

# Progressive Damage Modeling In Fiber-Reinforced Materials

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## Outline

Overview of Damage Model for Fiber-Reinforced Materials

Example

## Damage Model for Fiber-Reinforced Materials: Overview

- The damage in the material is anisotropic
- Four different failure modes are taken into account: fiber tension, fiber compression, matrix tension, matrix compression
- The behavior of the undamaged material is linearly elastic
- The model must be used with elements with a plane stress formulation (plane stress, shell, continuum shell, and membrane elements)
- Post-localization response is regularized (Crack Band Model)
- The model can be used in conjunction with a viscous regularization scheme to improve the convergence rate in the softening regime

## Damage Initiation (Hashin's criteria)

MODE I: fiber tension

$$f_I = \left( \frac{\hat{\sigma}_{11}}{X^T} \right)^2 + \alpha \left( \frac{\hat{\sigma}_{12}}{S^L} \right)^2, \quad \text{where } 0 \leq \alpha \leq 1 \quad (1)$$

MODE III: matrix tension

$$f_{III} = \left( \frac{\hat{\sigma}_{22}}{Y^T} \right)^2 + \left( \frac{\hat{\sigma}_{12}}{S^L} \right)^2 \quad (3)$$

MODE II: fiber compression

$$f_{II} = \left( \frac{\hat{\sigma}_{11}}{X^C} \right)^2 \quad (2)$$

MODE IV: matrix compression

$$f_{IV} = \left( \frac{\hat{\sigma}_{22}}{2S^T} \right)^2 + \left[ \left( \frac{Y^C}{2S^T} \right)^2 - 1 \right] \frac{\hat{\sigma}_{22}}{Y^C} + \left( \frac{\hat{\sigma}_{12}}{S^L} \right)^2 \quad (4)$$

$S^L, S^T, X^T, X^C, Y^T, Y^C$  – material strengths

**Damaged Material Response (Matzenmiller, Lubliner, Taylor)****□ Effective Stress**

$$\hat{\boldsymbol{\sigma}} = \mathbf{M} \boldsymbol{\sigma} \quad (1)$$

$\mathbf{M}$  – damage operator

$$\mathbf{M} = \begin{bmatrix} \frac{1}{1-d_f} & 0 & 0 \\ 0 & \frac{1}{1-d_m} & 0 \\ 0 & 0 & \frac{1}{1-d_s} \end{bmatrix} \quad (2)$$

$d_f, d_m, d_s$  – damage variables

## Damaged Material Response, cont.

### □ Damaged Compliance Matrix:

$$\mathbf{H} = \begin{bmatrix} \frac{1}{(1-d_f)E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{(1-d_m)E_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_s)G} \end{bmatrix} \quad (1)$$

$E_1, E_2, G_s$  – undamaged moduli

$\nu_{12}, \nu_{21}$  – Poisson's ratios

### □ Damaged Elasticity Matrix:

$$\mathbf{C} = \frac{1}{D} \begin{bmatrix} (1-d_f)E_1 & (1-d_f)(1-d_m)\nu_{21}E_1 & 0 \\ (1-d_f)(1-d_m)\nu_{12}E_2 & (1-d_m)E_2 & 0 \\ 0 & 0 & D(1-d_s)G \end{bmatrix} \quad (2)$$

where  $D = 1 - (1-d_f)(1-d_m)\nu_{12}\nu_{21} > 0$

$$d_s = 1 - (1-d_{ft})(1-d_{fc})(1-d_{mt})(1-d_{mc})$$

## Damaged Material Response, cont.

### □ Gibbs Free Energy:

$$G = \frac{1}{2E_1} \left( \frac{\langle \sigma_{11} \rangle^2}{1-d_{ft}} + \frac{\langle -\sigma_{11} \rangle^2}{1-d_{fc}} \right) + \frac{1}{2E_2} \left( \frac{\langle \sigma_{22} \rangle^2}{1-d_{mt}} + \frac{\langle -\sigma_{22} \rangle^2}{1-d_{mc}} \right) - \frac{\nu_{12}\sigma_{11}\sigma_{22}}{E_1} + \frac{\sigma_{12}^2}{G(1-d_s)} \quad (1)$$

where  $\langle \rangle$  is Macauley bracket operator, defined for every  $\alpha \in \Re$  as  $\langle \alpha \rangle = \frac{(\alpha + |\alpha|)}{2}$

