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Title	<b>Classification of Statistical Channel Models for Local Multipoint Distribution Service Using Antenna Height and Directivity</b>	
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Re:	802.16 Physical Layer Task Group Call for Contributions: Modeling Issues, Session #5	
Abstract	<p>The antenna height and directivity are used in the classification of statistical channel models for local multipoint distribution service (LMDS). When directional antennas are used and the antennas are sufficiently high providing a line-of-sight (LOS) propagation path between the transmitter and receiver, the statistics of the radio channel can be described by the lognormal or Nakagami-m distribution. The channel statistics fits the Rician distribution, if omni-directional antennas are used. The directivity of antennas sets the direction of arrival signals and limits the number of propagation paths. When omni-directional antennas are used and at least one of the antennas is lower than the surrounding obstacles, and hence an LOS propagation path is absent, the radio channel is described by the statistics of the Rayleigh distribution. The above-mentioned channel models can be used to evaluate the performance of LMDS systems.</p>	
Purpose	To provide an input to the specific area ‘‘Channel model’’.	
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# Classification of Statistical Channel Models for Local Multipoint Distribution Service Using Antenna Height and Directivity

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## Introduction

There are a number of statistical channel models available in the technical literature, which need to be classified or further developed before they can be used to evaluate the performance of broadband wireless systems operating at centimeter or millimeter wave frequencies. Local multi-point distribution service (LMDS) is one such system, which has received much attention lately as a viable alternative to existing standard broadcasting, cable, and satellite services.

The heights and directivities of antennas play very important roles in LMDS propagation characteristics. The use of high antennas enables line-of-sight (LOS) radio-wave propagation between a base station and a subscriber station. The directivity of antennas sets the direction of arrival signals and limits the number of propagation paths. The proper use of directional antennas minimizes multi-path effects, such as fading and delay, and interference from unexpected sources.

The mechanisms and characteristics of radio propagation depend strongly on the height and directivity of antennas. Therefore, we use the antenna height and directivity to classify statistical channel models for LMDS systems in this paper. The relationship between data rate, signal-to-noise ratio, and propagation impairments is presented and discussed as well.

## Antenna Height and Directivity Dependence of Channel Models

If the transmitter and receiver antennas are both lower than their surrounding obstacles, then no LOS propagation paths or direct wave components exist. An LOS propagation path may also be absent when one of the transmitter and receiver antennas is lower than the obstacles. Assume that an omni-directional antenna is used at least at the receiver end. There are a large number of scatters or obstacles (reflecting, diffracting, and scattering radios) that contribute to the received signal at a receiver in the shadow region, as in an urban propagation environment. The received fields will be zero-mean and the amplitude  $R$  of received signal components will have a Rayleigh distribution with probability density function

$$p(r) = \frac{r}{\mathbf{s}^2} \exp\left(-\frac{r^2}{2\mathbf{s}^2}\right) \quad (1)$$

where  $r \geq 0$  and  $\mathbf{s}^2$  is the variance. The phase of the signal components is uniformly distributed in the interval from 0 to  $2\mathbf{p}$ .

If the transmitter and receiver antennas are higher than the buildings, trees, and other obstacles between the transmitter and the receiver and an omni-directional antenna is used at least at one end of the radio link, then there will be a major constant direct wave, i.e., LOS component, contributing to the total received signal, together with the Rayleigh fading components. Even if one of the antennas is lower than its surrounding obstacles, there may still exist an LOS propagation path. In this case the radio channel fits the Rician model with the probability density function

$$p(r) = \frac{r}{\mathbf{s}^2} \exp\left(-\frac{r^2 + s^2}{2\mathbf{s}^2}\right) I_0\left(\frac{rs}{\mathbf{s}^2}\right) \quad (2)$$

where  $r \geq 0$ ,  $s^2$  represents the constant (or specular) component of the power, and  $I_0(\cdot)$  denotes the zero-order modified Bessel function of the first kind. Note that the Rician distribution becomes the Rayleigh distribution when  $s = 0$ .

