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Programs for Computation of Velocities and Streamlines on a Blade-to-Blade Surface of a Turbomachine

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This paper describes Fortran programs that give the solution to the two-dimensional, subsonic, nonviscous flow problem on a blade-to-blade surface of revolution of a turbomachine. Flow may be axial, radial, or mixed. There may be a change in stream channel thickness in the through-flow direction. Either single, tandem, or slotted blades may be handled as well as blade rows with splitter vanes. Also, small regions may be magnified to give more detail where desired, such as around a leading or trailing edge or through a slot. The method is based on a finite difference solution of the stream function equations. Numerical examples are shown to illustrate the type of blades which can be analyzed, and to show results which can be obtained. Results are compared with experimental data.

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INTRODUCTION

In the analysis and design of blades for compressors or turbines, it is desirable to obtain the velocity distribution through the passage and particularly over the blade surface. The current trend is to smaller diameter compressors and turbines with fewer stages and fewer blades per stage. This results in widely spaced blades with less of the passage within a guided channel between the blades. These trends also tend to increase diffusion. Higher diffusion can be tolerated with a tandem blade or a slotted blade. However, these types of blades also have poorly defined guided channels. The velocity distribution is readily obtained within a guided channel by stream filament techniques (1);[1] however, other techniques are required to extend the solution to the unguided portion of the channel.

For the unguided portion of the passage,

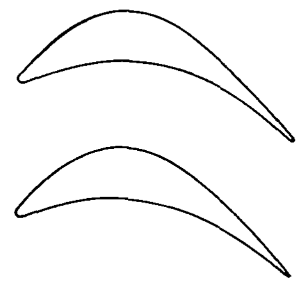


Fig.1 Typical single blade row

finite-difference methods have been used to solve the stream function ewuation. Stanitz (2,3) obtained finite-difference solutions for compressible flows through turbomachinery without the use of a computer. Kramer (4,5) has obtained finite-difference solutions for incompressible flow through centrifugal pumps, using a computer for the solution of the finite-difference equations for the stream function. More recently, FORTRAN programs have been written by the authors to perform, in addition to the solution of the finite-difference equations, the calculation of the coefficients and the differentiation of the stream functions to obtain the velocities for compressible flow (6-8).

This paper describes three programs for the computation of velocities and streamlines on the blade-to-blade surface of a turbomachine. These programs may be used with any type of single or tandem bladed turbomachine (compressor, turbine, or pump) and the flow may be axial, radial, or mixed. Numerical examples are shown to illustrate the type of blades that can be analyzed and to show results that can be obtained. Results are compared with experimental data.

[1] Underlined numbers in parentheses designate References at the end of the paper.

NOMENCLATURE

- b = stream channel thickness normal to meridional streamline (Fig.3)
- m = meridional streamline distance (Figs.2 and 3)
- r = radius from axis of rotation (Fig.3)
- u = stream function
- W = fluid velocity relative to blade
- w = mass flow per blade flowing through stream channel
- α = angle between meridional streamline and axis of rotation (Fig.3)
- θ = angular coordinate (Fig.2)
- ρ = density
- ω = rotational speed

Subscripts

- cr = critical velocity
- m = meridional component
- θ = tangential component

GENERAL DESCRIPTION OF PROGRAMS

The first program [2DCP, reference (6)] gives the ideal flow solution for a single, unslotted blade row (Fig.1). Since the stream function used is a function of meridional streamline distance, m, and angular coordinate θ (Fig.2) flow

