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Propagation of plane waves at the interface of an elastic solid half-space and a microstretch thermoelastic diffusion solid half-space

Abstract

The problem of reflection and refraction phenomenon due to plane waves incident obliquely at a plane interface between uniform elastic solid half-space and microstretch thermoelastic diffusion solid half-space has been studied. It is found that the amplitude ratios of various reflected and refracted waves are functions of angle of incidence, frequency of incident wave and are influenced by the microstretch thermoelastic diffusion properties of the media. The expressions of amplitude ratios and energy ratios are obtained in closed form. The energy ratios have been computed numerically for a particular model. The variations of energy ratios with angle of incidence are shown for thermoelastic diffusion media in the context of Lord-Shulman (L-S) (1967) and Green-Lindsay (G-L) (1972) theories. The conservation of energy at the interface is verified. Some particular cases are also deduced from the present investigation.

Keywords

Microstretch, thermoelastic diffusion solid, plane wave, wave propagation, amplitude ratios, energy ratios.

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1 INTRODUCTION

Theory of microstretch continua is a generalization of the theory of micropolar continua.The theory of microstretch elastic solids has been introduced by Eringen [7–10]. This theory is a special case of the micromorphic theory. In the framework of micromorphic theory, a material point is endowed with three deformable directors. When the directors are constrained to have only breathing-type microdeformations, then the body is a microstretch continuum [10]. The material points of these continua can stretch and contract independently of their translations and rotations. A microstretch continuum is a model for a Bravais lattice with its basis on the atomic level and two-phase dipolar solids with a core on the macroscopic level. Composite materials reinforced with chopped elastic fibers, porous media whose pores are filled with gas or inviscid liquid, asphalt, or other elastic inclusions and solid–liquid crystals, etc., are examples of microstretch solids. The theory is expected to find applications in the treatment of the mechanics of composite materials reinforced with chopped fibers and various porous materials.

Eringen [9] developed the theory of microstretch thermoelastic solids and derived the equations of motions, constitutive equations, and boundary conditions for thermo-microstretch fluids and obtained the solution of the problem for acoustical waves in bubbly liquids. During the last four decades, wide spread attention has been given to thermoelasticity theories which admit a finite speed for the propagation of a thermal field. Lord and Shulman [18] reported a new theory based on a modified Fourier's Law of heat conduction with one relaxation time. A more rigorous theory of thermoelasticity by introducing two relaxation times has been formulated by Green and Lindsay (G-L) [13]. A survey article of various representative theories in the range of generalized thermoelasticity have been brought out by Hetnarski and Ignaczak [14].

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases(e.g. xexon) and other light isotopes(e.g. carbon) for research purposes. In most of the applications, the concentration is calculated using what is known as Fick's law. This is a simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of temperature on this interaction. However, there is a certain degree of coupling with temperature and temperature gradients as temperature speeds up the diffusion process. The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment.

Nowacki[19-22] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [6] and Olesiak and Pyryev [23], respectively, discussed the theory of thermodiffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer. They studied the influence of cross effects arising from the coupling of the fields of temperature, mass diffusion and strain due to which the thermal excitation results in additional mass concentration and that generates additional fields of temperature. Gawinecki and Szymaniec [11] proved a theorem about global existence of the solution for a nonlinear parabolic thermoelastic diffusion problem. Gawinecki et al. [12] proved a theorem about existence, uniqueness and regularity of the solution for the same problem. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Sherief et al. [24] on the basis of the variational principle equations, under restrictive assumptions on the elastic coefficients. Due to the inherit complexity of the derivation of the variational principle equations, Aouadi [2] proved this theorem in the Laplace transform domain, under the assumption that the functions of the problem are continuous and the inverse Laplace transform of each is also unique. Sherief and Saleh [25] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Kumar and Kansal [16] developed the basic equation of anisotropic thermoelastic diffusion based upon Green-Lindsay model.

Borejko [4] discussed the reflection and transmission coefficients for three-dimensional plane waves in elastic media. Wu and Lundberg [28] investigated the problem of reflection and transmission of the energy of harmonic elastic waves in a bent bar. Sinha and Elsibai [27] discussed the reflection and refraction of thermoelastic waves at an interface of two semi-infinite media with two relaxation times. Singh [26] studied the reflection and refraction of plane waves at a liquid/thermo-microstretch elastic solid interface. Kumar and Pratap [15] discussed the reflection of plane waves in a heat flux dependent microstretch thermoelastic solid half space.

In the present paper, the reflection and refraction phenomenon at a plane interface between an elastic solid medium and a microstretch thermoelastic diffusion solid medium has been analyzed. In microstretch thermoelastic diffusion solid medium, potential functions are introduced to the equations. The amplitude ratios of various reflected and transmitted waves to that of incident wave are derived. These amplitude ratios are further used to find the expressions of energy ratios of various reflected and refracted waves to that of incident wave. The graphical representation is given for these energy ratios for different direction of propagation. The law of conservation of energy at the interface is verified.

2 BASIC EQUATIONS

Following Sherief et al. [24], Eringen [10] and Kumar & Kansal [17].The equations of motion and the constitutive relations in a homogeneous isotropic microstretch thermoelastic diffusion solid in the absence of body forces, body couples, stretch force, and heat sources are given by

$$\begin{aligned}&\left(\lambda+2\mu+K\right)\nabla\left(\nabla.\vec{u}\right)-\left(\mu+K\right)\nabla\times\nabla\times\vec{u}+K\nabla\times\vec{\varphi}-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T\\&-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C+\lambda_O\nabla\varphi^*=\rho\frac{\partial^2\vec{u}}{\partial t^2},\end{aligned}\tag{1}$$

$$\left(\alpha+\beta+\gamma\right)\nabla\left(\nabla.\vec{\varphi}\right)-\gamma\,\nabla\times\left(\nabla\times\vec{\varphi}\right)+K\nabla\times\vec{u}-2K\vec{\varphi}=\rho j\frac{\partial^2\vec{\varphi}}{\partial t^2},\tag{2}$$

$$\alpha_0\nabla^2\varphi^*+\nu_1\left(T+\tau_1\dot{T}\right)+\nu_2\left(C+\tau^1\dot{C}\right)-\lambda_1\varphi^*-\lambda_0\nabla.\vec{u}=\frac{\rho j_0}{2}\frac{\partial^2\varphi^*}{\partial t^2}\tag{3}$$

$$K^*\nabla^2T=\rho C^*\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+\beta_1T_0\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\nabla.\dot{\vec{u}}+\nu_1T_0\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\dot{\varphi}^*+aT_0\left(\dot{C}+\gamma_1\ddot{C}\right),\tag{4}$$

$$D\beta_2\varepsilon_{kk,ii}+D\nu_2\varphi^*_{\ ,ii}+Da\left(T+\tau_1\dot{T}\right)_{,ii}+\left(\dot{C}+\varepsilon\tau^0\ddot{C}\right)-Db\left(C+\tau^1\dot{C}\right)_{,ii}=0,\tag{5}$$

and constitutive relations are

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\varepsilon_{ijr}\varphi_r\right)-\beta_1(1+\tau_1\frac{\partial}{\partial t})T\delta_{ij}-\beta_2(1+\tau^1\frac{\partial}{\partial t})C\delta_{ij}+\lambda_o\delta_{ij}\varphi^*,\tag{6}$$

$$m_{ij} = \alpha\varphi_{r,r}\delta_{ij} + \beta\varphi_{i,j} + \gamma\varphi_{j,i} + b_0\varepsilon_{mji}\varphi^*_{,m}, \tag{7}$$

$$\lambda^*_i = \alpha_0\varphi^*_{,i} + b_0\varepsilon_{ijm}\varphi_{j,m}, \tag{8}$$

where

$\lambda, \mu, \alpha, \beta, \gamma, K, \lambda_o, \lambda_1, \alpha_o, b_o,$ are material constants, ρ is the mass density , $\vec{u} = (u_1, u_2, u_3)$ is the displacement vector and $\vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$ is the microrotation vector, φ^* is the microstretch scalar function, T and T_0 are the small temperature increment and the reference temperature of the body chosen such that $|T/T_0| \ll 1,$ C is the concentration of the diffusion material in the elastic body. K^* is the coefficient of the thermal conductivity, C^* the specific heat at constant strain, D is the thermoelastic diffusion constant. a, b are, respectively, coefficients describing the measure of thermodiffusion and of mass diffusion effects, $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t1}$, $\beta_2 = (3\lambda + 2\mu + K)\alpha_{c1}$, $\nu_1 = (3\lambda + 2\mu + K)\alpha_{t2}$, $\nu_2 = (3\lambda + 2\mu + K)\alpha_{c2}$, α_{t1}, α_{t2} are coefficients of linear thermal expansion and α_{c1}, α_{c2} are the coefficients of linear diffusion expansion. j is the microintertia, j_o is the microinertia of the microelements, σ_{ij} and m_{ij} are components of stress and couple stress tensors respectively, λ^*_i is the microstress tensor, $e_{ij}\left(=\frac{1}{2}\left(u_{i,j} + u_{j,i}\right)\right)$ are components of infinitesimal strain, e_{kk} is the dilatation, δ_{ij} is the Kronecker delta, τ^0, τ^1 are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$ and τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. Here $\tau_0 = \tau^0 = \tau_1 = \tau^1 = \gamma_1 = 0$ for Coupled Thermoelasitc (CT) model, $\tau_1 = \tau^1 = 0,$ $\varepsilon = 1, \gamma_1 = \tau_0$ for Lord-Shulman (L-S) model and $\varepsilon = 0,$ $\gamma_1 = \tau^0$ where $\tau^0 > 0$ for Green-Lindsay (G-L) model.

In the above equations, a comma followed by a suffix denotes spatial derivative and a superposed dot denotes the derivative with respect to time respectively.

The basic equations in a homogeneous isotropic elastic solid are written as

$$\left(\lambda^e + \mu^e\right)\nabla.\nabla\vec{u}^e + \mu^e\nabla^2\vec{u}^e = \rho^e\frac{\partial^2\vec{u}^e}{\partial t^2} \tag{9}$$

where λ^e, μ^e are Lame's constants, u^e_i are the components of the displacement vector $\vec{u}^e$, ρ^e is density corresponding to the isotropic elastic solid.

