Projection Selection Algorithms for Discrete Tomography

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Problem statement

- In binary tomography [1], the goal is to reconstruct homogeneous objects from a limited set of their (usually 2-10) projections.
- The small number of projections gives a relatively high freedom in choosing the directions to take projections with.
- The aim of our work is to determine, if the choice of projection angles can affect the result of reconstruction, and if the result of reconstruction can be improved only by using the proper projections.

Projection selection strategies

- Our approach for examining the problem, was trying to improve the reconstruction of a set of phantom images, performed by a given reconstruction algorithm [2], only by choosing better projections with four different angle selection strategies.
- The angle selection algorithms are based on performing a high number of reconstructions from different sets of projections of the phantoms, and choosing the directions belonging to the best reconstruction.

Naive angle selection

Equiangular angle sets are uniquely determined by a p number of projections and an α starting angle:

$$S(\alpha, p) = \{\alpha + i\frac{180°}{p} \mid i = 0, \ldots, p-1\}$$

- The naive angle selection strategy chooses a predetermined projection angle set, without taking the object to be reconstructed into account.
  - For a specified p number of projections use the $S(0°, p)$ angle set.
- Since, in a typical application this method is used for choosing projections, our basic goal is to improve reconstructions performed from these kind of projection sets.

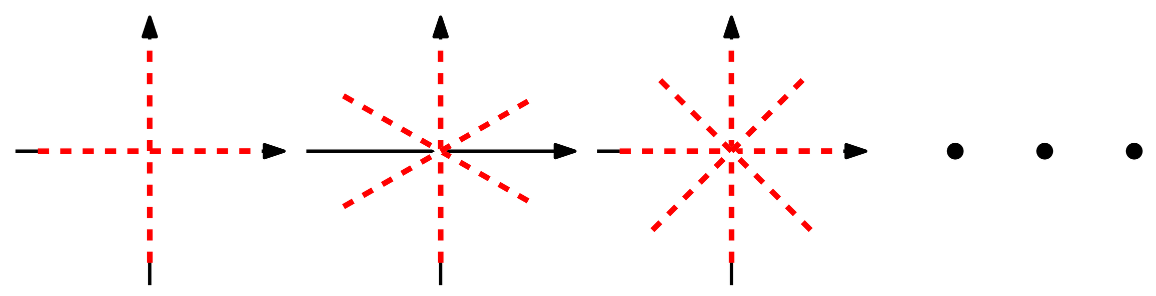

Figure 1: Angle sets produced by the naive angle selection strategy.

Equiangular search

- Search among the equiangular projection sets.
  - With every specified p number of projections, reconstruct the images from different equiangular projection sets defined by $S(\alpha, p)$, with integer α starting angles ranging from $0°$ to $\left\lceil \frac{180}{p} - 1 \right\rceil^{\circ}$.
  - After performing the reconstructions select the angle set that has the best corresponding reconstruction.