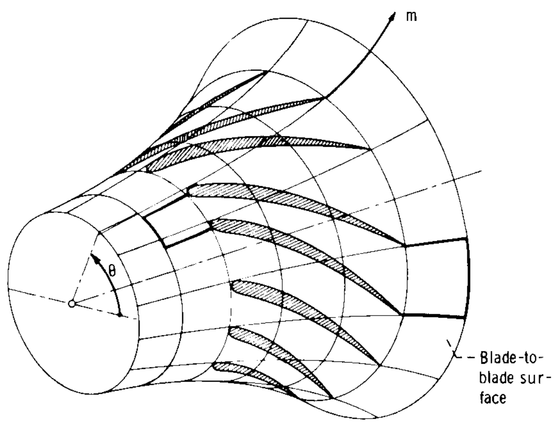


Fig.2 Blade-to-blade surface of revolution showing m-θ coordinates

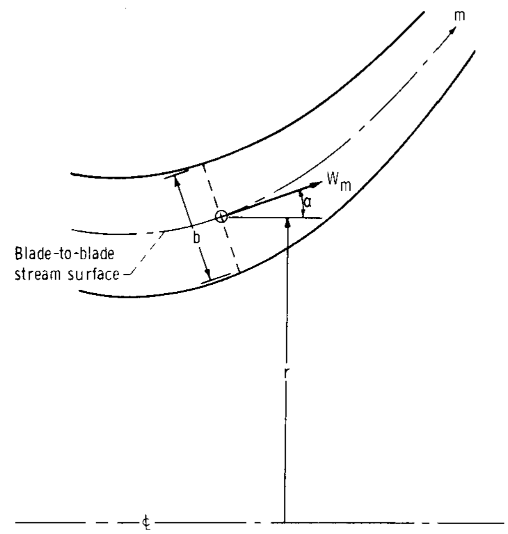


Fig.3 Stream channel in meridional flow plane

through the blade row may be axial, radial, or mixed. Flow may be compressible or incompressible and the machine may be rotating or stationary. Stationary cascades may be circular or straight (infinite). Stream channel thickness normal to the meridional streamline may vary with respect to the streamline distance, m (Fig.3).

The second program [TANDEM, reference (7)] gives the same type of solution for a tandem of slotted blade (Fig.4). Tandem blades may be either overlapping or non-overlapping in the meridional flow direction. Tandem blades are considered overlapping when the leading edge of the rear blade is further upstream than the trailing edge of the front blade. A single blade row with slots through the blades, Fig.4(b), or a single blade row with one set of splitter vanes (Fig.5), is analyzed as a tandem blade machine.

The third program [MAGNFY, reference (8)] obtains a magnified solution about the leading or trailing edge of a single or tandem blade, Fig. 6(a). It may also be used in the slot region of a tandem or slotted blade, Fig.6(b). A magnified solution is required since finite-difference techniques give a solution for velocities only at the mesh points of a grid between the blades. Due to storage limitations of the computer, grid spacing is generally too large to give the desired detail around small leading and trailing edge radii and within slot regions. Yet, it is in these areas where geometry, and thus velocities, change most rapidly. Therefore, a technique is required which reduces the mesh size in order to give a more detailed solution.

Input

All three programs require somewhat the same input. In general, it consists of blade geometry, stream channel geometry, gas constants and flow conditions, and a description of the solution region. Any consistent set of units, e.g. English or SI units, may be used.

Blade geometry is given as coordinates of the blade surfaces in the m-θ plane (Fig.2). Blade chord, stagger, and leading and trailing edge radii are also required.

The geometry of the stream channel is given in the meridional (m-r) plane (Fig.3). Coordinates of the stream channel centerline are required, as well as the thickness distribution in the meridional flow direction. Other input includes stream channel blade-to-blade weight flow, wheel speed, and average inlet and outlet flow angles.

The tandem blade program requires that the user also specify the portion of blade-to-blade weight flow which passes over the top of the rear or tandem blade, i.e., through the slot.

In order to use the magnification program, the user must first run either 2DCP or TANDEM. The input to MAGNFY then consists of two things: the input to 2DCP or TANDEM, plus boundary values which are obtained as output from one of these programs. A magnification factor is also required.