The stress- strain relation in isotropic elastic medium are given by

$$t_{ij}{}^e = 2\mu^e\, e_{ij}{}^e + \lambda^e\, e_{kk}{}^e\, \delta_{ij}, \tag{10}$$

where $e_{ij}^{e}=\left(\frac{1}{2}\left(u_{i,j}^{e}+u_{j,i}^{e}\right)\right)$ are components of the strain tensor, e_{kk}^{e} is the dilatation.

3 FORMULATION OF THE PROBLEM

We consider an isotropic elastic solid half-space (M_1) lying over a homogeneous isotropic, microstretch generalized thermoelastic diffusion solid half-space (M_2). The origin of the cartesian coordinate system (x_1,x_2,x_3) is taken at any point on the plane surface (interface) and x_3-axis point vertically downwards into the microstretch thermoelastic diffusion solid half-space. The elastic solid half-space (M_1) occupies the region $x_3<0$ and the region $x_3>0$ is occupied by the microstretch themoelastic diffusion solid half-space (M_2) as shown in Fig.1. We consider plane waves in the x_1-x_3 plane with wave front parallel to the x_2-axis. For two-dimensional problem, we have

$$\vec{u}=(u_1,0,u_3)\,,\ \vec{\varphi}=(0,\varphi_2,0)\,,\ \vec{u}^{e}=(u_1^{e},0,u_3^{e}) \tag{11}$$

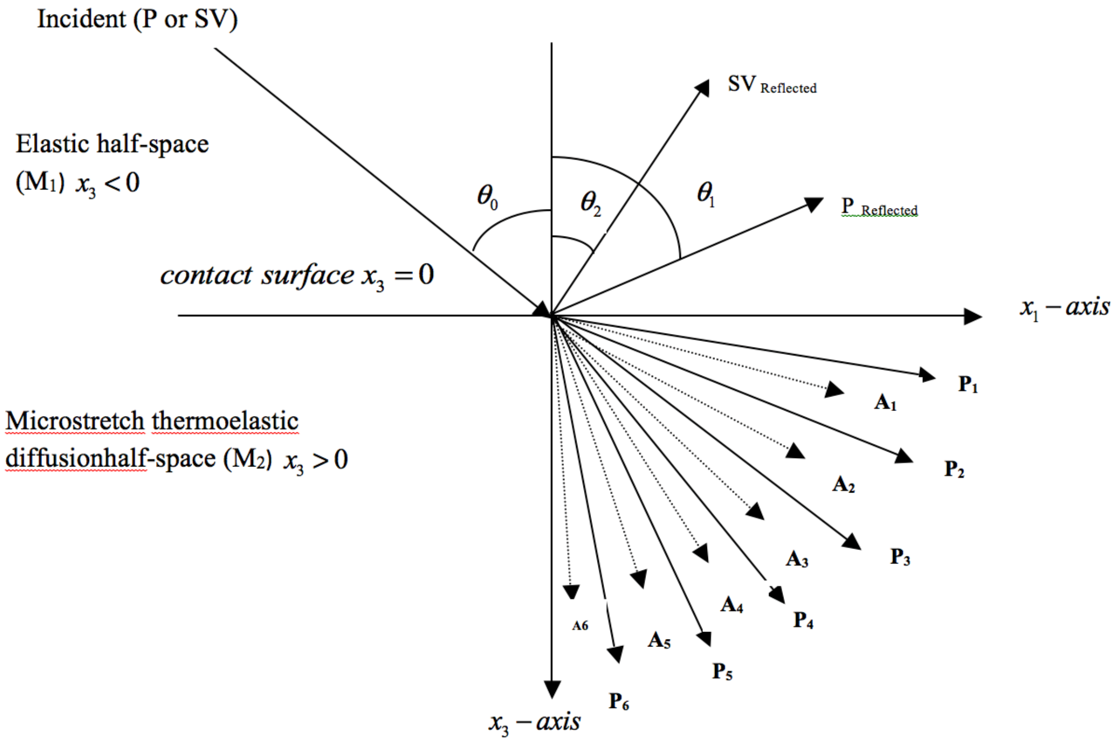

Figure 1 Geometry of the Problem

We define the following dimensionless quantities

$$\begin{gathered}\left(x_1^{'},x_3^{'}\right)=\frac{\omega^*}{c_1}\left(x_1,x_3\right),\left(u_1^{'},u_3^{'}\right)=\frac{\rho c_1\omega^*}{\beta_1 T_o}\left(u_1,u_3\right),t_{ij}^{'}=\frac{t_{ij}}{\beta_1 T_o},\\ t_{ij}^{e^{'}}=\frac{t_{ij}^{e}}{\beta_1 T_o},T^{'}=\frac{T}{T_o},t^{'}=\omega^* t,\tau_o^{'}=\omega^*\tau_o,\tau^{0'}=\omega^*\tau^0\\ \left(u_1^{e^{'}},u_3^{e^{'}}\right)=\frac{\rho c_1\omega^*}{\beta_1 T_o}\left(u_1^{e},u_3^{e}\right),\tau_1^{'}=\omega^*\tau_1,\tau^{1'}=\omega^*\tau^1,\\ \varphi^{*^{'}}=\frac{\rho c_1^2}{\beta_1 T_o}\varphi^*,\lambda_i^{*^{'}}=\frac{\lambda_i^*\omega^*}{c_1\beta_1 T_o},\varphi_2^{'}=\frac{\rho c_1^2}{\beta_1 T_o}\varphi_2,C^{'}=\frac{\beta_2 C}{\rho^2 c_1^2}\\ m_{ij}^{'}=\frac{\omega^*}{c_1\beta_1 T_o}m_{ij},\end{gathered} \tag{12}$$

where

$\omega^*=\dfrac{\rho C^* c_1^2}{K^*}$, $c_1^2=\dfrac{\lambda+2\mu+K}{\rho}$, ω^* is the characteristic frequency of the medium,

Upon introducing the quantities (12) in equations (1)-(5), with the aid of (11) and after suppressing the primes, we obtain

$$\delta^2\frac{\partial e}{\partial x_1}+\left(1-\delta^2\right)\nabla^2 u_1-\zeta_1^*\frac{\partial\varphi_2}{\partial x_3}-\tau_t^1\frac{\partial T}{\partial x_1}-\zeta_2^*\tau_c^1\frac{\partial C}{\partial x_1}+\zeta_3^*\frac{\partial\varphi^*}{\partial x_1}=\frac{\partial^2 u_1}{\partial t^2} \tag{13}$$

$$\delta^2\frac{\partial e}{\partial x_3}+\left(1-\delta^2\right)\nabla^2 u_3+\zeta_1^*\frac{\partial\varphi_2}{\partial x_1}-\tau_t^1\frac{\partial T}{\partial x_3}-\zeta_2^*\tau_c^1\frac{\partial C}{\partial x_3}+\zeta_3^*\frac{\partial\varphi^*}{\partial x_3}=\frac{\partial^2 u_3}{\partial t^2} \tag{14}$$

$$\zeta_1\nabla^2\varphi_2+\zeta_2\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-\zeta_3\varphi_2=\frac{\partial^2\varphi_2}{\partial t^2} \tag{15}$$

$$\nabla^2 T=\tau_t^0\frac{\partial T}{\partial t}+\ell_1^*\tau_e^0\frac{\partial e}{\partial t}+\ell_2^*\tau_e^0\frac{\partial\varphi^*}{\partial t}+\ell_3^*\tau_c^0\frac{\partial C}{\partial t} \tag{16}$$

$$q_1^*\nabla^2 e+q_4^*\nabla^2\varphi^*+q_2^*\tau_t^1\nabla^2 T+\tau_f^0\frac{\partial C}{\partial t}-q_3^*\tau_c^1\nabla^2 C=0 \tag{17}$$

$$\left(\delta_1^2\nabla^2-\chi_1^*\right)\varphi^*-\chi_2^* e+\chi_3^*\tau_t^1 T+\chi_4^*\tau_c^1 C=\frac{\partial^2\varphi^*}{\partial t^2} \tag{18}$$

where

$$\zeta_1 = \frac{\gamma}{j\rho c_1^2}, \zeta_2 = \frac{K}{j\rho\omega^{*2}}, \zeta_3 = \frac{2K}{j\rho\omega^{*2}}, \zeta_1^* = \frac{K}{\rho c_1^2}, \zeta_2^* = \frac{\rho c_1^2}{\beta_1 T_0}, \zeta_3^* = \frac{\lambda_0}{\rho c_1^2}, \delta^2 = \frac{\lambda+\mu}{\rho c_1^2}$$

$$\ell_1^* = \frac{T_0\beta_1^2}{\rho K^*\omega^*}, \ell_2^* = \frac{\beta_1 T_0 \nu_1}{\rho K^*\omega^*}, \ell_3^* = \frac{\rho c_1^4 a}{\beta_2 K^*\omega^*},$$

$$q_1^* = \frac{D\omega^*\beta_1^2}{\rho c_1^4}, q_2^* = \frac{D\omega^*\beta_2 a}{\beta_1 c_1^2}, q_3^* = \frac{Db\omega^*}{c_1^2}, q_4^* = \frac{D\nu_2\beta_2\omega^*}{\rho c_1^4}$$

$$\chi_1^* = \frac{2\lambda}{\rho j_0\omega^{*2}}, \chi_2^* = \frac{2\lambda_0}{\rho j_0\omega^{*2}}, \chi_3^* = \frac{2\nu_1 c_1^2}{j_0\beta_1\omega^{*2}}, \chi_4^* = \frac{2\nu_2\rho c_1^4}{j_0\beta_1\beta_2 T_0\omega^{*2}}, \delta_1^2 = \frac{c_2^2}{c_1^2}, c_2^2 = \frac{2\alpha_0}{\rho j_0},$$

$$\tau_t^1 = 1+\tau_1\frac{\partial}{\partial t}, \tau_c^1 = 1+\tau^1\frac{\partial}{\partial t}, \tau_f^0 = 1+\varepsilon\tau^0\frac{\partial}{\partial t}, \tau_t^0 = 1+\tau_0\frac{\partial}{\partial t},$$

$$\tau_e^0 = 1+\varepsilon\tau_0\frac{\partial}{\partial t}, \tau_c^0 = 1+\gamma_1\frac{\partial}{\partial t}, e = \frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}, \nabla^2 = \frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}$$