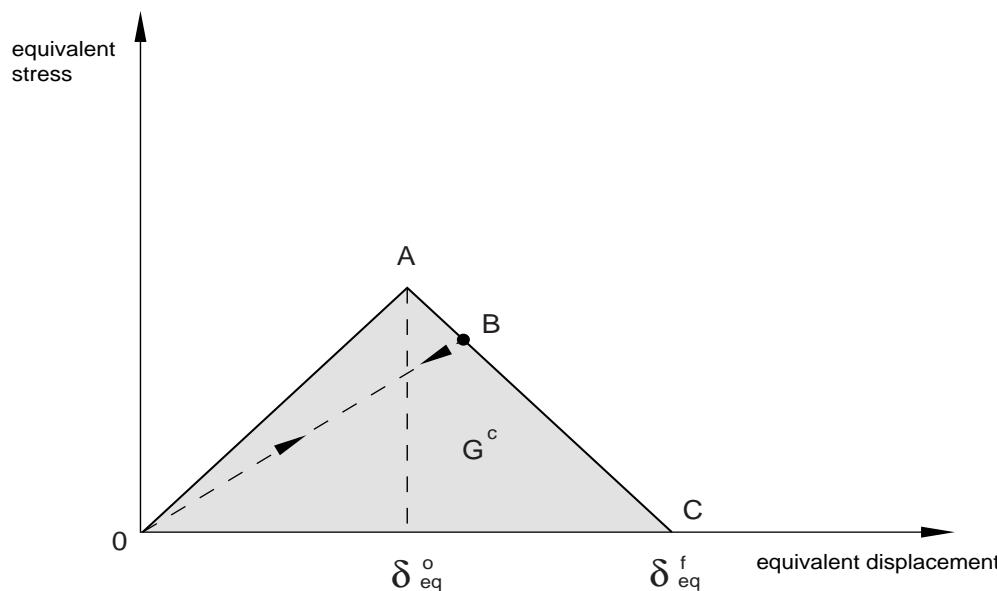
### □ Energy Dissipation:

$$\dot{D} = \sum_{i=1}^{number \ modes} Y_i \dot{d}_i \geq 0 \quad (2)$$

where  $Y_i = \frac{\partial G}{\partial d_i}$  are thermodynamic forces

## Damage Evolution

- The modeling approach is a generalization of that used to model cohesive elements, which is based on the work of C. Davila and P. Camanho
- The evolution law is based on the energy dissipated during the process
- Linear material softening is assumed



$$d = \frac{\delta_{eq}^f (\delta_{eq} - \delta_{eq}^0)}{\delta_{eq} (\delta_{eq}^f - \delta_{eq}^0)} \quad (1)$$

Figure 1.

## Equivalent Displacements and Stresses

□ Fiber Tensile Mode

$$\delta_{eq}^{ft} = \sqrt{\langle \delta_{11} \rangle^2 + \alpha \delta_{12}^2}$$

□ Fiber Compressive Mode

$$\delta_{eq}^{fc} = \langle -\delta_{11} \rangle$$

□ Matrix Tensile Mode

$$\delta_{eq}^{mt} = \sqrt{\langle \delta_{22} \rangle^2 + \delta_{12}^2}$$

□ Matrix Compressive Mode

$$\delta_{eq}^{mc} = \sqrt{\langle -\delta_{22} \rangle^2 + \delta_{12}^2}$$

$$\sigma_{eq}^{ft} = \frac{\langle \sigma_{11} \rangle \langle \delta_{11} \rangle + \alpha \sigma_{12} \delta_{12}}{\delta_{eq}^{ft}}$$

$$\sigma_{eq}^{ct} = \frac{\langle -\sigma_{11} \rangle \langle -\delta_{11} \rangle}{\delta_{eq}^{fc}}$$

$$\sigma_{eq}^{mt} = \frac{\langle \sigma_{22} \rangle \langle \delta_{22} \rangle + \sigma_{12} \delta_{12}}{\delta_{eq}^{mt}}$$