Output

The principal output is similar for all three programs. It consists of the following:

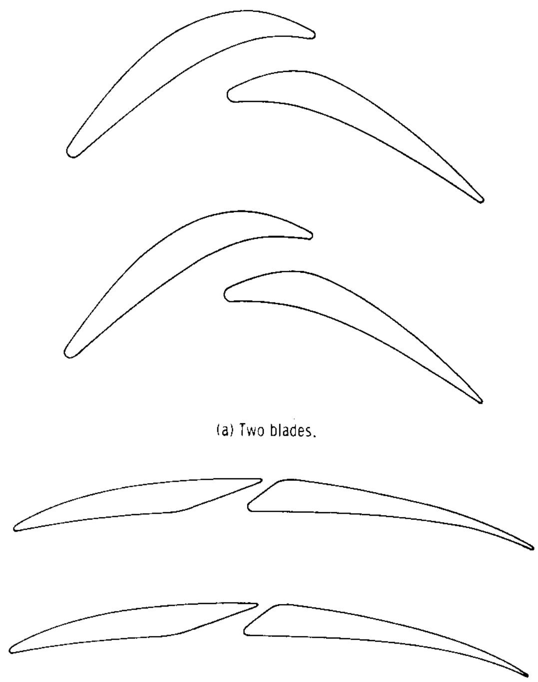
1 Stream function solution at each unknown mesh point in the solution region.

2 Coordinates of streamlines in the blade-to-blade passage, including all stagnation streamlines (except for MAGNFY).

3 Streamline plot (except for MAGNFY).

4 Velocities and flow angles at all mesh points in the solution region.

5 Blade surface velocities on all surfaces.



(a) Two blades.

(b) Slotted single blade.

Fig.4 Typical tandem blades

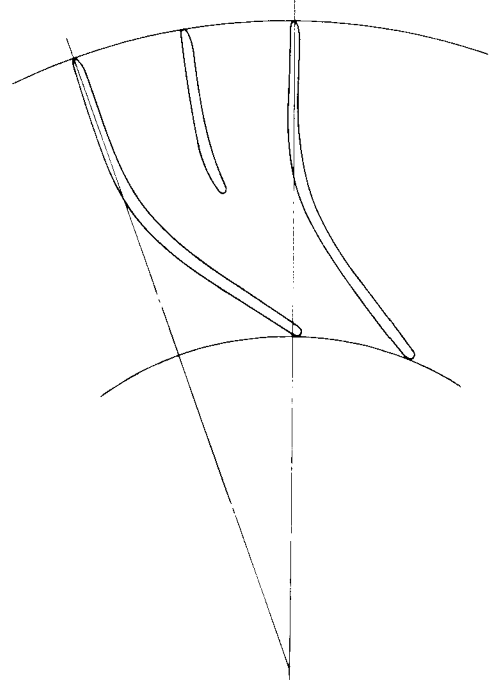


Fig.5 Mixed flow impeller blades with splitter vane

6 Plot of blade surface velocities.

In addition, all three programs generate less important geometrical output prior to computing the principal results listed in the foregoing. In all cases, the user can delete any output which he does not wish to obtain.

MATHEMATICAL ANALYSIS

The following assumptions are used in deriving equations for the three programs:

1 The flow is steady relative to the blade.

2 The fluid is a perfect gas, or else it is incompressible.

3 The fluid is nonviscous.

4 There is no loss of energy.

5 The flow is absolutely irrotational.

6 The blade-to-blade surface is a surface of revolution. (This does not exclude straight cascades, however.)

7 The velocity component normal to the blade-to-blade surface is zero.

8 The stagnation temperature is uniform across the inlet.

9 The velocity magnitude and direction are uniform along the upstream and downstream boundaries.

10 For tandem blades, the portion of flow between the front and rear blades is known.

11 The relative velocity is subsonic everywhere.

In order to make the analysis applicable to either axial, radial, or mixed flow turbomachines, the independent variables are chosen to be the meridional streamline distance, m, and the angular coordinate, θ. These coordinates are shown in Fig.2. A stream channel is defined by specifying a stream channel thickness, b, as shown in Fig.3.

