

## Propagation of $0\pi$ pulses in a gas of three-level atoms

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We have theoretically studied the pulsed regime of electromagnetically induced transparency. In particular, simulations of propagation of Gaussian and  $0\pi$  copropagating laser pulses in a medium consisting of three-level  $\Lambda$  atoms have been performed. It has been found that even at the two-photon resonance, the length of propagation for the  $0\pi$  pulses is much smaller than the one for the Gaussian probe pulses. Using the dark and bright basis, we explain this behavior. Some possible applications are discussed.

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### I. INTRODUCTION

Quantum coherence effects, such as coherent population trapping (CPT) [1] and electromagnetically induced transparency (EIT) [2–6], have been the focus of broad research activity for the past two decades, as they drastically change optical properties of media. For example, for EIT in cw and pulsed regimes [3–8], absorption practically vanishes. Media with excited coherence may display a high index of refraction without absorption [9]. The corresponding steep dispersion results in the ultraslow or fast propagation of light pulses [10–14] which can produce huge optical delays [14] and can be used for drastic modification of the phase-matching conditions for Brillouin scattering [15], four-wave mixing [16], controllable switching between bunching and antibunching [17], phase effects in EIT [18], dark-state polariton [19], storage and retrieval of pulses [20], and freezing of a light pulse [21]. EIT has led to several other coherent phenomena like enhancing the sensitivity of magnetometry [22,23], dynamical control [24,25], cavity QED [26], and optical switch [27–30]. It is possible to achieve manipulation of a coherent medium and optical pulses faster than the relaxation rates [31], enhanced nonlinear effects at a few-photon level [32,33], and to develop a bright source for efficient generation of far IR pulses [34,35].

EIT has been achieved in atomic [2,3,6] and molecular [36,37] gases, Bose-Einstein condensate [10], solid-state systems [38–40], metamaterials [41–43], and even in mechanical effect of light [44]. Physics of coherent effects in  $\Lambda$ -type three-level atoms is related to excitation of the maximum coherence between two ground states (in alkali-metal atoms, these are the hyperfine levels) under the condition when a special coherent state, the so-called dark state, is formed.

In general, the EIT has been studied in three-level  $\Lambda$ -type atomic systems, where a strong drive field and a weak probe field on adjacent transitions have been applied, and the fields typically have copropagating geometry to eliminate the Doppler effect [45]. However, the effects of laser pulses having arbitrary time dependence on the EIT have not yet been addressed.

Meanwhile, it is well known that the shape of optical pulses plays an important role in their propagation through a medium of two-level atoms. The propagation of the  $2n\pi$  pulses in a

two-level medium is a good example [46]. Short optical pulses with area given by

$$\theta = \int_{-\infty}^{\infty} \Omega(t') dt' = 2\pi n \quad (1)$$

can propagate through an atomic gas at the resonant frequency without absorption, where  $\Omega = \wp \varepsilon / \hbar$  is the Rabi frequency of the applied optical pulse,  $\wp$  is the dipole moments of the corresponding transition and  $\varepsilon$  is the electric field of pulse. Even weak optical pulses with pulse area equal to zero ( $n = 0$ ), the so-called  $0\pi$  pulses, have large propagation lengths. An interesting question arising in this regard is whether the propagation of pulses in EIT configuration can be improved by using the  $0\pi$  pulses. In this paper, we study these effects in detail. Employing the dressed-state basis, we show the mechanism of the pulse shape on EIT, and perform simulation to support the theoretical results.

Our results may find application as a method of coherent control of pulse propagation. Another interesting result is related to the spectrum change of the probe pulse during propagation through the EIT medium. The various frequencies of the probe pulse propagating through an EIT medium are influenced differently by nonlinear interaction resulting in strong modification of the pulse spectrum. We study the effect by using the  $0\pi$  pulse for this purpose. The  $0\pi$  pulse has a spectral hole that disappears while the  $0\pi$  pulse is propagating through the medium due to nonlinear interaction.

The organization of this paper is as follows. In Sec. II, we introduce a  $\Lambda$ -type three-level system. In Sec. II A, we simulate the effect of pulse shape on EIT and the possible control of pulse propagation through a three-level  $\Lambda$ -type medium. In Sec. IV, we discuss possible applications. Finally, in Sec. V we conclude by summarizing our results and give some future perspectives.

### II. MODEL

We consider an isotropic homogeneous medium consisting of three-level atoms in a  $\Lambda$  configuration. This system is driven by two copropagating laser pulses as shown in Fig. 1.  $r_{b,c}$  is the incoherent pumping rate, and  $\gamma_{b,c}$  is the spontaneous decay rate from  $|a\rangle$  to lower levels. The system is closed, so there is no decay or incoherent pumping outside these levels.

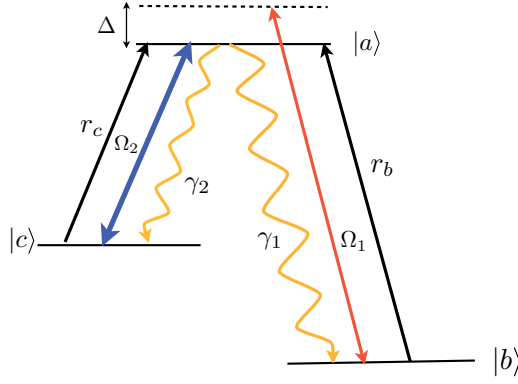


FIG. 1. (Color online) Scheme of  $\Lambda$ -type system for EIT. The system is driven by two classical fields and incoherent pumping. The pump field has resonant frequency while probe field has a detuning  $\Delta$  from the atomic resonance.

Note that the drive field has a frequency resonant with the transition between atomic levels  $|a\rangle$  and  $|c\rangle$ , while the probe field has a frequency detuning  $\Delta$  from the resonance. The two-photon resonance condition important for EIT can be achieved simply by tuning the probe field to the resonance ( $\Delta = 0$ ). Levels  $|b\rangle$  and  $|c\rangle$  are states that have lifetimes longer than the probe and drive pulse durations. Level  $|a\rangle$  is an electronic excited state that is coupled to level  $|b\rangle$  through the probe field and to level  $|c\rangle$  through the drive field. Note that the transition between levels  $|b\rangle$  and  $|c\rangle$  is dipole forbidden. In experimental schemes, the states  $|b\rangle$  and  $|c\rangle$  can be the hyperfine states  $5S_{1/2}(F=1)$  and  $5S_{1/2}(F=2)$  of  $^{87}\text{Rb}$ , respectively. Then the  $|a\rangle$  is the  $5P_{1/2}(F=2)$ .

