

Project Planning And Scheduling Using PERT And CPM Techniques With Linear Programming: Case Study

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Abstract: Completing a project on time and within budget is not an easy task. Project planning and scheduling plays a central role in predicting both the time and cost aspects of a project. This study is aimed at finding trade-off between the cost and minimum expected time that will be required to complete the building project. The data on the cost and duration of activities involved were obtained Angel Estates and Construction Ltd., a construction company based in Ashanti region, Ghana. Both critical path method (CPM) and project evaluation and review technique (PERT) were used for the analysis. The activities underwent crashing of both the time and cost using linear programming, this paved way for the determination of critical path. Further analysis revealed that the shortest possible time for the completion of the analyzed building project is 40 days instead of the expected duration of 79 days. This means that through proper scheduling of activities, the expected completion time was reduced by 39 days. The additional cost associated with the reduction in timing is GH¢1,887.22 which increases the initial expected cost required to complete the project from GH¢57,156.35 to GH¢59,043.57.

Keywords: Construction planning, Critical Path Method, Project Evaluation and Review Technique, Linear programming, crashing.

1. INTRODUCTION

Completing a project on time and within budget is not an easy task. In spite of advances in the field of project management today, most projects in Ghana today face cost and time over-runs which increases with the increase in complexity of the project involved. A large number of factors contribute to delays that comprise primarily of contractor delays, client delays, consultant delays, labor related delays and various other external delays. These delays in long run causes time overrun, cost overrun, dispute, arbitration, total abandonment and litigation. This is because some project activities are critical in the sense that delay in their commencement will delay the overall project completion time. Therefore proper planning and scheduling of project is important to overcome this problem. A project can be defined as a set of a large number of activities or jobs that are performed in a certain sequence determined logically or technologically and it has to be completed within a specified time and cost whilst meeting the performance standards. The project could be the development of a software program, the building of a house or an office building, development of a new drug, a marketing campaign for a new product, and many others. Network models are conventional means of finding the most skilful way to link a number of activities directly or indirectly in order to satisfy supply and demand requirements at different activity locations and project scheduling. For many years, two approaches that have been proven to be useful for planning, scheduling and controlling construction projects have been the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT). These techniques enables project managers to evaluate the early and late times at which activities can start and finish, calculate activity float (slack), define critical activities, and evaluate the impact of changes in duration, logical relations and cost on the overall project duration. Both CPM and PERT are network based techniques and therefore help in programming and monitoring the progress of the stages involved so that the project is completed within the deadline. In doing this, it specifies the part of the project that are crucial which if delayed beyond the normal time would increase the completion time of the project as a

whole. It further assists in allocating resources, such as labor and equipment and thus helps to make the total cost of the building project a minimum by finding the optimal trade-off between various costs and time involved [1]. Although PERT and CPM differ to some extent in terminology and in the construction of the network, their objectives are the same. Furthermore, the analysis used in both techniques is very similar. The major difference is that, in CPM activity times are assumed proportional to the amount of resources allocated to them, and by changing the level of resources the activity times and the project completion time can be varied. Thus CPM assumes prior experience with similar projects from which the relationships between resources and job times are available. On the other hand, PERT incorporates uncertainties in activity times in its analysis. It determines the probabilities of completing various stages of the project by specified deadlines. It also calculates the expected time to complete the project. An important and extremely useful by product of PERT analysis is its identification of various "bottlenecks" in a project. In other words, it identifies the activities that have high potential for causing delays in completing the project on schedule. Thus, even before the project has started, the project manager knows where he or she can expect delays. The manager can then take the necessary preventive measures to reduce possible delays so that the project schedule is maintained. Actually, both techniques, PERT and CPM, were developed almost simultaneously. Project managers often encounter the problem of having to shorten the scheduled completion time in order to expedite the execution of a project. Reducing the project duration can be achieved by adding more resources to the performance of the activity in the form of overtime, resources or by assigning additional labor. This managerial decision of additional resources, overtime and labor will however increases the overall cost of the project thus trimming down the project duration of activities on critical path. This concept of project management which involves investment of extra budget in order to minimize the duration to meet the targeted date is known as crashing. The objective of expediting project by crashing total project duration is helpful so that delays can be recovered and liquidated damages can be avoided. The aim is always to

strike balance between the cost and time and to obtain an optimum schedule. This study is therefore aimed at exploring the time and cost of various activities involved in a prototype 3-bed room house project at Kumasi, Ghana in order to determine the optimal completion time using CPM and PERT techniques with linear programming.