$$\sigma_{eq}^{mc} = \frac{\langle -\sigma_{22} \rangle \langle -\delta_{22} \rangle + \sigma_{12} \delta_{12}}{\delta_{eq}^{mc}}$$

where

$$\delta_{ij} = L_c \epsilon_{ij}$$

and  $L_c$  – is the characteristic length

## Damage Evolution: Procedure

- Evaluate the initiation criterion,  $f_I$
- Compute equivalent displacement and stress,  $\delta_{eq}, \sigma_{eq}$
- Compute equivalent displacement and stress at the onset of damage

$$\delta_{eq}^0 = \frac{\delta_{eq}}{\sqrt{f_I}} \quad \text{and} \quad \sigma_{eq}^0 = \frac{\sigma_{eq}}{\sqrt{f_I}}$$

- Store  $\delta_{eq}^0$  and  $\sigma_{eq}^0$
- Compute equivalent displacement at full damage

$$\delta_{eq}^f = \frac{2G_f^I}{\sigma_{eq}^0}$$

- Update the damage variable

$$d_I = \max\left(d_{I,OLD}, \frac{\delta_{eq}^f (\delta_{eq} - \delta_{eq}^0)}{\delta_{eq} (\delta_{eq}^f - \delta_{eq}^0)}\right)$$

## Mesh Dependence

### □ Example: uniaxial tension

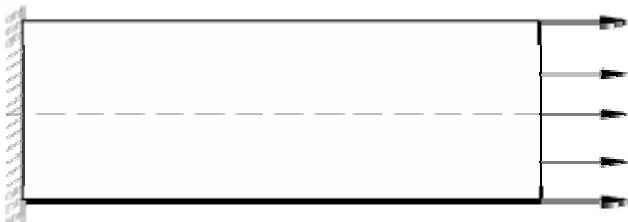


Figure 1. Bar subjected to axial load

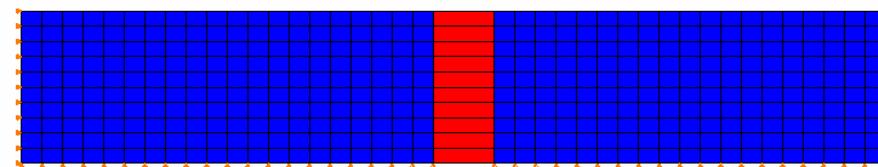
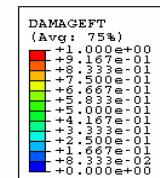


Figure 2. Localization of deformation

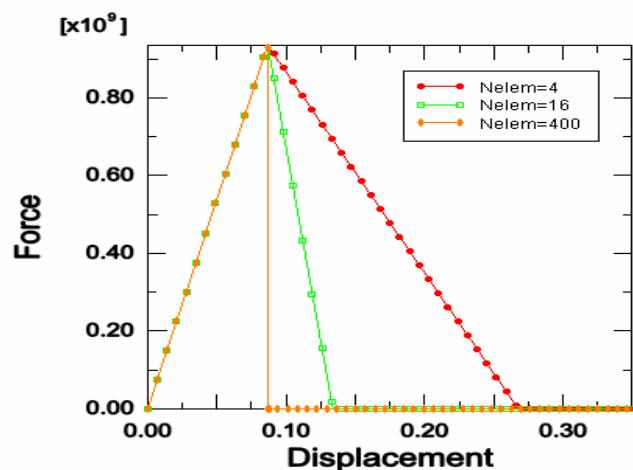


Figure 3. Force-displacement for different discretizations

- the energy dissipated is specified per unit volume
- the results are mesh dependent
- deformation localizes into one layer of elements

## Mesh Dependency, cont.

- The energy dissipated is regularized using Crack Band Model (Bazant and Oh)
  - The energy dissipated,  $G_f$ , is expressed per unit area instead of per unit volume
  - Characteristic length,  $L_c$ , is introduced, computed as

$$L_c = \sqrt{A_l}, \text{ where } A_l \text{ is the area at an integration point}$$

