Prompt Detection of Changepoint in the Operation of Networked Systems*

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Abstract. Detection of network problems is an important step in automating network management. Early detection of performance degradation can alleviate the last moment hassel of network managers. This paper focuses on a statistical method aiming at detecting changepoint as quickly as possible using Bayes factor along with the binary segmentation procedure modified for fast detection. Computer simulation verifies the effectiveness and correctness of the proposed approach.

Keywords: Bayes factor, binary segmentation, changepoint detection, network management, non-homogeneous Possion process.

1 Introduction

For detection and diagnosis of changepoint statistical analysis method has been successfully applied to a variety of networked and distributed systems. The rapid growth of networked and distributed systems throughout the workplace has given rise to the problems difficult to manage with the expertise of human operators in modern information technology field. There is an urgent need for automating the management functions to reduce the operation and management cost. Early detection of performance degradation can alleviate the last moment hassel of network mangers.

A sudden jump in the behavior of a system is a changepoint which can be estimated from the input data using the maximum likelihood estimator [2]. We need a formulation of changepoint for detecting even a slight change. In this paper the main goal is fast detection that is an important requirement for reducing potential negative impact on the network services and users. The detection problem is formulated as a changepoint problem using Bayes factor. The computation of the Bayes factor, however, gets more complex as the number of nodes increases.

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To overcome the computational difficulties, we explore the binary segmentation method proposed in [3]. Our approach for detecting changepoint is mainly developing formulas by Bayesian viewpoint. The proposed detection method is designed to be sensitive to slight changes in the operation characteristics of a network, and predict the changepoint at the change point of the shape parameter values. Computer simulation verifies the effectiveness and correctness of the proposed approach.

2 Preliminaries

In the non-homogeneous Poisson process (NHPP), the interarrival time is neither independent nor identically distributed. A random variable of special interest is $N(t)$, the number of failures in the time interval $(0, t]$. The intensity function of a counting process $\{N(t), t \geq 0\}$ is defined as $\lambda(t) = m'(t)$, where $m(t)$ is the mean number of failures in the time interval $(0, t]$, often called the mean value function. The power law process for the reliability growth of repairable systems has the intensity function,

$$\lambda(t) = \alpha\beta t^{\beta-1}, \alpha > 0, \beta > 0, t > 0, \tag{1}$$

where α is a scale parameter and β is a shape parameter [1].

The likelihood function for the data $D_{(0,T]}$ with the model of $\lambda(\cdot)$ given in (1) can be written as

$$L(\alpha, \beta | D_{(0,T]}) = \left(\prod_{i=1}^{n} \lambda(t_i) \right) \exp\left(- \int_0^T \lambda(t) dt \right), \tag{2}$$

where $D_{(0,T]} = \{t_1, \cdots, t_{n+1} | 0 < t_1 \leq \cdots \leq t_n < T, T = t_{n+1}\}$. See [3].

For the data $D_{(0,T]}$ of failure times, the intensity functions are given respectively by

$$M_0 : \lambda(t) = \alpha_0\beta_0 t^{\beta_0 - 1}, 0 < t \leq T, \ \textbf{vs.}\ M_1 : \lambda(t) = \begin{cases} \alpha_1\beta_1 t^{\beta_1 - 1}, 0 < t < \tau, \\ \alpha_1\beta_2 t^{\beta_2 - 1}, \tau \leq t < T, \end{cases}$$

where τ is a changepoint.

3 The Proposed Changepoint Detection Method

We use the Bayes factor for comparing and testing the no-changepoint model and single changepoint model. The test is based on calculating the Bayes factor B_{10} for the single changepoint model M_1 against the no-changepoint model M_0.

The detection procedure of the changepoint consists of the following steps:

Step 1: For the complete data, compute the Bayes factor B_{10}.
Step 2: When $B_{10} > 1$, estimate a changepoint $\hat{\tau}$ for the given data.