Basic Definitions

1. Activity Time: The time to complete a particular activity. The activity time can be deterministic or uncertain (probabilistic). When the activity time is deterministic, the activity is completed in a constant time. When the activity time is probabilistic, the activity is completed in a random time value that may have a certain probability distribution.
2. Critical Path: A path that has the longest total activity time.
3. Critical Activity: Any activity on the critical path.
4. Deterministic Project: A project that all activities finish in constant times.
5. End Activity: The activity that is not a predecessor of any other activity is called an end activity of the project.
6. Earliest finish (EF): this is the earliest possible time that an activity can be finished (= earliest start time + activity completion time).
7. Earliest start (ES): this is the earliest possible time that an activity can begin. All immediate predecessors must be finished before an activity can start.
8. Immediate Predecessor: The immediate predecessors of an activity are the activities that must immediately precede the activity.
9. Latest finish (LF): this is the latest time that an activity can be finished and not delay the completion time of the overall project (= latest start time + activity completion time). As with start times, the activity is critical if the earliest finish and latest finish times are the same.
10. Latest start (LS): this is the latest time that an activity can begin and not delay the completion time of the overall project. If the earliest start and latest start times are the same then the activity is critical.
11. Path: A sequence of activities in a project leading from the start activity to the completion activity of the project.
12. Precedence Relation: It tells which activity must be completed before another can be started.
13. Probabilistic Project: A project that has activities finish in uncertain times.
14. Slack time: this is the difference between the earliest start time and the latest start time (which in turn is equal to the difference between the latest start time and the latest finish time), i.e. Slack = LS-ES = LF-EF
15. Start Activity: The activity that has no immediate predecessor is called a start activity of the project.
16. Optimistic time: This is the shortest possible time in which the activity can be completed, and assumes that everything has to go perfect.
17. Most-likely time: This is the most likely time in which the activity can be completed under normal circumstances.
18. Pessimistic time: This is the longest possible time the activity might need, and assumes a worst-case scenario.

2. MATERIALS AND METHOD

In order to schedule the activities in the network we require an estimate of how much time each activity should take when it is done in the normal way. These estimates are provided by the project manager of Angel Estates and Construction Ltd., a construction company based in Ashanti region, Ghana. Table 1 shows the break down description of activities involved for the construction process of a 3-bed room house at Nkoransa, Off the Santasi-Obuasi Road, Kumasi. The construction activity begins with activity A and ends with activity N and shows the distribution of the project activities relative to the actual number of days to complete individual activity and their respective cost implication in terms of Ghana cedis of 3-bed room building. The costs are basically labour costs based on the assumption that materials are already available for use. This is because, once the materials are available, the reduction in number of days to complete a particular activity will only be affected by the cost of hiring additional labour.

Table1. Description of activities involved for the construction process of a house by Angel Estates and Construction Ltd.

Activity Code	Activity Description	Normal cost (GH¢)	Immediate Predecessors	Estimated Duration
A	Site clearing	1,841.52	—	2 days
B	Foundation	1,264.34	A	4 days
C	Block Laying	5,281.16	B	10 days
D	Roofing	4,165.82	C	6 days
E	Plumbing	5,017.92	C	4 days
F	Electrical work	5,268.11	E	5 days
G	Plastering	6,389.43	D	7 days
H	Fixing up of doors and windows	3,590.55	E, G	9 days
I	Ceiling	4,099.73	C	7 days
J	Flooring	4,239.48	F, I	8 days
K	Interior Fixtures	4,764.31	J	4 days
L	Exterior fixtures	3,388.74	J	5 days 2 days
M	Painting	4,153.91	H	6 days
N	Landscaping	3,691.33	K, L	

Based on the table we can now construct a network diagram, shown in Figure 1.