In the rotating-wave approximation, the semiclassical time-dependent interaction Hamiltonian that describes the atom-laser coupling for this  $\Lambda$  system is given [2] by

$$H_{\text{int}} = -\hbar(\Omega_1|a\rangle\langle b| + \Omega_2|a\rangle\langle c| - \Delta|a\rangle\langle a| - \Delta|c\rangle\langle c| + \text{H.c.}), \quad (2)$$

where  $\Omega_1 = \wp_{ab}\varepsilon_1/\hbar$  and  $\Omega_2 = \wp_{ac}\varepsilon_2/\hbar$  are the Rabi frequencies of probe (drive) field,  $\wp_{ab}(\wp_{ac})$  are the dipole moments of the transition  $|a\rangle \leftrightarrow |b\rangle$  ( $|a\rangle \leftrightarrow |c\rangle$ ), and  $\varepsilon_1$  and  $\varepsilon_2$  are the applied electric fields of probe and pump pulse, respectively.

The time evolution of the density matrix is given by the master equation [2]

$$\dot{\rho} = -\frac{i}{\hbar}[H_{\text{int}}, \rho] + L\rho, \quad (3)$$

where  $L\rho = L_1\rho + L_2\rho + L_3\rho + L_4\rho$  describes spontaneous emission and incoherent pumping, and is given by

$$L_1\rho = -\frac{\gamma_1}{2}[\sigma_1^+\sigma_1\rho + \rho\sigma_1^+\sigma_1 - 2\sigma_1\rho\sigma_1^+], \quad (4)$$

$$L_2\rho = -\frac{\gamma_2}{2}[\sigma_2^+\sigma_2\rho + \rho\sigma_2^+\sigma_2 - 2\sigma_2\rho\sigma_2^+], \quad (5)$$

$$L_3\rho = -\frac{r_b}{2}[\sigma_1\sigma_1^+\rho + \rho\sigma_1\sigma_1^+ - 2\sigma_1^+\rho\sigma_1], \quad (6)$$

$$L_4\rho = -\frac{r_c}{2}[\sigma_2\sigma_2^+\rho + \rho\sigma_2\sigma_2^+ - 2\sigma_2^+\rho\sigma_2]. \quad (7)$$

Here  $\sigma_1 = |b\rangle\langle a|$ ,  $\sigma_2 = |c\rangle\langle a|$  are the atomic transition operators.

The dynamical evolution of the density matrix elements is given by

$$\dot{\rho}_{bb} = i\Omega_1^*\rho_{ab} - i\Omega_1\rho_{ba} + \gamma_b\rho_{aa} - r_b\rho_{bb}, \quad (8)$$

$$\dot{\rho}_{cc} = i\Omega_2^*\rho_{ac} - i\Omega_2\rho_{ca} + \gamma_c\rho_{aa} - r_c\rho_{cc}, \quad (9)$$

$$\dot{\rho}_{ab} = -\Gamma_{ab}\rho_{ab} + i\Omega_1(\rho_{bb} - \rho_{aa}) + i\Omega_2\rho_{cb}, \quad (10)$$

$$\dot{\rho}_{ca} = -\Gamma_{ca}\rho_{ca} + i\Omega_2^*(\rho_{aa} - \rho_{cc}) - i\Omega_1^*\rho_{cb}, \quad (11)$$

$$\dot{\rho}_{cb} = -\Gamma_{cb}\rho_{cb} - i\Omega_1\rho_{ca} + i\Omega_2^*\rho_{ab}, \quad (12)$$

where

$$\Gamma_{ab} = \frac{\gamma_1 + \gamma_2 + r_b}{2} - i\Delta \equiv \gamma_{ab} - i\Delta, \quad (13)$$

$$\Gamma_{ca} = \frac{\gamma_1 + \gamma_2 + r_c}{2} \equiv \gamma_{ac}, \quad (14)$$

$$\Gamma_{cb} = \frac{r_b + r_c}{2} - i\Delta \equiv \gamma_{cb} - i\Delta \quad (15)$$

are the relaxation rates, and the condition for close system is given by  $\rho_{aa} + \rho_{bb} + \rho_{cc} = 1$ .

#### A. Basis of dark and bright states

To understand the effect of different pulse time dependence on EIT, we can take advantage of the so-called dark- and bright-state bases. To simplify our calculation, we consider two-photon resonance and the Rabi frequency to be real. The interaction Hamiltonian for the  $\Lambda$  system can be written as

$$H_{\text{int}} = -\hbar \begin{pmatrix} 0 & \Omega_1 & \Omega_2 \\ \Omega_1 & 0 & 0 \\ \Omega_2 & 0 & 0 \end{pmatrix}. \quad (16)$$

The state vector in this bare basis is

$$|\psi\rangle = a|a\rangle + b|b\rangle + c|c\rangle, \quad (17)$$

where  $a$ ,  $b$ , and  $c$  are the probability amplitudes. Let  $u$  represent these amplitudes:

$$u = \begin{pmatrix} a \\ b \\ c \end{pmatrix}. \quad (18)$$

The evolution of the state vector obeys the Schrödinger equation,

$$|\dot{\psi}\rangle = -\frac{i}{\hbar}\hat{H}_{\text{int}}|\psi\rangle. \quad (19)$$

Thus the equations of motion for the amplitudes become

$$\dot{u} = -\frac{i}{\hbar}H_{\text{int}}u, \quad (20)$$

which is

$$\begin{pmatrix} \dot{a} \\ \dot{b} \\ \dot{c} \end{pmatrix} = -i \begin{pmatrix} \Omega_1 b + \Omega_2 c \\ \Omega_1 a \\ \Omega_2 a \end{pmatrix}. \quad (21)$$

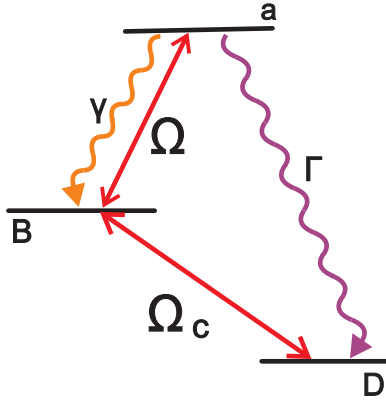


FIG. 2. (Color online) Effective scheme in the dark-bright basis.