Step 3: After a changepoint, as new data are sequentially added, compute the Bayes factor B_{10}.
Step 4: When $B_{10} > 1$, estimate a new changepoint $\hat{\tau}$ for the given data.

The Bayes factor B_{10} for the changepoint model against the no-changepoint model is

$$B_{10}(D_{(0,T]}) = B_1/B_0, \tag{3}$$

where

$$\begin{aligned}
B_1 = &\int_{[0,t_1)}\int_0^\infty\int_0^\infty \frac{2}{(1+\beta_1)^3}\cdot\frac{2\beta_2^n}{(1+\beta_2)^3}\cdot\frac{[\prod_{i=1}^n t_i)]^{\beta_2}}{K_1}d\beta_1 d\beta_2\frac{1}{T}d\tau\\
&+\sum_{j=1}^{n-1}\int_{[t_j,t_{j+1})}\int_0^\infty\int_0^\infty \frac{2\beta_1^j}{(1+\beta_1)^3}\cdot\frac{2\beta_2^{n-j}}{(1+\beta_2)^3}\cdot\frac{q_j}{K_1}d\beta_1 d\beta_2\frac{1}{T}d\tau\\
&+\int_{[t_n,T)}\int_0^\infty\int_0^\infty \frac{2\beta_1^n}{(1+\beta_1)^3}\cdot\frac{2}{(1+\beta_2)^3}\cdot\frac{[\prod_{i=1}^n t_i]^{\beta_1}}{K_1}d\beta_1 d\beta_2\frac{1}{T}d\tau,\\
B_0 = &\int_0^\infty \frac{2\beta_0^n}{(1+\beta_0)^3}\cdot\frac{[\prod_{i=1}^n t_i]^{\beta_0}}{(T^{\beta_0}+\nu)^{n+\xi}}d\beta_0.
\end{aligned}$$

Here, $K_1 = (\tau^{\beta_1}+T^{\beta_2}-\tau^{\beta_2}+\nu)^{n+\xi}$ and $q_j = \left[\prod_{i=1}^j t_i\right]^{\beta_1}\cdot\left[\prod_{i=j+1}^n t_i\right]^{\beta_2}$, for $j = 1,\cdots,n-1$.

In this procedure if the Bayes factor $B_{10} > 1$, estimate a changepoint $\hat{\tau}$ by using

$$\hat{\tau} = E(\tau|D_{(0,T]}) = A/B, \tag{4}$$

where

$$\begin{aligned}
A = &\int_{[0,t_1)}\int_0^\infty\int_0^\infty \frac{2}{(1+\beta_1)^3}\cdot\frac{2\beta_2^n}{(1+\beta_2)^3}\cdot\frac{[\prod_{i=1}^n t_i]^{\beta_2}}{K_1}d\beta_1 d\beta_2\tau d\tau\\
&+\sum_{j=1}^{n-1}\int_{[t_j,t_{j+1})}\int_0^\infty\int_0^\infty \frac{2\beta_1^j}{(1+\beta_1)^3}\cdot\frac{2\beta_2^{n-j}}{(1+\beta_2)^3}\cdot\frac{q_j}{K_1}d\beta_1 d\beta_2\tau d\tau\\
&+\int_{[t_n,T)}\int_0^\infty\int_0^\infty \frac{2\beta_1^n}{(1+\beta_1)^3}\cdot\frac{2}{(1+\beta_2)^3}\cdot\frac{[\prod_{i=1}^n t_i]^{\beta_1}}{K_1}d\beta_1 d\beta_2\tau d\tau,\\
B = &\int_{[0,t_1)}\int_0^\infty\int_0^\infty \frac{2}{(1+\beta_1)^3}\cdot\frac{2\beta_2^n}{(1+\beta_2)^3}\cdot\frac{[\prod_{i=1}^n t_i]^{\beta_2}}{K_1}d\beta_1 d\beta_2 d\tau\\
&+\sum_{j=1}^{n-1}\int_{[t_j,t_{j+1})}\int_0^\infty\int_0^\infty \frac{2\beta_1^j}{(1+\beta_1)^3}\cdot\frac{2\beta_2^{n-j}}{(1+\beta_2)^3}\cdot\frac{q_j}{K_1}d\beta_1 d\beta_2 d\tau\\
&+\int_{[t_n,T)}\int_0^\infty\int_0^\infty \frac{2\beta_1^n}{(1+\beta_1)^3}\cdot\frac{2}{(1+\beta_2)^3}\cdot\frac{[\prod_{i=1}^n t_i]^{\beta_1}}{K_1}d\beta_1 d\beta_2 d\tau.
\end{aligned}$$

