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# Properties of charged Higgs bosons

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## Properties of charged Higgs bosons

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In theories with more scalar multiplets than the minimal Weinberg-Salam model, there will exist charged physical Higgs particles, in addition to the neutral Higgs particle. We examine the theory and phenomenology of such a situation as a guide for future experimental searches.

### I. INTRODUCTION

One of the crucial ingredients in the gauge theory of weak and electromagnetic interactions is the Higgs phenomena,<sup>1</sup> where scalar particles are introduced to give masses to the gauge particles through spontaneous symmetry breaking. Roughly speaking, in the processes of spontaneous symmetry breaking, the massless gauge bosons absorb the scalar particles to become massive vector bosons. However, not all the scalar particles will be absorbed by gauge particles, and the presence of these physical scalar particles, which are left over after the symmetry breaking, is one of the important characteristics of this type of spontaneously broken gauge theories. Thus the experimental search for such Higgs particles will be a good test for the validity of the gauge-model description of weak and electromagnetic interactions.

Generally, the structure of the Higgs particles depends on the model, and the experimental consequences will vary for different models. In the popular Weinberg-Salam model,<sup>2</sup> the structure of the Higgs system is rather simple. After the spontaneous symmetry breaking, only one neutral Higgs particle survives, and its phenomenology has been discussed by J. Ellis *et al.*<sup>3</sup> Even in this simple situation, the experimental search for such a neutral Higgs particle is very difficult because of the wide range of possible masses for the Higgs scalar. However, in the other models, more complicated Higgs structures are needed for various reasons. In particular, charged Higgs particles might exist and may lead to a phenomenology richer than that of the neutral Higgs particle. In view of the importance of finding hard evidence for the gauge-theory description of weak and electromagnetic interactions, it is desirable to search all possible signals for the existence of the Higgs particles. In this paper, we want to study the properties of general Higgs particles and their experimental consequences. Since it is impossible and also impractical to cover all possible Higgs structures, we will study a few simple cases and extract the general properties of Higgs particles as a guide to

the possibilities that are present when the content of a gauge group includes many Higgs particles.

In Sec. II, we will study the possible Higgs structures within the  $SU(2) \times U(1)$  group. These Higgs structures are complicated enough to illustrate the general features of the Higgs particles. In Sec. III, we will outline the phenomenology of Higgs particles in models with more than one physical Higgs particle. Most of the discussion will center on charged Higgs particles.

### II. STRUCTURES OF THE HIGGS SYSTEM

In this section, we will discuss the general features of the Higgs structure in various models. First, we will review the Higgs system in the Weinberg-Salam model. Then, various Higgs structures in models based on the  $SU(2) \times U(1)$  group will be studied. Most of the features discussed here will also be applicable to models based on groups more complicated than  $SU(2) \times U(1)$ .

In the Weinberg-Salam model, the Higgs particles are chosen to be in a doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

with respect to the gauge group  $SU(2) \times U(1)$  and after the spontaneous symmetry breaking, only one neutral Higgs particle,

$$2^{-1/2}(\phi^{0r} + \phi^{0i}),$$

survives. In this model, there are several special properties which are consequences of this particular simple structure of the Higgs particles.

(a) The mass of the neutral gauge boson is related to the charged gauge boson by

$$m_z^2/m_w^2 = 1/\cos^2\theta_w,$$

where  $\theta_w$  is the Weinberg angle. This property is characteristic of the doublet Higgs structure and will hold in the case where there are more than one Higgs doublet as long as there are no Higgs particles in the other representations.

(b) The mass of the Higgs particle is bounded from below and possibly from above. Weinberg

and Linde<sup>4</sup> have shown that the quartic coupling  $\lambda$  which governs the Higgs-particle mass ( $m_H^2 = \sqrt{2}\lambda/G_F$ ) cannot be made arbitrarily small in a stable vacuum. This leads to a bound

$$m_H^2 \geq 7 \text{ GeV}, \quad (1)$$

using  $\sin^2\theta_w \approx 0.25$ . Several people have observed that unless  $m_H \leq 0.3 \sim 1$  TeV the weak interactions would become strong (e.g.,  $\lambda > 1$ ) and the perturbative expansion for radiative corrections would break down. While it has not been proved that such a breakdown is ruled out, this situation would probably induce large corrections to lowest order relations such as the one in (a).

