International Electronic Journal of Geometry
Volume 6 No.1 pp. 159-169 (2013) ©IEJG

PROJECTIVE CURVATURE TENSOR OF A SEMI-SYMMETRIC METRIC CONNECTION IN A KENMOTSU MANIFOLD

AJIT BARMAN AND U. C. DE

(Communicated by Cihan ÖZGÜR)

Abstract. The object of the present paper is to study a Kenmotsu manifold admitting a semi-symmetric metric connection whose projective curvature tensor satisfies certain curvature conditions.

1. Introduction

The product of an almost contact manifold M and the real line R carries a natural almost complex structure. However if one takes M to be an almost contact metric manifold and suppose that the product metric G on $M \times R$ is Kaehlerian, then the structure on M is cosymplectic [12] and not Sasakian. On the other hand Oubina [15] pointed out that if the conformally related metric $e^{2t}G$, t being the coordinate on R, is Kaehlerian, then M is Sasakian and conversely.

In [19], S. Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold M, the sectional curvature of plane sections containing ξ is a constant, say c. If $c > 0$, M is a homogeneous Sasakian manifold of constant sectional curvature. If $c = 0$, M is the product of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature. If $c < 0$, M is a warped product space $R \times_f C^n$. In 1971, Kenmotsu studied a class of contact Riemannian manifolds satisfying some special conditions [14]. We call it Kenmotsu manifold. Kenmotsu manifolds have been studied by J.B. Jun , U.C. De and G. Pathak [13], C. Özgür and U.C. De [16], U.C. De and G. Pathak [9], A. Yıldiz, U.C. De and B.E. Acet [22] and others.

H.A. Hayden [11] introduced semi-symmetric linear connections on a Riemannian manifold and this was further developed by K. Yano [20], K. Amur and S.S. Pujar [1], M. Prvanović [17], U.C. De and S.C. Biswas [8], A. Sharfuddin and S.I. Hussain [18], T.Q. Binh [3], F.Ö. Zengin and S.A. Uysal and S.A. Demirbag [26], S.K. Chaubey and R.H. Ojha ([6], [7]), H.B. Yılmaz [23] and others.

Date: Rceived: September 22, 2012 and Accepted: November 9, 2012.
2000 *Mathematics Subject Classification.* 53C15, 53C25.
Key words and phrases. Kenmotsu manifold, projective curvature tensor, ξ-projectively flat, quasi-projectively flat, ϕ-projectively flat, η-Einstein manifold.

Let M be an n-dimensional Riemannian manifold of class C^∞ endowed with the Riemannian metric g and D be the Levi-Civita connection on (M^n, g).

A linear connection ∇ defined on (M^n, g) is said to be semi-symmetric [10] if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y, \tag{1.1}$$

where η is a 1-form and ξ is a vector field given by

$$\eta(X) = g(X,\xi), \tag{1.2}$$

for all vector fields $X \in \chi(M^n)$, $\chi(M^n)$ is the set of all differentiable vector fields on M^n.

A semi-symmetric connection ∇ is called a semi-symmetric metric connection [11] if it further satisfies

$$\nabla g = 0. \tag{1.3}$$

A relation between the semi-symmetric metric connection ∇ and the Levi-Civita connection D on (M^n, g) has been obtained by K. Yano [20] which is given by

$$\nabla_X Y = D_X Y + \eta(Y)X - g(X,Y)\xi. \tag{1.4}$$

We also have

$$(\nabla_X \eta)(Y) = (D_X \eta)Y - \eta(X)\eta(Y) + \eta(\xi)g(X,Y). \tag{1.5}$$

Further, a relation between the curvature tensor R of the semi-symmetric metric connection ∇ and the curvature tensor K of the Levi-Civita connection D is given by

$$\begin{aligned} R(X,Y)Z = K(X,Y)Z + \alpha(X,Z)Y - \alpha(Y,Z)X +\\ g(X,Z)QY - g(Y,Z)QX, \end{aligned} \tag{1.6}$$

where α is a tensor field of type (0,2) and Q is a tensor field of type (1,1) which is given by

$$\alpha(Y,Z) = g(QY,Z) = (D_Y \eta)(Z) - \eta(Y)\eta(Z) + \frac{1}{2}\eta(\xi)g(Y,Z). \tag{1.7}$$