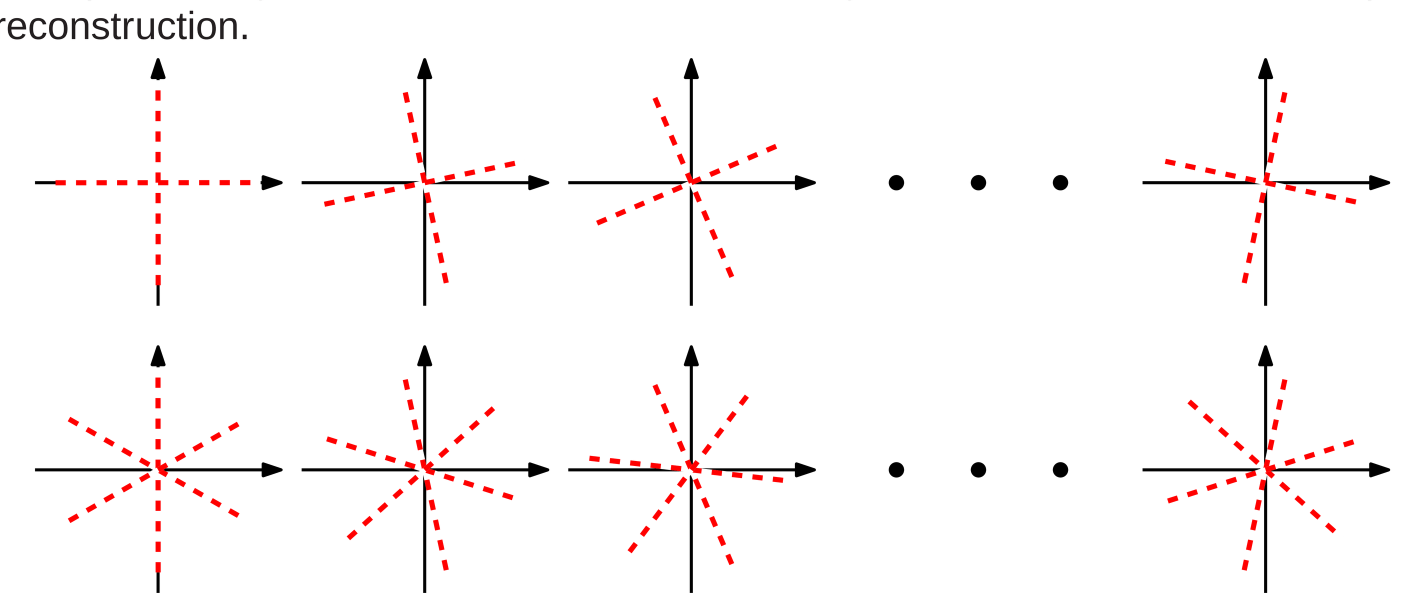

Figure 2: Example of angle sets examined by the equiangular angle searching strategy.

Greedy angle testing

- A greedy algorithm, that chooses projection angles based on local decisions.
  - For each image start with one single predetermined projection angle.
  - In each iteration add the projection to the current ones that brings the biggest improvement to the reconstruction.
  - The result is a list of projection angles ordered in decreasing significance.

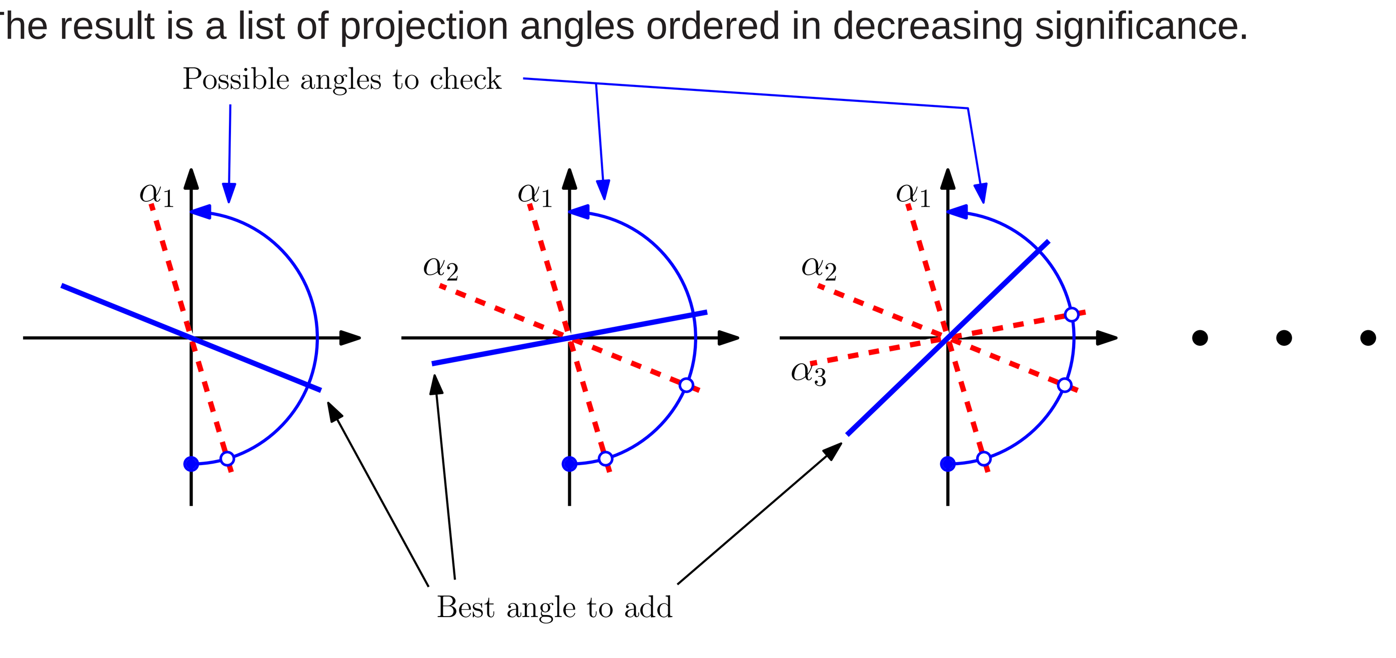

Figure 3: Process of the greedy angle selection strategy.

Altering angles with simulated annealing

- Taken that $\mathbf{x}$ is a reconstructed result and $\mathbf{x}^*$ is the expected reconstruction, the error of the $\mathbf{x}$ reconstruction can be measured by its Relative Mean Error (RME) value:

$$RME(\mathbf{x}) = \frac{\|\mathbf{x} - \mathbf{x}^*\|_2^2}{\|\mathbf{x}^*\|_2^2}$$

- Use the RME value as an energy function and try to minimize it with simulated annealing [3] (assuming that the variables are the projection angles).

Results

We evaluated the results with two different approaches:

- Visual observation.
- Comparing the accuracy of the reconstructions by their Relative Mean Error (RME) values.