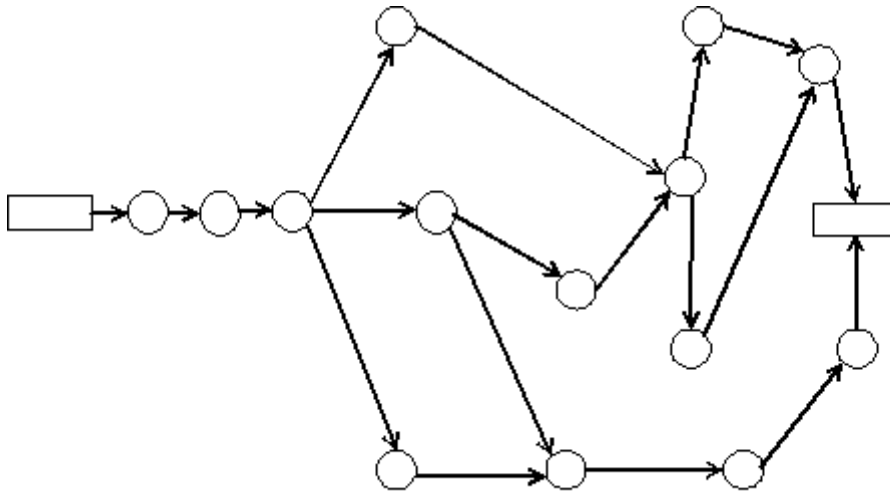


Figure1. Network diagram for construction of new house by Angel Estates and Construction Lt

The critical path method (CPM) is a very efficient scheduling procedure for larger projects. The starting and finishing times of each activity if no delays occur anywhere in the project are called the earliest start time and the earliest finish time of the activity. Suppose ES_j is the earliest time activity j can start, and EF_j is the earliest time activity j can finish. Clearly, the earliest an activity can finish is the earliest time it can start plus its duration. For example, if the earliest activity D can start is time 8 and its duration is 12, then the earliest D can finish is time 20. In general, if d_j is the duration of activity j , we have

$$EF_j = ES_j + t_j \quad (1)$$

If activity i is an immediate predecessor of activity j , then activity j cannot start until activity i finishes. In fact, activity j cannot start until all of its immediate predecessors have finished, so the earliest time activity j can start is the maximum of the earliest finish times of its immediate predecessors:

$$ES_j = \max(EF_i) \quad (2)$$

Where the maximum is over all immediate predecessors i of activity j . We use equations (1) and (2) to find the earliest start and earliest finish times of all activities, beginning with the fact that the earliest start time of the Start node is 0. A final product of these calculations is that we automatically obtain the project completion time. It is the earliest start of the Finish node.

$$\text{Project completion time} = ES_{\text{Finish}} \quad (3)$$

The reason is that as soon as we reach the Finish node, the entire project is complete. This calculation of the earliest start and finish times through equations (1) to (3) is usually called the forward pass of the CPM algorithm. The reason for this term is that we do the calculations in forward chronological order of activities. To find the critical activities and critical path, we need to find two other equations. Let LS_j and LF_j be the latest time activity j can start and the latest time it can finish without increasing the project completion time. Again, we have analogous to equation (1)

$$LS_j = LF_j - t_j \quad (4)$$

Suppose activity j is an immediate successor of activity i .

Then activity i must be finished before activity j can start. In fact, the latest time activity i can finish is the minimum of the latest start times of all its successors:

$$LF_i = \min(LS_j) \quad (5)$$

Where the minimum is over all immediate successors j of activity i . We use equations (4) and (5) to calculate the latest start times and latest finish times for all activities, beginning with the fact that the latest finish time for the Finish node is the project completion time. This set of calculations is called backward pass of the CPM algorithm because we work through the activities in backward chronological order. We then calculate the slack also known as float of each activity j as the difference between the latest start time and earliest start time of activity j .