Now we introduce a new basis that involves the so-called dark and bright states, namely, the bright state

$$|B\rangle = \frac{\Omega_1|b\rangle + \Omega_2|c\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}}, \quad (22)$$

and the dark state

$$|D\rangle = \frac{-\Omega_2|b\rangle + \Omega_1|c\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}}. \quad (23)$$

In this basis, the state vector is

$$|\psi\rangle = a|a\rangle + B|B\rangle + D|D\rangle. \quad (24)$$

Rewriting the linear relationship between these two bases in matrix form, we obtain

$$\begin{pmatrix} |a\rangle \\ |B\rangle \\ |D\rangle \end{pmatrix} = S \begin{pmatrix} |a\rangle \\ |b\rangle \\ |c\rangle \end{pmatrix}, \quad (25)$$

where

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\Omega_1}{\Omega} & \frac{\Omega_2}{\Omega} \\ 0 & -\frac{\Omega_2}{\Omega} & \frac{\Omega_1}{\Omega} \end{pmatrix} \quad (26)$$

is the rotation matrix, and  $\Omega = \sqrt{\Omega_1^2 + \Omega_2^2}$ .

Let  $v$  present the probability amplitudes of the new basis,

$$v = \begin{pmatrix} a \\ B \\ D \end{pmatrix}. \quad (27)$$

We obtain

$$v = (S^T)^{-1}u = Su \quad (28)$$

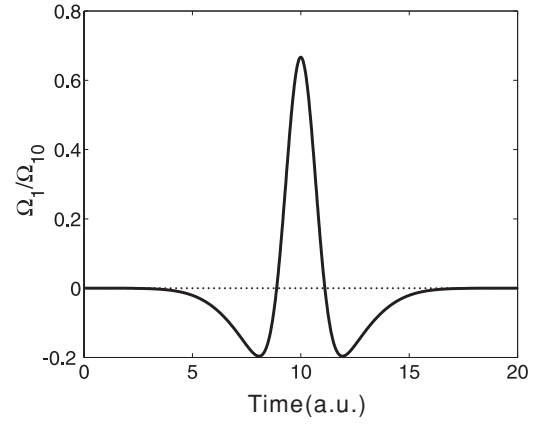
or

$$u = S^{-1}v, \quad (29)$$

where  $S^T$  is the transposed matrix of  $S$ .

After straightforward calculations, replacing  $u$  by  $v$  in Eq. (20), and taking into consideration that  $\Omega_i$  are functions of time, we obtain

$$\dot{v} = -\frac{i}{\hbar} H_{\text{eff}} v, \quad (30)$$


 FIG. 3. Time dependence of the pulse shape of the  $0\pi$  pulse with parameters  $T_1 = 1$  and  $T_2 = 3$ .

where

$$H_{\text{eff}} = SH_{\text{int}}S^{-1} - iS\dot{S}^{-1} \quad (31)$$

is the effective interaction Hamiltonian in the new basis. Substituting Eqs. (16) and (26) into Eq. (31), we obtain

$$H_{\text{eff}} = -\hbar \begin{pmatrix} 0 & \Omega & 0 \\ \Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - i\hbar \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Omega_c \\ 0 & -\Omega_c & 0 \end{pmatrix}. \quad (32)$$

Now it is clear that  $\Omega$  is the effective Rabi frequency of coupling between the bright state,  $|B\rangle$ , and state  $|a\rangle$ , and

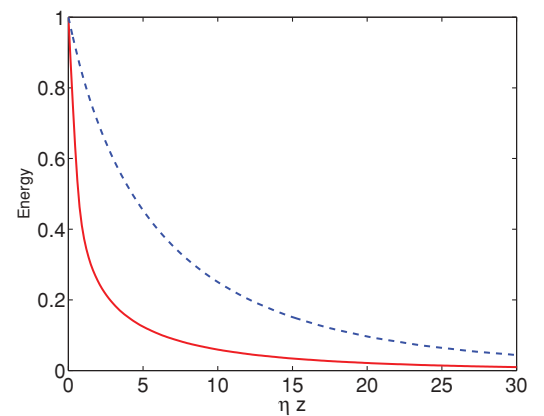
$$\Omega_c = \frac{\dot{\Omega}_1\Omega_2 - \dot{\Omega}_2\Omega_1}{\Omega^2} \quad (33)$$

is the Rabi frequency of coupling between the dark and bright states.

Thus the  $\Lambda$  system as shown in Fig. 1 becomes the effective system shown in Fig. 2, where

$$\gamma = \frac{\Omega_1^2\gamma_b + \Omega_2^2\gamma_c}{\Omega^2}, \quad (34)$$

$$\Gamma = \frac{\Omega_2^2\gamma_b + \Omega_1^2\gamma_c}{\Omega^2} \quad (35)$$


 FIG. 4. (Color online) Dependence of pulse energy on optical density. Gaussian pulse (red solid line) and  $0\pi$  pulse (blue dashed line).

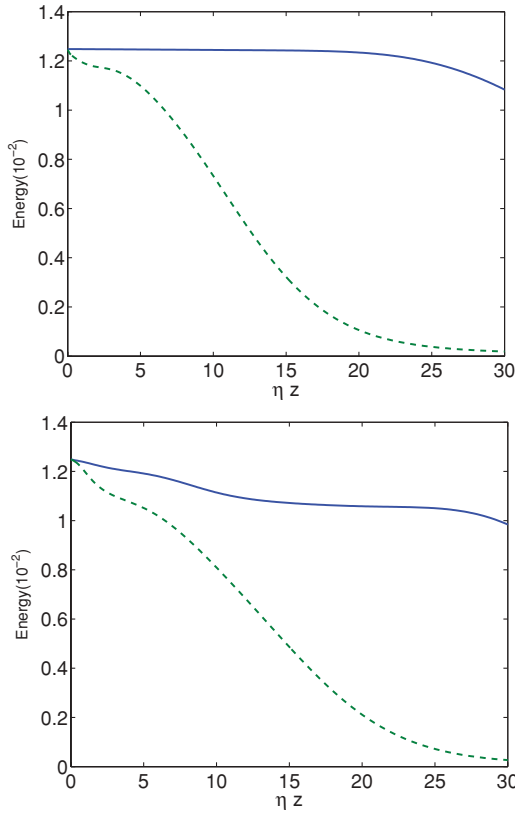


FIG. 5. (Color online) Energy of  $\Omega_1$  vs position in the medium. Comparison for different driving fields  $\Omega_2$ :  $\Omega_2$  is a Gaussian pulse (solid line) and  $\Omega_2$  is a  $0\pi$  pulse (dashed line). (Upper panel)  $\Omega_1$  is a Gaussian pulse. (Lower panel)  $\Omega_1$  is a  $0\pi$  pulse.

are the effective decay rates from  $|a\rangle$  to the bright and dark states, correspondingly.

It can be seen that when the Rabi frequencies  $\Omega_1$  and  $\Omega_2$  have the same time dependence (i.e., the drive and probe fields are matched pulses), the Rabi frequency  $\Omega_c$  is zero. There is no interaction between the bright and dark states; the effective system can be viewed as a two-level system. Once the entire population is trapped in the dark state, there is no absorption or excitation; this is the CPT phenomenon. On the other hand, when the probe and drive pulses have different time dependence, there is coupling between dark and bright

states that leads to absorption of the probe beam and excitation of level  $|a\rangle$ .

