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PROPAGATION OF MONOCHROMATIC WAVES IN AN INITIALLY STRESSED INFINITE MICROPOLAR ELASTIC PLATE

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Summary

The object of this paper is to investigate the propagation of monochromatic waves in an initially stressed infinite micropolar elastic plate. The "initial stress" is considered in the light of Cauchy theory. The problem of propagation of waves is reduced to the solution of symmetric and anti-symmetric vibrations.

1. Introduction

The papers mentioned below show an increasing interest in problems of propagation of monochromatic waves in the general Cosserat medium, wherein the deformation of a body is described by two vectors independent of each other, namely by the displacement vector $\boldsymbol{u}(\boldsymbol{x}, t)$ and the rotation vector $\omega(\boldsymbol{x}, t)$.

The propagation of plane waves in an infinite micropolar medium was discussed by V. A. Palmov [1]; the propagation of rotation waves in an infinite medium was the subject of a paper by W. Nowacki [2]. S. Kaliski, J. Kapelewski and C. Rymarz devoted a paper [3] to the problem of propagation of surface waves in a micropolar medium. W. Nowacki and W. K. Nowacki [4] devoted another paper to the propagation of monochromatic waves in an infinite elastic plate.

In this paper we are concerned with the propagation of monochromatic waves in an initially stressed infinite micropolar elastic plate of uniform thickness. The initial stress is discussed in the light of Cauchy theory. It was shown much earlier by Cauchy [5] that the presence of initial stress alters the classical stress-strain relations which hold for an initially unstressed body. Guided by the so-called "structure theory" he deduced the relations between the components of stress and strain which automatically involved the components of initial stress. Paria [6] has developed the Cauchy theory of initial stress and has applied it to the problems of plain strain in elastic materials.

In the present paper, the initial stress theory of Cauchy is coupled with the theory of couple stress and the stress-strain relations are deduced. The symmetric and anti-symmetric vibrations are considered as a reduction to the propagation of monochromatic waves. The results obtained in this paper completely agree with the results obtained in [4] in absence of the initial stress.

2. Basic Equations

Let us first consider the equations describing an elastic micropolar medium [7], [8]. An elastic, homogeneous, isotropic and centri-symmetric body will be the object of our subsequent discussions. Under the effect of external loadings, displacement and rotation field $\boldsymbol{u}(\boldsymbol{x}, t)$ and $\boldsymbol{\omega}(\boldsymbol{x}, t)$ respectively, will form in such a body.

The state of strain is described by asymmetric tensors, the strain tensor v_{ji} and the curvature-twist tensor χ_{ji}:

$$(2.1) \qquad v_{ji} = u_{i,j} - \varepsilon_{kji}\omega_k\,, \quad \chi_{j,i} = \omega_{ij}\,.$$

The state of stress is defined, in turn, by the following two asymmetric tensors: the stress tensor σ_{ji} and the couple-stress tensor μ_{ji}. The relation between the state of strain and that of stress is described by the relations

$$(2.2) \qquad \begin{aligned} \sigma_{ji} &= (\mu + \alpha)\, v_{ji} + (\mu - \alpha)\, v_{ij} + \lambda v_{kk}\delta_{ij}\,, \\ \mu_{ji} &= (\nu + \varepsilon)\,\chi_{ji} + (\nu - \varepsilon)\,\chi_{ij} + \beta\chi_{kk}\delta_{ij}\,, \quad i, j = 1, 2, 3\,. \end{aligned}$$

The quantities μ, λ, α, β, ν, ε denote the material constants. The equations of motion in presence of a couple-stress can be written as

$$(2.3) \qquad \sigma_{ij,j} + X_i - \varrho\ddot{u}_i = 0\,, \quad \varepsilon_{ijk}\sigma_{jk} + \mu_{ji,j} + Y_i - J\ddot{\omega}_i = \,,$$

where X_i is the component of the body force and Y_i stands for the component of the body couple, and ϱ stands for the density and J for the rotational inertia.