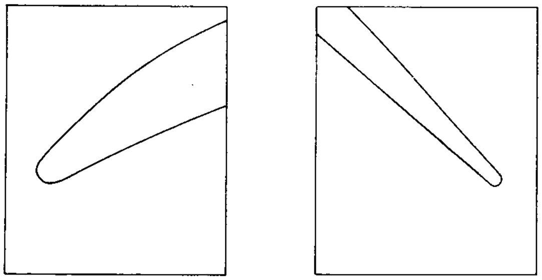
For the mathematical formation of the problem the stream function is used. The stream function u satisfies

$$\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial m^2} - \frac{1}{r^2}\frac{1}{\rho}\frac{\partial \rho}{\partial \theta}\frac{\partial u}{\partial \theta} + \left[\frac{\sin\alpha}{r} - \frac{1}{b\rho}\frac{\partial (b\rho)}{\partial m}\right]\frac{\partial u}{\partial m} = \frac{2b\rho}{w}\sin\alpha \qquad (1)$$

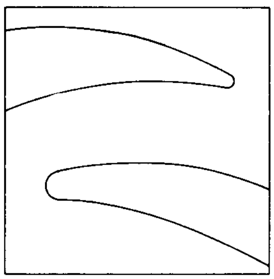
This may be obtained from equation 12(9) of reference (9) by letting $u = -\psi/w$, where ψ is the stream function as defined in reference (9). The stream function u has the value 0 on the upper surface of a blade and 1 on the lower surface of a blade.

Also, the derivatives of the stream function satisfy

$$\frac{\partial u}{\partial m} = -\frac{b\rho}{w} W_\theta \qquad (2)$$



(a) Leading or trailing edge.



(b) Slot region.