**B. Basis of diagonalized Hamiltonian**

A simple physical picture can be gained in an alternative basis of the so-called dressed states that can be obtained by diagonalizing the interaction Hamiltonian. The transformation matrices between the bare-state basis and the dressed-state basis are given by

$$P^{-1} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\Omega_2}{\Omega} & \frac{\Omega_1}{\sqrt{2}\Omega} & -\frac{\Omega_1}{\sqrt{2}\Omega} \\ -\frac{\Omega_1}{\Omega} & \frac{\Omega_2}{\sqrt{2}\Omega} & -\frac{\Omega_2}{\sqrt{2}\Omega} \end{pmatrix}, \quad (36)$$

$$P = \begin{pmatrix} 0 & \frac{\Omega_2}{\Omega} & -\frac{\Omega_1}{\Omega} \\ \frac{1}{\sqrt{2}} & \frac{\Omega_1}{\sqrt{2}\Omega} & \frac{\Omega_2}{\sqrt{2}\Omega} \\ \frac{1}{\sqrt{2}} & -\frac{\Omega_1}{\sqrt{2}\Omega} & -\frac{\Omega_2}{\sqrt{2}\Omega} \end{pmatrix}. \quad (37)$$

The dressed states are given by

$$\begin{pmatrix} |\varphi_0\rangle \\ |\varphi_+\rangle \\ |\varphi_-\rangle \end{pmatrix} = P \begin{pmatrix} |a\rangle \\ |b\rangle \\ |c\rangle \end{pmatrix} = \begin{pmatrix} \frac{\Omega_2}{\Omega}|b\rangle - \frac{\Omega_1}{\Omega}|c\rangle \\ \frac{1}{\sqrt{2}}|a\rangle + \frac{\Omega_1}{\sqrt{2}\Omega}|b\rangle + \frac{\Omega_2}{\sqrt{2}\Omega}|c\rangle \\ \frac{1}{\sqrt{2}}|a\rangle - \frac{\Omega_1}{\sqrt{2}\Omega}|b\rangle - \frac{\Omega_2}{\sqrt{2}\Omega}|c\rangle \end{pmatrix} \quad (38)$$

and the state vector is

$$|\psi\rangle = \varphi_0|\varphi_0\rangle + \varphi_+|\varphi_+\rangle + \varphi_-|\varphi_-\rangle. \quad (39)$$

Let  $w$  represent the amplitudes in the dressed-state basis,

$$w = \begin{pmatrix} \varphi_0 \\ \varphi_+ \\ \varphi_- \end{pmatrix}, \quad (40)$$

then we have

$$w = Pu \quad \text{or} \quad u = P^{-1}w. \quad (41)$$

The evolution of the state vector obeys the same Schrödinger equation. Similarly, replacing  $u$  in Eq. (20) by  $w$ , and taking

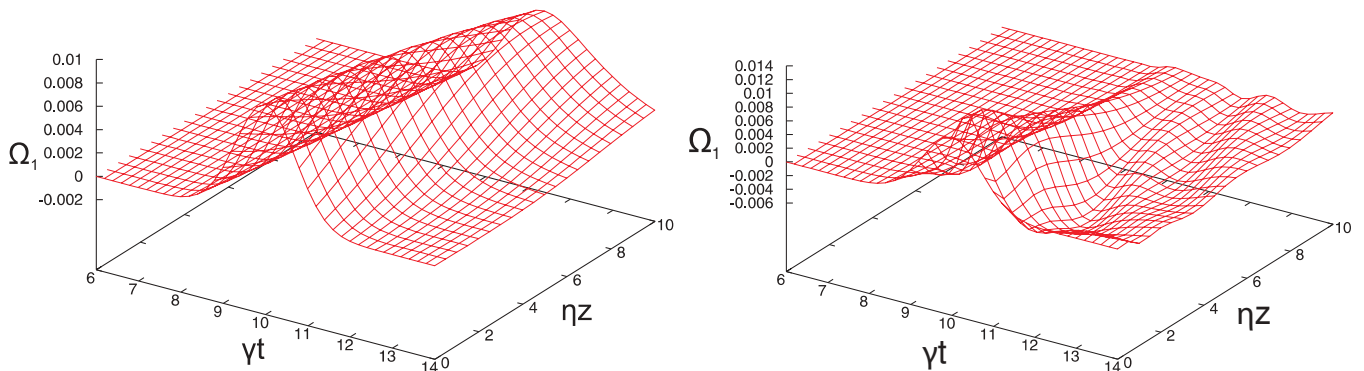


FIG. 6. (Color online) Propagation of the Gaussian probe pulse. (Upper panel)  $\Omega_2$  is a Gaussian pulse. (Lower panel)  $\Omega_2$  is a  $0\pi$  pulse.

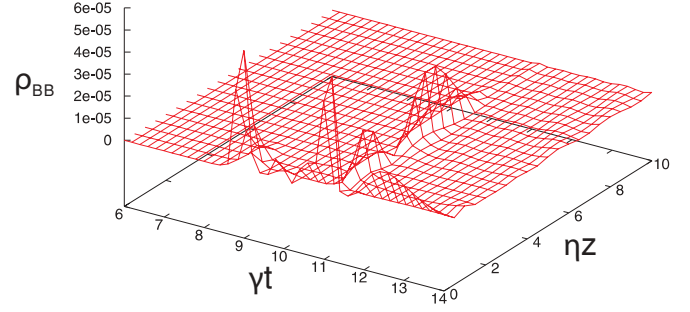
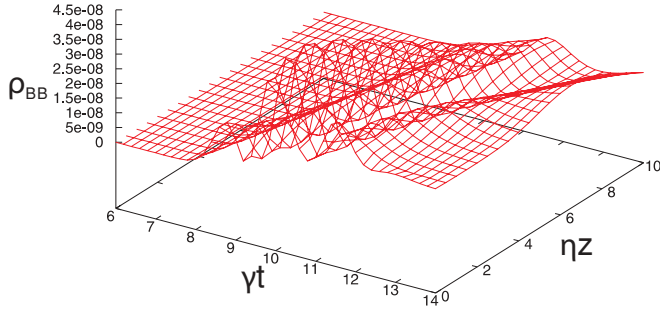


FIG. 7. (Color online) Time-space dependence of population in the bright state. (Upper panel)  $\Omega_2$  is a Gaussian pulse. (Lower panel)  $\Omega_2$  is a  $0\pi$  pulse.

into consideration the time dependence of the Rabi frequency, we obtain

$$\dot{w} = -\frac{i}{\hbar} H'_{\text{eff}} w, \quad (42)$$

where

$$H'_{\text{eff}} = P H_{\text{int}} P^{-1} - i P \dot{P}^{-1} \quad (43)$$

is the effective interaction Hamiltonian in the new basis. We obtain

$$H'_{\text{eff}} = \hbar \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Omega & 0 \\ 0 & 0 & -\Omega \end{pmatrix} - \frac{i}{\sqrt{2}} \hbar \begin{pmatrix} 0 & \Omega_c & -\Omega_c \\ -\Omega_c & 0 & 0 \\ \Omega_c & 0 & 0 \end{pmatrix}, \quad (44)$$

where  $\Omega_c$  is the Rabi frequency between these dressed states, the same as in Eq. (33).