To minimize the interference level and other multi-path propagation effects, directional high-gain antennas are recommended for LMDS systems. If an LOS propagation path exists, along with a limited number of multi-path components, then the lognormal distribution applies, as expressed by

$$p(r) = \frac{1}{\sqrt{2\pi}\mathbf{s}r} \exp\left(-\frac{\ln^2[r/\bar{r}]}{2\mathbf{s}^2}\right) \quad (3)$$

where  $\bar{r}$  is the mean of random variable  $\ln r$  that has a Gaussian distribution and that represents the amplitude of overall received signal, and  $\mathbf{s}^2$  is the variance of  $\ln r$ .

The Nakagami  $m$ -distribution is frequently used in the modeling of radio channels. It may apply to LMDS systems with a proper choice of the parameter  $m$ . For example, this distribution becomes the Rayleigh distribution when  $m = 1$ . It applies to cases where there is a large number of multi-path components as well as to cases where this number is limited. In addition, it does not require the use of omni-directional antennas. It is written as

$$p(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} \exp\left(-\frac{mr^2}{\Omega}\right) \quad (4)$$

where  $\Omega = E(R^2)$  denotes the second moment of random variable  $R$ ,  $E(\cdot)$  denotes mathematical expectation,  $\Gamma(m)$  is the gamma function, and  $m$  is defined as  $m = \Omega^2 / E[(R^2 - \Omega)^2] \geq 1/2$ .

## Complementary Cumulative Distribution of Relative Power Level

The complementary cumulative distribution function  $CF(x)$  is defined as

$$CF(x) = 1 - F(x) \quad (5)$$

where

$$F(x) = \int_{-\infty}^x p(u) du \quad (6)$$

is the cumulative distribution function.

When there is an LOS propagation path along with a few non-LOS propagation paths, then the lognormal distribution does a good job of characterizing the LMDS radio channel. In this case, the function  $CF(x)$  of relative power level can be written as

$$CF(\ln r) = \frac{1}{2} - \frac{1}{2} \operatorname{erf} \left( \frac{\ln(r/\bar{r})}{\sqrt{2\mathbf{s}}} \right) \quad (7)$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt \quad (8)$$

is the error function. Figure 1 presents a plot of  $100 \times CF(x)$  for different values of standard deviation  $\mathbf{s}$  (in dB) for the relative power level. Note that  $x$  has been shown on the vertical axis. In addition, a similar plot for the case where  $x$  is Rayleigh distributed has been shown in Fig. 1 for comparisons. The result for Rayleigh distribution is calculated by

$$CF(\mathbf{g}) = \exp(-\mathbf{g}/\bar{\mathbf{g}}) \quad (9)$$

where  $\mathbf{g} = r^2 E_b / N_0$  is introduced for convenience,  $E_b / N_0$  is the signal-to-noise ratio (SNR) per bit, and  $\bar{\mathbf{g}} = 2\mathbf{s}^2 E_b / N_0$ .

Derived from definition (5) and equation (I.4) of [1],  $CF(x)$  for Nakagami- $m$  distribution is expressed by

$$CF(\mathbf{g}) = \exp \left( -\frac{m\mathbf{g}}{\bar{\mathbf{g}}} \right) \sum_{n=0}^{m-1} \frac{(m\mathbf{g}/\bar{\mathbf{g}})^n}{n!} \quad (10)$$

for integer values of  $m$ . Clearly, (10) reduces to the result for Rayleigh distribution for  $m = 1$ . Some additional results are presented in Fig. 2 for this case.

For a propagation environment including both LOS and non-LOS propagation paths, a hybrid of lognormal and Rayleigh distributions should be applied to the LMDS radio channel. The Nakagami- $m$  distribution, or its modified version, is also expected to do a good job of describing the LMDS channel. As indicated by the study [2], the Rician distribution may fail to characterize LMDS radio channels, because the use of high-gain antennas limits the number of propagation paths.

