Michele Sbacchi

Università di Palermo
Dipartimento Storia e Progetto nell'Architettura
Palazzo Larderia
C.so Vittorio Emanuele 188
sbacchi@architettura.unipa.it

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Research

Projective Architecture

Abstract. Michele Sbacchi investigates the real influence of the notion of projection on architectural design before and during the age of Guarini. He takes into consideration concepts such as light and shadow, abstract line, plane, section, projective geometry and perspective. To do this he looks at the ideas of Gregorius Saint Vincent, Alberti, Guarini, Desargues and de l'Orme, among others.

Introduction

In his *Problema Austriacum* of 1648, Gregorius Saint Vincent, a rather famous Jesuit and mathematician, gives an extraordinary allegory of projection. The frontispiece of his book displays a projection of sunlight which, although it passes through a square body, projects, in fact, as a circle on the ground (fig. 1). On a ray of light appears the motto: *Mutat quadrata rotundis*.

Then in the preface he further clarifies his mythological view:

> *Nihil in humano stabile, nec raro Dominos mutant orbis. Ut traiectos per quadrum radios in orbem deduces Quadrata rotundis mutat Sol, ita prosper adversis…* [1647: III].

Gregorius's life-long scientific concern was the problem of squaring the circle. His coupling this problem with the fascinating issue of projection provides exceptional evidence of the deeply symbolic meaning that projections had, especially in the seventeenth century. Gregorius explained the "miracle" of *quadratura* as a divine projection; the anamorphists during that very epoch dwelled upon the same theme: clearly both were fascinated by the "transformation" achieved through projection.

The verb "to project" comes from the Latin *proicere*, literally "to throw forth." The word forth (*pro*), suggests that the idea of future, as the temporal realm of these operations, is, in some way, involved.

The architectural relevance is manifest: architectural "projects," as such, and "projections," as understood in the terminology of architectural drawing, are both the domain of architects. Whereas in English the common root of "project" and "projection" is partly lost because the term "project" is often replaced by "design", it survives in most Latin languages. In French, for instance, *projet* and *projection* are the relevant terms. Despite the linguistic differences,[1] the idea of projection clearly lurks behind both these words, but whereas it is immediately conjured up in the use of the word "projection", it is, in fact, forgotten in the common understanding and use of the term "project."

In this article I shall try to investigate what the real influence of the notion of projection is, both in literal and metaphorical terms, on architectural design. If we want to make a pun, my aim is to see how "projective" a project is.[2] I will also briefly consider the importance of the projective plane, as the paper-made realm of architectural manipulations.

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Fig. 1. Gregorius Saint Vincent, *Problema Austriacum* [1648], frontispiece

Light and shadow

The close relationship between the idea of projection and the idea of drawing is paradigmatically fixed in the legend about the origin of painting reported by Pliny.[3] He tells the story of a Corinthian maiden who, on the occasion of the departure of her lover, wanted something to remember him by. She illuminated his face with a lamp and traced the profile of his shadow on the wall. Drawing, then, was born as the marking edge between shadow and light; moreover, it came out of a projection. Quintilian also refers to the birth of painting as the primordial act of drawing around cast shadows.[4]

Apart from the mythological realm, the light/shadow dichotomy has an important place in art and architectural theories. I might quote Alberti who, almost literally in keeping with Pliny's legend, made *circumscriptio* and *receptio luminis* two of the three basic principles of painting in his *De Pictura* [bk. II, 31]. It is also well known that Daniele Barbaro translated the Vitruvian term *Scenographia* as *Sciographia*, rather than "Perspective," thus making "shadowed drawing" one of the three basic forms of architectural representation.[5] No less interesting is the way by which Henrich Füssli built an evolutionary theory of art entirely grounded on the idea of shadows. For him art evolved from shadow-like images (*sciagrammi*) towards more complex forms to reach maturity with fully colored forms of art. The same concept was adopted by Thomas Kirk and by many others and constituted the basis for the triumph of polychrome architecture [Füssli 1801: 10; Middleton 1985]. At the very root of art, then, we find "projection" as a primary act.