It can be seen that when the Rabi frequencies are independent of time or the drive and probe fields are matched pulses, the second term in Eq. (44) is zero, and there is no coupling between the dressed states. These two bases give us different forms of the effective Hamiltonian that both lead us to the same conclusion: the different time dependence of drive and probe pulses introduces interaction between the dark and bright states and between the dressed states.

### C. Effects of field propagation

For an optically thin medium, the dynamic equations (8)–(12) are enough to analyze the behavior of the system. Although, for an optically thick medium such as a long rubidium vapor cell or a solid-state medium, the propagation of the probe and drive fields should be taken into account. Under the slowly varying envelope approximation [47], Maxwell's equations can be written as

$$\frac{\partial \Omega_1}{\partial z} + \frac{1}{c} \frac{\partial \Omega_1}{\partial t} = i \eta_1 \rho_{ab}, \quad (45)$$

$$\frac{\partial \Omega_2}{\partial z} + \frac{1}{c} \frac{\partial \Omega_2}{\partial t} = i \eta_2 \rho_{ac}, \quad (46)$$

where

$$\eta_i = \frac{3N\lambda_i^2\gamma_i}{8\pi} \quad (47)$$

are the corresponding coupling constants ( $i = 1, 2$ ),  $N$  is the density of atoms, and  $\lambda_i$  is the wavelength of the corresponding

transition. These ordinary differential Eqs. (8)–(12), and the partial differential Eqs. (45) and (46) with proper initial ( $t = 0$ ) and boundary conditions ( $z = 0$ ), determine the evolution of the system.

Considering the probe and drive pulses  $\Omega_1(t)$ ,  $\Omega_2(t)$  are long enough, we can rewrite Eqs. (10) and (12) as

$$\rho_{cb} = -\frac{\Omega_2 \Omega_1}{\Gamma_{ab} \Gamma_{cb} + |\Omega_2|^2}, \quad (48)$$

$$\rho_{ab} = -i \frac{\gamma_{cb}}{|\Omega_2|^2} \Omega_1 + \frac{1}{|\Omega_2|^2} \frac{\partial \Omega_1}{\partial t}. \quad (49)$$

In the case where absorption is negligible and  $\gamma_{cb} \simeq 0$ , by plugging Eq. (49) into Eq. (45), we obtain

$$\frac{\partial \Omega_1}{\partial z} + \frac{1}{V_g(z, t)} \frac{\partial \Omega_1}{\partial t} = 0, \quad (50)$$

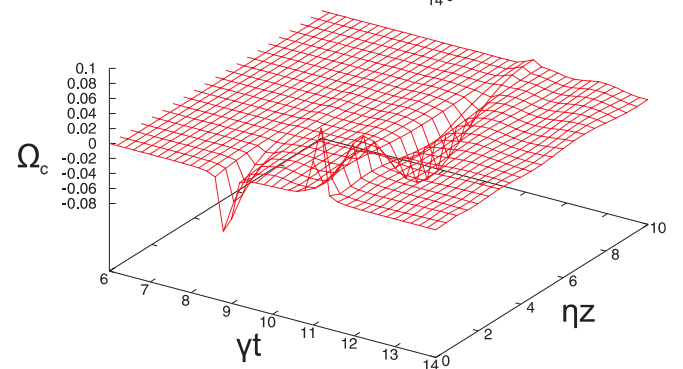
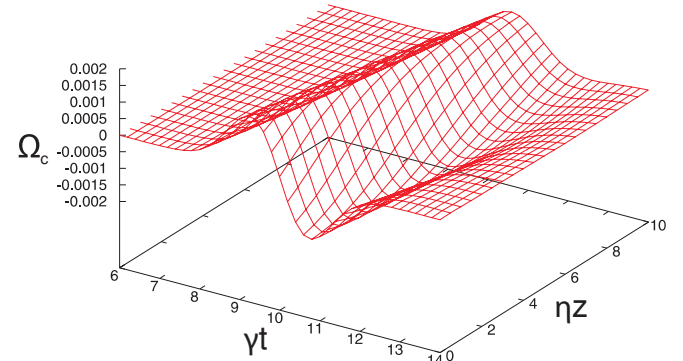


FIG. 8. (Color online) Time-space dependence of coupling between the bright and dark states. (Upper panel)  $\Omega_2$  is a Gaussian pulse. (Lower panel)  $\Omega_2$  is a  $0\pi$  pulse.



where  $V_g = c/(1 + c\eta_1/|\Omega_2|^2) \simeq |\Omega_2|^2/\eta_1$ . The solution of Eq. (50) is given by

$$\Omega_1(z,t) = \Omega_1 \left( z - \int_{-\infty}^t V_g(z,t') dt' \right). \quad (51)$$

In the case where  $\gamma_{cb}$  is small, the propagation equation is given by

$$\frac{\partial \Omega_1}{\partial z} + \frac{1}{V_g(z,t)} \frac{\partial \Omega_1}{\partial t} = -\frac{\gamma_{cb}\eta_1\Omega_1}{\gamma_{ab}\gamma_{cb} + |\Omega_2|^2}. \quad (52)$$

For small delay times, smaller than the duration of the drive pulse, such that

$$|\Omega_2|^2 \gg \frac{z}{V_g} \frac{\partial |\Omega_2|^2}{\partial t}, \quad (53)$$

we can write the solution as

$$\Omega_1(z,t) = \Omega_1 \left[ z - \int_{-\infty}^t V_g(z,t') dt' \right] \exp \left[ -\frac{\gamma_{cb}\eta_1 z}{\gamma_{ab}\gamma_{cb} + |\Omega_2|^2} \right],$$

where we can see the modification of the probe pulse shape due to the delay of the probe pulse by the group velocity and due to absorption.