$$\text{Slack of activity } j = LS_j - ES_j \quad (6)$$

The idea behind slack is that if an activity has a positive slack, then that activity has some room to maneuver i.e. it could start a bit later without delaying the project. In fact, its duration could increase by the amount of its slack without delaying the project [3]. However, if an activity has zero slack, then any increase in its duration necessarily delays the whole project. Therefore, the critical path consists of activities with zero slack. Table 2 shows the computation

Activity Code	Earliest Start (ES)	Earliest Finish (EF)	Latest Start (LS)	Latest Finish (LF)	Slack (LS-ES)	Critical
A	0	2	0	2	0	Yes
B	2	6	2	6	0	Yes
C	6	16	6	16	0	Yes
D	16	22	20	26	4	No
E	16	20	16	20	0	Yes
F	20	25	20	25	0	Yes
G	22	29	26	33	4	No
H	29	38	33	42	4	No
I	16	23	18	25	2	No
J	25	33	25	33	0	Yes
K	33	37	34	38	1	No
L	33	38	33	38	0	Yes
M	38	40	42	44	4	No
N	38	44	38	44	0	Yes

Table 2. The earliest start, earliest finish, latest start, latest finish times and slack activities of a building construction project in Kumasi.

In CPM, all the activities time estimates are single values with the assumption that activity time are known with certainty by using a single activity time estimate. In reality however, because of uncertainties in carrying out each activity, the activity time estimate cannot be certain [4]. Therefore, using PERT probabilistic method which follows beta distribution, three time estimates were subsequently used to estimate the expected time (mean) and variance of the distribution. Expected time is the weight average of the three time estimates (optimistic (a), pessimistic (b) and most likely time (m)). Then, it becomes necessary to know how realistic this will be by estimating the probability of achieving this scheduled date. Table 3 shows the variations in the estimates as the affect the construction activities. Using the formulae:

$$\text{Expected time (mean)} \bar{x} = \frac{a+4m+b}{6} \text{ and Variance } \sigma = \left(\frac{b-a}{6}\right)^2$$

Table 3 Project Activity, Predecessor, optimistic estimate (a), most likely estimate (m) and pessimistic estimate (b) of a building construction project at Kumasi, Ashanti region

Activity	Predecessors	Optimistic Estimate (a)	Most Likely Estimate (m)	Pessimistic Estimate (b)	Mean	Variance
A	—	1	2	3	2	1/9
B	A	2	3.5	8	4	4
C	B	6	9	18	10	1
D	C	4	5.5	10	6	4/9
E	C	1	4.5	5	4	1
F	E	4	4	10	5	1
G	D	5	6.5	11	7	4
H	E, G	5	8	17	9	1
I	C	3	7.5	9	7	1
J	F, I	3	9	9	8	0
K	J	4	4	4	4	1
L	J	1	5.5	7	5	1/9
M	H	1	2	3	2	4/9
N	K, L	5	5.5	9	6	4/9

Although the project is estimated to be completed within 44 days there is no guarantee that it will actually be completed within the 44 days. If by some circumstances various activities take longer than their expected time, the project might not be completed within the desired schedule. Therefore, it will be useful to know the probability that the project deadline will be met. The first step is to find the variance and standard deviation of the total time along

critical path, which is equal to the sum of the variances of activity times on the critical path. Thus

$$\text{var } T = 1/9 + 1 + 4 + 4/9 + 1 + 1 + 1 + 4/9 = 9.0$$

$$\text{std. dev } T = \sqrt{(\text{var } T)} = \sqrt{9} = 3.0$$

We then compute the probability that the project will be completed assuming the deadline is say 47 days (i.e. the estimated completion + standard deviation). Therefore, the

probability of meeting the deadline given the three simplifying approximations using the normal distribution is:

$$\begin{aligned} \text{Prob}\{T \leq 47\} &= \text{Prob}\left\{\frac{T - 44}{3} \leq \frac{47 - 44}{3}\right\} \\ &= \text{Prob}\{Z \leq 1.0\} \\ &= 1 - \text{Prob}\{Z > 1.0\} = 1 - 0.1587 = 0.84 \end{aligned}$$

Thus, there is 84% chance to that the critical path will be completed in less than 47 days.