We introduce the potential functions $\phi \; and \; \psi$ through the relations

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}, \tag{19}$$

in the equations (13)-(18), we obtain

$$\nabla^2\phi - \tau_t^1 T - \zeta_2^*\tau_c^1 C + \zeta_3^*\varphi^* = \ddot{\phi}, \tag{20}$$

$$\left(1-\delta^2\right)\nabla^2\psi + \zeta_1^*\varphi_2 = \ddot{\psi}, \tag{21}$$

$$\left(\zeta_1\nabla^2 - \zeta_3\right)\varphi_2 - \zeta_2\nabla^2\psi = \ddot{\varphi}_2, \tag{22}$$

$$\nabla^2 T = \tau_t^0\dot{T} + \tau_e^0\left(\ell_1^*\nabla^2\dot{\phi} + \ell_2^*\dot{\varphi}^*\right) + \ell_3^*\tau_c^0\dot{C}, \tag{23}$$

$$q_1^*\nabla^4\phi + q_4^*\nabla^2\varphi^* + q_2^*\tau_t^1\nabla^2 T - q_3^*\tau_c^1\nabla^2 C + \tau_f^0\dot{C} = 0 \tag{24}$$

$$\left(\delta_1^2\nabla^2 - \chi_1^*\right)\varphi^* - \chi_2^*\nabla^2\phi + \chi_3^*\tau_t^1 T + \chi_4^*\tau_c^1 C = \ddot{\varphi}^*, \tag{25}$$

For the propagation of harmonic waves in $x_1 - x_3$ plane, we assume

$$\left\{\phi,\psi,T,C,\varphi^*,\varphi_2\right\}(x_1,x_3,t)=\left\{\bar{\phi},\bar{\psi},\bar{T},\bar{C},\bar{\varphi}^*,\bar{\varphi}_2\right\}e^{-i\omega t} \tag{26}$$

where ω is the angular frequency

Substituting the values of $\phi,\psi,T,C,\varphi^*,\varphi_2$ from equation (26) in the equations (20)-(25), we obtain

$$\left(\nabla^2+\omega^2\right)\bar{\varphi}-\tau_t^{"}\bar{T}+\zeta_3^*\bar{\varphi}^*-\zeta_2^*\tau_c^{"}\bar{C}=0, \tag{27}$$

$$\left(\left(1-\delta^2\right)\nabla^2+\omega^2\right)\bar{\psi}+\zeta_1^*\bar{\varphi}_2=0, \tag{28}$$

$$\zeta_2\nabla^2\bar{\psi}+\left(-\omega^2-\zeta_1\nabla^2+\zeta_3\right)\bar{\varphi}_2=0, \tag{29}$$

$$\ell_1^*\tau_e^{10}\nabla^2\bar{\varphi}+\left(\tau_t^{10}-\nabla^2\right)\bar{T}+\ell_2^*\tau_e^{10}\bar{\varphi}^*+\ell_3^*\tau_c^{10}\bar{C}=0, \tag{30}$$

$$q_1^*\nabla^4\bar{\varphi}+q_4^*\nabla^2\varphi^*+q_2^*\tau_t^{"}\nabla^2\bar{T}+\left(\tau_f^{10}-q_3^*\tau_c^{"}\nabla^2\right)\bar{C}=0; \tag{31}$$

$$-\chi_2^*\nabla^2\bar{\varphi}+\chi_3^*\tau_t^{"}\bar{T}+r_1\bar{\varphi}^*+r_2\bar{C}=0, \tag{32}$$

where

$$r_1=\delta_1^2\nabla^2-\chi_1^*+\omega^2, r_2=\chi_4^*(1-i\omega), \tau_t^{11}=(1-i\omega\tau_1), \tau_c^{11}=(1-i\omega\tau^1),$$
$$\tau_e^{10}=-i\omega(1-i\omega\varepsilon\tau_0), \tau_t^{10}=-i\omega(1-i\omega\tau_0), \tau_c^{10}=-i\omega(1-i\omega\gamma), \tau_f^{10}=-i\omega(1-i\omega\varepsilon\tau^0)$$

The system of equations (27), (30)-(32) has a non-trivial solution if the determinant of the coefficients $\left[\bar{\varphi},\bar{T},\bar{\varphi}^*,\bar{C}\right]^T$ vanishes, which yields to the following polynomial characteristic equation

$$\nabla^8+B_1\nabla^6+B_2\nabla^4+B_3\nabla^2+B_4=0 \tag{33}$$

where

$$B_i=A_i/A \;\; for\, (i=1,2,3,4), A=g_1^*-a_{14}g_{14}^*, A_1=g_2^*+g_1^*\omega^2-a_{12}g_6^*+a_{13}g_9^*-a_{14}g_{12}^*,$$
$$A_2=g_3^*+g_2^*\omega^2-a_{12}g_7^*+a_{13}g_{10}^*-a_{14}g_{13}^*, A_3=g_4^*+g_3^*\omega^2-a_{12}g_8^*+a_{13}g_{11}^*, A_4=g_4^*\omega^2,$$

and

$$g_1^* = -\delta_1^2 a_{46},$$
$$g_2^* = a_{23}a_{46} - a_{24}a_{43} + \delta_1^2(a_{32}a_{46} + a_{45} + a_{34}a_{42}), g_4^* = a_{45}(a_{22}a_{33} + a_{23}a_{32}),$$
$$g_3^* = -a_{33}(a_{22}a_{46} + a_{24}a_{42}) - a_{23}(a_{32}a_{46} + a_{45} + a_{34}a_{42}) + a_{43}(a_{24}a_{32} - a_{22}a_{34}) - \delta_1^2 a_{32}a_{45},$$
$$g_6^* = \delta_1^2(a_{31}a_{46} + a_{41}a_{34}),$$
$$g_7^* = a_{33}(-a_{21}a_{46} - a_{24}a_{41}) - a_{23}(a_{31}a_{46} + a_{34}a_{41}) + (a_{24}a_{31} - a_{21}a_{34})a_{43} - \delta_1^2 a_{31}a_{45},$$
$$g_8^* = a_{45}(a_{23}a_{31} + a_{21}a_{33}), g_9^* = a_{24}a_{41} + a_{21}a_{46},$$
$$g_{10}^* = -a_{21}(a_{32}a_{46} + a_{45} + a_{34}a_{42}) + a_{22}(a_{31}a_{46} + a_{34}a_{41}) + a_{24}(a_{31}a_{42} - a_{32}a_{41})$$
$$g_{11}^* = a_{45}(a_{21}a_{32} - a_{22}a_{31}), g_{12}^* = -(a_{23}a_{41} + a_{21}a_{43}) + \delta_1^2(a_{31}a_{42} - a_{41}a_{32}), a_{45} = \tau_f^{10}, a_{46} = q_3^*\tau_c^{11}$$
$$g_{13}^* = a_{33}(a_{22}a_{41} - a_{21}a_{42}) + a_{23}(a_{32}a_{41} - a_{31}a_{42}) + a_{43}(a_{21}a_{32} - a_{22}a_{31}), g_{14}^* = \delta_1^2 a_{41}, a_{11} = \nabla^2 + \omega^2, a_{21} = -\chi_2^*,$$
$$a_{31} = \ell_1^*\tau_e^{10}, a_{41} = q_1^*, a_{12} = -\tau_t^{11}, a_{22} = \chi_3^*\tau_t^{11}, a_{32} = \tau_t^{10}, a_{42} = q_2^*\tau_t^{11}, a_{13} = \zeta_3^*, a_{23} = \chi_1^* - \omega^2, a_{43} = q_4^*\nabla^2,$$
$$a_{33} = \ell_2^*\tau_e^{10}, a_{14} = -\zeta_2^*\tau_c^{11}, a_{24} = r_2, a_{34} = \ell_3^*\tau_c^{10}, a_{44} = (a_{45} - a_{46}\nabla^2),$$

The general solution of equation (33) can be written as

$$\bar{\varphi} = \sum_{i=1}^{4} \bar{\varphi}_i \tag{34}$$

where the potentials $\bar{\varphi}_i,\ i = 1,2,3,4$ are solutions of wave equations, given by

$$\left[\nabla^2 + \frac{\omega^2}{V_i^2}\right]\bar{\varphi}_i = 0,\ i = 1,2,3,4 \tag{35}$$

Here $\left(V_i^2, i = 1,2,3,4\right)$ are the velocities of four longitudinal waves, that is, longitudinal displacement wave (LD), mass diffusion wave (MD), thermal wave (T) and longitudinal microstretch wave (LM) and derived from the roots of the biquadratic equation in V^2, given by

$$\left(B_4V^8 - B_3\omega^2V^6 + B_2\omega^4V^4 - B_1\omega^6V^2 + \omega^8\right) = 0 \tag{36}$$

Making use of equation (34) in the equations (27), (30)-(32) with the aid of equations (26) and (35), the general solutions for φ, T, φ^* *and* C are obtained as

$$(\varphi, T, \varphi^*, C) = \sum_{1}^{4} (1, k_{1i}, k_{2i}, k_{3i}) \bar{\varphi}_i \tag{37}$$

where

$$k_{1i} = (g_6^* \omega^6 - g_7^* \omega^4 V_i^2 + g_8^* \omega^2 V_i^4) / k^d, k_{2i} = -(g_9^* \omega^6 + g_{10}^* \omega^4 V_i^2 + g_{11}^* \omega^2 V_i^4) / k^d,$$
$$k_{3i} = (-g_{14}^* \omega^8 + g_{12}^* \omega^6 V_i^2 - g_{13}^* \omega^4 V_i^4) / (V_i^2 k^d), k^d = (g_1^* \omega^6 + g_2^* \omega^4 V_i^2 + g_3^* \omega^2 V_i^4 + g_4^* V_i^6),\ i = 1,2,3,4$$