Fig.6 Regions for magnified solution

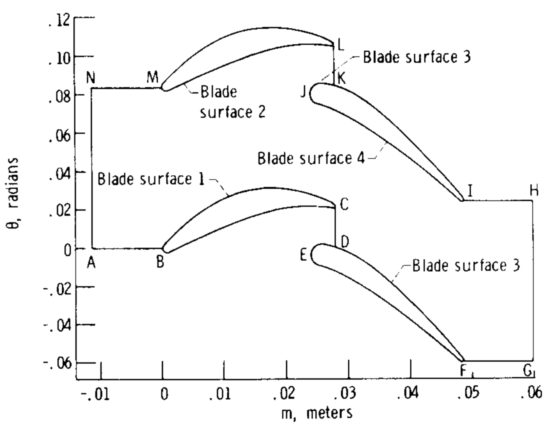


Fig.7 Blade-to-blade flow region for tandem axial turbine rotor

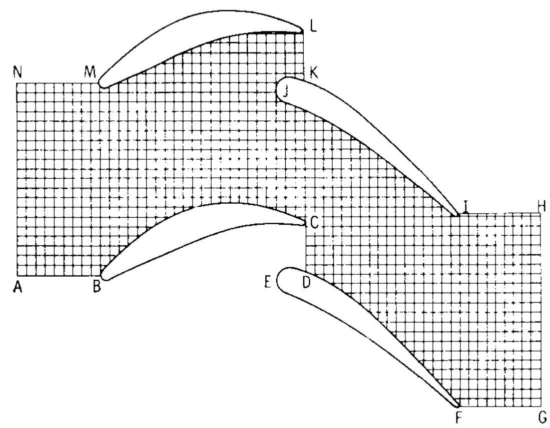


Fig.8 Mesh used for tandem axial turbine rotor

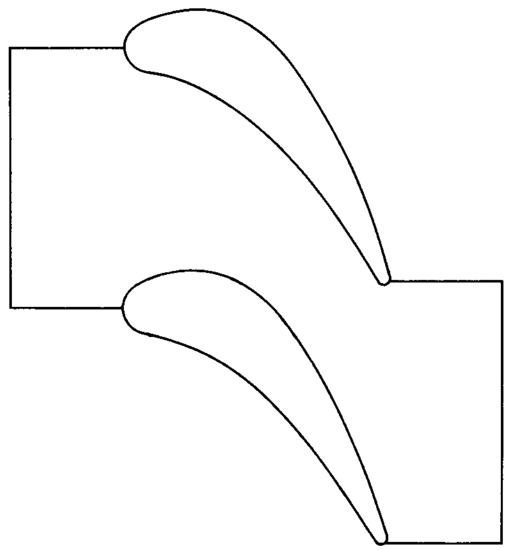


Fig.9 Single blade row axial stator showing solution region

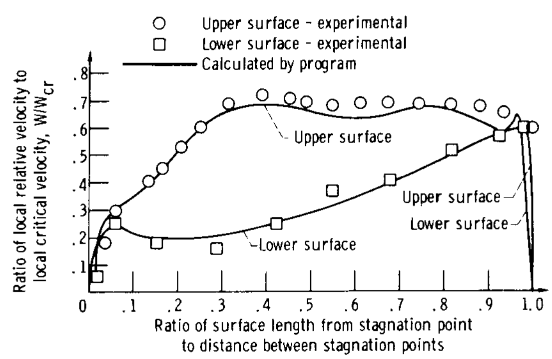


Fig.10 Blade surface velocities on axial stator

$$\frac{\partial u}{\partial \theta} = \frac{b \rho r}{w} W_m \qquad (3)$$

For the solution of equation (1), a finite region is considered. This is shown in Fig.7 as boundary ABCDFGHIJKLMNA. The solution to equation (1) is determined by boundary conditions along all sides of this region. Three types of boundary conditions are given.

1 Along any blade surface, the value of the stream function is known.

2 Along the upstream and downstream boundaries (AN and GH in Fig.7), the flow is uniform and the flow angle is known.

3 Along the remaining upper and lower boundaries, the flow along the upper boundary is the same as the flow along the lower boundary.

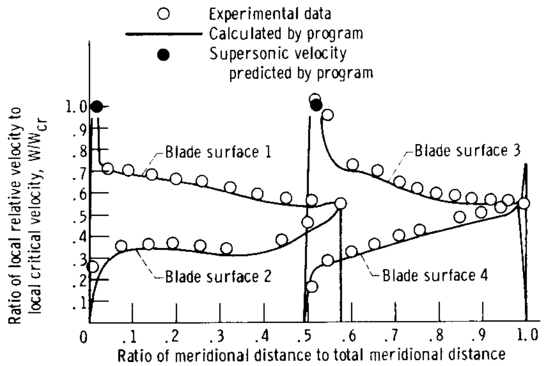


Fig.11 Blade surface velocities on tandem axial turbine

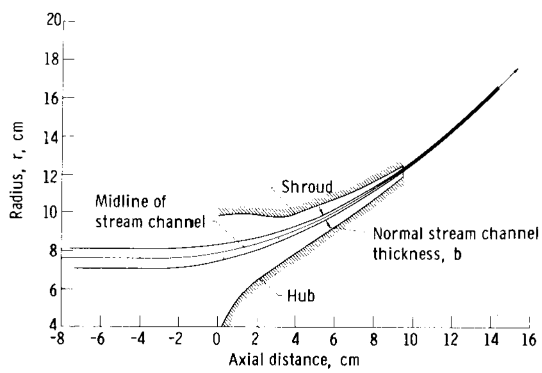


Fig.12 Hub-shroud profile of mixed flow pump impeller, showing meridional section of stream tube

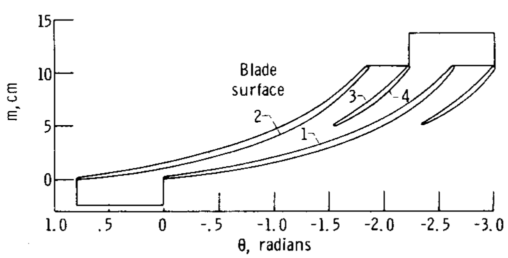


Fig.13 Blade-to-blade flow region for mixed flow pump impeller

The solution to equation (1) with these boundary conditions is obtained by solving finite-difference equations as described in references (6-8). For this purpose, a mesh region (Fig.8) is generated automatically by the programs based on mesh spacing given as input. After computing a numerical solution to equation (1) in a given flow region, the velocity at any point can be computed from equations (2) and (3) by using numerical differentiation. The streamlines are located by the contours of equal stream-function values.