## Error Rate Performance for Uncoded Transmission

Having introduced  $\mathbf{g} = r^2 E_b / N_0$ , one can write (1) as

$$p(\mathbf{g}) = \frac{1}{\bar{\mathbf{g}}} \exp \left( -\frac{\mathbf{g}}{\bar{\mathbf{g}}} \right) \quad (11)$$

to facilitate the evaluation of bit error rate (BER) performance. Similarly, the Nakagami  $m$ -distribution can be written as

$$p(\mathbf{g}) = \frac{m^m}{\Gamma(m)\bar{\mathbf{g}}^m} \mathbf{g}^{m-1} \exp\left(-\frac{m\mathbf{g}}{\bar{\mathbf{g}}}\right) \quad (12)$$

with  $\bar{\mathbf{g}} = \Omega E_b / N_0$ .

The average bit-error probability rate  $P$  for uncoded transmission can then be evaluated by

$$P = \int_0^{\infty} P(\mathbf{g}) p(\mathbf{g}) d\mathbf{g} \quad (13)$$

where  $P(\mathbf{g})$  is the bit error rate of a modulation scheme without coding.

Some typical plots of the average  $P$  as a function of mean SNR are presented in Figs. 3 and 4. For binary phase-shift keying (BPSK) modulation,  $P(\mathbf{g})$  is expressed as

$$P(\mathbf{g}) = \frac{1}{2} \operatorname{erfc}(\sqrt{\mathbf{g}}) \quad (14)$$

where

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-t^2) dt \quad (15)$$

is the complementary error function. The expression for  $P(\mathbf{g})$  for differential quadrature phase-shift keying (DQPSK) modulation is written [3] as

$$P(\mathbf{g}) = \exp(-2\mathbf{g}) \left[ \frac{1}{2} I_0(\sqrt{2\mathbf{g}}) + \sum_{m=1}^{\infty} (\sqrt{2} - 1)^m I_m(\sqrt{2\mathbf{g}}) \right] \quad (16)$$

where  $I_m(\cdot)$  denotes the  $m$ -th-order modified Bessel function of the first kind.

The plots of  $P$  as a function of the mean SNR  $\bar{\mathbf{g}}$  for the Nakagami- $m$  fading channel are presented in Fig. 3. For  $m=1$ , the radio channel suffers from Rayleigh fading. As  $m$  increases, the error rate performance improves. However, the improvement becomes less significant when  $m$  is larger than ten. On the other hand, the channel performance is worse than the Rayleigh case for  $m < 1$ . In addition, BPSK modulation outperforms DQPSK modulation.

For the lognormal distribution, the plots of  $P$  as a function of the mean SNR,  $\bar{r}$ , are presented in Fig. 4. The performance of the radio channel improves as  $\mathbf{s}$  decreases. Again, BPSK modulation performs better than DQPSK.

## Relationship Between Data Rate and SNR

The data rate  $R_{dB}$  in dB bits per second relates to the SNR,  $E_b / N_0$ , in dB per bit and received power  $P_r$  in dBW by [4]

$$R_{dB} = P_r - N_0 - (E_b / N_0) \quad (17)$$

where  $N_0$  is the thermal noise power density at the receiver front end in dBW/Hz and  $N_0$  in W/Hz is given by

$$N_0 = k_B T_0 \quad (18)$$

where  $k_B = 1.38 \times 10^{-23}$  Ws/K is Boltzmann's constant and  $T_0$  is the noise temperature in Kelvin.

By using the standard radar equation, the received power  $P_r$  is expressed as

$$P_r = P_t + G_t + G_r - L_S - L_p \quad (19)$$

where  $P_t$  is the transmitted power from a transmitting antenna,  $G_t$  and  $G_r$  are the gains of the transmitter and receiver antennas, respectively,  $L_S$  is the receiver system loss, and  $L_p$  is the propagation path loss defined by the ratio of radiated to received power for isotropic antennas.

Clearly,  $R_{dB}$  (describing the link capacity) decreases as the propagation loss  $L_p$  increases for given LMDS system parameters  $P_t$ ,  $G_t$ ,  $G_r$ ,  $N_0$ , and  $L_S$ , and for a required  $E_b / N_0$ . Also,  $E_b / N_0$  (describing the quality of service) decreases as the propagation loss  $L_p$  increases for a given  $R_{dB}$ . Furthermore,  $R_{dB}$  decreases as  $E_b / N_0$  increases for given system parameters and propagation loss.