3. RESULTS AND DISCUSSION

We now investigate how much it would cost to reduce the expected project duration further down from 44 days. The assumed deadline for the construction company and the maximum duration the company is able to increase its resources for the project is 40 days. The question that any

operation research analyst will ask is, if extra money is spent to expedite the project, what is the optimal way of attempting to meet the target of the completion time. CPM provides an excellent procedure for investigating time–cost trade-offs, so we will use this approach, that is the process of crashing by linear programming. This special measure may include using overtime, hiring additional workers, using special time-saving materials, special equipment, and so forth [5]. Table 5 shows the project activity, days (normal and crash), cost (normal and cash) and the computations of the crash cost for each activities.

$$\text{Crash cost}(r) = \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal time} - \text{Crash time}} = \frac{\Delta C}{\Delta t} \quad (7)$$

Table 4. Crashing data for time-cost analysis for the construction activities of Angel Estates and Construction Ltd.

Activity Code	Normal		Crash		ΔC	Δt	$r = \frac{\Delta C}{\Delta t}$
	Time (Days)	Cost (GH¢)	Time (Days)	Cost (GH¢)			
A	2	1,841.52	1	2,433.37	591.85	1	591.85
B	4	1,264.34	2	2,113.74	849.40	2	424.70
C	10	5,281.16	7	7,049.89	1,768.73	3	589.58
D	6	4,165.82	4	4,996.76	830.94	2	415.47
E	4	5,017.92	3	5,597.21	579.29	1	579.29
F	5	5,268.11	3	6,978.68	810.57	2	405.29
G	7	6,389.43	4	8,281.45	1,892.02	3	630.67
H	9	3,590.55	6	5,352.24	1,761.69	3	587.23
I	7	4,099.73	5	4,987.63	887.90	2	443.95
J	8	4,239.48	6	5,316.11	1,076.63	2	538.32
K	4	4,764.31	3	5,221.87	457.56	1	457.56
L	5	3,388.74	3	4,709.21	1,320.47	2	660.24
M	2	4,153.91	1	4,780.65	626.74	1	626.74
N	6	3,691.33	3	4,968.25	1,276.92	3	425.64

We formulate the problem as a linear programming model as follows: Let Z be the total cost of crashing activities. The problem then is to minimize Z, subject to the constraint that the construction duration must be less than or equal to the deadline desired by the project manager. The decision variables x_i = reduction in the duration of activity i due to crashing this activity, for $i = A, B, \dots, N$. y_{Finish} = construction duration, i.e. the time at which the finish node in the project network is reached. To help the linear programming model assign the appropriate value to y_{FINISH} , given the values of x_A, x_B, \dots, x_N , it is expedient to introduce into the model the following additional variables y_i = start time activity i (for $i = B, C, \dots, N$) given the values of x_A, x_B, \dots, x_N (No such variable is needed for activity that begins the project as it is automatically assigned a value of 0.) By treating the Finish node as another activity (even though with zero duration), as we now will do, this definition of y_i for activity Finish also fits the definition of y_{Finish} given above. The start time of each activity (including Finish) is directly related to the start time and duration of each of its immediate predecessors as summarized below. For each activity (B, C, ..., N, Finish) and each of its immediate predecessors, Start time of this activity (start time duration) for this immediate predecessor. Using the normal times from Table, the duration of each activity is given by the equation: Duration of activity $i =$ its normal time $- x_i$.

To illustrate these relationships, consider activity C in the construction network in figure 1.

The immediate predecessor of activity C:

Activity B, which has a duration = $4 - x_B$

Relationship between these activities:

$$y_C \geq y_B + 4 - x_B$$

Thus, activity C cannot start until activity B starts and then completes its duration of $4 - x_B$.

Now consider activity H, which has two immediate predecessors.

Immediate predecessors of activity H:

Activity E, which has duration = $4 - x_E$.

Activity G, which has duration = $7 - x_G$.