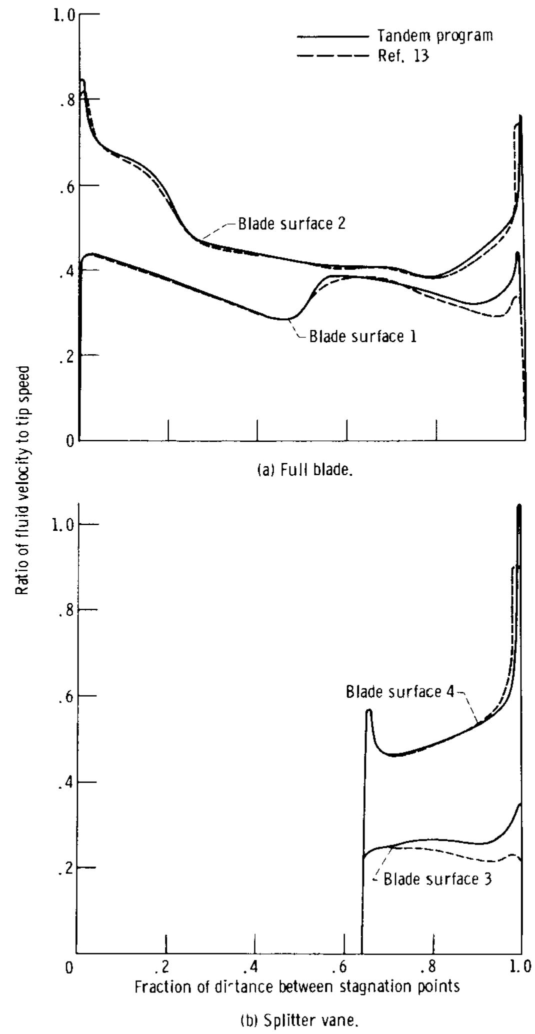


Fig.14 Velocity distribution for mixed flow pump impeller

NUMERICAL EXAMPLES

Four examples are given to illustrate the use of the programs:

1 A single-blade-row axial stator run on 2DCP.

2 A tandem-blade axial turbine rotor run on TANDEM.

3 A mixed flow impeller with splitter vanes run on TANDEM.

4 The leading edge of the rear blade of a tandem axial turbine rotor, run on MAGNFY.

Axial Turbine Stator

The first example is the mean blade section of a stator (Fig.9), for a turbine built at Lewis Research Center (10). The velocities are plotted against blade surface length in Fig.10. Also on the figure are experimental data obtained from the investigation described in reference (10). Com-

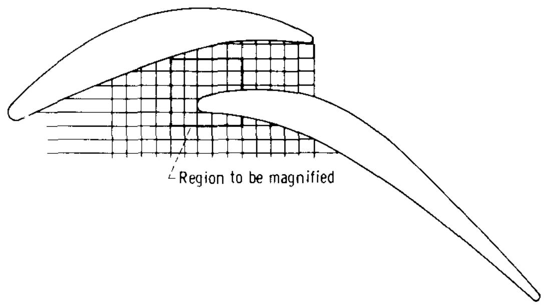


Fig.15 Tandem axial turbine with region to be magnified

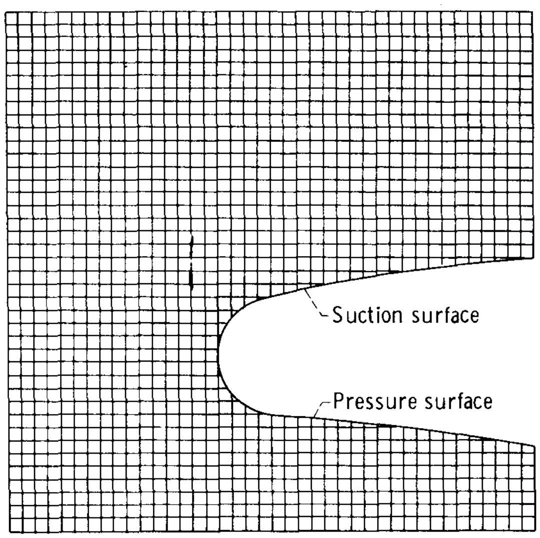


Fig.16 Magnified region of Fig.15 with reduced mesh size

puted values agree well with experimental data for this example.

Tandem Axial Turbine Rotor

This example is a two-dimensional cascade of turbine rotor blades currently undergoing testing at the Lewis Research Center. This blade is a modified version of the design reported in reference (11). It has a blunt leading edge on the rear blade in order to achieve a converging channel, and it has a wider slot than that reported in reference (11).