The propagation loss  $L_p$  can be expressed as

$$L_p = L_{fs} + L_R + L_{at} + L_o + L_m \quad (20)$$

where  $L_{fs}$  is free-space loss,  $L_R$  is the loss due to rain,  $L_{at}$  is the loss due to atmospheric gases,  $L_o$  is the loss due to obstruction by terrain obstacles, buildings, etc., and  $L_m$  accounts for the loss or fading due to multipaths such as atmospheric refraction, reflections of the propagation environment, and interference from unexpected sources.

The free-space propagation loss  $L_{fs}$  [5] is calculated by

$$L_{fs} = 92.4 + 20 \log_{10} f + 20 \log_{10} d \quad (21)$$

where  $f$  in GHz is the frequency and  $d$  in kilometers is the distance between transmitter and receiver antennas. The rain loss  $L_R$ , which is severe and must be considered for LMDS systems, can be calculated, for example, by using the model in [6]. Recently, power-law parameters of rain specific attenuation, i.e., rain attenuation (loss)

per kilometer, for an expanded set of raindrop size distributions have been found [7] so that the rain attenuation may be computed more accurately. The methods for estimating  $L_{at}$ , which is less severe than  $L_R$ , can be traced in [6].

The height and directivity of antennas are important in computing  $L_o$  and  $L_m$ . By installing high antennas,  $L_o$  can be made negligible or at least minimized. Also,  $L_m$  can be minimized by using directional high-gain antennas properly. Some methods for calculating the effects of  $L_o$  and  $L_m$  may be found in [6]. Models for vegetation attenuation are available in [8]. A contribution addressing two formulations for multiple diffraction by building and trees was presented in [9].

## Summary

We have classified statistical channel models for LMDS systems on the basis of antenna height and directivity.

When at least one of the two antennas are sufficiently high providing an LOS propagation path between the transmitter and receiver, the statistics of the LMDS radio channel should be described by one of the Rician, Nakagami-m, and lognormal distributions, depending on the directivity of the antennas. The channel statistics of LMDS radios using directional antennas should be characterized by lognormal or Nakagami-m distribution. A radio channel for an LMDS system using omni-directional antennas may be evaluated by the statistics of the Rician distribution.

If neither antenna is higher than the surrounding obstacles and no LOS propagation path is present, then the statistics of the radio channel should be described by the Rayleigh distribution when the antennas are omni-directional.

Using the above-mentioned models, it becomes straightforward to evaluate the performance of an LMDS channel. Plots of complementary cumulative distribution of relative power level and average bit error rate performance were presented.

The relationship between data rate and average SNR was presented and discussed. The use of high antennas with directional high-gain can minimize propagation impairments and, therefore, increase LMDS communication link's capacity and quality of service.

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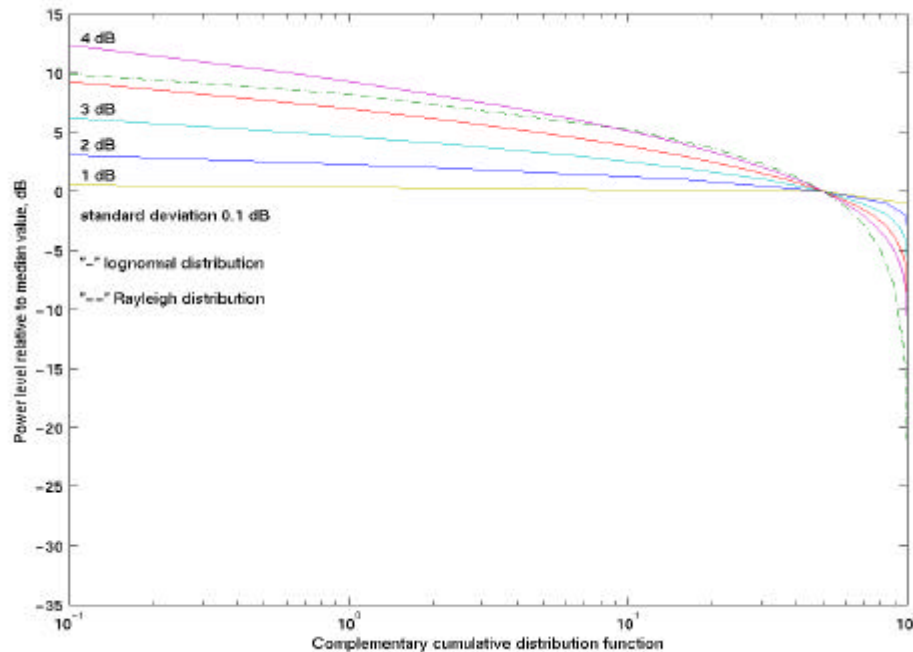