Interest in shadows was a widespread phenomenon during the sixteenth and seventeenth centuries even beyond art theories. Studies on both astronomy and perspective had to refer back to the projection of shadows to establish their internal laws. The writer who placed the greatest emphasis on the importance of the projections of shadows was Biagio Pelacani da Parma who combined shadows (previously reserved to astronomy) with optics (see [Da Costa Kaufmann 1975: 266]). In this regard, the name of Giordano Bruno inevitably comes to mind as the common ontological reference for all these diverse interests. The disciplines involved with problems of graphic representation thus focussed on shadows. This is hardly surprising if we think that shadows and mirrored images are the only two forms of "natural representation":[6] they are respectively the products of the two opposite optical phenomena of total reflection and total absorption of light rays and thus the two ways by which nature can duplicate and represent itself without human intervention.[7] In light of these considerations it is easier to understand that Pliny's legend stigmatizes the edge and the passage from natural representation to man-made representation: from shadow to outline. It is a legend of origin: it portrays the very first step of human representation. From it a fully man-made representation will develop – a representation where "projection" happens on designated surfaces.

Abstract line and plane

If we abandon the archaic realm we have dealt with so far, we note that, in our understanding of drawing, the section of the visual rays – the Albertian *intersecatio* – can hardly be regarded as real or natural; it is, in fact, a highly abstract and artificial operation, where the "projective plane" takes the place of the material surfaces, which constituted the support for primordial drawing. A virtual and abstract mental idea substitutes a material object. Of course such a thing was impossible in the Arcadian gracefulness of Pliny's scene, but this was due to a peculiar reason: in that realm, drawing

was identified with a real line. It describes the verisimilar outline of the object. It has, therefore a strict link with reality. Lines, however – and this is the key – can, in varying degrees, also be abstract.

Another legend can help us to grasp the notion of abstract line. It is the legend reported by Diogenes Laertius and Plutarch, and it ties the discovery of abstract lines to the name of Thales and his attempt to measure the Pyramids.[8] As the story goes, Thales ingeniously thought of comparing the height of the Pyramid and the length of the shadow cast with those of a vertical object such as a stick: the shadow of the Pyramid and that of the stick would be proportional. He therefore constructed two virtual triangles out of the vertical axes of the objects, the lengths of their shadows and the line linking the apex of the objects and the extremes of the shadows. Through this simple observation Thales solved a problem that had long challenged the ancients: how to measure an unreachable object. The device that he employed is his famous theorem on the similar triangles that still carries his name. Yet the legend is important for another reason: Thales conceived of lines in abstract terms. His two triangles contain projective, "virtual" lines such as the one linking the apex of the pyramid to the end of its shadow cast on the ground. Thus he inaugurated the geometry of abstract lines.[9]

With the conception of abstract line we can move more easily towards that idea of projective plane that I suggested earlier. Yet another notion is necessary: that of "section". It too is significantly missing in Pliny's tale or in the tradition of *skiagraphia*. We note that, in that natural context, light rays are not sectioned, they are merely interrupted by the presence either of an object or a human body. In fact, the notion of abstract line implies, consequently, the rather more elaborate idea of virtual abstract plane. The virtual plane, in turn, permits us to reconsider the somewhat simple process of shadow casting, in the form in which we have been dealing with so far. For a virtual plane, in contrast to an object, allows a simple but fundamental operation for architectural drawing: the cutting of light rays. The idea of section thus comes to life. "Natural" projection and "artificial" section complete so the apparatus of graphic representation.

The "place" of projection becomes, then, an ideal surface, no longer necessarily a material object. It will be the transparent surface suggested by Alberti: *...non altrimenti che essa fosse di vetro translucente...*(*De Pictura,* bk. I, ch. 12).

The role of the projective plane within the making of architectural design is interestingly emphasized by Alberti and also deeply linked to his notion of composition. As we know, Alberti does not use the term *compositio* in *De re aedificatoria*, the closest term being the rather different one of *concinnitas*: for him, *concinnitas* intrinsically regulates architecture whereas ornaments belong to a complementary beauty. The significant absence of the term *compositio* together with a statement contained in *De Pictura*, substantiate his position. In *De Pictura* Alberti states that architects borrow ornaments from painters, who were entitled to deal with *compositio*: "the architect took from the painter architraves, capitals, bases, columns and all the other fine features of building."[10] Alberti therefore does not neglect the notion of composition, but he relegates it to the two-dimensional realm of pictorial representation. Furthermore he suggests a borrowing procedure from the two-dimensional plane of painters to the three-dimensional reality of architecture. As Hubert Damish has written: "The kinship between the notions of composition and concinnitas is thus established: both refer to the way in which the parts of the same body, of the same object relate and match, But while a body belongs to a three-dimensional space, the notion of composition is valid only

insofar as it is ascribed to the two-dimensional projective plane."[11] *Concinnitas* is for bodies what *compositio* is for their projections. The idea of projection is yet present elsewhere in Alberti's theory, as in his parallel of projected ornaments with alphabetic letters. In this case the procedure is different, since the projection is used as far as the outline is concerned. Rather than establishing a hierarchy of disciplines or a literal procedure of borrowing between them, Alberti was seeking the realm, in a very strict sense, of architectural design. Quite clearly, for him, the projective plane assumes a basic role, for it is the *locus* of architecture as the textual space is the *locus* of writing, "For Alberti no architecture is possible if not born on paper, through the function of projection and transcription that the drawing assumes."[12] Paper, then, nothing but paper, is, ultimately, the way of being and the material status of an architectural project.