- The post-localization stress-displacement response is computed correctly

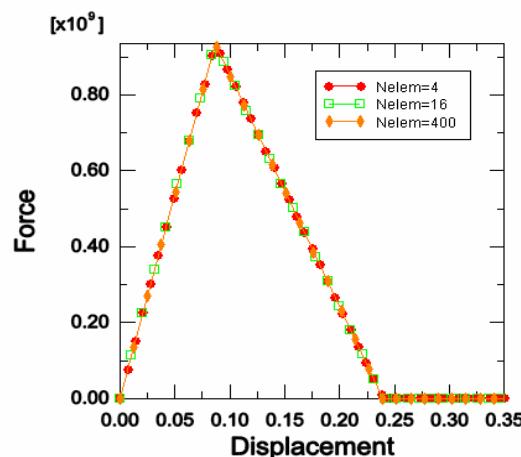


Figure 1.

## Viscous Regularization

- Generalization of the Duvaut-Lions regularization model

$$\dot{d}_{ft}^v = \frac{1}{\eta_{ft}}(d_{ft} - d_{ft}^v) \quad \dot{d}_{fc}^v = \frac{1}{\eta_{fc}}(d_{fc} - d_{fc}^v) \quad (1a-b)$$

$$\dot{d}_{mt}^v = \frac{1}{\eta_{mt}}(d_{mt} - d_{mt}^v) \quad \dot{d}_{mc}^v = \frac{1}{\eta_{mc}}(d_{mc} - d_{mc}^v) \quad (2a-b)$$

$\eta_{ft}$ ,  $\eta_{fc}$ ,  $\eta_{mt}$ ,  $\eta_{mc}$  - are viscosities

$d_{ft}^v$ ,  $d_{fc}^v$ ,  $d_{mt}^v$ ,  $d_{mc}^v$  - are used to compute damaged stiffness matrix

## Viscous Regularization, cont.

### □ Updating “Regularized” Damage Variables

$$d_I^v \Big|_{t_0 + \Delta t} = \frac{\Delta t}{\eta_I + \Delta t} d_I \Big|_{t_0 + \Delta t} + \frac{\eta_I}{\eta_I + \Delta t} d_I^v \Big|_{t_0}, \quad \text{where I denotes a damage mode} \quad (1)$$

### □ Jacobian

$$\frac{\partial \sigma}{\partial \epsilon} = C_d + \sum_I \frac{\partial C_d}{\partial d_I^v} \frac{\partial d_I}{\partial \epsilon} \frac{\Delta t}{\eta_I + \Delta t} \quad (2)$$

$$\frac{\partial d_I^v}{\partial d_I} = \frac{\Delta t}{\eta_I + \Delta t}$$

## Viscous Regularization, cont.

### □ Damage Energy

$$\Delta E_D = \frac{(\mathbf{C}_d \boldsymbol{\varepsilon})|_{t_0} + (\mathbf{C}_d \boldsymbol{\varepsilon})|_{t_0+\Delta t}}{2} \Delta \boldsymbol{\varepsilon} - \psi|_{t_0+\Delta t} + \psi|_{t_0} \quad (1)$$

$\psi$  – free energy

### □ Viscous Energy Dissipation

$$\Delta E_V = \frac{(\mathbf{C}_d \boldsymbol{\varepsilon})|_{t_0} + (\mathbf{C}_d \boldsymbol{\varepsilon})|_{t_0+\Delta t}}{2} \Delta \boldsymbol{\varepsilon} - \frac{(\mathbf{C}_d^0 \boldsymbol{\varepsilon})|_{t_0} + (\mathbf{C}_d^0 \boldsymbol{\varepsilon})|_{t_0+\Delta t}}{2} \Delta \boldsymbol{\varepsilon} \quad (2)$$

$$\mathbf{C}_d^0 = \mathbf{C}_d^0(d_I)$$

$$\mathbf{C}_d = \mathbf{C}_d(d_I^v)$$

## Output

### □ Initiation Criteria Variables

- HSNFTCRT – tensile fiber Hashin's criterion
- HSNFCCRT – compressive fiber Hashin's criterion
- HSNMTCRT – tensile matrix Hashin's criterion
- HSNCMCCRT – compressive matrix Hashin's criterion