In theories with many Higgs particles, these bounds may be violated. For example, for  $SU(2) \times U(1)$  with many doublets of Higgs particles, the lower bound Eq. (1) only holds for the heaviest Higgs particles. Likewise, small mixing angles could delay the onset of nonperturbative effects for masses somewhat above 1 TeV. We note that two "natural" ranges in general theories for masses correspond to  $\lambda \sim O(e^4)$  which gives  $m_H \sim 1-10$  GeV or  $\lambda \sim O(e^2)$  with  $m_H \sim m_w$ .

(c) The coupling of the Higgs to the fermions conserves parity, CP, and fermion flavors, and the coupling strength is proportional to the fermion mass. This property follows from the fact that the fermions get their masses from Higgs scalars which are in a single irreducible representation with respect to the gauge group and will not be true if fermions obtain their masses from Higgs scalars in the reducible representation or if fermions get their masses from both Higgs and bare-mass terms. To see that the coupling is proportional to the fermion masses we note that in this model there is no gauge-invariant bare-mass term because all the left-handed fermions are in doublets and all the right-handed fermions are in the singlets. Thus fermion masses can only come from their coupling to the Higgs scalar given by

$$f_i \bar{\psi}_{iL} \phi \psi_{iR} + \text{H.c.} \quad (2)$$

After spontaneous symmetry breaking, the Higgs scalar picks up a vacuum expectation value given by

$$\langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (3)$$

where  $v$  is related to the Fermi constant  $G_F$  by

$$v = (\sqrt{2} G_F)^{-1/2}. \quad (4)$$

This will generate the fermion mass term

$$f_i \frac{v}{\sqrt{2}} \bar{\psi}_{iL} \psi_{iR} + \text{H.c.}, \quad (5)$$

with the identification  $m_i = f_i v / \sqrt{2}$ , where  $m_i$  is the mass of the fermion  $i$ . The mass referred to is the

Lagrangian mass, not that determined in phenomenological quark models. We can rewrite the Higgs-particle coupling as

$$L_{\bar{\psi}\psi\phi} = m_i 2^{3/4} \sqrt{G_F} \bar{\psi}_{iL} \phi^0 \psi_{iR} + \text{H.c.} \quad (6)$$

Thus the Higgs-particle coupling to the fermions is stronger for heavy leptons and heavy quarks for the ordinary light fermions.

Now we will discuss the possible extension of Higgs structure in models based on  $SU(2) \times U(1)$  group. We will assume that the left-handed components of the known fermions ( $e, \mu, \nu_e, \nu_\mu, u, d, s, c$ ) are all in the doublets, but the right-handed component could be either in the doublet or in the singlet. Even though we will study only a few simple cases, they are enough to illustrate the general features.

#### A. Singlet and doublet Higgs particles

We will concentrate on the effect of the additional singlet Higgs particle on the theory. Here we will distinguish between three different cases depending on the charges carried by the singlet Higgs particle.

(1) Neutral singlet: From the charge assignment  $Q = T_3 + Y/2$  where  $Y$  is the weak hypercharge, the neutral singlet must have  $Y = 0$  and cannot couple to any gauge bosons. But it can contribute to the mass of the fermions whose right-handed components and left-handed components have the same quantum number. This will have the effect that the property (b) mentioned before will be lost.

(2) Integrally charged singlet: From the charge conservation, this kind of Higgs particle cannot develop a vacuum expectation value. It can couple possible left-handed singlet fermions to the right-handed fermion with different charge.

(3) Fractionally charged singlet: This type of Higgs particle can couple quarks to leptons (singlet to singlet, doublet to doublet) and will carry both lepton number and baryon number. Since it cannot develop a vacuum expectation value, the baryon and lepton number will be conserved.

None of these three cases seem to serve any purpose.

#### B. Two Higgs doublets

The interesting case is where both doublets have the same charge assignment so that they both couple to the fermions as in the one-doublet case. With

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \quad (7)$$

the most general charge-conserving vacuum expectation values are

$$\langle \phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \quad (8)$$

It turns out that at least for some finite ranges of the parameters in the Higgs potential, these charge-conserving vacuum expectation values are the stable minimum of the Higgs potential.