Not insignificantly Guarini, the most "projective" of the architects, also emphasized the role of paper as the primary support of architectural design to the point of almost identifying the two things. Several times throughout his treatise he recalls the paper-like essence of architectural drawing:

> Drawing, or idea according to Vitruvius, has three parts, the first of which is called Ichnografia, which is a description and expression *on paper* of what will be occupied by the building, which is drawn in plan; the second is called Orthografia or Elevation, and it is the description and expression *on paper* of the elevation of one of its sides"... .[13]
>
> ...Ichnografia being a description *on paper* of buildings… .[14]

He also points out the somewhat crude fact that, if an actual product should be attributed to architects, it will be the "paper of the project," rather than buildings: "the architect does not build walls, nor roofs, nor machines, nor statues, nor doors, nor locks, nor bricks."[15] He consistently regards the production of drawings on paper as the basic activity of architects. This awareness brought him to devote a considerable part of his treatise to topics such as drawing instruments, the use of paper, the making of ink, etc. He says quite explicitly: "the instruments used by Architecture proper in order to direct the Arts subject to it are few because they are only those which serve to draw and represent its ideas *on paper*."[16] Lomazzo, not concerned with building practise, made this process more magical and "envisaged the form emerging from paper as a ghost materializing."[17] It is probably worth going back to the emphasis Alberti places on the idea of "ornaments composed through projection". Alberti's conception was represented, with an interesting shift, by Claude Bragdon in a booklet, which carries the meaningful title of *Projective Ornament* [Bragdon 1915]. Bragdon's "projective ornaments" are superficial decorative patterns composed in a two-dimensional plane. Acting in a post-projective geometry epoch, Bragdon was concerned with exploiting the possibilities of projections for his decorative purposes. The *projectivity* of Bragdon's ornaments lays with the way in which they are begotten – i.e., by means of a broad use of projective geometry. The rule of *compositio*, rendered even stronger by the possibilities opened up by projective geometry, returns. "Architecture composes through painting" as in Hubert Damish's reading of Alberti (and is reinforced by projective geometry, we could add, thus acknowledging Bragdon).

Projective geometry

Bragdon's use of projective geometry is interesting in its peculiarity, since projective geometry, somewhat strangely, even paradoxically, is taken as a compositional device (in

a fine way, I would add). Projective geometry was in fact meant to follow rather different paths, almost avoiding uses such as Bragdon's. It was conceived by Gérard Desargues as a deliberate attempt to create a universal rule – a *maniere universelle* as Desargues's very title indicates – applicable to the different realms in which graphic representation was required. Clear evidence of this intention is, paradoxically, the very difficulty that master masons found in using it, because of its extreme abstruseness.[18] Desargues, and his pupil Abraham Bosse, a painter who devoted himself to defending and promulgating his master's theories, took the discipline as a totally neutral tool which, then, could transcend the specificity of the different disciplines.[19] It is significant that currently projective geometry is often contrasted to metric geometry, highlighting the fact that its peculiarity consists in allowing planar transformations which are valid despite numerical attributes.[20] Quite symptomatic of Desargues's and Bosse's rational attitude is their treatment of projections on irregular surfaces (see [Baltrusaitis 1969: 71] (fig. 2).

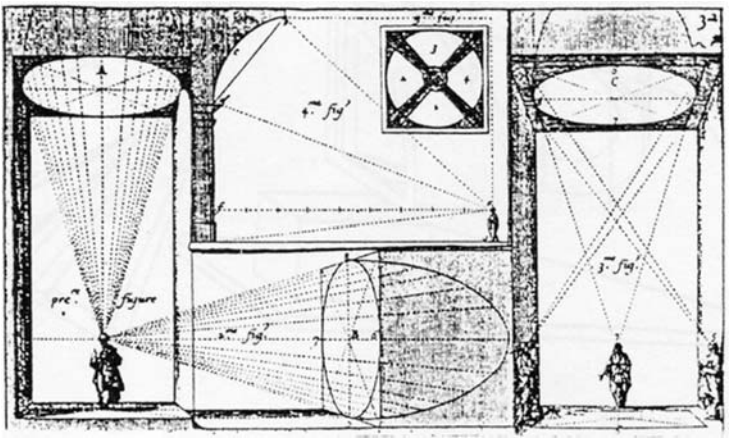

Fig. 2. The rational treatment of perspective from *Moyen universel de pratiquer la perspective sur les tableaux, ou les surfaces irregulieres* [Bosse 1653]