From (1.6) and (1.7), we obtain

$$\begin{aligned} \tilde{R}(X,Y,Z,W) = \tilde{K}(X,Y,Z,W) - \alpha(Y,Z)g(X,W) +\\ \alpha(X,Z)g(Y,W) - g(Y,Z)\alpha(X,W) +\\ g(X,Z)\alpha(Y,W), \end{aligned} \tag{1.8}$$

where

$$\tilde{R}(X,Y,Z,W) = g(R(X,Y)Z,W), \quad \tilde{K}(X,Y,Z,W) = g(K(X,Y)Z,W). \tag{1.9}$$

The Projective curvature tensor is an important tensor from the differential geometric point of view. Let M be a $(2n+1)$-dimensional Riemannian manifold. If there exists a one-to-one correspondence between each coordinate neighbourhood of M and a domain in Euclidian space such that any geodesic of the Riemannian manifold corresponds to a straight line in the Euclidean space, then M is said to be locally projectively flat. For $n \geq 1$, M is locally projectively flat if and only if the projective curvature tensor P vanishes. Here the projective curvature tensor P with respect to the semi-symmetric metric connection is defined by

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(Y,Z)X - S(X,Z)Y], \tag{1.10}$$

From (1.10), it follows that

$$\tilde{P}(X,Y,Z,W) = \tilde{R}(X,Y,Z,W) - \frac{1}{2n}[S(Y,Z)g(X,W) - S(X,Z)g(Y,W)], \tag{1.11}$$

and

$$\tilde{P}(X,Y,Z,W) = g(P(X,Y)Z,W), \tag{1.12}$$

for X, Y, Z , $W \in \chi(M)$, where S is the Ricci tensor with respect to the semi-symmetric metric connection. In fact M is projectively flat if and only if it is of constant curvature [21]. Thus the projective curvature tensor is the measure of the failure of a Riemannian manifold to be of constant curvature.

In this paper we study the projective curvature tensor on Kenmotsu manifold with respect to the semi-symmetric metric connection. The paper is organized as follows : After introduction in section 2, we give a brief account of the Kenmotsu manifolds. In section 3, we investigate the quasi-projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection and we prove that the manifold is an η-Einstein manifold. Section 4 is devoted to study ξ-projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection. Section 5 deals with ϕ-projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection. Finally, we study $P.S = 0$ in a Kenmotsu manifold with respect to the semi-symmetric metric connection.

2. Kenmotsu Manifolds

Let M be an $(2n+1)$-dimensional almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g) consisting of a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g on M satisfying [4]

$$\phi^2(X) = -X + \eta(X)\xi, \;\; g(X,\xi) = \eta(X), \tag{2.1}$$

$$\eta(\xi) = 1, \;\; \phi(\xi) = 0, \;\; \eta(\phi(X)) = 0, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.3}$$

for all vector fields X ,Y on M. If an almost contact metric manifold satisfies

$$(D_X\phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.4}$$

then M is called a Kenmotsu manifold [14]. From the above relations, it follows that

$$D_X\xi = X - \eta(X)\xi, \tag{2.5}$$

$$(D_X\eta)(Y) = g(X, Y) - \eta(X)\eta(Y). \tag{2.6}$$

Moreover the curvature tensor K and the Ricci tensor $\tilde{S}$ of the Kenmotsu manifold with respect to the Levi-Civita connection satisfies

$$K(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

$$K(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.8}$$

$$K(X, \xi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.9}$$

$$\tilde{S}(\phi X, \phi Y) = \tilde{S}(X, Y) + 2n\eta(X)\eta(Y), \tag{2.10}$$

$$\tilde{S}(X, \xi) = -2n\eta(X). \tag{2.11}$$

We state the following lemma which will be used in the next section:

Lemma 2.1. [14] *Let M be an η-Einstein Kenmotsu manifold of the form $S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$. If $b = constant(or, a = constant)$, then M is an Einstein one.*

3. Quasi-Projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection

Definition 3.1. A Kenmotsu manifold is said to be quasi-projectively flat with respect to the semi-symmetric metric connection if

$$g(P(X,Y)Z,\phi W)=0. \tag{3.1}$$

Definition 3.2. A Kenmotsu manifold is said to be an η-Einstein manifold if its Ricci tensor $\tilde{S}$ of the Levi-Civita connection is of the form

$$\tilde{S}(X,Y)=ag(X,Y)+b\eta(X)\eta(Y), \tag{3.2}$$

where a and b are smooth functions on the manifold.

Using (1.7), (2.2) and (2.6) in (1.6), we obtain

$$\begin{aligned}R(X,Y)Z=K(X,Y)Z-3g(Y,Z)X+3g(X,Z)Y+\\ 2\eta(Y)\eta(Z)X-2\eta(X)\eta(Z)Y+\\ 2g(Y,Z)\eta(X)\xi-2g(X,Z)\eta(Y)\xi.\end{aligned} \tag{3.3}$$

Using (1.9) in (3.3), we get

$$\begin{aligned}\tilde{R}(X,Y,Z,W)=\tilde{K}(X,Y,Z,W)-3g(Y,Z)g(X,W)+3g(X,Z)g(Y,W)+\\ 2\eta(Y)\eta(Z)g(X,W)-2\eta(X)\eta(Z)g(Y,W)+\\ 2g(Y,Z)\eta(X)\eta(W)-2g(X,Z)\eta(Y)\eta(W).\end{aligned} \tag{3.4}$$

Contracting X in (3.3), we have

$$S(Y,Z)=\tilde{S}(Y,Z)-2(3n-1)g(Y,Z)+2(2n-1)\eta(Y)\eta(Z). \tag{3.5}$$

Putting $Z=\xi$ in (3.5) and using (2.11), (2.1) and (2.2), we obtain

$$S(Y,\xi)=-4n\eta(Y). \tag{3.6}$$

Again contracting Y and Z in (3.5), it follows that

$$r=\tilde{r}-2n(6n-1). \tag{3.7}$$

where r and $\tilde{r}$ are the scalar curvature with respect to the semi-symmetric metric connection and the Levi-Civita connection respectively.
Putting $X=\phi X$ and $Y=\phi Y$ in (1.11) and using (1.12), we get

$$\begin{aligned}g(P(\phi X,Y)Z,\phi W)=\tilde{R}(\phi X,Y,Z,\phi W)-\\ \frac{1}{2n}[S(Y,Z)g(\phi X,\phi W)-S(\phi X,Z)g(Y,\phi W)].\end{aligned} \tag{3.8}$$

We begin with the following:

Lemma 3.1. *Let M be a $(2n+1)$-dimensional Kenmotsu manifold. If M satisfies*

$$g(P(\phi X,Y)Z,\phi W)=0,\quad X,Y,Z,W\in\chi(M), \tag{3.9}$$

then M is an η-Einstein manifold.

Proof: Using (3.9) in (3.8), we have

$$\tilde{R}(\phi X,Y,Z,\phi W)=\frac{1}{2n}[S(Y,Z)g(\phi X,\phi W)-S(\phi X,Z)g(Y,\phi W)]. \tag{3.10}$$

Again using (3.4) and (3.5) in (3.10), it follows that

$$\begin{aligned}\tilde{K}(\phi X,Y,Z,\phi W)=\frac{1}{n}g(Y,Z)g(\phi X,\phi W)-\frac{1}{n}g(\phi X,Z)g(Y,\phi W)-\\ \frac{1}{n}\eta(Y)\eta(Z)g(\phi X,\phi W)+\\ \frac{1}{2n}[\tilde{S}(Y,Z)g(\phi X,\phi W)-\tilde{S}(\phi X,Z)g(Y,\phi W)].\end{aligned} \tag{3.11}$$