We can redefine the Higgs fields such that only one doublet will develop a vacuum expectation value. The solution is

$$\begin{aligned} \phi'_1 &= \cos\alpha \phi_1 + \sin\alpha \phi_2, \\ \phi'_2 &= -\sin\alpha \phi_1 + \cos\alpha \phi_2, \end{aligned} \quad (9)$$

with

$$\cot\alpha = \frac{v_1}{v_2}.$$

It is easy to see that

$$\langle \phi'_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

and

$$\langle \phi'_2 \rangle_0 = 0, \quad (10)$$

where

$$v = (v_1^2 + v_2^2)^{1/2}.$$

Therefore, the spontaneous symmetry breaking and fermion mass generation are completely controlled by the doublet  $\phi'_1$ , while the other doublet  $\phi'_2$  is just ordinary scalar particles. It is then clear that property (a) of Weinberg-Salam model still holds and the coupling of the doublet  $\phi'_1$  to the fermions will satisfy property (b). But the coupling of  $\phi'_2$  to the fermions is completely unconstrained and the coupling of the neutral component of  $\phi'_2$  to the fermions will not conserve parity and fermion flavor. The physical effects due to this doublet  $\phi'_2$  become totally arbitrary and very difficult to estimate. A few remarks are in order.

(1) It is possible to impose some discrete symmetry such that all the neutral-Higgs-particle couplings conserve fermion flavors. As an example of this let us consider the  $SU(2) \times U(1)$  model with two doublets with a discrete symmetry such that  $\phi_1(\phi_2)$  gives mass to the charged  $\frac{2}{3}(-\frac{1}{3})$  quarks. After diagonalization of the mass matrices the Higgs-particle couplings to quarks are of the form

$$\begin{aligned} H &= 2^{3/4} \sqrt{G_F} \{ H^* (\cot\alpha \bar{u}_R M_u U d_L - \tan\alpha \bar{u}_L U M_d d_R) + \text{H.c.} \\ &\quad - \sqrt{2} H_1^0 (\cot\alpha \bar{u}_L M_u \gamma_5 u + \tan\alpha \bar{d}_L M_d \gamma_5 d) + \sqrt{2} H_2^0 [(\cos\phi + \cot\alpha \sin\phi) \bar{u} M_u u + (\cos\phi - \tan\alpha \sin\phi) \bar{d} M_d d] \\ &\quad + \sqrt{2} H_3^0 [(\cot\alpha \cos\phi - \sin\phi) \bar{u} M_u u - (\sin\phi + \tan\alpha \cos\phi) \bar{d} M_d d] \}. \end{aligned} \quad (11)$$

Where  $u$  ( $d$ ) is a vector of all the charged  $\frac{2}{3}$  ( $-\frac{1}{3}$ ) quarks,  $M_u$  and  $M_d$  are the diagonal mass matrices,  $U$  is the unitary matrix that describes the quark couplings to the  $W^+$ , and the angle  $\phi$  describes the mixing of two of the neutral Higgs particles and is determined by the parameters of the Higgs potential. We see that due to the presence of the mixing angles, the coupling strengths are essentially arbitrary, although they are still governed roughly by the quark masses.

(2) The generalization to the case of three or more Higgs doublets is straightforward. One interesting case is where some kind of permutation symmetry<sup>6</sup> can be imposed such that all the Higgs coupling to the leptons are proportional to the mass of the heaviest lepton

$$\sim m_L \sqrt{G_F} \bar{l}_i l_i \phi^a + \text{H.c.}, \quad (12)$$

where  $m_L$  is the mass of the heaviest lepton,  $l_i$  are

the lepton fields. Similarly, the Higgs-particle coupling to the quarks will be proportional to the heaviest quark mass. So in this type of model, the Higgs-particle couplings to the ordinary light fermions are much larger than those in the standard Weinberg-Salam model and the physical effects due to this type of Higgs particle will be more accessible experimentally.

This type of two or more doublet Higgs structure has been suggested in connection with muon-number violation and CP violation. However, these effects are very difficult to estimate due to the uncertainties in the masses of the Higgs scalars and/or the Higgs-particle couplings to the fermions.