Desargues dedicated an entire treatise to this topic [Bosse 1653] but the variations that the curve or inclined surface of a vault produce on representation are absolutely emptied of their mystic and symbolic power. Whereas Niceron, Scott and many others were fascinated by these irregularities and made them the object of their anamorphic art, Desargues and Bosse did just the opposite, treating distorted representations as ordinary, plain cases. The contrast between the highly symbolic way of dealing of the anamorphists and the cool rational attitude of Desargues and Bosse is even more striking if we consider that they developed their disciplines during the same years, and that both Desargues and Bosse were very close to people like Niceron and the circle of the Minims which was a centre point of anamorphic art.

But it must be borne in mind that the idea of projection went, in that period, through a major change, which, although it originated within the realm of graphic representation would have, as I will try to show, a substantial architectural twist. Up to the Renaissance, perspective was essentially intended to reproduce the natural process of human vision: the use of the same term of *perspectiva* for both optics and perspective is symptomatic of this fact. Similarity, or, better, verisimilitude had firstly to be sought. Consistently, the symmetrical correspondence between the elements of real objects and those of their relevant images was not recognized. Objects, as rendered in perspective,

were considered altered (*digradati* was the term frequently used). Attention was indeed paid to what was changed by projection. Even Descartes admitted, with something like regret, that correspondence was not always observed between reality and perspective: "Following the rules of perspective we often better represent circles with ovals and squares with lozenges rather than with other squares … so that often, to be more perfect as images and to better represent the object, they have not to resemble it at all" [Descartes 1637: 113]. As J. V. Field and Jeremy Gray put it, "Emphasis was then upon what has been changed by the 'projection'" [Field & Gray 1987: 28]. Desargues, differently, introduced a new way of considering the process. His notion of invariance was the root of an understanding in which attention was focussed upon the elements which remained unchanged. This attitude can easily be grasped by looking at one of his few drawings of a human shape contained in Bosse's expanded version of his *Perspective*. But this change cannot be attributed to Desargues's contribution alone. Very probably it was the outcome of developments in several fields and certainly a substantial part was played by stereotomy. Indeed, as we will see later, the theorization of the art of stone-cutting was made on the basis that "projection" implied a symmetrical correspondence, in the full meaning of the word.

Aside from the work already mentioned, Desargues wrote a book on sundials [1640a], a treatise on stereotomy [1636] and a draft for a book on conic sections [1639]. His studies, though original, did not come from an isolated enterprise, but arose from that very prolific scientific community that was the "mathematical" Paris[21] of the seventeenth century. At that time within the Parisian *intellighenzia* there was close contact between scholars of geometry and architects, with a leading role played by Jesuits and Minims, as we have already seen. Desargues's conceptual contribution to projective geometry consists mainly in the notion of "point at infinity" and "line at infinity," to which later Poncelet added the "plane at infinity". These are developments of the idea of vanishing point originally formulated in 1600 by Guido Ubaldo Del Monte as *punctum cuncursus*.[22] The striking contrast between Desargues's mathematization of the notion of "point at infinity" and the still symbolic understanding of his contemporaries can be clearly grasped by looking at Pietro Accolti's *Lo Inganno degli Occhi* in which, as late as in 1625, he is still talking about *Occhio del Sole* (Eye of the Sun) (fig. 3).[23]

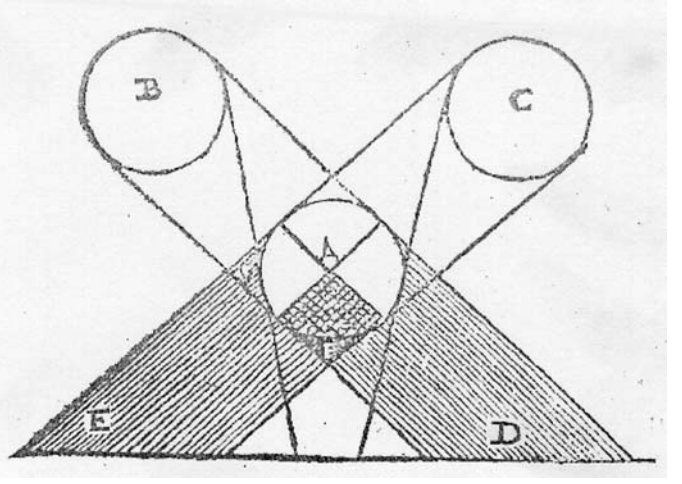

Fig. 3. Projective bands. Pietro Accolti, *Lo Inganno degli Occhi* [1625: 143]