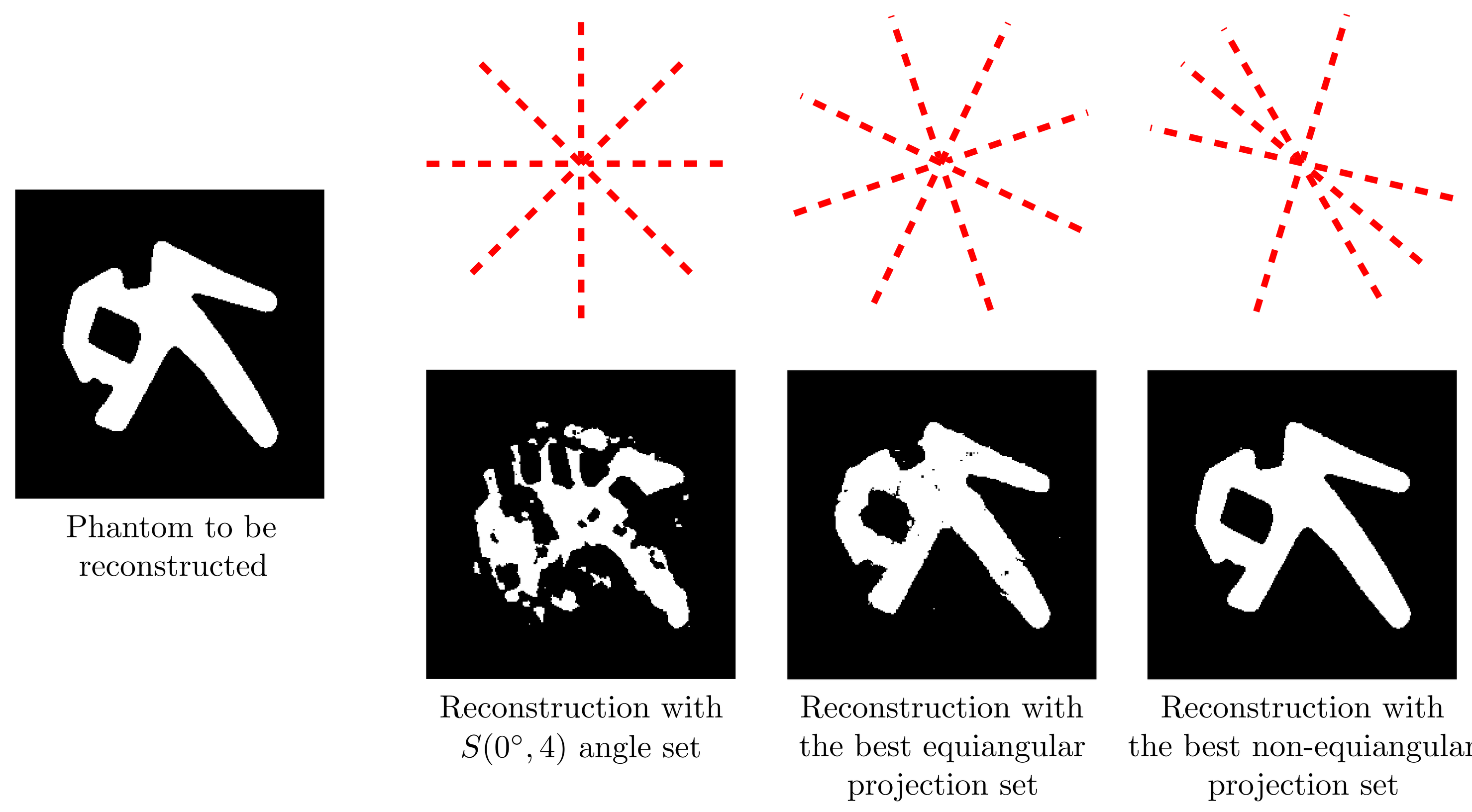

Figure 4: Reconstructions of a selected phantom image, performed from different sets of their projections. (Red dashed lines indicate the directions of the projection rays.)

Table 1: RME values of the best reconstructions of a phantom image, produced by the different angle selection strategies.

Proj.Num.	2	3	4	5	6
Naive	0.7742	0.5836	0.4255	0.1512	0
EquiAng	0.5475	0.1691	0.0500	0.0001	0
Greedy	0.5756	0.1530	0.0117	0	
AltAng	0.5268	0.1617	0.0029	0	

Conclusion

- The result of a binary tomographic reconstruction algorithm can strongly rely on the choice of directions used to take projections with.
- One can reach significant improvement in the reconstruction by finding the optimal equiangular projection sets, and even more improvement can be reached by omitting the assumption of equiangularity.
- Results indicate that this direction-dependency is the intrinsic property of each reconstructed object that – in special cases – can be used for improving the reconstruction without taking additional projections.

References

[1] Herman, G.T., Kuba, A., (Eds.).: *Discrete Tomography: Foundations, Algorithms and Applications*, Birkhäuser, Boston, 1999.

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[3] Metropolis, N., A. Rosenbluth, A. T. Rosenbluth, M., Teller, E.: *Equation of state calculation by fast computing machines*, J. Chem. Phys. **21** (1953), 1087–1092.

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