### □ Damage Variables

- DAMAGEFT – tensile fiber damage
- DAMAGEFC – compressive fiber damage
- DAMAGEMT – tensile matrix damage
- DAMAGEMC – compressive matrix damage
- DAMAGESHR - shear damage

### □ Status

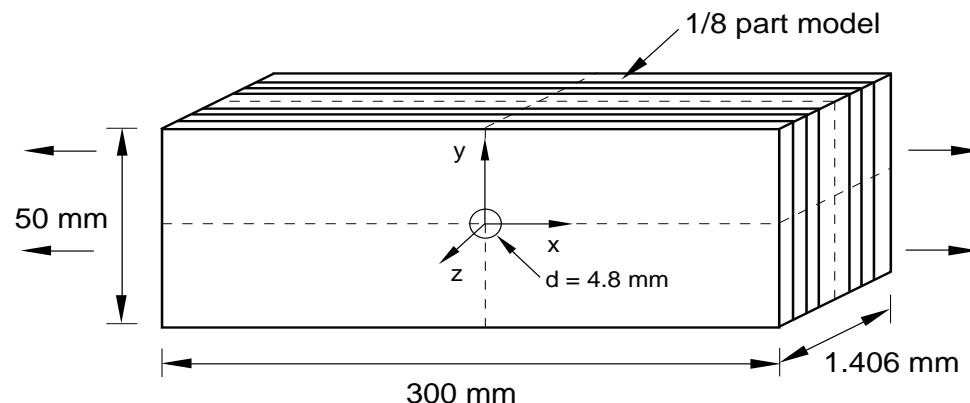
- STATUS – element status (1 – present, 0 – removed)

### □ Energies

- Damage energy (ALLDMD, DMENER, ELDMD, EDMDDEN)
- Viscous regularization (ALLCD, CENER, ELCD, ECDDEN)

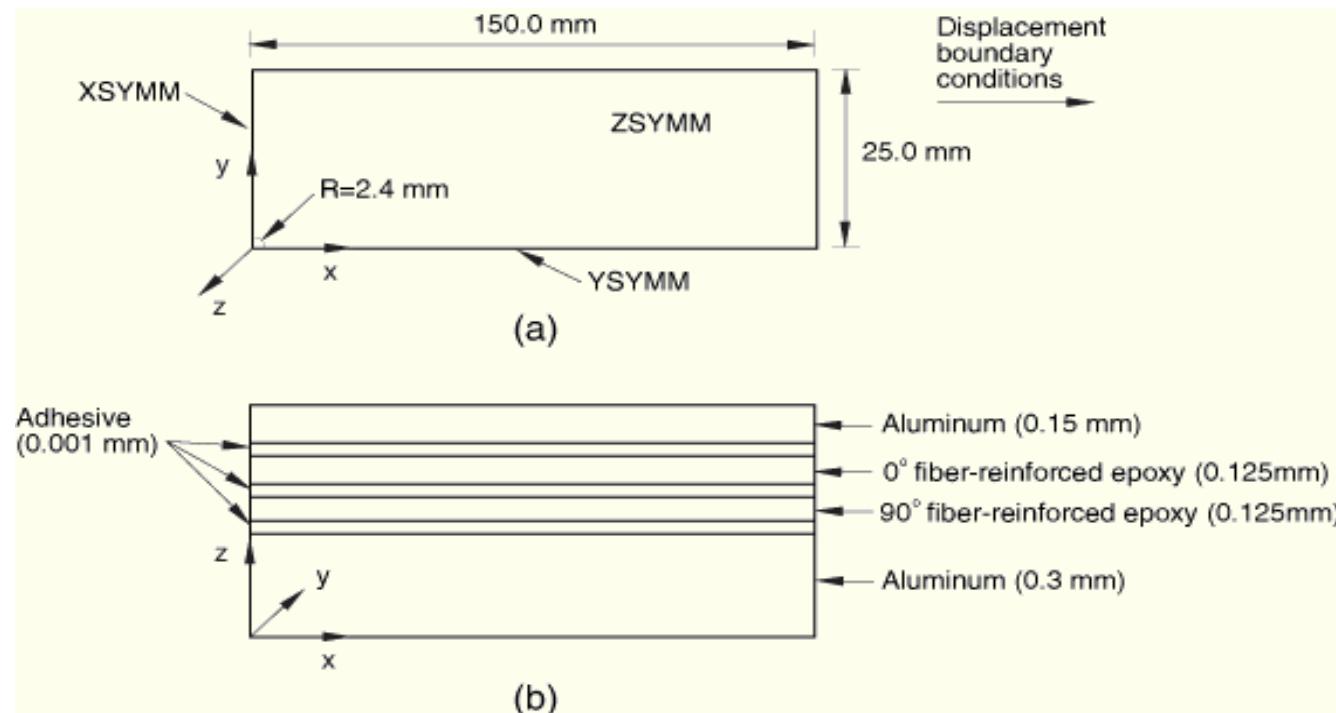
## Example: Failure of Blunt Notched Fiber Metal Laminates