Later, as is well known, the process of rationalization was completed by Monge, who took up these works and systematized them in his descriptive geometry, which, almost unchanged, we still use today. Apart from the merely mathematical notations, what interests me is that projective geometry, in the Desargues-Bosse version, is a discipline both graphically and conceptually centred around the two notions of projection and section, which we have singled out before as foundations of the idea of drawing.[24] But fascination with the possibilities of projections at that time led to a further development of the ancient discipline of conic sections, which is tied to the name of Apollonius. In this regard Desargues is again a key figure: his concern for this subject and his *Brouillon Projet* which, alongside Pascal's *Essay on Conics*, is a seminal contribution in the field, cannot be taken as coincidental. Using again the idea of projection, both Desargues and Pascal at that time visualized the three conics – hyperbola, parabola and ellipse – as projections of the circle. Their studies testify to an uncommon interest paid by French scholars in speculating about the generations of curves, especially when generated by the intersection of different surfaces. Is it likely that architects' experimentations in domes migrated to a broader context? Probably the real common root and stimulus must be sought within the very realm of architecture itself, namely in the vaulting-stereotomy tradition. The making of vaults was, indeed, carried by French architects and master masons to an unbelievably high degree of virtuosity. Vaults and all their manifold derivates such as domes, pendentives, *trompes*, suspended stairs were experimented in the most various shapes, with a spectacular display of unadorned intradoses. This virtuosity gave birth to the more theoretical science of stereotomy – literally "the cutting of solids" – concerned with the possibility of taking control over stone cutting through a two-dimensional representation. The highest possible degree of precision was one of the key requisites. But the real obsession was in the possibility of "making two-dimensional", objects which are three-dimensional, almost making the attempt to unfold them on paper.[25] Stereotomy was a real gymnasium of projections and transformations, in which, as I have already pointed out, the idea of correspondence between drawings and objects was a major concern.

There are, then, enough reasons to think that at that time, especially through Desargues's synthesis, the idea of "projectivity" acted as a common channel linking different disciplines, all more or less on the periphery of architecture. The relationship between Desargues's work on conic sections and his work on perspective has already been demonstrated. Even more striking is the relationship between shadow theory in perspective, conic sections and sun-dialling, this latter being another discipline whose connection with architecture is notoriously sanctioned by Vitruvius's inclusion of it in Book IX of his treatise. The link between these disciplines is so unquestionable that it went, at that time, even to the point of actual confusion: Simon Stevin's book on sun-dialling was translated in Latin as *De Sciagraphia*; John Wells entitled his treatise on sun dialling *Sciagraphia or the Art of Shadows*. Jonas Moore made the connection even more explicit in his "Epistle to the Reader", the introduction to the 1659 English translation of Desargues's work on sun-dials:

> Dyalling I accompt one kind of Perspective, for that glorious Body the Sun, the Eye of the world, traceth out the lines and hour-points by his Diurnal Course, and upon the resubjected Plane by the laws of Picture, Scenographically delineates the Dyal.

He also relates sun-dialling to conic sections:

> ... this point B [the tip of the gnomon, whose shadow marks the time], you must imagine to be the center of the Earth (for the vast distance of the Sun, maketh the space betwixt the Center and superficies of the Earth to be insensible) and from it at all times of the year the Sun in its course forms two Cones, whose apex is the point B, that next the Sun termed *Conus luminosus* or the light Cone, the other whereof our Author makes use, termed *Conus umbrosus* the dark Cone, now this dark Cone, if by any three points equally distant from the Apex B, the Cone be cut, the section will be a Circle parallell to the Equinoctial: And thereby, as the Author shews many wayes, the position of the Axis or Gnomon may be found out, and the Dyal easily made.