Fig. 1. Complementary cumulative distribution function of relative power level for lognormal distribution. Similar result for Rayleigh distribution is included.



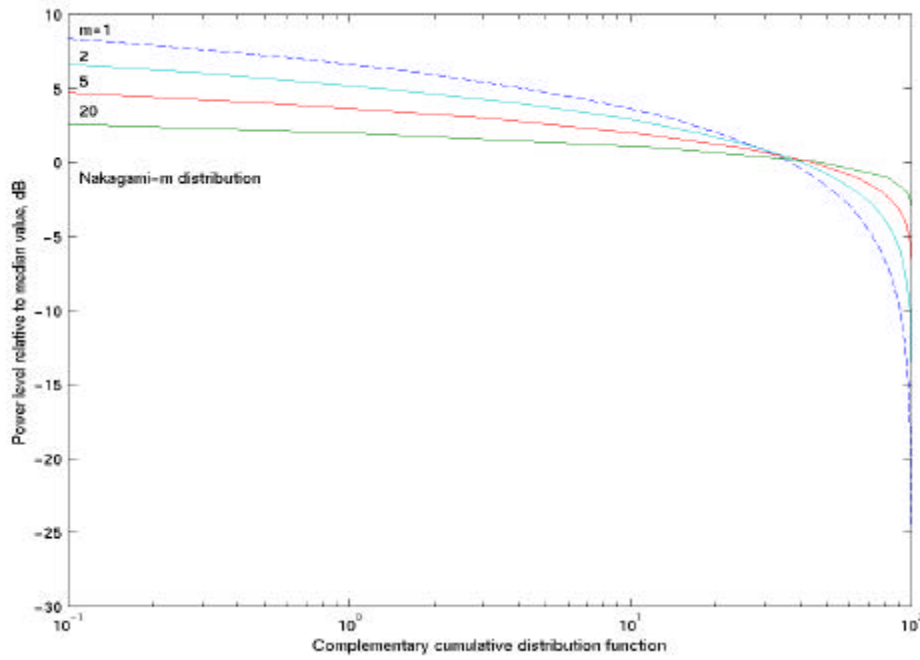


Fig. 2. Complementary cumulative distribution function of relative power level for Nakagami m-distribution.