4 Simulation Results

Each simulation dataset consists of two parts for different shape parameter values of the intensity function of the power law process. We expect to detect a changepoint around the point where the shape parameter value changes. The hyperparameters for the prior of the shape parameter are fixed as $(\xi, \nu) = (1, 1)$.

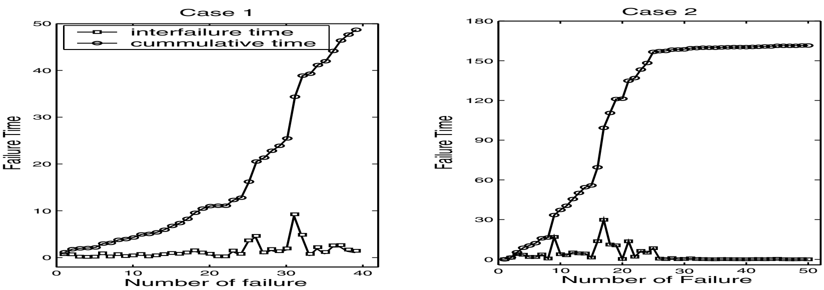

Fig. 1. The plots of simulated datasets with a single changepoint

Case 1. We fix the scale parameter as $\alpha = 0.5$. The datasets are generated with $\beta = 1.0$ and $\beta = 0.6$, respectively. Each dataset size is 20. Since the maximum likelihood estimates of (α, β) are $(0.471, 0.951)$ and $(0.507, 0.624)$ respectively, it seems that the data are fairly well generated. The time interval of the entire data is $(0, 48.185]$. We detect a changepoint using Bayes factor. For the complete data in $(0, 48.185]$, the Bayes factor is 1.902 by (3) and the estimated changepoint is 12.218 by (4), where it is located between the 23^{th} and 24^{th} observation. We want to build a detection system for sequential data. Thus we assume to detect a change at the 25^{th} observation. From the first to the 25^{th} observation, Bayes factor is 4.620, and a changepoint is 11.986 in $(0, 15.734]$. Note that the position of the changepoint estimated with only the 25 incomplete observations is still same as that with the complete data. This demonstrates that the proposed approach can promptly and correctly detect the change.

Case 2. Scale parameter $\alpha = 1.0$, shape parameter $\beta = 0.6$ and 2.0, complete data size: 50, MLEs of (α, β): $(1.160, 0.616)$, $(1.103, 1.887)$, time interval of complete data: $(0, 161.471]$, Bayes factor: 1.591×10^{21}, changepoint: 156.633 (between the 24^{th} and 25^{th} observation), change detection: 30^{th} observation, B_{10} and $\hat{\tau}$ with the first to the 30^{th} observation: 10.683 and 152.863 (between the 24^{th} and 25^{th} observation).

5 Conclusion

In this paper we have proposed a Bayesian approach for detecting changepoint with the power law process of the NHPP. As new data added sequentially, we immediately detect changepoint as soon as Bayes factor becomes greater than 1.

Computer simulation verified the effectiveness and correctness of the proposed approach. This procedure of changepoint detection is applicable when the complete data is unavailable.

References

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