### □ Plate Geometry



## Example: Failure of Blunt Notched Fiber Metal Laminates

### □ Through-thickness view of the laminate



## Example: Results

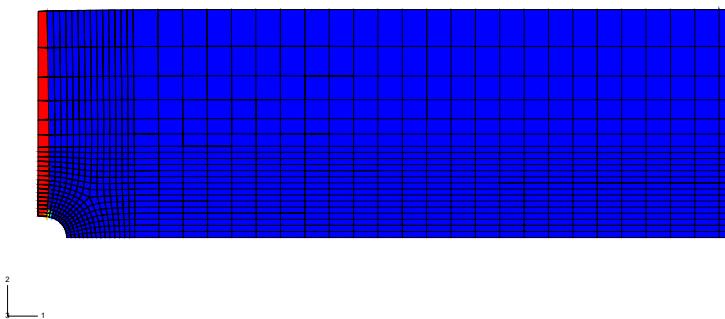


Figure 1. Fiber damage pattern

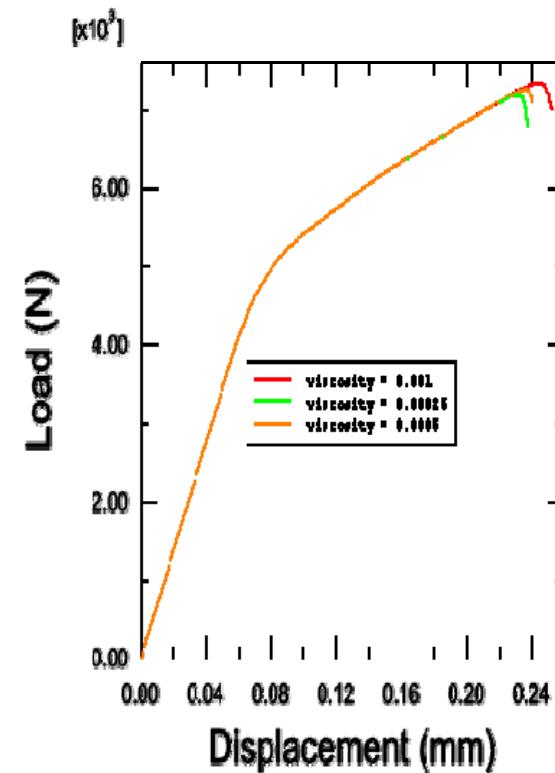


Figure 2. Load-displacement curve

## Example: Results

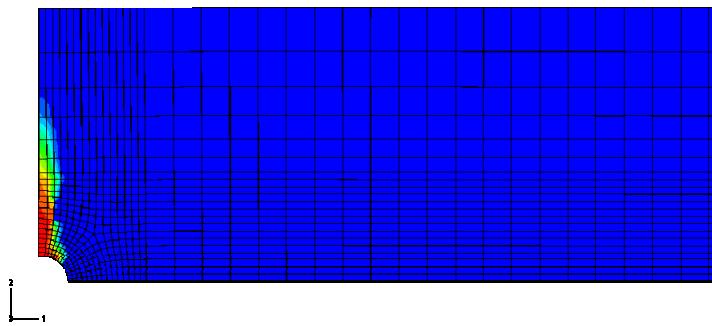


Figure 1. Matrix tension damage pattern