Light, shadow, projection, section: the very ingredients of Pliny's legend are used now in a context in which science and myth are strictly bound together. How these concepts migrated into the very heart of architectural design is clearly shown by the cases of Philibert de l'Orme and Guarino Guarini, not by coincidence both linked to the French milieu. De l'Orme is well known as the first theorist of stereotomy; Guarini's involvement with astronomy, geometry and mathematics is notorious. It is also noteworthy that he taught mathematics in Paris for four years. Yet their concern for disciplines like sun dialling, stereotomy and conic sections went beyond the mere treatment of complementary topics; they were melded into the design method itself. Robin Evans has demonstrated quite clearly how the idea of correspondence and that of projection play a central role in de l'Orme's Chapel of Anet. Guarini is an even more striking example. From his drawings, so full of projections of ceilings or vaults, emerges a deep concern with the architectural "correspondence of elements" between plan and elevation. Yet the idea of projecting one element onto the other is used by him horizontally as well. As Manfredo Tafuri has pointed out, the generation of undulating pilasters in Guarini's church of S. Maria della Divina Provvidenza in Lisbon is the mere consequence of the projection of a twisted column onto the wall: "If a pilaster is nothing but a column projected onto the plane, once chosen the model of a twisted column – present in S. Maria della Divina Provvidenza as a minor order – pilasters will follow their undulated proceeding."[26]

His church of S. Anne La Royale is a further example of "projectivity in plan", and in this case the projection is a central one (fig. 4).

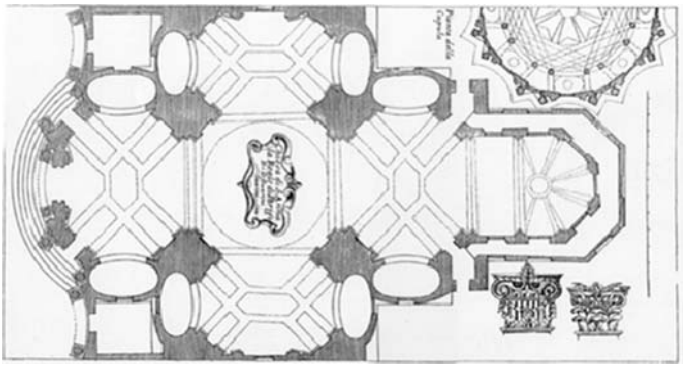

Fig. 4. Guarino Guarini, plan of the church of St Anne La Royale, Paris, *Architettura Civile* [1737]

Guarini will make no mystery of these procedures, For him, quite clearly, a plan is a projection *tout court*: "In projection. therefore Ortografia" [Guarini 1737, bk. IV, I]. And architecture itself, for him, is divided in the somewhat projective categories of *Icnografia*, *Ortografia Elevata* and *Ortografia Gettata*. The definitions of *Ortografia Elevata* and *Ortografia Gettata* are full explications of his design process. On *Ortografia Elevata*:

> The architect has to speculate two kinds of orthography; one which presumes the plane and from it elevates its drawing; a second one which does not presume any drawing in plan, but what is drawn 'above' which has to be later cast onto the plane and see which part is occupied by it; yet orthographies are two, one elevated, one depressed, we will talk about the first in the following treatise...[27]

On *Ortografia Gettata*:

> This orthography is opposed to the previous both by name and by its way of operating; for whereas in the former plane surfaces are elevated by perpendicular lines to give them body and so forming the fabric, this latter on the contrary reduces by perpendicular lines the bodies which are suspended above and reduces them on the plane to unfold their surface…[28]
>
> ...this is why this Ortografia has been experimented, which indeed puts their surfaces on the plane and shapes them like they are above.[29]

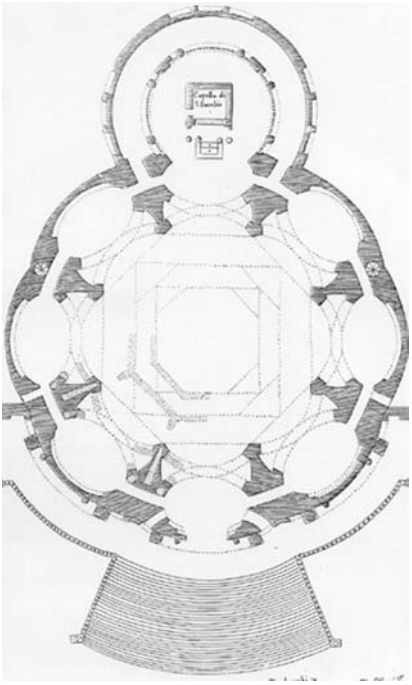

Fig. 5. Guarino Guarini, project for the Sanctuary of Oropa, from *Architettura Civile* [1737]

His dealing with architectural representation is consistent with this idea of "plan surfaces which elevate with perpendicular lines and form the fabric." He has a predilection for drawing in section and especially for constraining in the plan all the elements which will later "explode" in elevation. His plans[30] for the church of S. Lorenzo and for the Oropa sanctuary (fig. 5), in which all the elements of the dome and even its section are projected in one single drawing, are remarkable outcomes of his theory.