### III. NUMERICAL SIMULATIONS

It is known that the so-called  $2n\pi$  pulses [46] can propagate through a medium of two-level atoms without absorption. In particular, we are interested in propagation of weak optical pulses with the so called  $0\pi$  pulses that have larger propagation length in comparison with the Gaussian pulses that have small area.

In our simulations, we chose the Rabi frequency of the  $0\pi$  probe pulse to have the following time dependence:

$$\Omega_1 = \Omega_{10} \left( e^{-\left(\frac{t-10}{T_1}\right)^2} - \frac{T_1}{T_2} e^{-\left(\frac{t-10}{T_2}\right)^2} \right), \quad (54)$$

where  $T_1$  and  $T_2$  are parameters. The shape of this probe pulse is showed in Fig. 3. It can be considered as the envelope of two Gaussian pulses. The first one has larger amplitude and

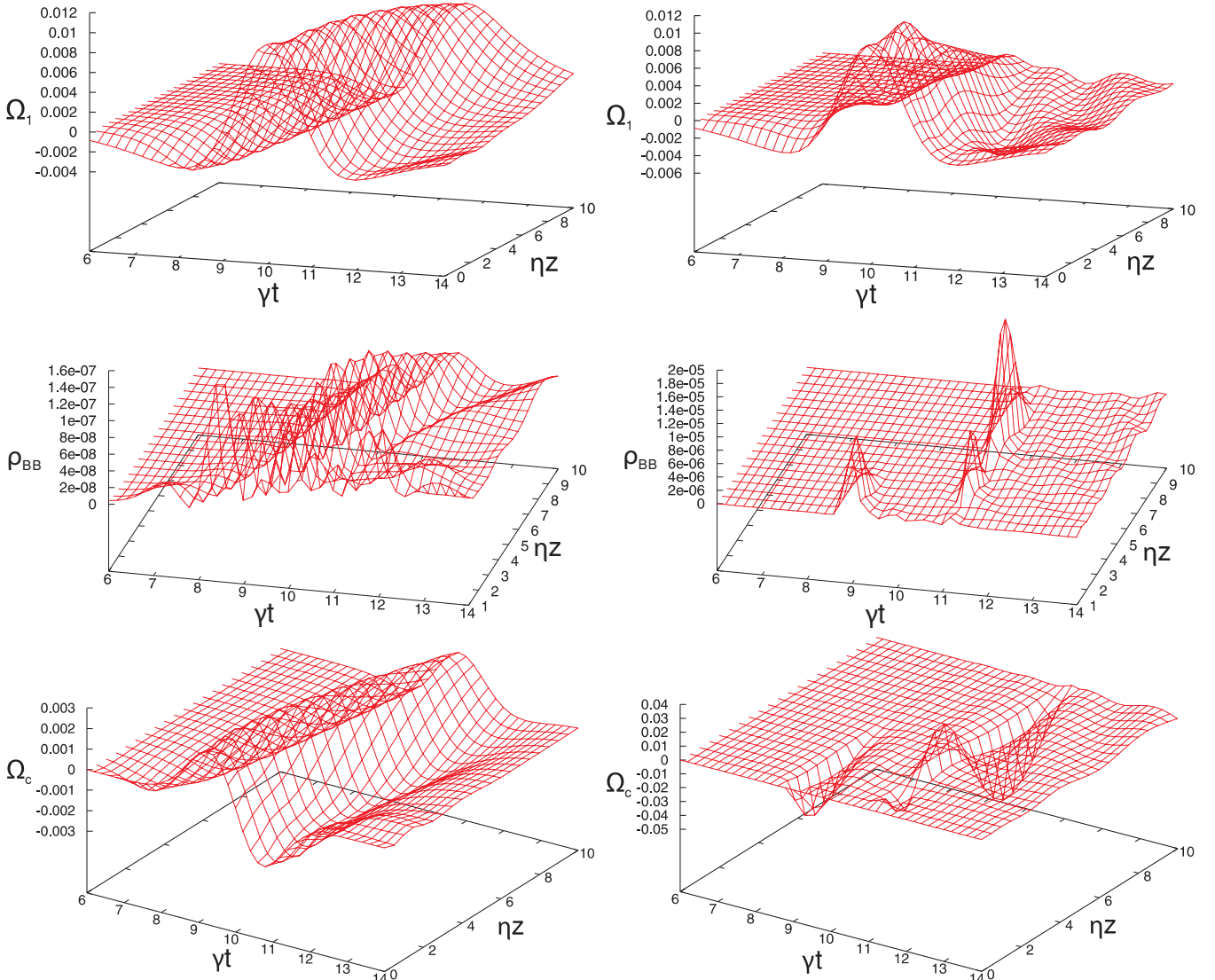


FIG. 9. (Color online) Propagation of the  $0\pi$  probe pulse, (Left panel)  $\Omega_2$  is Gaussian pulse and (Right panel)  $\Omega_2$  is  $0\pi$  pulse.

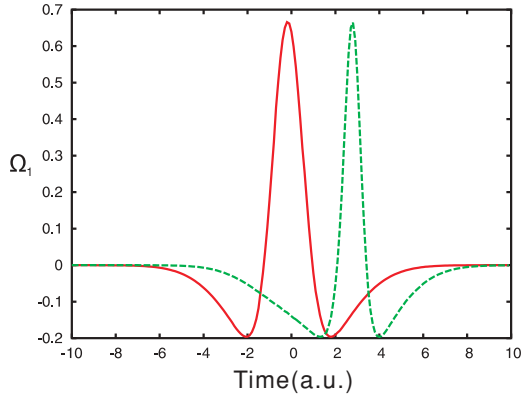


FIG. 10. (Color online) The probe pulse snapshots at different locations during propagation in a three-level  $\Lambda$  medium. Area of the pulse is initially zero (solid curve), after some distance, the pulse (dashed line) is delayed, and its area is not zero.

shorter duration  $T_1$ , the second one has smaller amplitude and longer duration  $T_2$ . In our simulations,  $T_1 = 1$ ,  $T_2 = 3$ , and the area of the pulse given by Eq. (54) is

$$\begin{aligned} \theta &= \int_{-\infty}^{\infty} \Omega(t') dt' = \Omega_{10} \int_{-\infty}^{\infty} \left( e^{-(t'-10)^2} - \frac{1}{3} e^{-(t'-\frac{10}{3})^2} \right) dt' \\ &= \Omega_{10} (\sqrt{\pi} - \sqrt{\pi}) = 0 \times \pi. \end{aligned} \quad (55)$$

In Fig. 4, the propagation of the  $0\pi$  pulses is shown in comparison with the Gaussian pulse in the form

$$\Omega_2 = \Omega_{20} e^{-\left(\frac{t-10}{T_2}\right)^2}, \quad (56)$$

where  $\Omega_{10}$  and  $\Omega_{20}$  are the initial amplitudes of the probe and drive pulses before the pulses enter the medium. Clearly, the absorption of the Gaussian pulses is larger than the  $0\pi$  pulse, and its propagation length is several times shorter.