The system of equations (28)-(29) has a non-trivial solution if the determinant of the coefficients $\left[\bar{\psi}, \bar{\varphi}_2\right]^T$ vanishes, which yields to the following polynomial characteristic equation

$$\nabla^4 + A^* \nabla^2 + B^* = 0 \tag{38}$$

where

$$A^* = \left(\omega^2 \zeta_1 + \zeta_1^* \zeta_2 - \left(1 - \delta^2\right)\left(\zeta_3 + \omega^2\right)\right) \Big/ \left(1 - \delta^2\right) \zeta_1, B^* = \omega^2 \left(\omega^2 - \zeta_3\right) \Big/ \left(1 - \delta^2\right) \zeta_1$$

The general solution of equation (38) can be written as

$$\bar{\psi} = \sum_{i=5}^{6} \bar{\psi}_i \tag{39}$$

where the potentials $\bar{\psi}_i,\ i = 1,2$ are solutions of wave equations, given by

$$\left[\nabla^2 + \frac{\omega^2}{V_i^2}\right] \bar{\psi}_i = 0,\ i = 5,6 \tag{40}$$

Here $\left(V_i^2, i = 5,6\right)$ are the velocities of two coupled transverse displacement and microrotational (CD I, CD II) waves and derived from the root of quadratic equation in V^2, given by

$$\left(B^* V^4 - A^* \omega^2 V^2 + \omega^4\right) = 0 \tag{41}$$

Making use of equation (39) in the equations (28)-(29) with the aid of equations (26) and (40), the general solutions for ψ *and* φ_2 are obtained as

$$\left\{\psi, \varphi_2\right\} = \sum_{i=5}^{6} \left\{1, n_{1i}\right\} \bar{\psi}_i \tag{42}$$

where

$$n_{1i} = \frac{\zeta_2 \omega^2}{\left(\zeta_3 - \omega^2\right) V_i^2 + \zeta_1 \omega^2} \quad for\ i = 5,6$$

Applying the dimensionless quantities (12) in the equation (9) with the aid of (11) and after suppressing the primes, we obtain

$$\left(\frac{\alpha^{e^2} - \beta^{e^2}}{c_1^2}\right)\left(\frac{\partial e^e}{\partial x_1}\right) + \frac{\beta^{e^2}}{c_1^2} \nabla^2 u_1^e = \ddot{u}_1^e \tag{43}$$

$$\left(\frac{\alpha^{e^2} - \beta^{e^2}}{c_1^2}\right)\left(\frac{\partial e^e}{\partial x_3}\right) + \frac{\beta^{e^2}}{c_1^2} \nabla^2 u_3^e = \ddot{u}_3^e \tag{44}$$

where

$$e^e = \left(\frac{\partial u_1^e}{\partial x_1} + \frac{\partial u_3^e}{\partial x_3}\right)$$

and

$\alpha^e = \sqrt{\left(\lambda^e + 2\mu^e\right)/\rho^e}$, $\beta^e = \sqrt{\mu^e/\rho^e}$ are velocities of longitudinal wave (P-wave) and transverse wave (SV-wave) corresponding to M_1, respectively.

The components of u_1^e and u_3^e are related by the potential functions as:

$$u_1^e = \frac{\partial \varphi^e}{\partial x_1} - \frac{\partial \psi^e}{\partial x_3}\ ,\ u_3^e = \frac{\partial \varphi^e}{\partial x_3} + \frac{\partial \psi^e}{\partial x_1}\ , \tag{45}$$

where φ^e and ψ^e satisfy the wave equations as

$$\nabla^2 \varphi^e = \frac{\ddot{\varphi}^e}{\alpha^2}\ ,\ \nabla^2 \psi^e = \frac{\ddot{\psi}^e}{\beta^2}\ , \tag{46}$$

and $\alpha = \alpha^e / c_1$, $\beta = \beta^e / c_1$.

4 REFLECTION AND REFRACTION

We consider a plane harmonic wave (P or SV) propagating through the isotropic elastic solid half-space and is incident at the interface $x_3 = 0$ as shown in Fig.1. Corresponding to each incident wave, two homogeneous waves (P and SV) are reflected in an isotropic elastic solid and six inhomogeneous waves (LD, MD, T, LM, CD I and CD II) are transmitted in isotropic microstretch thermoelastic diffusion solid half-space.

In elastic solid half-space, the potential functions satisfying equation (46) can be written as

$$\phi^e = A_0^e e^{\left\{i\omega\left(x_1 \sin\theta_0 + x_3 \cos\theta_0\right)/\alpha - t\right\}} + A_1^e e^{\left\{i\omega\left(x_1 \sin\theta_1 + x_3 \cos\theta_1\right)/\alpha - t\right\}} \tag{47}$$

$$\psi^e = B_0^e e^{\left\{i\omega\left(x_1 \sin\theta_0 + x_3 \cos\theta_0\right)/\beta - t\right\}} + B_1^e e^{\left\{i\omega\left(x_1 \sin\theta_2 + x_3 \cos\theta_2\right)/\beta - t\right\}} \tag{48}$$

The coefficients A_0^e (B_0^e), A_1^e and B_1^e are amplitudes of the incident P (or SV), reflected P and reflected SV waves respectively.

Following Borcherdt [3], in a homogeneous isotropic microstretch thermoelastic diffusion half-space, potential functions satisfying equations (35) and (40) can be written as

$$\left(\varphi, T, \varphi^*, C\right) = \sum_{i=1}^{4} \left\{1, k_{1i}, k_{2i}, k_{3i}\right\} B_i e^{(\vec{A}_i . \vec{r})} e^{\left\{i(\vec{P}_i . \vec{r} - \omega t)\right\}} \tag{49}$$

$$\left(\psi, \phi_2\right) = \sum_{i=5}^{6} \{1, n_{ip}\} B_i\, e^{(\vec{A}_i . \vec{r})} e^{\left\{i(\vec{P}_i . \vec{r} - \omega t)\right\}} \tag{50}$$

The coefficients B_i, $i = 1,2,3,4,5,6$ are the amplitudes of refracted waves. The propagation vector $\vec{P}_i$, $i = 1,2,3,4,5,6$ and attenuation $\vec{A}_i$ factor ($i = 1,2,3,4,5,6$) are given by

$$\vec{P}_i = \xi_R \hat{x}_1 + dV_{iR} \hat{x}_3, \quad \vec{A}_i = -\xi_I \hat{x}_1 - dV_{iI} \hat{x}_3, \quad i = 1,2,3,4,5,6 \tag{51}$$

where

$$dV_i = dV_{iR} + i\, dV_{iI} = \text{p.v.} \left(\frac{\omega^2}{V_i^2} - \xi^2 \right)^{1/2}, \quad i = 1,2,3,4,5,6 \tag{52}$$

and

$\xi = \xi_R + i\xi_I$ is the complex wave number. The subscripts R and I denote the real and imaginary parts of the corresponding complex number and p.v. stands for the principal value of the complex quantity derived from square root. $\xi_R \geq 0$ ensures propagation in positive x_1-direction. The complex wave number ξ in the microstretch thermoelastic diffusion medium is given by

$$\xi = \left|\vec{P}_i\right| \sin\theta_i' - i\left|\vec{A}_i\right| \sin(\theta_i' - \gamma_i) \quad i = 1,2,3,4,5,6 \tag{53}$$

where γ_i , $i=1,2,3,4,5,6$ is the angle between the propagation and attenuation vector and $\theta_i^{'}$, $i=1,2,3,4,5,6$ is the angle of refraction in medium II.

5 BOUNDARY CONDITIONS

The boundary conditions are the continuity of stress and displacement components, vanishing of the gradient of temperature, mass concentration, the tangential couple stress and microstress components. Mathematically these can be written as

Continuity of the normal stress component

$$t_{33}^{e}=t_{33} , \tag{54}$$

Continuity of the tangential stress component

$$t_{31}^{e}=t_{31} , \tag{56}$$

Continuity of the tangential displacement component

$$u_1^{e}=u_1 , \tag{57}$$

Continuity of the normal displacement component

$$u_3^{e}=u_3 , \tag{58}$$

Vanishing the gradient of temperature

$$\frac{\partial T}{\partial x_3}=0 , \tag{59}$$

Vanishing the mass concentration

$$\frac{\partial C}{\partial x_3}=0 , \tag{60}$$

Vanishing of the tangential couple stress component

$$m_{32}=0 \tag{61}$$

Vanishing of the microstress component

$$\lambda_3^* = 0 \tag{62}$$

Making the use of potentials given by equations (47)-(50), we find that the boundary conditions are satisfied if and only if

$$\xi_R = \frac{\omega \sin\theta_0}{V_0} = \frac{\omega \sin\theta_1}{\alpha} = \frac{\omega \sin\theta_2}{\beta} \tag{63}$$

and

$$\xi_I = 0. \tag{64}$$

where

$$V_0 = \begin{cases} \alpha, \ for \ \ incident\, P - wawe \\ \beta, \ for \ \ incident\, SV - wawe \end{cases} \tag{65}$$

It means that waves are attenuating only in x_3-direction. From equation (53), it implies that if $\left|\vec{A}_i\right| \neq 0$, then $\gamma_i^{'} = \theta_i'$, $i = 1,2,3,4,5,6$, that is, attenuated vectors for the six refracted waves are directed along the x_3-axis.