Let us consider an elastic plate of uniform thickness $2h$ which is subjected to a uniform initial tension K whose direction is parallel to the plane faces of the plate. Let the origin of coordinates be taken at a point of the middle plane of the plate and the x-axis be drawn in the direction of the initial tension. The axis of y is normal to the middle plane and the z-axis is normal to the xy-plane. Then the components of the initial stress are

$$(2.4) \qquad \sigma^0_{11} = K\,, \quad \sigma^0_{22} = \sigma^0_{33} = \sigma^0_{12} = \sigma^0_{21} = \sigma^0_{23} = \sigma^0_{32} = \sigma^0_{31} = \sigma^0_{13} = 0\,,$$

where K is constant, and the faces of the plate are located in the planes $y = \pm h$.

The additional stresses developed in the plate due to the deformation are denoted by

(2.5)
$$\sigma'_{11}, \sigma'_{21}, \sigma'_{31}, \sigma'_{12}, \sigma'_{22}, \sigma'_{32}, \sigma'_{13}, \sigma'_{23}, \sigma'_{33}\,;$$
$$\mu'_{11}, \mu'_{21}, \mu'_{31}, \mu'_{12}, \mu'_{22}, \mu'_{32}, \mu'_{13}, \mu'_{23}, \mu'_{33}\,.$$

We shall restrict ourselves to two dimensional incremental deformations in the xy-plane. We shall determine the displacement components (u, v) for the case of anti-symmetric and symmetric oscillations by applying the Cauchy theory of initial stress [5]. The corresponding values of the traction components which must be applied at the faces $y = \pm h$ to cause these types of oscillations shall also be determined. They will vanish for free oscillations.

According to the Cauchy theory of initial stress [5, p. 110, equations (39)], the equations (2.3) can be transformed in absence of both body forces and body couples to

(2.6)
$$\sigma'_{ji,j} - (\varrho + \varrho')\,\ddot{u}_i = 0\,, \quad \varepsilon_{ijk}\sigma'_{jk} + \mu'_{ji,j} - J\ddot{\omega}_i = 0\,,$$

where $(\varrho + \varrho')$ is the density of the material in the strained state.

According to the Cauchy theory, the boundary forces corresponding to the stress system σ'_{11} etc. are to be referred to the displaced boundary. However, if the deformation (u, v) is small, we may refer the forces to the initial boundary. Thus assuming that the deformation (u, v) is small, the boundary conditions [5, p. 110, equations (40)] become

(2.7)
$$\sigma'_{11}l_1 + \sigma'_{21}m_1 + \sigma'_{31}n_1 = \bar{x}, \quad \sigma'_{12}l_1 + \sigma'_{22}m_1 + \sigma'_{32}n_1 = \bar{y}\,,$$
$$\sigma'_{13}l_1 + \sigma'_{23}m_1 + \sigma'_{33}n_1 = \bar{z}\,;$$
$$\mu'_{11}l_1 + \mu'_{21}m_1 + \mu'_{31}n_1 = X\,, \quad \mu'_{12}l_1 + \mu'_{22}m_1 + \mu'_{32}n_1 = Y\,,$$
$$\mu'_{13}l_1 + \mu'_{23}m_1 + \mu'_{33}n_1 = Z\,,$$

where (l_1, m_1, n_1) are the direction cosines of the normal to the initial boundary and $(\bar{x}, \bar{y}, \bar{z})$ and (X, Y, Z) are the corresponding surface tractions at any point of the boundary.