### A. Two-photon resonance with $0\pi$ pulse

To answer the interesting question of whether the propagation of pulses in EIT configuration can be improved by using  $0\pi$  pulses instead of Gaussian pulses, we have performed simulation for a medium with optical density  $3N\lambda^2 z / (8\pi)$ , where  $z$  is the length of the medium and  $N$  is the density of  $^{87}\text{Rb}$  atomic gas.

Figure 5 shows the dependence of the probe pulse energy on position, with  $z = 10$ ,  $\eta = 3$ ,  $\rho_{bb} = 1$ . We take  $\Omega_1$  to be either a Gaussian pulse (Fig. 5 Upper) or a  $0\pi$  pulse (Fig. 5 Lower) and for each case we take the driving field  $\Omega_2$  to be either a Gaussian (Fig. 5 solid lines) or a  $0\pi$  pulse (Fig. 5 dashed lines). The shape of the probe pulse is

$$\Omega_1 = \Omega_{p0} e^{-\left(\frac{t-10}{T_1}\right)^2} \quad (57)$$

for the Gaussian pulse. The driving pulse has an amplitude of  $\Omega_{20} = 5$ . To compare the results, we consider these two types of drive pulses to have the same energy, and we introduce the coefficient 1.51 for the  $0\pi$  pulse initial amplitude. In the top figure, where  $\Omega_1$  is a Gaussian pulse, it shows that by switching the driving pulse between these two shapes, the probe pulse will have large absorption for the  $0\pi$  pulse and nearly no absorption for the Gaussian pulse.

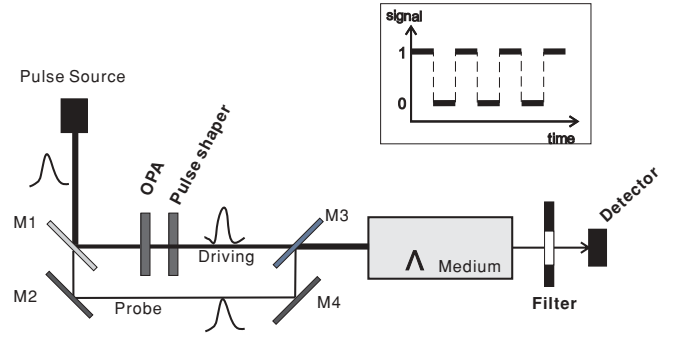


FIG. 11. All-optical switch.

When the probe and drive pulses are both Gaussian we have perfect EIT, and there is no absorption of the probe field. When the probe is a  $0\pi$  pulse and the drive a Gaussian, we have some absorption but practically we have almost the same EIT as we had for two Gaussian pulses. However, when both the probe and the drive are  $0\pi$  pulses we have no EIT. This was quite unanticipated, as we expected not to observe EIT with two pulses of different shapes but to observe it with two pulses of, matching,  $0\pi$  shape. In Fig. 6, we plot propagation of the probe fields for Gaussian and  $0\pi$  drive fields. One can see that, for the case of Gaussian drive field, the probe field propagates without absorption, meanwhile for  $0\pi$  drive field, the probe field absorbs very quickly. To gain physical insight, we plot the population of the bright state for these cases in Fig. 7, and we also plot the coupling between bright and dark states in Fig. 8. It is clearly seen that for the case of the  $0\pi$  drive field, the population of the bright state is much larger (see Fig. 7) and the coupling between bright and dark states is also much stronger. Surprisingly, for the case of  $0\pi$  probe pulse, we observe similar behavior. We have a better propagation for the Gaussian drive than for  $0\pi$  drive pulse. Intuitively, one would expect that for matching pulses, in this case, they are  $0\pi$  pulses; the case where the probe field and drive field are  $0\pi$  pulses should be better than the case where one pulse is Gaussian. But as one can see in Fig. 9, the physical reason for such counterintuitive behavior is the strong dispersion of the resonant medium that the probe pulse experiences during propagation. It is interesting to see the evolution of the pulses that have a spectral hole. Such pulses can propagate through the medium according

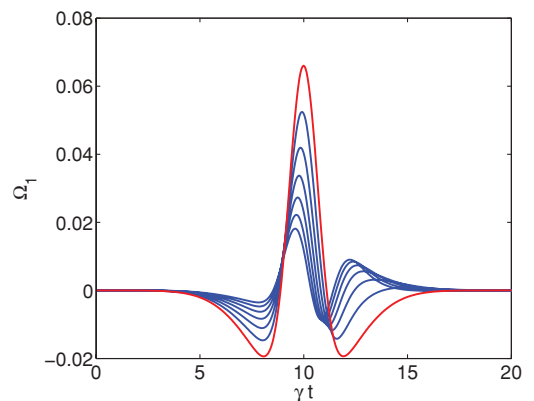


FIG. 12. (Color online) Dependence of pulse shape on time at different positions inside medium.

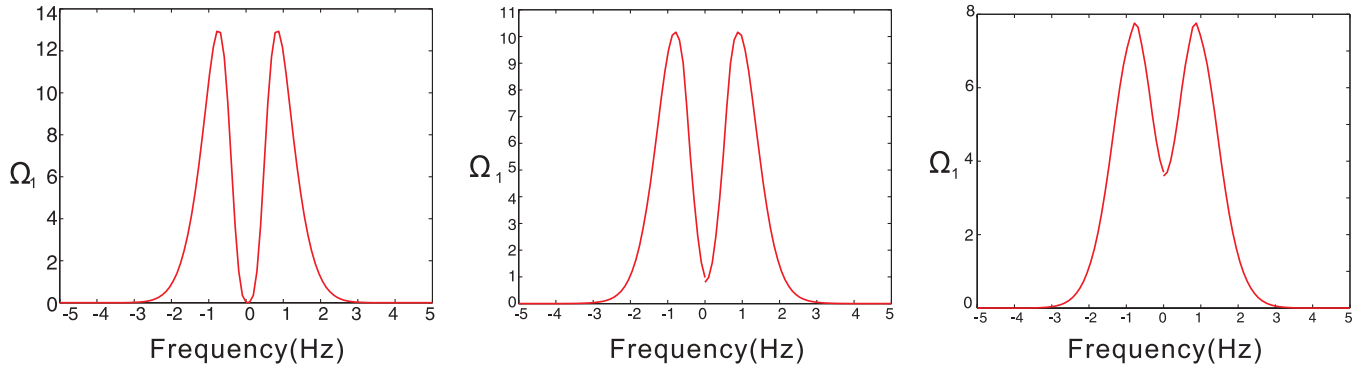


FIG. 13. (Color online) Spectrum of probe pulse  $\Omega_1$ . (L) Input pulse spectrum. (M) Pulse spectrum in the middle of medium. (R) Output pulse spectrum.

to Eq. (51), and as it is shown in Fig. 10. It causes the delay of the probe pulse and some reshaping of the pulse. Even initially, before entering the medium, the probe and drive pulses have been matched due to strong dispersion; the probe delays from the drive pulse and it gives rise to coupling between dark and bright states. The level of this coupling is different for different shapes of the pulses. It turns out that the Gaussian pulses are more tolerant of reshaping and delays, i.e., why the Gaussian drive pulses give us better propagation results.