Guarini's case is indicative of the notion that the projective dimension of architectural design is not a mere representational constraint. The continuous shifting from the three-dimensional being of architecture and the two-dimensional being of architectural projects – performed constantly during the design process – can strongly orient design choices. The technique of projection, which allows this shifting, makes the 'conceiving of architecture' a highly peculiar process – even from this specific point of view. Therefore, the 'construction of the project' is a process much different from the actual 'making of architecture', this latter taking place in a realm in which the three dimensionality is unquestioned.

Notes

1. I will not deal here with this interesting linguistic shift nor with the rise of the notion of design because this would lead us out of my topic. See [Frampton 1986].
2. I owe the term "projectivity", with this non-mathematical meaning, to Dalibor Vesely.
3. Pliny the Elder (Gaius Plinius Secundus), *Naturalis Historia*, book XXXV; see [Rosenblum 1957].
4. : *... non esset pictura nisi quae lineas modo extremas umbrae quam corpora in sole fecissent, circumscriberet* [Quintilian, *De Institutione Oratoriae*, bk. X, chap. ii, 7].
5. [Barbaro 1556: 30]; see also Perrault's notorious remark in [Perrault 1979: 10].
6. The development and the conceptual background of outline representation has been masterfully investigated in [Rosenblum 1956]. See also [Ottavi Canina 1982] and [Rykwert 1980: 366ff].
7. It is quite puzzling in this respect that Lomazzo, acknowledging these two extreme positions, divided perspective in *Ottica*, *Sciographia* and *Specularia*. It is equally puzzling that by the time he wrote the *Trattato della Pittura* Lomazzo was blind. Also interesting is the definition of mirror as "Figuratam, per Esemplare" in the *Vocabolario degli Accademici della Crusca* (Naples, 1747, vol. IV, p. 372). See also [Oechslin 1983].
8. See [Serres 1982 : 85], [Evans 1986] and [Meserve 1983: 222-223]. Less legendary discoveries, made through the same principle, are those of Aristarchus, who made a similar attempt to measure the distances from the earth of the sun and the moon, and of Eratosthenes, who did the same with the circumference of the earth.
9. A distinction has therefore to be made between two types of lines in architectural drawing: those representing actual bodies and those having no corresponding elements in reality.
10. Alberti, *De Pictura*, II, 26 [1972: 60]: *Nam architectus quidem epistilia, capitula, bases, columnas fastigiaque et huiusmodi ceteras omnes aedificiorum laudes, ni fallor, ab ipso tantum pictore sumpsit*.
11. [Damish 1986] ; Savignat draws the same conclusions about Alberti. : "*...la composition de la forme architecturale n'est alors qu'un assemblage de lignes, de figures sur la surface de la feuille de dessin*" [Savignat 1983 : 63].
12. [Damish 1986]. Quite interesting in this regard are Louis Marin's speculations in "Les voies de la carte" [Marin 1982] and in the chapter "Utopiques de la carte" [Marin 1973].
13. *Il Disegno, o Idea secondo Vitruvio, ha tre parti, delle quali la prima dicesi Ichnografia, che è la descrizione, ed espressione* in carta *di quello, che deve occupare la fabbrica, che si disegna nel piano; l'Ortografia, o Alzato chiamasi la seconda, che è la descrizione ed espressione* in carta *della elevazione di una sua Faccia: ...* [Guarini 1737: bk. II, intr.] (emphasis mine).
14. *Essendo la Ichnografia ... una descrizione degli edifici* sulla carta [Guarini 1737: bk. II, intr] (emphasis mine).