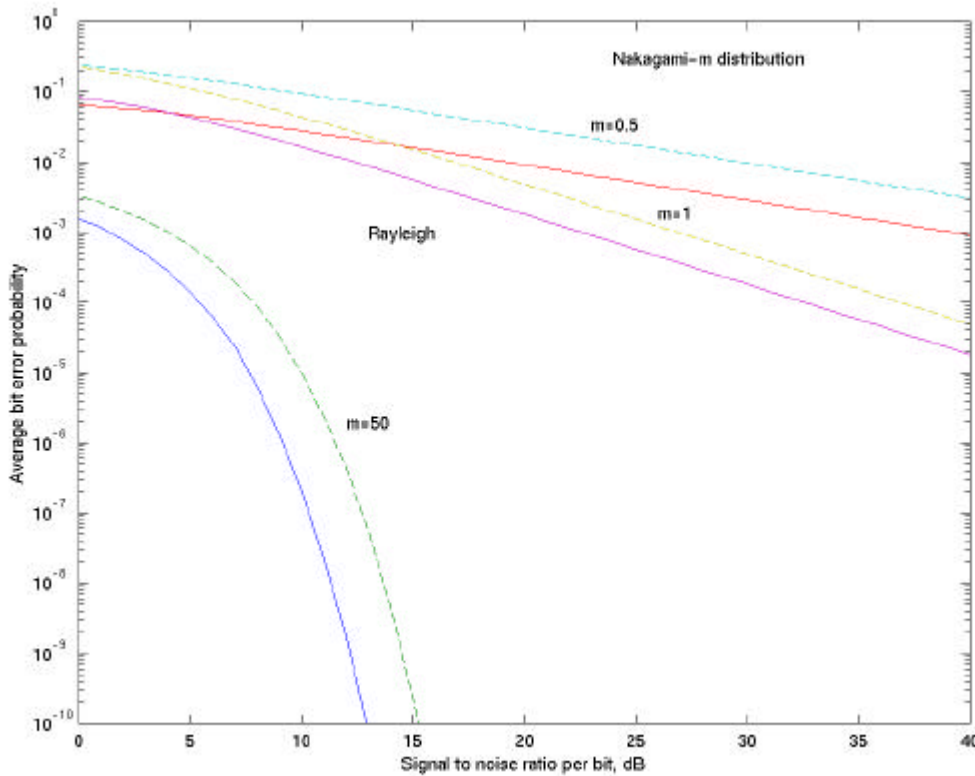


Fig. 3. Average bit error probability of Nakagami m-distribution channel for uncoded transmission as a function of the mean signal to noise ratio per bit; solid lines used for BPSK modulation and dashed lines for DQPSK modulation.

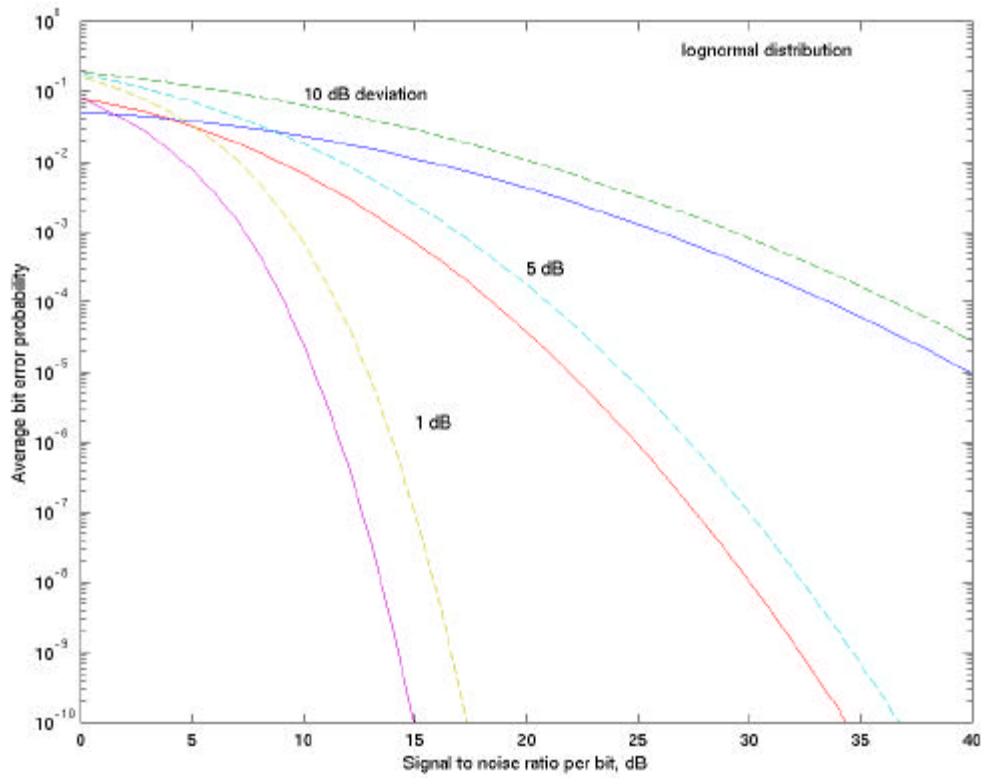


Fig. 4. Average bit error probability of lognormal distribution channel for uncoded transmission as a function of the mean signal to noise ratio per bit; solid lines used for BPSK modulation and dashed lines for DQPSK modulation.