#### IV. APPLICATIONS OF OBTAINED RESULTS

##### A. Control of propagation

It has been shown above that when the drive pulse is a  $0\pi$  pulse the probe pulse has larger absorption, and when the probe pulse is a Gaussian pulse, the probe pulse passes through the medium practically without absorption. This allows for an all-optical switch. It can be achieved by controlling the shape of the drive pulse in Fig. 5.

The scheme is shown in Fig. 11. A Gaussian drive pulse with frequency resonant to the transition  $|a\rangle - |b\rangle$  is generated by a laser system. Then the pulse interacts with a partially transparent mirror  $M1$  that allows for a small amount of the pulse energy to pass through and to reflect the most of the pulse energy. The part that passes through the mirror is used as the probe pulse, and the reflected part is used as the drive pulse. The probe pulse is fully reflected by mirrors  $M2$  and  $M4$ . The drive pulse passes through an optical parametric amplifier (OPA) and a pulse shaper. The OPA changes the frequency of the drive pulse to be resonant to the transition between  $|a\rangle$  and  $|c\rangle$ . The pulse shaper controls the pulse shape. Two paths for the probe and drive pulses are adjusted to have the same distance, so they can reach the mirror  $M3$  simultaneously, and then they combine with the drive pulse by semitransparent mirror  $M3$ , which lets the drive pulse fully pass through and lets the probe pulse fully be reflected.

If we change the drive pulse shape to be a  $0\pi$  pulse, the probe pulse has a large absorption and cannot pass through the  $\Lambda$  medium. In this case, the detector can detect nothing. This is equivalent to the output of signal 0. If the drive pulse shape is the Gaussian, then the probe pulse propagates through this medium. In this case, the detector can detect the probe pulse, which is equivalent to the output signal of 1. Using

the controlled pulse shaper, we are able to control whether the probe pulse can pass through the medium or not and generate any sequences of the probe pulses. This can have application to all-optical computing. This is a new way to control the propagation, namely, by using different shapes of drive pulses; we can control the transparency of the medium for the probe pulses. The most interesting is to have nearly 100% absorption. In this situation, the change of the drive pulse allows us to implement an all-optical switching. The advantage of our proposal is that we do not need a mechanical part to physically switch the drive pulse on or off as has been done in [48], where they have successfully made the EIT-based optical switch by turning on or off of the drive field periodically, but just continuously change the drive pulse shape by using the pulse shaper that introduces phases to different spectral components of the drive pulse.

##### B. Hole-filling effect

The  $0\pi$  probe pulse shapes on different spatial positions with detuning  $\Delta = 0$  are shown in Fig. 12. Here we use the following parameters:  $\Omega_{10} = 0.1, \Omega_{20} = 5, \rho_{bb} = 1, \gamma = 0.1$ . Due to the stimulated Raman process, the high-intensity part of the probe pulse is more influenced by the nonlinear interaction than a relatively weak part. This leads to a modification of the intense part of the pulse with respect to a weaker part, and influence on the pulse shape. As we have mentioned, when the input  $0\pi$  pulse has an initial area equal to zero, any change of area is easy to observe because the area is related to the some frequency of the probe pulse that has zeroth amplitude. Change of the pulse area means that this “zeroth” amplitude changes; this process can be referred to as a “hole-filling effect.” It is interesting to study the change of the pulse area with the change of pulse shape. This influence can be shown more clearly in the dependence of the pulse shape on time at different positions inside the medium, as shown in Fig. 12.

To make it more clear, in Fig. 13 we show the spectrum of the probe pulse at three different positions: at the initial input, at the middle of the medium, and at the output. It can be seen that initially there is a spectral hole in the middle of pulse spectrum, which is related to the fact that the input is the  $0\pi$  pulse and it has the zero area. While the pulse is propagating through the medium, the spectral hole is partially filled; the longer distance it passes, the more the hole is filled, which can



be seen from the second and third figures. By the level of hole filling, one can obtain the density of the atoms or molecules in the medium.

It is interesting to see the evolution of the pulse that has a spectral hole. Such a pulse can propagate through the medium according to Eq. (51), and as is shown in Fig. 10. One can see that due to time delay the area of the pulse changes, as is shown in Fig. 13. We can also see the change of the spectrum of the probe pulse during its propagation.

These results suggest a new approach to Raman spectroscopy that extends application of the stimulated Raman scattering to media which has a lot of scattering. The stimulated Raman scattering has an advantage of a relaxing phase-matching condition which is difficult or even impossible to meet under the condition of strong scattering. This technique is compatible and can be applied to microscopy.

## V. CONCLUSION

In conclusion, we present a theoretical study of electromagnetically induced transparency with  $0\pi$  pulses. We simulate the propagation of  $0\pi$  laser pulses through a medium of three-level  $\Lambda$  atoms, and compare with Gaussian copropagating pulses. We find that even on two-photon resonance the absorption of the  $0\pi$  pulses is significantly greater than that of the Gaussian pulses. We use the dark and bright bases to explain the behavior, and discuss possible applications.

In this paper we study the effects of pulse shape on the efficiency of electromagnetically induced transparency. Since a single  $0\pi$  pulse experiences less absorption than a Gaussian pulse of the same energy, we researched whether using  $0\pi$  pulses instead of Gaussians as probe pulses in EIT will result in less absorption. It turned out that this is not the case; in fact, it is quite the opposite. When using  $0\pi$  pulses as the probes in EIT absorption is increased. The explanation of this counterintuitive result is elegantly illustrated in the dressed-state basis. The time dependence in the  $0\pi$  pulse results in an interaction between the dark and the bright states which decouples the dark-state population and hence increases absorption. The effect may find a possible application in developing an all-optical switch.

Future work includes studying the effects of propagation of  $0\pi$  pulses in the spectral domain [49], as such pulses have applications for nonlinear spectroscopy in scattering media [50].

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