Using equations (47)-(50) in the boundary conditions (54)-(62) and with the aid of equations (19), (45), (63)-(65), we get a system of eight non-homogeneous equations which can be written as

$$\sum_{j=1}^{8} d_{ij} Z_j = g_i \tag{66}$$

where $Z_j = \left|Z_j\right| e^{i\psi_j^*}, \left|Z_j\right|, \psi_j^*, j = 1,2,3,4,5,6,7,8$ represents amplitude ratios and phase shift of reflected P-, reflected SV-, refracted LD-, refracted MD-, refracted T-, refracted LM-, refracted CD I -, refracted CD II - waves to that of amplitude of incident wave, respectively.

$$d_{11} = \left[2\mu^e \left(\frac{\xi_R}{\omega}\right)^2 - \rho^e c_1^2 \right], d_{12} = \left[2\mu^e \left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_\beta}{\omega}\right) \right], d_{17} = \left[(2\mu + K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_5}{\omega}\right) \right],$$

$$d_{18} = \left[(2\mu + K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_6}{\omega}\right) \right], d_{21} = 2\mu^e \left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_\alpha}{\omega}\right), \ d_{22} = \mu^e \left[\left(\frac{dV_\beta}{\omega}\right)^2 - \left(\frac{\xi_R}{\omega}\right)^2 \right],$$

$$d_{27}=\left[\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_5}{\omega}\right)^2-\frac{Kn_{5p}}{\omega^2}\right],\ d_{28}=\left[\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_6}{\omega}\right)^2-\frac{Kn_{6p}}{\omega^2}\right]$$

$$d_{31}=\left(\frac{\xi_R}{\omega}\right),\ d_{32}=-\left(\frac{dV_\beta}{\omega}\right), d_{37}=\left(\frac{dV_5}{\omega}\right),\ d_{38}=\left(\frac{dV_6}{\omega}\right)\ d_{41}=-\left(\frac{dV_\alpha}{\omega}\right),\ d_{42}=\left(\frac{\xi_R}{\omega}\right),$$

$$d_{47}=-\left(\frac{\xi_R}{\omega}\right), d_{48}=-\left(\frac{\xi_R}{\omega}\right),\ d_{5j}=0\quad for\ (j=1,2,7,8), d_{6j}=0\quad for\ (j=1,2,7,8)$$

$$d_{7j}=0\ \ for\ (j=1,2),\ d_{7j}=b_o k_{2(j-2)}\left(\frac{\xi_R}{\omega}\right)\ for\ (j=3,4,5,6),$$

$$d_{7j}=\gamma\left(\frac{dV_{j-2}}{\omega}\right)n_{(j-2)p}\ \ for\ (j=7,8)$$

$$d_{8j}=0\ \ for\ (j=1,2)\quad d_{8j}=-b_o n_{(j-2)p}\left(\frac{\xi_R}{\omega}\right)\ for(j=7,8),$$

$$d_{8j}=\alpha_O k_{2(j-2)}\left(\frac{dV_{j-2}}{\omega}\right)\ for\ (j=3,4,5,6)$$

$$d_{1j}=\left[\lambda\left(\frac{\xi_R}{\omega}\right)^2+\rho c_1^2\left(\frac{dV_j}{\omega}\right)^2+\rho c_1^2\left(\frac{k_{1j}\tau_t^{"}+k_{3j}\tau_c^{"}}{\omega^2}\right)-\frac{\lambda_0 k_{2j}}{\omega^2}\right]\quad for\ (j=3,4,5,6),$$

$$d_{2j}=(2\mu+K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_j}{\omega}\right)\quad for\ (j=3,4,5,6), d_{3j}=-\left(\frac{\xi_R}{\omega}\right)\ for(j=3,4,5,6),$$

$$d_{4j}=-\left(\frac{dV_{j-2}}{\omega}\right)\quad for\ (j=3,4,5,6), d_{5j}=k_{1(j-2)}\left(\frac{dV_{j-2}}{\omega}\right)\ for\ (j=3,4,5,6),$$

$$d_{6j}=k_{3(j-2)}\left(\frac{dV_{j-2}}{\omega}\right)\ for\ (j=3,4,5,6)$$

$$\left(\frac{dV_\alpha}{\omega}\right)=\left[\frac{1}{\alpha^2}-\left(\frac{\xi_R}{\omega}\right)^2\right]^{1/2}=\left[\frac{1}{\alpha^2}-\left(\frac{\sin\theta_0}{V_0}\right)^2\right]^{1/2},$$

$$\left(\frac{dV_\beta}{\omega}\right)=\left[\frac{1}{\alpha^2}-\left(\frac{\xi_R}{\omega}\right)^2\right]^{1/2}=\left[\frac{1}{\beta^2}-\left(\frac{\sin\theta_0}{V_0}\right)^2\right]^{1/2}\ \left(\frac{dV_j}{\omega}\right)=p.v.\left[\frac{1}{V_j^2}-\left(\frac{\sin\theta_0}{V_0}\right)^2\right]^{1/2},$$

$$for\ (j=1,2,3,4,5,6)$$

Here p.v. is evaluated with restriction $dV_{jI} \geq 0$ to satisfy decay condition in the microstretch thermoelastic diffusion medium. The coefficients g_i, *for* $(i = 1,2,3,4,5,6,7,8)$ on the right side of the equation (66) are given by

(i) For incident P-wave

$$g_i = (-1)^i d_{i1}, \ \ for\ (i = 1,2,3,4) \qquad g_i = 0\,,\ for\ (i = 5,6,7,8), \tag{67}$$

(ii)For incident SV-wave

$$g_i = (-1)^{i+1} d_{i2}, \ \ for\ (i = 1,2,3,4) \qquad g_i = 0\,,\ for\ (i = 5,6,7,8), \tag{68}$$

Now we consider a surface element of unit area at the interface between two media. The reason for this consideration is to calculate the partition of energy of the incident wave among the reflected and refracted waves on the both sides of surface. Following Achenbach [1], the energy flux across the surface element, that is, rate at which the energy is communicated per unit area of the surface is represented as

$$P^* = t_{lm}\, l_m\, \dot{u}_l \tag{69}$$

Where t_{lm} is the stress tensor, l_m are the direction cosines of the unit normal $\hat{l}$ outward to the surface element and $\dot{u}_l$ are the components of the particle velocity. The time average of P^* over a period, denoted by $< P^* >$, represents the average energy transmission per unit surface area per unit time. Thus, on the surface with normal along x_3-direction, the average energy intensities of the waves in the elastic solid are given by

$$< P^{*e} > = \mathrm{Re}< t >^e_{31} .\mathrm{Re}\ (\dot{u}^e_1) + \mathrm{Re}< t >^e_{33} .\mathrm{Re}\ (\dot{u}^e_3) \tag{70}$$

Following Achenbach [1], for any two complex functions f and g, we have

$$< \mathrm{Re}\ (f).\mathrm{Re}\ (g) > = \tfrac{1}{2}\, \mathrm{Re}\ (f\,\overline{g}) \tag{71}$$

The expressions for energy ratios E_i, $i = 1, 2$ for the reflected P and reflected SV are given by

$$E_i = -\frac{< P_i^{*e} >}{< P_0^{*e} >} \quad for\ (i = 1,2) \tag{72}$$

where

$$< P_1^{*e} > = \frac{\omega^4 \rho^e c_1^2}{2\alpha} |Z_1|^2 \operatorname{Re}(\cos\theta_1),\ < P_2^{*e} > = \frac{\omega^4 \rho^e c_1^2}{2\beta} |Z_2|^2 \operatorname{Re}(\cos\theta_2)$$

and

(i)For incident P- wave

$$< P_0^{*e} > = -\frac{\omega^4 \rho^e c_1^2 \cos\theta_0}{2\alpha}, \tag{73}$$

(ii)For incident SV- wave

$$< P_0^{*e} > = -\frac{\omega^4 \rho^e c_1^2 \cos\theta_0}{2\beta}, \tag{74}$$

are the average energy intensities of the reflected P-, reflected SV-, incident P- and incident SV- waves respectively. In equation (72), negative sign is taken because the direction of reflected waves is opposite to that of incident wave.

For microstretch thermoelastic diffusion medium, the average energy intensities of the waves on the surface with normal along x_3-direction, are given by

$$\begin{aligned} < P_{ij}^* > = & \operatorname{Re} < t >_{31}^{(i)} .\operatorname{Re}(\dot{u}_1^{(j)}) + \operatorname{Re} < t >_{33}^{(i)} .\operatorname{Re}(\dot{u}_3^{(j)}) \\ & + \operatorname{Re} < m >_{32}^{(i)} .\operatorname{Re}(\dot{\phi}_2^{(j)}) + \operatorname{Re} < \lambda_3^* >^{(i)} .\operatorname{Re}(\dot{\varphi}^{*(j)}) \end{aligned} \tag{75}$$

The expressions for the energy ratios $E_{ij}\ for\ (i, j = 1,2,3,4,5,6)$ for the refracted waves are given by

$$E_{ij} = \frac{< P_{ij}^* >}{< P_0^{*e} >}\ \ for\ (i, j = 1,2,3,4,5,6)\ , \tag{76}$$

where

$$<P_{ij}^{*}> = -\frac{\omega^4}{2}\mathrm{Re}\left[(2\mu+K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_i}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right)+\left\{\begin{array}{l}\lambda\left(\frac{\xi_R}{\omega}\right)^2+\rho c_1^2\left(\frac{dV_i}{\omega}\right)^2+\\ \rho c_1^2\left(\frac{k_{1i}\tau_t''+k_{3i}\tau_c''}{\omega^2}\right)-\frac{\lambda_0 k_{2i}}{\omega^2}\end{array}\right\}\left(\frac{d\bar{V}_j}{\omega}\right)+\frac{\alpha_o k_{2i}\bar{k}_{2j}\omega^{*2}}{\rho c_1^4\omega^2}\left(\frac{dV_i}{\omega}\right)\right]Z_{i+2}\bar{Z}_{j+2},$$

$$for\ (i,j=1,2,3,4)$$

$$<P_{ij}^{*}> = -\frac{\omega^4}{2}\mathrm{Re}\left[\begin{array}{l}-\left(\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_i}{\omega}\right)^2+\frac{Kn_{ip}}{\omega^2}\right)\left(\frac{d\bar{V}_j}{\omega}\right)+\\ (2\mu+K)\left(\frac{dV_i}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right)+\frac{\omega^{*2}\gamma n_{ip}\bar{n}_{jp}}{\rho c_1^4\omega^2}\left(\frac{dV_i}{\omega}\right)\end{array}\right]Z_{i+2}\bar{Z}_{j+2},\ for\ (i,j=5,6)$$