The stress-displacement relations of Cauchy [5, p. III, equations (41)] when the incremental deformation is in the xy-plane and the plate is in a state of initial stress defined by the components (2.4), can be obtained as

(2.8)
$$\sigma'_{11} = (3\mu + K)\frac{\partial u}{\partial x} + (\mu - K)\frac{\partial v}{\partial y},$$
$$\sigma'_{22} = \mu\left(\frac{\partial u}{\partial x} + 3\frac{\partial v}{\partial y}\right),$$
$$\sigma'_{33} = \mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

$$\sigma'_{21} = K\frac{\partial v}{\partial x} + \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) - \alpha\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + 2\alpha\omega_3,$$

$$\sigma'_{12} = K\frac{\partial v}{\partial x} + \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + \alpha\left(\frac{\partial v'}{\partial x} - \frac{\partial u}{\partial y}\right) - 2\alpha\omega_3,$$

$$\sigma'_{31} = \sigma'_{32} = \sigma'_{13} = \sigma'_{23} = 0;$$

$$\mu'_{11} = \mu'_{12} = \mu'_{21} = \mu'_{22} = \mu'_{33} = 0,$$

$$\mu'_{13} = (\nu + \varepsilon)\frac{\partial \omega_3}{\partial x},$$

$$\mu'_{31} = (\nu - \varepsilon)\frac{\partial \omega_3}{\partial x},$$

$$\mu'_{23} = (\nu + \varepsilon)\frac{\partial \omega_3}{\partial y},$$

$$\mu'_{32} = (\nu - \varepsilon)\frac{\partial \omega_3}{\partial y},$$

where μ is the rigidity of the material.

The density of the material in the strained state is given by $(\varrho + \varrho') = \varrho/(1 + e)$, where e denotes dilatation. In terms of displacements

(2.9) $$(u, v), \quad \text{it is} \quad e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}.$$

Introducing (2.8) and (2.9) into (2.6) and neglecting small quantities of the second order, the equations of motion in terms of the displacement components u and v become

(2.10) $$(3\mu + K)\frac{\partial^2 u}{\partial x^2} + (2\mu - \alpha)\frac{\partial^2 v}{\partial x\,\partial y} + (\mu + \alpha)\frac{\partial^2 u}{\partial y^2} + 2\alpha\frac{\partial \omega_3}{\partial y} = \varrho\frac{\partial^2 u}{\partial t^2},$$

$$(K + \mu + \alpha)\frac{\partial^2 v}{\partial x} + (2\mu - \alpha)\frac{\partial^2 u}{\partial x\,\partial y} + 3\mu\,.\frac{\partial^2 v}{\partial y^2} - 2\alpha\frac{\partial \omega_3}{\partial y} = \varrho\frac{\partial^2 v}{\partial z^2},$$

$$(\nu + \varepsilon)\left[\frac{\partial^2 \omega_3}{\partial x^2} + \frac{\partial^2 \omega_3}{\partial y^2}\right] - 4\alpha\omega_3 + 2\alpha\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = J\frac{\partial^2 \omega_3}{\partial t^2}.$$

3. Boundary conditions in terms of displacements

The boundary conditions (2.7) can be obtained in terms of the displacements u and v by substituting for σ'_{11} etc., from (2.8) in the conditions (2.7), Let us assume that a monochromatic wave propagates in an elastic plate of thickness $2h$ along the

x-axis. We assume that the edges of the layers $y = \pm h$ are free of stresses and for the boundaries $y = \pm h$ of the plate $l_1 = 0$, $m_1 = \pm 1$, $n_1 = 0$. The following conditions should be satisfied on these edges:

$$(3.1)\qquad \sigma'_{21} = (K + \mu - \alpha)\frac{\partial v}{\partial x} + (\mu + \alpha)\frac{\partial u}{\partial y} + 2\alpha\omega_3 = 0\,, \quad \text{for} \quad y = \pm h\,,$$

$$\sigma'_{22} = \mu\left(\frac{\partial u}{\partial x} + 3\frac{\partial v}{\partial y}\right) = 0\,, \qquad \text{for} \quad y = \pm h\,,$$

$$\mu'_{23} = (\nu + \varepsilon)\frac{\partial \omega_3}{\partial y} = 0\,, \qquad \text{for} \quad y = \pm h\,.$$