The blade shape in m-θ coordinates and the blade-to-blade solution regions are shown in Figs. 7 and 8. Results for blade surface W/W_{cr} are plotted in Fig.11, where comparison is made with experimental data for the Lewis turbine cascade (12). There is close agreement between computed and experimental values on all four blade surfaces. Fig.11 indicates supersonic velocities at the leading edges of both blades. The presence, but not

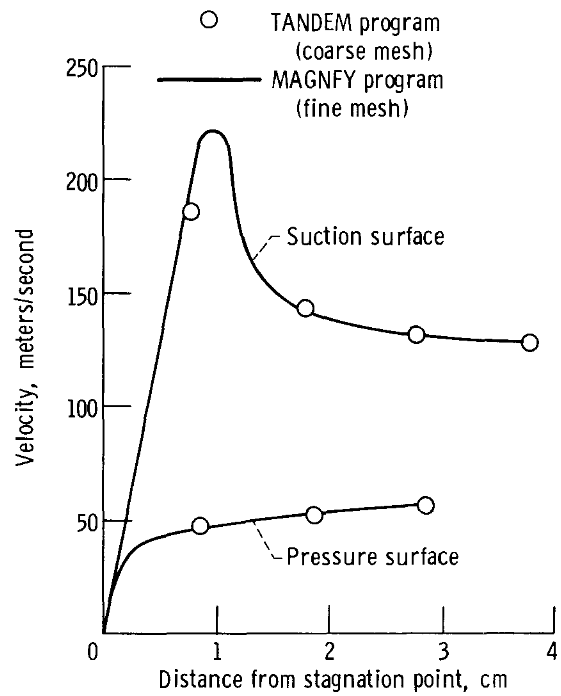


Fig.17 Comparison of velocities from coarse mesh and fine mesh solutions

the magnitude, of such velocities is predicted by the program. (All three programs operate in this manner when supersonic flow occurs over a blade surface.)

Mixed Flow Impeller with Splitter Vanes

This example is a comparison with an example from reference (13). In that reference a similar stream function analysis was made on the same impeller, but the mesh was set up graphically and the finite difference coefficients were calculated by hand. The resulting equations were then solved by relaxation on a computer.

The hub-shroud profile of the impeller is shown in Fig.12. The blade shape in m-θ coordinates, and the blade-to-blade solution region are shown in Fig.13. In Fig.14, the blade surface velocities obtained by TANDEM are compared with those obtained originally in reference (13). There is good agreement over most of the blades. Minor discrepancies are due to slight differences in the boundary conditions, weight flow split, and downstream flow angle. The exact magnitudes of the peaks near the leading and trailing edges are uncertain because of the small radii compared to the mesh spacing.

Leading Edge of Tandem Blade Turbine Rotor

Flow about the leading edge of the rear blade of a tandem axial turbine rotor blade (11) has been analyzed to illustrate the use of MAGNFY.

The entire blade (Fig.15) was first analyzed using TANDEM with the mesh size shown. Due to computer storage limitations, this was as fine a mesh as could be obtained with TANDEM. However, more detail was desired for velocities between adjacent mesh points on the leading edge of the tandem blade. Therefore, MAGNFY was used in order to reduce the mesh spacing in the region around the leading edge by a factor of 8, as shown in Fig.16.

Blade surface velocities from both TANDEM and MAGNFY are plotted in Fig.17. The original velocities obtained using the coarse mesh are denoted by circles in Fig.17. The MAGNFY output are plotted as a solid line. This illustrates the need for a finer grid in some parts of the solution region. The figure shows that the velocities denoted by circles do not define the velocity curve adequately.

CONCLUDING REMARKS

Three general purpose programs for blade-to-blade analysis of velocities in turbomachines have been described. These programs are described in detail in references (6-8), including input forms, listing of program, and program documentation. The source programs are available through COSMIC (Computer Software Management and Information Center), Computer Center, University of Georgia, Athens, Georgia 30601. Program numbers can be obtained from the authors.

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