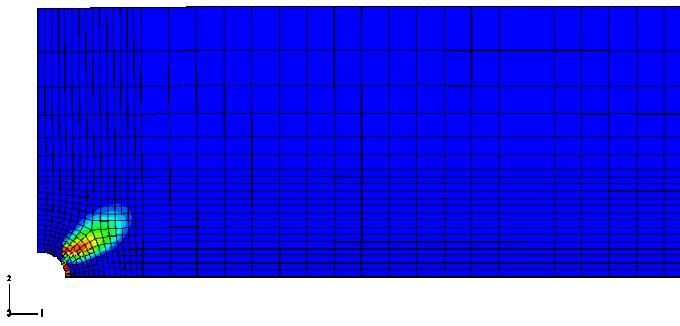


Figure 2. Matrix compression damage pattern

## Example: Energy dissipation

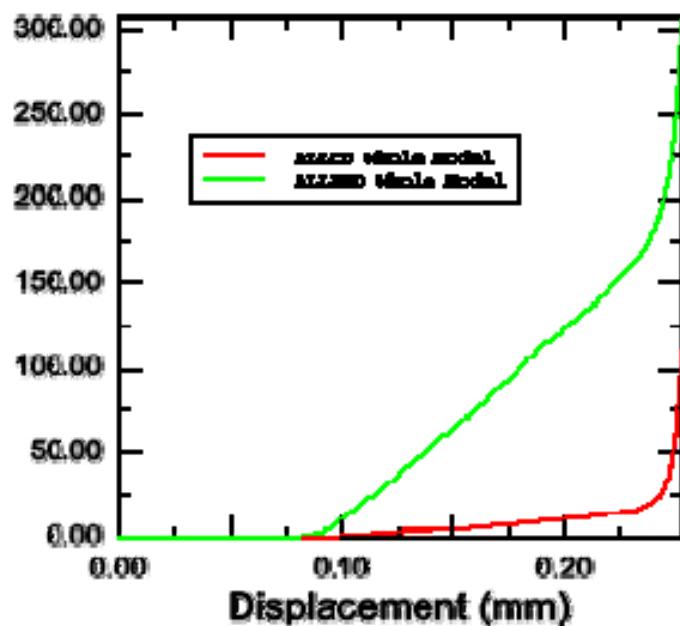


Figure 1.

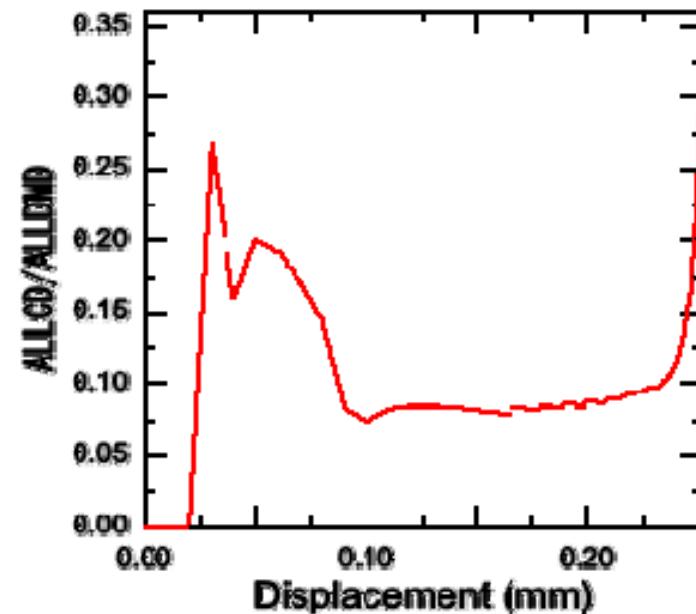


Figure 2.

## Example: Results

### □ Blunt Notch Strength (MPa) for Different Viscosities

Numerical Results (SC8R, 0° angle)			Experimental Results (De Vries, 2001)
$\eta_f=0.001$	$\eta_f=0.0005$	$\eta_f=0.00025$	
466.6	461.3	454.2	446