4. Modified Lamb's waves

Expressing the displacements by the potentials ϕ, ψ,

$$(4.1)\qquad u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial y}, \quad \vartheta = \frac{\partial\phi}{\partial y} + \frac{\partial\psi}{\partial x},$$

we can derive from the system of equations (2.10) the following equations:

$$(4.2)\qquad (K + 3\mu)\frac{\partial^2\phi}{\partial x^2} + 3\mu\frac{\partial^2\phi}{\partial y^2} - \varrho\frac{\partial^2\phi}{\partial t^2} = 0\,,$$

$$(K + \mu + \alpha)\frac{\partial^2\psi}{\partial x^2} + (\mu + \alpha)\frac{\partial^2\psi}{\partial y^2} - 2\alpha\omega_3 - \varrho\frac{\partial^2\psi}{\partial t^2} = 0\,,$$

$$(\nu + \varepsilon)\left[\frac{\partial^2\omega_3}{\partial x^2} + \frac{\partial^2\omega_3}{\partial y^2}\right] - 4\alpha\omega_3 + 2\alpha\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) = J\frac{\partial^2\omega_3}{\partial t^2}.$$

Eliminating from the last two equations first ω_3 and then ψ, we get

$$(4.3)\qquad \left\{\left[(K + \mu + \alpha)\frac{\partial^2}{\partial x^2} + (\mu + \alpha)\frac{\partial^2}{\partial y^2} - \varrho\frac{\partial^2}{\partial t^2}\right]\left[(\nu + \varepsilon)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \right.\right.$$
$$\left.\left. - 4\alpha - J\frac{\partial^2}{\partial t^2}\right] + 4\alpha^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right\}(\psi, \omega_3) = 0\,.$$

The first equation of (4.2) describes the longitudinal wave, while equation (4.3) describes the modified transverse wave in an initially stressed elastic plate of uniform thickness.

The solutions of equations (4.2), and (4.3) will be sought for the form

$$(4.4)\qquad (\phi, \psi, \omega_3) = [\phi^*(y), \psi^*(y), \omega^*(y)]\, e^{i(lx - \omega t)}\,.$$

These solutions are

$$(4.5)\qquad \begin{aligned} \phi^*(y) &= A \operatorname{sh} \delta' y + B \operatorname{ch} \delta' y\,, \quad \delta'^2 = [l^2(K + 3\mu) - \varrho\omega^2]/3\mu \\ \psi^*(y) &= C \operatorname{sh} \lambda_1 y + D \operatorname{ch} \lambda_1 y + E \operatorname{sh} \lambda_2 y + F \operatorname{ch} \lambda_2 y\,, \\ \omega^*(y) &= C' \operatorname{sh} \lambda_1 y + D' \operatorname{ch} \lambda_1 y + E' \operatorname{sh} \lambda_2 y + F' \operatorname{ch} \lambda_2 y\,. \end{aligned}$$

The following notation have been introduced:

$$(4.6)\qquad \sigma_2 = \frac{\omega}{c_2}, \quad c_2 = \left(\frac{\mu + \alpha}{\varrho}\right)^{1/2}, \quad \sigma_4 = \frac{\omega}{c_4}, \quad C_4 = \left(\frac{\nu + \varepsilon}{J}\right)^{1/2},$$

$$\nu^2 = \frac{4\alpha}{\nu + \varepsilon}, \quad \eta^2 = \frac{4\alpha^2}{(\nu + \varepsilon)(\mu + \alpha)},$$

$$\lambda_{1,2}^2 = \frac{1}{2}\left\{\left[l^2\left(2 + \frac{K}{\mu + \alpha}\right) + \nu^2 - \eta^2 - \sigma_2^2 - \sigma_1^2\right] \pm \left(\left[l^2\left(2 + \frac{K}{\mu + \alpha}\right) + \right.\right.\right.$$
$$\left. + \nu^2 - \eta^2 - \sigma_2^2 - \sigma_1^2\right]^2 - 4\left[l^4\left(1 + \frac{K}{\mu + \alpha}\right) - \right.$$
$$\left.\left.\left. - l^2\sigma_2^2 + l^2\nu^2\left(1 + \frac{K}{\mu + \alpha}\right) - \nu^2\sigma_2^2 - l^2\eta^2 - l^2\sigma_4^2\left(1 + \frac{K}{\mu + \alpha}\right)\right]\right)^{1/2}\right\}.$$

Since the quantities λ_1^2, λ_2^2 have to be positive (this follows from the postulate that the phase velocities be real), we have $\omega^2 > 4\alpha/J$. The equations $(4.2)_2$ and $(4.3)_3$ are connected through $(4.5)_2$ and $(4.5)_3$ respectively.