Relationships between these activities:

$$y_H \geq y_E + 4 - x_E$$

$$y_H \geq y_G + 7 - x_G$$

These inequalities together say that activity H cannot start until both of its predecessors finish. By including these relationships for all the activities as constraints, we obtain the complete linear programming model given below.

$$\begin{aligned} \text{Minimize } Z &= 591.85x_A + 424.70x_B + 589.58x_C + 415.47x_D \\ &+ 579.29x_E + 405.29x_F + 630.67x_G \\ &+ 587.23x_H + 443.95x_I + 538.32x_J \\ &+ 457.56x_K + 660.24x_L + 626.74x_M \\ &+ 425.64x_N \end{aligned}$$

Subject to:

Maximum reduction constraints:

$$x_A \leq 1, x_B \leq 2, x_C \leq 3, x_D \leq 2, x_E \leq 1, x_F \leq 2, x_G \leq 3, \\ x_H \leq 3, x_I \leq 2, x_J \leq 2, x_K \leq 1, x_L \leq 2, x_M \leq 1, x_N \leq 3,$$

Non-negativity constraints:

$$x_A \geq 0, x_B \geq 0, x_C \geq 0, x_D \geq 0, x_E \geq 0, x_F \geq 0, \\ x_G \geq 0, x_H \geq 0, x_I \geq 0, x_J \geq 0, x_K \geq 0, x_L \geq 0, x_M \geq 0, x_N \geq 0, \\ y_B \geq 0, y_C \geq 0, y_D \geq 0, y_E \geq 0, y_F \geq 0, y_G \geq 0, y_H \geq 0, y_I \\ \geq 0, y_J \geq 0, y_K \geq 0, y_L \geq 0, y_M \geq 0, y_N \\ \geq 0, y_{FINISH} \geq 0$$

Start time constraints:

$$y_B + x_A \geq 2 \quad y_C - y_B + x_B \geq 4 \\ y_D - y_C + x_C \geq 10 \quad y_E - y_C + x_C \geq 10 \\ y_F - y_E + x_E \geq 4 \quad y_G - y_D + x_D \geq 6 \\ y_H - y_E + x_E \geq 4 \quad y_H - y_G + x_G \geq 7 \\ y_I - y_C + x_C \geq 10 \quad y_J - y_F + x_F \geq 3 \\ y_J - y_I + x_I \geq 7 \quad y_K - y_J + x_J \geq 6 \\ y_L - y_J + x_J \geq 6 \quad y_M - y_H + x_H \geq 9 \\ y_N - y_K + x_K \geq 4 \quad y_N - y_L + x_L \geq 5$$

$$y_{FINISH} - y_M + x_M \geq 2 \quad y_{FINISH} - y_N + x_N \geq 6$$

Projection duration constraint:

$$y_{FINISH} \leq 40$$

Which means that the project manager need to crash the project time by 4 days. Using LINDO optimization software, the solution of the mathematical model is show in Table 5.

Table 5. Solution for the linear programming model for crashing construction activities of Angel Estates and Construction Ltd.

Activity	Start Time (y _i)	Time Reduction (x _i)	Finish Time
A	0	0	2
B	2	0	6
C	6	0	16
D	16	0	22
E	16	0	20
F	20	2	23
G	22	0	29
H	29	0	38
I	16	0	23
J	23	2	29
K	30	0	34
L	29	0	34
M	38	0	40
N	34	0	40

Finish Time = 40 days TotalCost = GH¢ 59,043.57

Further analysis revealed that the shortest possible time for the completion of the analyzed building project is 40 days instead of the expected duration of 79 days. This means that through proper scheduling of activities, the expected completion time was reduced by 39 days. The additional cost associated with the reduction in timing is GH¢1,887.22 which increases the initial expected cost required to complete the project from GH¢57,156.35 to GH¢59,043.57.

4. CONCLUSION

This paper addressed the problem of the application of

project scheduling in a building construction by Angel Estates and Construction Ltd in Kumasi was studied as CPM and PERT methods. Using CPM, our building project will take 44days to complete the project whereas the actual project could take. By crashing the project activities by means of linear programming to 40days the project can be completed 10% decrease in the project duration but achieving this, 3.30% additional cost has to be incurred, which is quite appreciable. From the results we thus conclude that the schedule proposed by us provides much shorter completion time as compared to the actual time taken by the project and paves the way for use of CPM scheduling for building construction projects to be a lucrative and viable option

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