Let $\{e_1,...,e_{2n},\xi\}$ be a local orthonormal basis of vector fields in M, then $\{\phi e_1,...,\phi e_{2n},\xi\}$ is also a local orthonormal basis. Putting $X=W=e_i$ in (3.11) and summing over $i=1$ to $2n$, we get

$$\begin{aligned}\sum_{i=1}^{2n}\tilde{K}(\phi e_i,Y,Z,\phi e_i)=\frac{1}{n}\sum_{i=1}^{2n}g(Y,Z)g(\phi e_i,\phi e_i)-\frac{1}{n}\sum_{i=1}^{2n}g(\phi e_i,Z)g(Y,\phi e_i)-\\ \frac{1}{n}\sum_{i=1}^{2n}\eta(Y)\eta(Z)g(\phi e_i,\phi e_i)+\\ \frac{1}{2n}\sum_{i=1}^{2n}[\tilde{S}(Y,Z)g(\phi e_i,\phi e_i)-\tilde{S}(\phi e_i,Z)g(Y,\phi e_i)].\end{aligned} \tag{3.12}$$

From (3.12), we obtain

$$\tilde{S}(Y,Z)=(4n-2)g(Y,Z)-4n\eta(Y)\eta(Z). \tag{3.13}$$

Therefore, $\tilde{S}(Y,Z)=ag(Y,Z)+b\eta(Y)\eta(Z)$,

where $a=4n-2$ and $b=-4n$.

This result shows that the manifold is an η-Einstein manifold. This proves the Lemma .

In view of Lemma (3.1), we can state the following theorem :

Theorem 3.1. *If a Kenmotsu manifold is quasi-projectively flat with respect to the semi-symmetric metric connection, then the manifold is an η-Einstein manifold.*

Since a and b are both constant, by Lemma (2.1), we get the following:

Corollary 3.1. *If a Kenmotsu manifold is quasi-projectively flat with respect to the semi-symmetric metric connection, then the manifold is an Einstein manifold.*

4. ξ -Projectively flat and ϕ-Projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection

Let C be the Weyl conformal curvature tensor of a $(2n+1)$-dimensional manifold M. Since at each point $p \in M$ the tangent space $\chi_p(M)$ can be decomposed into the direct sum $\chi_p(M) = \phi(\chi_p(M)) \oplus L(\xi_p)$, where $L(\xi_p)$ is an 1-dimensional linear subspace of $\chi_p(M)$ generated by ξ_p. Then we have a map:

$$C : \chi_p(M) \times \chi_p(M) \times \chi_p(M) \longrightarrow \phi(\chi_p(M)) \oplus L(\xi_p).$$

It may be natural to consider the following particular cases:

(1)$C : \chi_p(M) \times \chi_p(M) \times \chi_p(M) \longrightarrow L(\xi_p)$, i.e, the projection of the image of C in $\phi(\chi_p(M))$ is zero.

(2)$C : \chi_p(M) \times \chi_p(M) \times \chi_p(M) \longrightarrow \phi(\chi_p(M))$, i.e, the projection of the image of C in $L(\xi_p)$ is zero.

$$C(X,Y)\xi = 0. \tag{4.1}$$

(3)$C : \phi(\chi_p(M)) \times \phi(\chi_p(M)) \times \phi(\chi_p(M)) \longrightarrow L(\xi_p)$, i.e, when C is restricted to $\phi(\chi_p(M)) \times \phi(\chi_p(M)) \times \phi(\chi_p(M))$, the projection of the image of C in $\phi(\chi_p(M))$ is zero. This condition is equivalent to

$$\phi^2 C(\phi X, \phi Y)\phi Z = 0. \tag{4.2}$$

Here the cases 1, 2 and 3 are conformally symmetric, ξ-conformally flat and ϕ-conformally flat respectively. The cases (1) and (2) were considered in [5] and [24] respectively. The case (3) was considered in [25] for the case M is a K-contact manifold. Furthermore in [2], the authors studied contact metric manifolds satisfying (3). Analogous to the definition of ξ-conformally flat and ϕ-conformally flat, we give the folowing definitions :