The general problem of propagation of waves may be reduced to the solution of two simple problems, that is, to the consideration of symmetric and antisymmetric vibrations.

a) Symmetric vibration

We shall first consider the symmetric vibrations characterized by the symmetry of the displacement u and stresses σ'_{11}, σ'_{22} and μ'_{23} with respect to the plane $y = 0$. In this case we have to put in the expressions (4.5): $A = D = F = D' = F' = 0$. In view of the coupling of the equations $(4.2)_2$ and $(4.2)_3$, we have

$$(4.7)\qquad C' = K_1 C \quad \text{and} \quad E' = K_2 E\,,$$

where

$$K_r = \frac{1}{P}\left[\sigma_2^2 + \lambda_r^2 - l^2\left(1 + \frac{K}{\mu + \alpha}\right)\right], \quad r = 1, 2\,, \quad p = \frac{2\alpha}{\mu + \alpha}.$$

Expressing the boundary conditions (3.1) by the functions ϕ^*, ψ^* and ω^*, we obtain a system of three homogeneous equations. Equating to zero the determinant of this system, we arrive at the characteristic equation

$$(4.8)\quad \frac{\operatorname{tgh}(\delta' h)}{\operatorname{tgh}(\lambda_1 h)} = \frac{3\delta'^2 - l^2}{2l^2(K + 2\mu)\,\delta'\lambda_1(K_2 - K_1)} \cdot \left[a_1' K_2 - a_2' K_1 \cdot \frac{\lambda_1}{\lambda_2} \cdot \frac{\operatorname{tgh}(\lambda_2 h)}{\operatorname{tgh}(\lambda_1 h)}\right],$$

where $a_r' = (K + \mu - \alpha)\, l^2 + (\mu + \alpha)\,\lambda_r^2 - 2\alpha K_r$, $r = 1, 2$. The quantity $c = \omega/l$ is the phase velocity sought for. From the transcendental equation (4.8), we obtain an infinite number of roots of l. To each of these roots there corresponds a definite form of vibrations. For $\lambda = \mu$ and $K \to 0$, the equation (4.8) corresponds exactly to the same equation obtained by [4]. Also for $\alpha \to 0$ (which corresponds to the classical theory of elasticity) the equation (4.8) reduces to

$$(4.9)\quad \frac{\operatorname{tgh}(lh\sqrt{(1 - c^2/\hat{c}_1^2)})}{\operatorname{tgh}(lh\sqrt{(1 - c^2/\hat{c}_2^2)})} = \frac{(2 - c^2/\hat{c}_2^2)^2}{4\sqrt{((1 - c^2/\hat{c}_1^2)(1 - c^2/\hat{c}_2^2))}},$$

$$\hat{c}_1 = \left(\frac{3\mu}{\varrho}\right)^{1/2}, \quad \hat{c}_2 = \left(\frac{\mu}{\varrho}\right)^{1/2},$$

which is the same equation for Lamb's waves [9] for $\lambda = \mu$. Hence $\hat{c}_1 = \sqrt{(3)}\hat{c}_2$.