$$<P_{i5}^{*}> = -\frac{\omega^4}{2}\mathrm{Re}\left[-(2\mu+K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_i}{\omega}\right)\left(\frac{d\bar{V}_5}{\omega}\right)+\left\{\begin{array}{l}\lambda\left(\frac{\xi_R}{\omega}\right)^2+\rho c_1^2\left(\frac{dV_i}{\omega}\right)^2\\ +\rho c_1^2\left(\frac{k_{1i}\tau_t''+k_{3i}\tau_c''}{\omega^2}\right)-\frac{\lambda_0 k_{2i}}{\omega^2}\end{array}\right\}\left(\frac{\bar{\xi}_R}{\omega}\right)-\frac{b_o\omega^{*2}\bar{n}_{5p}k_{2i}}{\rho c_1^4\omega^2}\left(\frac{\xi_R}{\omega}\right)\right]Z_{i+2}\bar{Z}_7,$$

$$for\ (i=1,2,3,4)$$

$$<P_{i5}^{*}> = -\frac{\omega^4}{2}\mathrm{Re}\left[\begin{array}{l}-\left(\mu\left(\frac{\xi_R}{\omega}\right)^2-(\mu+K)\left(\frac{dV_i}{\omega}\right)^2+\frac{Kn_{ip}}{\omega^2}\right)\left(\frac{d\bar{V}_5}{\omega}\right)\\ +(2\mu+K)\left(\frac{dV_i}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right)-\frac{\omega^{*2}\gamma n_{ip}\bar{n}_{5p}}{\rho c_1^4\omega^2}\left(\frac{dV_i}{\omega}\right)\end{array}\right]Z_{i+2}\bar{Z}_7,\ for\ (i=5,6)$$

$$<P_{i6}^{*}> = -\frac{\omega^4}{2}\mathrm{Re}\left[-(2\mu+K)\left(\frac{\xi_R}{\omega}\right)\left(\frac{dV_i}{\omega}\right)\left(\frac{d\bar{V}_6}{\omega}\right)+\left\{\begin{array}{l}\lambda\left(\frac{\xi_R}{\omega}\right)^2+\rho c_1^2\left(\frac{dV_i}{\omega}\right)^2\\ +\rho c_1^2\left(\frac{k_{1i}\tau_t''+k_{3i}\tau_c''}{\omega^2}\right)-\frac{\lambda_0 k_{2i}}{\omega^2}\end{array}\right\}\left(\frac{\bar{\xi}_R}{\omega}\right)-\frac{b_o\omega^{*2}\bar{n}_{6p}k_{2i}}{\rho c_1^4\omega^2}\left(\frac{\xi_R}{\omega}\right)\right]Z_{i+2}\bar{Z}_8,$$

$$for\ (i=1,2,3,4)$$

$$< P_{i6}^{*} > = -\frac{\omega^4}{2}\operatorname{Re}\left[\begin{array}{l} -\left(\mu\left(\frac{\xi_R}{\omega}\right)^2 - \left(\mu+K\right)\left(\frac{dV_i}{\omega}\right)^2 + \frac{Kn_{ip}}{\omega^2}\right)\left(\frac{d\bar{V}_6}{\omega}\right) \\ +(2\mu+K)\left(\frac{dV_i}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right) - \frac{\omega^{*2}\gamma n_{ip}\bar{n}_{6p}}{\rho c_1^4\omega^2}\left(\frac{dV_i}{\omega}\right) \end{array}\right] Z_{i+2}\bar{Z}_8, \ for\ (i=5,6)$$

$$< P_{5j}^{*} > = -\frac{\omega^4}{2}\operatorname{Re}\left[\left(\mu\left(\frac{\xi_R}{\omega}\right)^2 - \left(\mu+K\right)\left(\frac{dV_5}{\omega}\right)^2 + \frac{Kn_{5p}}{\omega^2}\right)\left(\frac{\bar{\xi}_R}{\omega}\right) + \left\{\left(2\mu+K\right)\left(\frac{dV_5}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\right\}\left(\frac{d\bar{V}_j}{\omega}\right) - \frac{\omega^{*2} b_o n_{5p}\bar{k}_{2j}}{\rho c_1^4\omega^2}\left(\frac{\xi_R}{\omega}\right)\right] Z_7\bar{Z}_{j+2},$$

$$for\ (j = 1,2,3,4)$$

$$< P_{5j}^{*} > = -\frac{\omega^4}{2}\operatorname{Re}\left[\begin{array}{l} -\left(\mu\left(\frac{\xi_R}{\omega}\right)^2 - \left(\mu+K\right)\left(\frac{dV_j}{\omega}\right)^2 + \frac{Kn_{5p}}{\omega^2}\right)\left(\frac{d\bar{V}_j}{\omega}\right) + \\ (2\mu+K)\left(\frac{dV_5}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right) + \frac{\gamma n_{5p}\bar{n}_{jp}\omega^{*2}}{\rho c_1^4\omega^2}\left(\frac{dV_5}{\omega}\right) \end{array}\right] Z_7\bar{Z}_{j+2}, \ for\ (j=5,6)$$

$$< P_{6j}^{*} > = -\frac{\omega^4}{2}\operatorname{Re}\left[\left(\mu\left(\frac{\xi_R}{\omega}\right)^2 - \left(\mu+K\right)\left(\frac{dV_6}{\omega}\right)^2 + \frac{Kn_{6p}}{\omega^2}\right)\left(\frac{\bar{\xi}_R}{\omega}\right) + \left\{\left(2\mu+K\right)\left(\frac{dV_6}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\right\}\left(\frac{d\bar{V}_j}{\omega}\right) - \frac{\omega^{*2} b_o n_{6p}\bar{k}_{2j}}{\rho c_1^4\omega^2}\left(\frac{\xi_R}{\omega}\right)\right] Z_8\bar{Z}_{j+2},$$

$$for\ (j = 1,2,3,4)$$

$$< P_{6j}^{*} > = -\frac{\omega^4}{2}\operatorname{Re}\left[\begin{array}{l} -\left(\mu\left(\frac{\xi_R}{\omega}\right)^2 - \left(\mu+K\right)\left(\frac{dV_j}{\omega}\right)^2 + Kn_{jp}\right)\left(\frac{d\bar{V}_j}{\omega}\right) + \\ (2\mu+K)\left(\frac{dV_6}{\omega}\right)\left(\frac{\xi_R}{\omega}\right)\left(\frac{\bar{\xi}_R}{\omega}\right) + \frac{\gamma n_{6p}\bar{n}_{jp}\omega^{*2}}{\rho c_1^4\omega^2}\left(\frac{dV_6}{\omega}\right) \end{array}\right] Z_8\bar{Z}_{j+2}, \ for\ (j=5,6)$$

The diagonal entries of energy matrix E_{ij} in equation (76) represent the energy ratios of the waves, whereas sum of the non-diagonal entries of E_{ij} give the share of interaction energy among all the refracted waves in the medium and is given by

$$E_{RR} = \sum_{i=1}^{6}\left(\sum_{j=1}^{6} E_{ij} - E_{ii}\right) \tag{77}$$

The energy ratios E_i, $i = 1, 2$, diagonal entries and sum of non-diagonal entries of energy matrix E_{ij} , that is, $E_{11}, E_{22}, E_{33}, E_{44}, E_{55}, E_{66}$ and E_{RR} yield the conservation of incident energy across the interface, through the relation

$$E_1 + E_2 + E_{11} + E_{22} + E_{33} + E_{44} + E_{55} + E_{66} + E_{RR} = 1 \tag{78}$$

6 NUMERICAL RESULTS AND DISCUSSION

The analysis is conducted for a magnesium crystal-like material. Following [8], the values of physical constants are

$$\lambda = 9.4\times10^{10}\mathrm{Nm}^{-2}, \mu = 4.0\times10^{10}\mathrm{Nm}^{-2},\ K = 1.0\times10^{10}\,Nm^{-2},$$
$$\rho = 1.74\times10^{3}\,Kgm^{-3}, j = 0.2\times10^{-19}\,m^{2}, \gamma = 0.779\times10^{-9}\,N$$

Thermal and diffusion parameters are given by

$$C^{*} = 1.04\times10^{3}\,JKg^{-1}K^{-1},\ \mathrm{K}^{*} = 1.7\times10^{6}\,Jm^{-1}s^{-1}K^{-1}, \alpha_{t1} = 2.33\times10^{-5}\mathrm{K}^{-1}, \alpha_{t2} = 2.48\times10^{-5}\mathrm{K}^{-1},$$
$$T_0 = .298\times10^{3}\mathrm{K}, \tau_1 = 0.01, \tau_0 = 0.02, \alpha_{c1} = 2.65\times10^{-4}\mathrm{m}^{3}\mathrm{Kg}^{-1}, \alpha_{c2} = 2.83\times10^{-4}\mathrm{m}^{3}\mathrm{Kg}^{-1},$$
$$a = 2.9\times10^{4}\mathrm{m}^{2}\mathrm{s}^{-2}\mathrm{K}^{-1}, b = 32\times10^{5}\mathrm{Kg}^{-1}\mathrm{m}^{5}\mathrm{s}^{-2}, \tau^{1} = 0.04,, \tau^{0} = 0.03, D = 0.85\times10^{-8}\,Kgm^{-3}s$$

and, the microstretch parameters are taken as

$$j_o = 0.19\times10^{-19}\,m^{2}, \alpha_o = 0.779\times10^{-9}\,N, b_o = 0.5\times10^{-9}\,N, \lambda_o = 0.5\times10^{10}\,Nm^{-2}, \lambda_1 = 0.5\times10^{10}\,Nm^{-2}$$

Following Bullen [5], the numerical data of granite for elastic medium is given by

$$\rho^{e} = 2.65\times10^{3}\,Kgm^{-3}, \alpha^{e} = 5.27\times10^{3}\,ms^{-1}, \beta^{e} = 3.17\times10^{3}\,ms^{-1}$$

The Matlab software 7.04 has been used to determine the values of energy ratios $E_i,\ i = 1,2$ and energy matrix $E_{ij},\ i,j = 1,2,3,4,5,6$ defined in the previous section for different values of incident angle (θ_o) ranging from 0^0 to 90^0 for fixed frequency $\omega = 2\times\pi\times100\,Hz$. Corresponding to incident P wave, the variation of energy ratios with respect to angle of incident have been plotted in Figures (2)-(10). Similarly, corresponding to SV waves, the variation of energy ratios with respect to angle of incident have been plotted in Figures (11)-(19). In all figures of microstretch thermoelastic diffusion medium the graphs for L-S and G-L theories are represented by the word MDLS and MDGL respectively.

