMF are shown. We observe that light is focused into the four targeted output ports with the desired amplitudes and phases. (L: lenses, F: filter, HWP: half wave plate, PBS: polarising beamsplitter, D: Iris diaphragm, FM: Flip Mirror, WP: Wollaston prism, BS: beamsplitter.)

FIG. 2. Control of two-photon interference among spatial-polarisation degrees of freedom (a) Two-photon interference: fitting (solid lines) and experiment (dots) for Fourier  $\mathcal{L}_F^{(1,2)}$ , Sylvester  $\mathcal{L}_{Sy}^{(1,2)}$ , and non-unitary  $\mathcal{L}_N^{(1,2)}$  transformations where the two-photon state is coupled to the (1,2) input pair. (b) Visibility pattern of four-dimensional Fourier (F), Sylvester (Sy) and non-unitary (N) transformation for all input-output combinations. This corresponds to 18 balanced 4x2 optical networks with fully controllable phase relations.

FIG. 3. Controlled coherent absorption (a) The linear network  $\mathcal{L}(\phi, \alpha)$  programmed in the MMF (Fig.1) emulates the following circuit: Photon pair enters a Mach-Zehnder (MZ) interferometer composed of a balanced beamsplitter and a lossy balanced phase-tunable beamsplitter (LTBS). Both the phase  $\phi$  between the two arms and the phase  $\alpha$  of the LTBS can be tuned at will. Light in each output port of the MZ interferometer is analysed via two balanced beamsplitters preceding an array of four photouncounters to measure the probability of two-photon survival at the targeted output ports. (b) Probability of two-photon survival at the targeted outputs: theory (solid lines) and experiment (dots). The blue dots are for  $\alpha = \pi/2$ , corresponding to an emulated lossless MZ interferometer. The corresponding probability of two-photon survival is independent of  $\phi$ . The red dots are for  $\alpha = \pi$ , corresponding to a lossy beamsplitter in which the probability of two-photon survival depends on the relative phase  $\phi$ . (c) Probability of two-photon survival as a function of  $\phi$  and  $\alpha$ , showing a transition from emulated lossless to lossy LTBS.

FIG. 4. Intensity image of a high-dimensional linear-optical network on the EMCCD. The SPDC light from both inputs is simultaneously distributed into 18 targeted outputs, 9 in each polarisation (H: Horizontal; V: Vertical).