Definition 4.1. A Kenmotsu manifold with respect to the semi-symmetric metric connection is said to be ξ-projectively flat if

$$P(X,Y)\xi = 0. \tag{4.3}$$

Definition 4.2. A Kenmotsu manifold is said to be ϕ-projectively flat with respect to the semi-symmetric metric connection if

$$g(P(\phi X, \phi Y)\phi Z, \phi W) = 0, \tag{4.4}$$

where $X, Y, Z, W \in \chi(M)$.

Putting $Z = \xi$ in (3.3) and using (2.1) and (2.2), it follows that

$$R(X,Y)\xi = K(X,Y)\xi + \eta(X)Y - \eta(Y)X. \tag{4.5}$$

Using (2.7) in (4.5), we obtain

$$R(X,Y)\xi = 2K(X,Y)\xi. \tag{4.6}$$

Putting $Z=\xi$ in (1.10), we have

$$P(X,Y)\xi = R(X,Y)\xi - \frac{1}{2n}[S(Y,\xi)X - S(X,\xi)Y]. \tag{4.7}$$

Using (3.6) and (4.6) in (4.7), we get

$$P(X,Y)\xi = 0. \tag{4.8}$$

Hence we can state the following theorem:

Theorem 4.1. *If a Kenmotsu manifold admits a semi-symmetric metric connection, then the Kenmotsu manifold is ξ -Projectively flat with respect to the semi-symmetric metric connection.*

Putting $Y=\phi Y$ and $Z=\phi Z$ in (3.8), we get

$$g(P(\phi X,\phi Y)\phi Z,\phi W) = g(R(\phi X,\phi Y)\phi Z,\phi W) - \frac{1}{2n}[S(\phi Y,\phi Z)g(\phi X,\phi W) - S(\phi X,\phi Z)g(\phi Y,\phi W)]. \tag{4.9}$$

Using (2.1), (2.2), (3.3) and (3.5) in (4.9), we have

$$g(P(\phi X,\phi Y)\phi Z,\phi W) = g(K(\phi X,\phi Y)\phi Z,\phi W) - \frac{1}{2n}[\tilde{S}(\phi Y,\phi Z)g(\phi X,\phi W) - \tilde{S}(\phi X,\phi Z)g(\phi Y,\phi W)] - \frac{1}{n}[g(\phi Y,\phi Z)g(\phi X,\phi W) - g(\phi X,\phi Z)g(\phi Y,\phi W)]. \tag{4.10}$$

Again using (4.4) in (4.10), we obtain

$$g(K(\phi X,\phi Y)\phi Z,\phi W) = \frac{1}{2n}[\tilde{S}(\phi Y,\phi Z)g(\phi X,\phi W) - \tilde{S}(\phi X,\phi Z)g(\phi Y,\phi W)] + \frac{1}{n}[g(\phi Y,\phi Z)g(\phi X,\phi W) - g(\phi X,\phi Z)g(\phi Y,\phi W)]. \tag{4.11}$$

Let $\{e_1,...,e_{2n},\xi\}$ be a local orthonormal basis of vector fields in M, then $\{\phi e_1,...,\phi e_{2n},\xi\}$ is also a local orthonormal basis. Putting $X=W=e_i$ in (4.11) and summing over $i=1$ to $2n$, we get

$$\sum_{i=1}^{2n} g(K(\phi e_i,\phi Y)\phi Z,\phi e_i) = \frac{1}{2n}\sum_{i=1}^{2n}[\tilde{S}(\phi Y,\phi Z)g(\phi e_i,\phi e_i) - \tilde{S}(\phi e_i,\phi Z)g(\phi Y,\phi e_i)] + \frac{1}{n}\sum_{i=1}^{2n}[g(\phi Y,\phi Z)g(\phi e_i,\phi e_i) - g(\phi e_i,\phi Z)g(\phi Y,\phi e_i)]. \tag{4.12}$$