Incident P-wave

Figs.2-10 depicts the variation of energy ratios with the angle of incidence (θ_0) for P waves.

Fig.2 exhibits the variation of energy ratio E_1 with the angle of incidence (θ_o). It shows that the values of E_1 for both cases MDLS and MDGL decrease with the increase in θ_o from 0^0 to 50^0 and then increase as θ_o increase further. **Fig.3** depicts the variation of energy ratio E_2 with θ_o and it shows nearly opposite behavior to the that of E_1, the values of E_2 increase with the increase in θ_o from 0^0 to 50^0 and then decrease monotonically within the range $50^0 \leq \theta_0 \leq 90^0$ for both the cases. **Fig.4** depicts the variation of energy ratio E_{11} with θ_o and it shows that the values of E_{11} for the case of MDGL are similar to MDLS but the corresponding values are different in magnitude. **Fig.5** exhibits the variation of energy ratio E_{22} with θ_o and it shows that the values of E_{22} for MDLS point toward the opposite oscillation with MDGL respectively within the range $10^0 \leq \theta_0 \leq 80^0$. In this case the value of E_{22} is magnified by 10^5.

Fig.6 depicts the variation of energy ratio E_{33} with θ_o and it indicates the values of E_{33} for the case of MDLS are very large as compared to the MDGL within the whole range of θ_o, though the maximum value of E_{33} can be noticed within the range $30^0 \leq \theta_0 \leq 40^0$ for both the cases. In this case the value of E_{33} is magnified by 10^6. **Fig.7** depicts the variation of energy ratio E_{44} with θ_o. It shows that the values of E_{44} for both cases MDLS and MDGL increase with the increase of θ_o from 0^0 to 70^0 and then decrease as θ_o increase further. In this case the value of E_{44} is magnified by 10^5. **Fig.8** exhibits the variation of energy ratio E_{55} with θ_o and it indicates the behavior of the graph is nearly equivalent to that of fig.4 but the corresponding values are different in magnitude. **Fig.9** shows the variations of E_{66} with θ_o and it indicates that the value of E_{66} for both MDLS and MDGL shows a small change within the range $0^0 \leq \theta_0 \leq 10^0$ and then increase sharply within the range $11^0 \leq \theta_0 \leq 65^0$ and decrease further. **Fig.10** shows the variation of interaction energy ratio E_{RR} with θ_o and it indicates the values of E_{RR} for the case of MDLS are less as compared to MDGL within the whole range of θ_o. The values of interaction energy decrease initially with the increase in values of θ_o and attain a minimum within the range of $50^0 \leq \theta_0 \leq 60^0$, for the rest of the range the graph of E_{RR} shows a smooth growth and attain a maximum value at the end of the range.

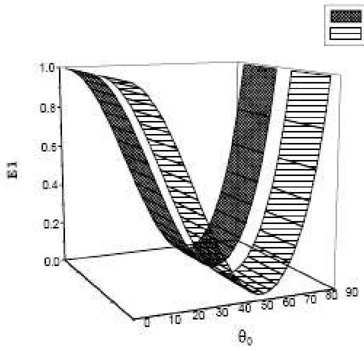

Figure 2 Variation of energy ratio E_1 w.r.t. angle of incidence P-wave

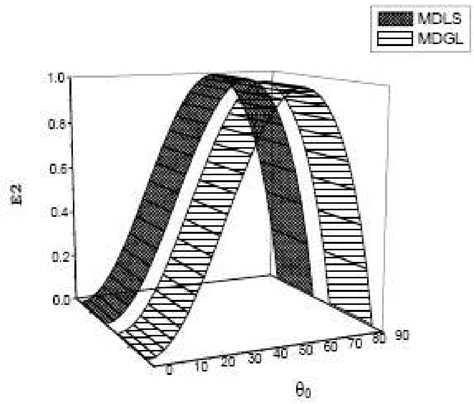

Figure 3 Variation of energy ratio E_2 w.r.t. angle of incidence P-wave

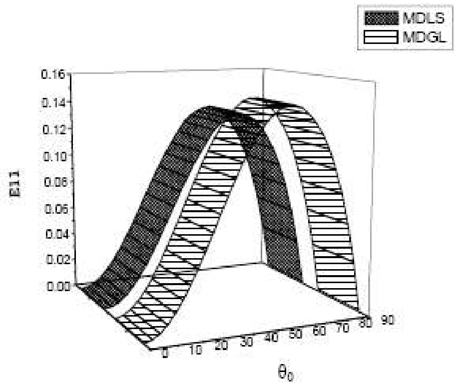

Figure 4 Variation of energy ratio E_{11} w.r.t. angle of incidence P-wave

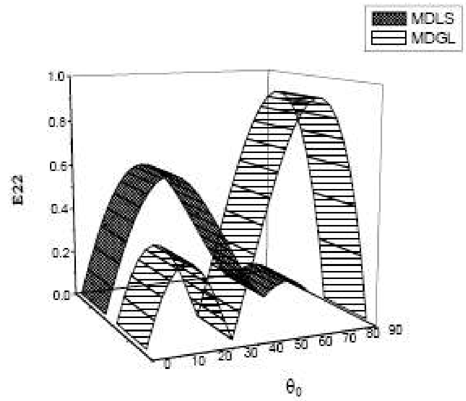

Figure 5 Variation of energy ratio E_{22} w.r.t. angle of incidence P-wave

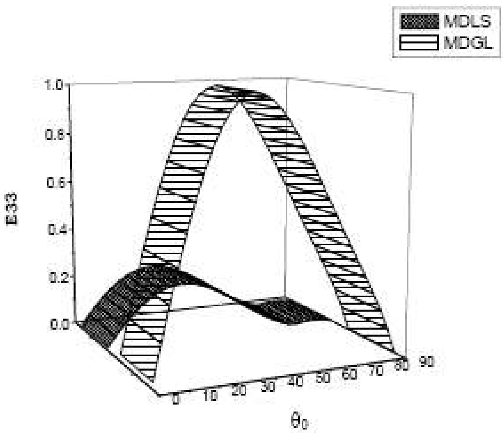

Figure 6 Variation of energy ratio E_{33} w.r.t. angle of incidence P-wave

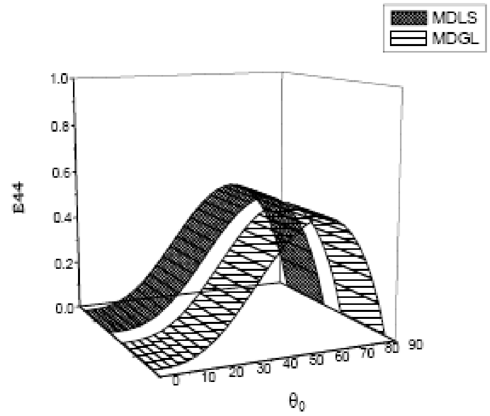

Figure 7 Variation of energy ratio E_{44} w.r.t. angle of incidence P-wave

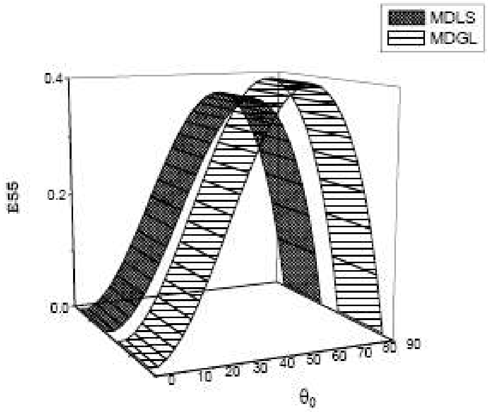

Figure 8 Variation of energy ratio E_{55} w.r.t. angle of incidence P-wave

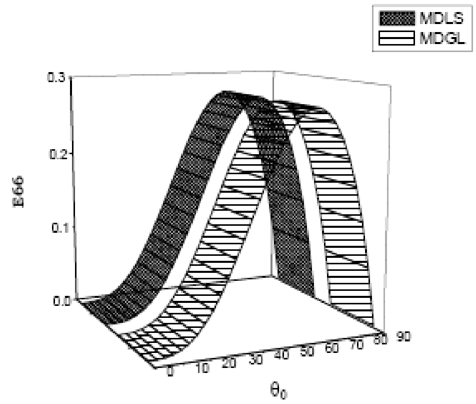

Figure 9 Variation of energy ratio E_{66} w.r.t. angle of incidence P-wave

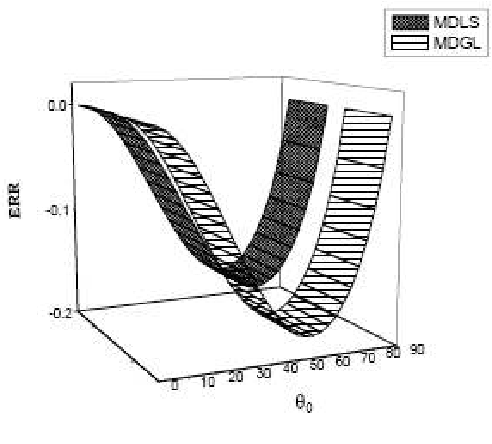

Figure 10 Variation of energy ratio E_{RR} w.r.t. angle of incidence P-wave

Incident SV-wave

Figs.11-19 depicts the variation of energy ratios with the angle of incidence (θ_0) for SV waves.