15. *l'architett non fabbrica muri, non tetti, non macchine, né statue, né porte, né serrature, né mattoni* [Guarini 1737: bk. I, chap. I, 8].
16. *…gli instrumenti di cui si serve l'Architettura per sé unicamente, in quanta dirige le Arti a sé soggette, son pochi, perché non sono se non quelli i quali servono per disegnare e rappresentare le sue idee* sulla carta [Guarini 1737: bk. I, chap. IV, 21] (emphasis mine).
17. George Hersey [1976: 85] referring to [Lomazzo 1590].
18. Yet it is fair to add that part of the incomprehensibility of Desargues's work is due to the odd botanical language that he adopted. For this, see the letter that Descartes wrote to Desargues to exhort him to use a more accessible language, in [Descartes 1936-1963: vol. III (1940), 228-229]. This fact has been recently considered in [Field & Gray 1987: 60-68].
19. See [Pérez Gómez 1985: 93ff.] and [Scolari 1984: 46].
20. "Projective Geometry: a branch of geometry that deals with the properties of geometric configurations that are unaltered by projective transformation and in which the notion of length does not appear." S.v. "Projective Geometry", *Webster's Third International Dictionary of the English Language* (Springfield, MA: Merriam Webster Inc. Publisher, 1986), p. 1814.
21. I am using this expression after the title of David Smith's booklet, *Historical-Mathematical Paris* (Paris: Les Presses Universitaires de France, 1925).
22. See [Cassina 1961: 306ff]. Meserve instead attributes to Kepler the notion of "point at infinity" [Meserve 1983: 45-47]. Quite correctly Field and Gray, although acknowledging Kepler's intuition, have recognized the conceptual gap between Kepler's and Desargues's concept; see [Field & Gray 1987: 87-89].
23. *... insegnandoci il testimonio del senso visivo manda l'ombre sue parallele al piano ... con la infinita distanza del luminoso degli opachi ... così restiamo capaci potersi all'occhio nostro, in disegnar far rappresentazione di quella precisa veduta di qualsivoglia dato corvo, esposto all'occhio (per così dire) del Sole quale ad esso Sole gli si rappresenta in veduta: onde siccome specolando intendiamo il Sole non vedere giammai alcuna ombra degli opachi, e superficie ch'egli rimiri e illustri ...* [Accolti 1625: 143].
24. See [Young 1930], especially "Introduction, Paragraph 2: Projection and Section, Correspondences."
25. For me it is not, therefore, coincidental that the idea of "beauty produced by precision of execution" has been extolled by a French scientist-architect such as Perrault. It must also be added that the ideology of precision in stereotomy was also brought about by the economic reason of using the material with the least possible amount of waste. For this see [Potié 1984].
26. *Se la lesena, infatti, non è altro che la colonna proiettatta sui piano, una volta scelto il modello della colonna tortile – presente nella Divina Provvidenza come ordine minore – le lesene dovranno seguirne l'andamento ondulato* [Tafuri 1970: 672, note 1]. Tafuri has also pointed out that projections operate in two other works by Guarini: the curved facade of the Annunziata church and the Tabernacle in Verona: *La nuova legge guida è proprio quella proiezione delle gerarchie dello spazio interno sui piano ... La meccanica combinatoria dei corpi geometrici si proietta, qui, sulla struttura discreta della parete inflessa e articolata* [Tafuri 1970: 669].
27. *DELLA ORTOGRAFIA ELEVATA. Due sorte di ortografia deve speculare l'architetto; l'una che presuppone il piano, e da esso solleva il suo disegno; l'altra che non presuppone alcun disegno sui piano, ma quello che si disegna in alto, che poi si deve gettare in piano, e vedere qual parte vien occupata da esso: però due sono le ortografie, una si dirà elevata, l'altra si chiamerà depressa; di questa ne scriveremo nel trattato seguente...* [Guarini 1737, bk. III, intr. (1968: 113)].
28. *DELLA ORTOGRAFIA GETTATA. Questa ortografia siccome è opposta nel suo titolo all'antecedente, così anche nel suo modo di operare; perché là dove in quella le superfizie piane si innalzano con linee perpendicolari, per dare a loco corpo, e formare la fabbrica, questa per lo contrario i corpi in alto sospesi con linee perpendicolari riduce in piano per istendere la loro superficie…* [Guarini 1737, bk. IV, intr. (1968: 288)]. The peculiar expression, *per dare a loro corpo*, to give them body, is noteworthy.

29. *...perciò è stata ritrovata questa Ortografia, che appunto mette le loro superfizie in piano, e le forma, come sono in alto, e sarebbero nel proprio loro luogo, di questa abbiamo a ragionare*. [Guarini 1737, bk. IV, intr. (1968: 288)].
30. The plan of S. Lorenzo raises the question about Guarini's use of the term *vestigium* to designate a plan. The same term occurs in another plan. Although the attribution of these captions to Guarini himself is quite doubtful, the term remarkably occurs in his treatise: *La Ortographia non è altro, secondo che provo nel nostro Euclide al tratt. 26 che una impressione, terminazione o vestigio notato nel piano di una superficie ad esso normale* [Guarini 1737: bk IV, I]. Aside from the Vitruvian connections, Guarini's *vestigium* is particularly puzzling if confronted with Desargues's argument about the substitution, in architecture, of the term "plan" with that of *assiette*.

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About the author

Michele Sbacchi is Associate Professor in architectonic and urban composition at the Faculty of Architecture of the University of Palermo. He is also a practicing architect.