From (4.12), it follows that

$$\tilde{S}(\phi Y, \phi Z) = 2(2n-1)g(\phi Y, \phi Z). \tag{4.13}$$

Using (2.3) and (2.10) in (4.13), we obtain

$$\tilde{S}(Y, Z) = 2(2n-1)g(Y, Z) - 2(3n-1)\eta(Y)\eta(Z). \tag{4.14}$$

Therefore, $\tilde{S}(Y, Z) = ag(Y, Z) + b\eta(Y)\eta(Z)$,

where $a = 2(2n-1)$ and $b = -2(3n-1)$.
We can state the following theorem :

Theorem 4.2. *If a Kenmotsu manifold is ϕ-projectively flat with respect to the semi-symmetric metric connection, then the manifold is an η-Einstein manifold.*

Since a and b are both constant, by Lemma (2.1), we get the following:

Corollary 4.1. *If a Kenmotsu manifold is ϕ-projectively flat with respect to the semi-symmetric metric connection, then the manifold is an Einstein manifold.*

5. Kenmotsu manifolds with respect to the semi-symmetric metric connection satisfying $P.S = 0$

In this section we consider Kenmotsu manifold with respect to the semi-symmetric metric connection M^{2n+1} satisfying condition

$$(P(U, Y).S)(Z, X) = 0$$

Then we have

$$S(P(U, Y)Z, X) + S(Z, P(U, Y)X) = 0. \tag{5.1}$$

Putting $U = \xi$ in (5.1), it follows that

$$S(P(\xi, Y)Z, X) + S(Z, P(\xi, Y)X) = 0. \tag{5.2}$$

Putting $X = \xi$ and using (3.5) and (3.6) in (1.10), we get

$$\begin{aligned} P(\xi, Y)Z = R(\xi, Y)Z - \frac{1}{2n}[\tilde{S}(Y, Z)\xi - 2(3n-1)g(Y, Z)\xi + \\ 2(2n-1)\eta(Y)\eta(Z)\xi + 4n\eta(Z)Y]. \end{aligned} \tag{5.3}$$

Again putting $X = \xi$ in (3.3) and using (2.8), we obtain

$$R(\xi, Y)Z = 2[\eta(Z)Y - g(Y, Z)\xi]. \tag{5.4}$$

Using (3.5), (3.6), (5.3) and (5.4) in (5.2), it follows that

$$\tilde{S}(Y,Z) = 2(n-1)g(Y,Z) + 2(1-2n)\eta(Y)\eta(Z). \tag{5.5}$$

Therefore, $\tilde{S}(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z)$,

where $a = 2(n-1)$ and $b = 2(1-2n)$.
We can state the following theorem :

Theorem 5.1. *If a Kenmotsu manifold with respect to the semi-symmetric metric connection satisfying $P.S = 0$, then the manifold is an η-Einstein manifold.*

Since a and b are both constant, by Lemma (2.1), we get the following:

Corollary 5.1. *If a Kenmotsu manifold with respect to the semi-symmetric metric connection satisfying $P.S = 0$, then the manifold is an Einstein manifold.*