Fig.11 represents the variation of energy ratio E_1 with θ_0 and it indicates that the values of E_1 for both cases MDLS and MDGL increase for smaller values of θ_0, whereas for higher values of θ_0, the values of E_1 decrease and finally become constant. It is noticed that the values of E_1 in case of MDGL remain more in comparison to the MDLS case. **Fig.12** shows the variation of energy ratio E_2 with θ_0. and it indicates that the values for both cases MDLS and MDGL decrease when $0 \le \theta_0 < 10$ and for $10 \le \theta_0 < 30$ the values of E_2 increases and for higher values of θ_0 the values E_2 become dispersionless. It is noticed that for smaller values of θ_0 the values of E_2 in case of MDLS remain more whereas for higher values of θ_0 reverse behavior occurs. **Fig.13** shows that the values of E_{11} for both cases MDLS and MDGL show an oscillatory behavior for initial values of θ_0, whereas for higher values of θ_0, the values of E_{11} become dispersionless. It is evident that that the values of E_{11} in case of MDLS remain more in comparison to the MDGL case. **Fig.14** exhibits the variation of energy ratio E_{22} with θ_0 and it indicates that the values of E_{22} oscillates for smaller values of θ_0 although for higher values of θ_0, the values of E_{22} become constant. In this case the value of E_{22} is magnified by 10^2. **Fig.15** depicts the variation of energy ratio E_{33} with θ_0 and it is noticed that the behavior and variation of E_{33} is similar as E_{22} with difference in their magnitude values. In this case the value of E_{33} is magnified by 10^2. **Figs.16-18** show the variation of energy ratio E_{44}, E_{55} and E_{66} with θ_0 and it is evident that the behavior and variation of E_{44}, E_{55} and E_{66} are similar as E_{11} whereas magnitude values of E_{44}, E_{55} and E_{66} are different from $E_{11.}$ and in all these three figure the magnitude of energy ratios are magnified by 10^2. **Fig.19** represents the variation of E_{RR} with θ_0 and it shows that the values of E_{RR} decrease for smaller values of θ_0 whereas for higher values of θ_0 the values of E_{RR} slightly increase. It is noticed that the values of E_{RR} in case of MDGL remain more for higher values of θ_0.

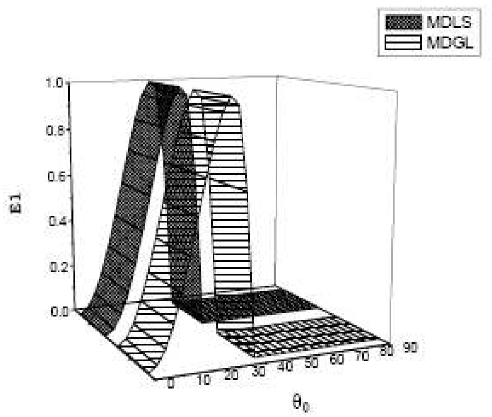

Figure 11 Variation of energy ratio E_1 w.r.t. angle of incidence

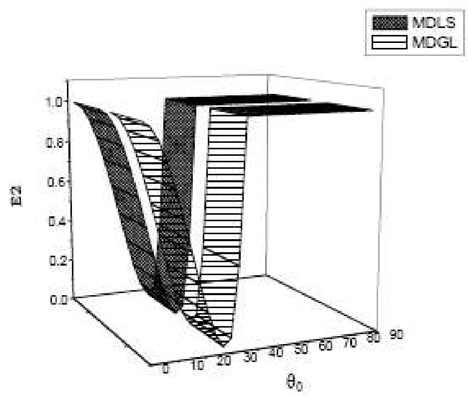

Figure 12 Variation of energy ratio E_2 w.r.t. angle of incidence

SV-wave

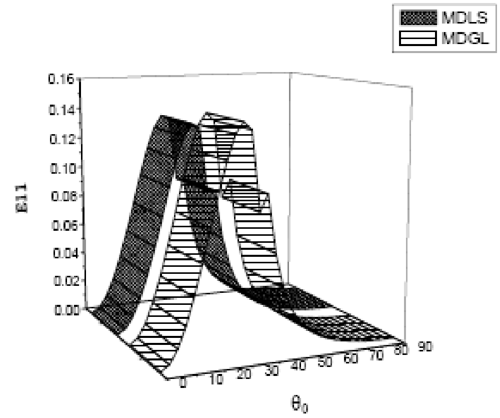

Figure 13 Variation of energy ratio E_{11} w.r.t. angle of incidence SV-wave

SV-wave

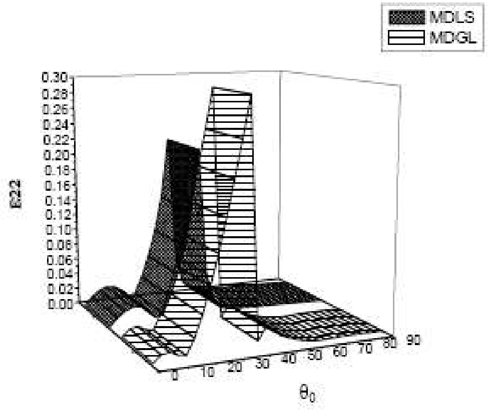

Figure 14 Variation of energy ratio E_{22} w.r.t. angle of incidence SV-wave

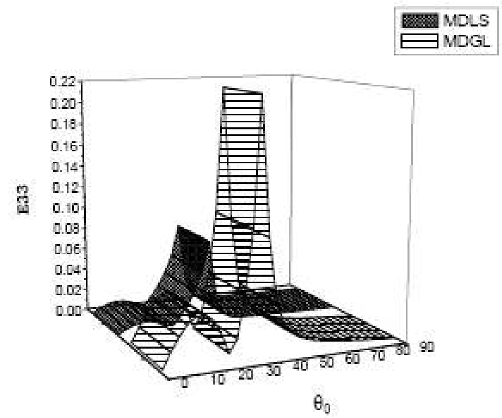

Figure 15 Variation of energy ratio E_{33} w.r.t. angle of incidence SV-wave

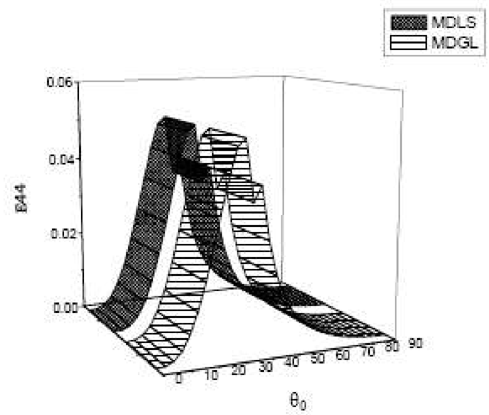

Figure 16 Variation of energy ratio E_{44} w.r.t. angle of incidence SV-wave

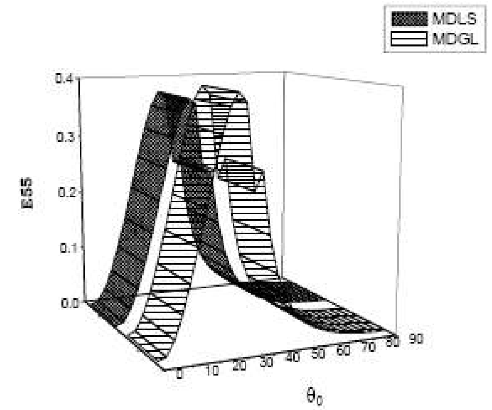

Figure 17 Variation of energy ratio E_{55} w.r.t. angle of incidence SV-wave

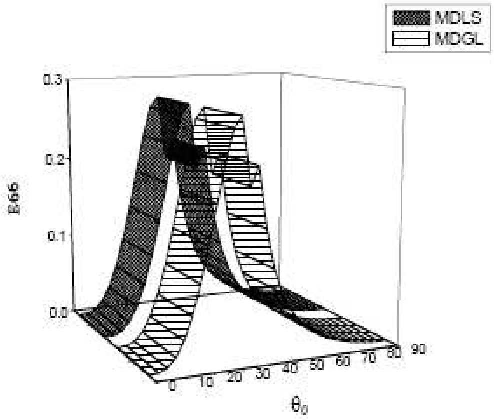

Figure 18 Variation of energy ratio E_{66} w.r.t. angle of incidence SV-wave

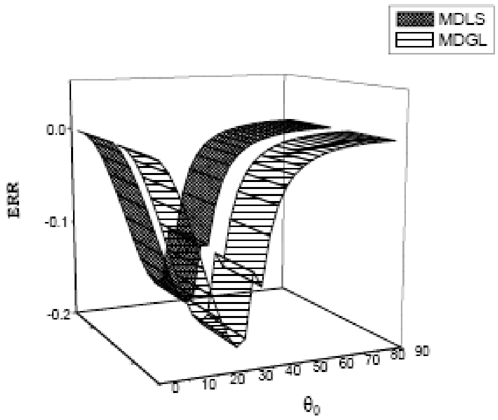

Figure 19 Variation of energy ratio E_{RR} w.r.t. angle of incidence SV-wave

7 CONCLUSION

In the present article, the phenomenon of reflection and refraction of obliquely incident elastic waves at the interface between an elastic solid half-space and a microstretch thermoelastic diffusion solid half-space has been studied. The six waves in microstretch thermoelastic diffusion medium are identified and explained through different wave equations in terms of displacement potentials. The energy ratios of different reflected and refracted waves to that of incident wave are computed numerically and presented graphically with respect to the angle of incidence.

From numerical results, we conclude that the effect of angle of incidence on the energy ratios of the reflected and refracted waves is significant. It is evident that, the values of energy ratios attained their optimum values within the range $40 \le \theta_0 < 60$ in almost all figures related to L-S and G-L theories. Moreover, in majority of cases, the magnitude of energy ratios for L-S theory are more as compared to G-L theory and vanishes at the grazing incidence. The sum of all energy ratios of the reflected waves, refracted waves and interference between refracted waves is verified to be always unity which ensures the law of conservation of incident energy at the interface.

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