Let us now consider two particular cases. We assume first that the wave length is small compared with the thickness of the plate $2h$. Then the quantities $\delta' h$, $\lambda_1 h$ and $\lambda_2 h$ are large such that it is plausible to assume the ratio of hyperbolic tangents as equal to one. Then

$$(4.10)\quad \frac{a_1' K_2}{K_2 - K_1} - \frac{a_2' K_1}{K_2 - K_1} = \frac{2l^2(K + 2\mu)\,\delta\lambda_1}{(3\delta'^2 - l^2)}.$$

The above equation coincides with the dispersional equation for the surface wave in a micropolar medium [3] for $\lambda = \mu$ and $K = 0$. For $\alpha \to 0$, $K \to 0$, we obtain from (4.9) the equation

$$(4.11)\quad (2 - c^2/\hat{c}_2^2)^2 = 4 \cdot \sqrt{(1 - c^2/\hat{c}_1^2)(1 - c^2/\hat{c}_2^2)},$$

which is the same characteristic equation for Rayleigh waves [10] for $\lambda = \mu$. For waves which are long compared with the thickness $2h$, the quantities $\delta' h$, $\lambda_1 h$, $\lambda_2 h$ are small and the hyperbolic tangents in (4.8) may be replaced by their arguments.

We have

$$(4.12)\quad 2l^2(K + 2\mu)\,\delta'^2(K_2 - K_1) = (\delta'^2 - l^2)(a_1' K_2 - a_2' K_1).$$

In the particular case $\alpha \to 0$ and $\lambda = \mu$, $K \to 0$, the result is

$$c = 2 \cdot \frac{\hat{c}_2}{\hat{c}_1}(\hat{c}_1^2 - \hat{c}_2^2)^{1/2}.$$

b) Antisymmetric vibrations

Let us now consider the particular case where the displacement and the stresses σ'_{11}, σ'_{22} and μ'_{23} are antisymmetric with respect to the plane $y = 0$. Then we have to put in the expressions (4.5),

$$B = C = E = C' = E' = 0 \quad \text{and} \quad D' = K_1 D\,, \quad F' = K_2 F\,.$$

Making use of the boundary conditions (3.1), we arrive at the transcendental equation

$$\text{(4.13)} \quad \left[\frac{a'_1 K_2 \lambda_2}{\operatorname{tgh}(\lambda_1 h)} - \frac{a'_2 K_1 \lambda_1}{\operatorname{tgh}(\lambda_2 h)}\right] \operatorname{tgh}(\delta' h) = \frac{2l^2(K + 2\mu)\, \delta' \lambda_1 \lambda_2 (K_2 - K_1)}{(3\delta'^2 - l^2)}\,,$$

which helps to determine the successive values of the parameter l. For $\alpha \to 0$, $K \to 0$, the equation (4.13) reduces to

$$\text{(4.14)} \quad \frac{\operatorname{tgh}(lh \sqrt{(1 - c^2/\hat{c}_1^2)})}{\operatorname{tgh}(lh \sqrt{(1 - c^2/\hat{c}_2^2)})} = \frac{4\,((1 - c^2/\hat{c}_1^2)(1 - c^2/\hat{c}_2^2))}{(2 - c^2/\hat{c}_2^2)^2}\,.$$

The equation (4.14) is exactly the same equation as obtained in [10] for $\lambda = \mu$.

If the wave length is large compared with the thickness of the plate $2h$, then expanding the hyperbolic tangents into a series and retaining only first two terms, we obtain the equation

$$\text{(4.15)} \left[\frac{a'_1 K_2}{\lambda_1^2 \left(1 - \dfrac{\lambda_1^2 h^2}{3}\right)} - \frac{a'_2 K_1}{\lambda_2^2 \left(1 - \dfrac{\lambda_2^2 h^2}{3}\right)}\right] \left(1 - \frac{\delta'^2 h^2}{3}\right) = \frac{2l^2(K + 2\mu)(K_2 - K_1)}{3\delta'^2 - l^2}\,.$$

The equation (4.15) permits to determine the phase velocity $c = \omega/l$ of the flexural wave.

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Souhrn

ŠÍŘENÍ MONOCHROMATICKÝCH VLN V PŘEDPJATÉ NEKONEČNÉ MIKROPOLÁRNÍ PRUŽNÉ DESCE

R. K. BERA

V článku se vyšetřuje šíření monochromatických vln v předpjaté nekonečné mikropolární pružné desce. Předpjetí je uvažováno ve smyslu Cauchyho teorie. Problém šíření vln je redukován na řešení symetrických a antisymetrických oscilací.

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