References

[1] Amur, K. and Pujar, S.S., On submanifolds of a Riemannian manifold admitting a metric semi-symmetric connection, Tensor, N.S., 32(1978), 35-38.
[2] Arslan, K., Murathan, C. and *Özgür*, C., On ϕ-conformally flat contact metric manifolds, Balkan J. Geom. Appl. (BJGA), 5(2)(2000), 1-7.
[3] Binh, T.Q., On semi-symmetric connection, Periodica Math. Hungerica, 21(2), 1990, 101-107.
[4] Blair, D.E., Contact manifolds in Riemannian geometry, Lecture Note in Mathematics, 509, Springer-Verlag Berlin, 1976.
[5] Cabrerizo, J.L., Fernandez, M., Fernandez, L.M. and Zhen, G. : On ξ-conformaly flat K-contact manifolds, Indian J. Pure and Applied Math., 28(1997), 725-734.
[6] Chaubey, S.K. and Ojha, R.H., On a semi-symmetric non-metric connection, Filomat 25:4 (2011), 19-27.
[7] Chaubey, S.K. and Ojha, R.H., On a semi-symmetric non-metric connection, Filmot 26:2 (2012), 63-69.
[8] De, U.C. and Biswas, S.C., On a type of semi-symmetric metric connection on a Riemannian manifold, Pub. De L Institut Math., N.S., Tome 61(75), 1997, 90-96.
[9] De, U.C. and Pathak, G., On 3-dimensional Kenmotsu manifolds, Indian J. Pure Applied Math., 35 (2004), 159-165.
[10] Friedmann, A. and Schouten, J.A., *Ü*ber die Geometric der halbsymmetrischen *Ü*bertragung, Math., Zeitschr., 21(1924), 211-223.
[11] Hayden, H.A., Subspaces of space with torsion, Proc. London Math. Soc., 34(1932), 27-50.
[12] Ianus, S. and Smaranda, D., Some remarkable structures on the product of an almost contact metric manifold with the real line, Papers from the National Coll. on Geometry and Topology, Univ. Timisoara, (1997),107-110.
[13] Jun, J.B. , De, U.C. and Pathak, G., On Kenmotsu manifolds, J. Korean Math. Soc., 42(2005), 435-445.
[14] Kenmotsu, K., A class of almost contact Riemannian manifolds, Tohoku Math. J., 24(1972), 93-103.
[15] Oubina, A., New classes of contact metric structures, Publ. Math. Debrecen, 32(3-4)(1985),187-193.
[16] *Özgür*, C. and De, U.C., On the quasi-conformal curvature tensor of a Kenmotsu manifold, Mathematica Pannonica, 17/2, (2006), 221-228.
[17] Prvanović, M., On pseudo metric semi-symmetric connections, Pub. De L Institut Math., Nouvelle serie, 18 (32), 1975, 157-164.
[18] Sharfuddin, A. and Hussain, S.I., Semi-symmetric metric connexions in almost contact manifolds, Tensor, N.S., 30(1976), 133-139.

[19] Tanno, S., The automorphism groups of almost contact Riemannian manifolds, Tohoku Math. j., 21(1969), 21-38.
[20] Yano, K., On semi-symmetric connection, Revue Roumaine de Math. Pure et Appliques, 15(1970), 1570-1586.
[21] Yano, K. and Bochner, S., Curvature and Betti numbers, Annals of Mathematics studies, 32(Princeton university press) (1953).
[22] Yıldiz, A., De, U.C. and Acet, B.E., On Kenmotsu manifolds satisfying certain curvature conditions, SUT J. of Math. 45, 2(2009), 89-101.
[23] Yılmaz, H.B., On weakly symmetric manifolds with a type of semi-symmetric non-metric connection, Annales Polonici Mathematici 102.3 (2011).
[24] Zhen, G., On conformal symmetric K-contact manifolds, Chinese Quart. J. Math., 7(1992), 5-10.
[25] Zhen, G., Cabrerizo, J. L., Fernandez, L. M. and Fernandez, M., The structure of a class of K-contact manifolds, Acta Math. Hungar, 82(4)(1999), 331-340.
[26] Zengin F. O., Uysal S. A. and Demirbag S. A., On sectional curvature of a Riemannian manifold with semi-symmetric metric connection, Ann. Polon. Math. 101(2011), 131-138.

Department of Mathematics, Kabi-Nazrul Mahavidyalaya, Sonamura-799181, Tripura, India.
E-mail address: ajitbarmanaw@yahoo.in

Department of Mathematics, University of Calcutta, Kolkata 700019, West Bengal, India.
E-mail address: uc_de@yahoo.com