### C. Doublet and triplet Higgs particles

For the case where the triplet has the weak hypercharge  $Y=1$ , the charge assignment is

$$\chi = \begin{pmatrix} \chi^+ \\ \chi^0 \\ \chi^- \end{pmatrix}. \quad (13)$$

The most general charge-conserving vacuum expectation values for the Higgs particles are given by

$$\langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{and} \quad \langle \chi \rangle_0 = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}. \quad (14)$$

It is easy to see that the neutral gauge boson obtains its mass only from the Higgs doublet  $\phi$  while the charged gauge boson obtains its mass from both the Higgs doublet and triplet,

$$M_W^2 = \frac{g^2}{4}(v_1^2 + 4v_2^2), \quad M_Z^2 = \frac{g^2 v_1^2}{4 \cos^2 \theta_W}. \quad (15)$$

Thus, instead of the property (a) of Weinberg-Salam model, we get an inequality  $M_W^2 > M_Z^2 \cos^2 \theta_W$ . This will make the neutral-current cross sections smaller than those in the Weinberg-Salam model.

The physical Higgs particles turn out to be some linear combinations of the doublet and the triplet; i.e., the charged Higgs particle is

$$h = \frac{-\sin \alpha \phi^+ + (1/\sqrt{2}) \cos \alpha \chi^+}{(\sin^2 \alpha + \frac{1}{2} \cos^2 \alpha)^{1/2}}$$

where

$$\tan \alpha = \frac{\sqrt{2} v_2}{v_1}, \quad (16)$$

and the neutral Higgs particle is

$$\begin{aligned} h_0 &= \cos \beta \frac{1}{\sqrt{2}} (\phi^0 + \phi^{0t}) + \sin \beta \chi^0, \\ h'_0 &= \sin \beta \frac{1}{\sqrt{2}} (\phi^0 + \phi^{0t}) + \cos \beta \chi^0, \end{aligned} \quad (17)$$

where the angle  $\beta$  depends on the Higgs-particle self-coupling.

In the case where the left-handed fermions are all in the doublets and the right-handed fermions are all in the singlets, the Higgs triplet  $\chi$  will not couple to the fermions. The coupling of the charged physical Higgs particle to the fermions will be very similar to property (b) of the Weinberg-Salam model except for the extra factor  $\tan \alpha$ , which will make this coupling arbitrary. However, the current data on the neutral-current cross sections, presumably requires  $v_2 \ll v_1$  or  $\tan \alpha \ll 1$  and will make this coupling small. The neutral-Higgs-particle coupling to the fermions will also be complicated by the presence of the mixing angle  $\beta$ .

If one changes the weak hypercharge of the triplet to  $Y=2$ , the charge assignment is then

$$\chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}.$$

In this case, both the Higgs triplet and doublet will contribute to the masses of the charged gauge bosons as well as the neutral gauge bosons and the ratio of the masses is unconstrained in this case. The presence of the doubly charged Higgs particle in this model might lead to another class of phenomenology. The doubly charged Higgs particles also exist in one of the  $SU(2) \times SU(2) \times U(1)$  models.

It is clear from the discussion in cases A, B, and C that for a Higgs structure more complicated than those in Weinberg-Salam model, the general features are as follows:

- (i) There will be charged Higgs particles as well as the neutral Higgs particles.
- (ii) The Higgs couplings to the fermions are not constrained except in the cases where some additional discrete symmetries are imposed.

We would like to emphasize that the presence of the charged Higgs particle is a very general phenomena, with the Weinberg-Salam model an exception. In particular, when one enlarges the gauge group to unify the strong and weak and electromagnetic interactions, the charged Higgs particles are present even for the minimal structure which is needed to break the symmetry appropriately.

There have been many models proposed that are based on groups more complicated than  $SU(2) \times U(1)$ . These theories always involve many Higgs particles, usually in several different group representations. Their diversity makes a general analysis rather difficult, but most of the features have already been seen in the discussion of  $SU(2) \times U(1)$  models. In particular, for the important case of the coupling to the fermions we again have two classes.

(1) The Higgs-particle coupling to the fermion is proportional to the mass of the fermion to which the Higgs particle couples and is of order of  $\sqrt{G_F} m_f$ , as suggested by the neutral-Higgs-particle coupling to the fermions in the Weinberg-Salam model. In this case, the Higgs-particle couplings are more important for the heavy fermions than the ordinary light fermions.

(2) The Higgs-particle couplings to the light fermions as well as the heavy fermions are proportional to the masses of the heavy fermions, i.e., of order of  $\sqrt{G_F} m_F$ , where  $m_F$  is the mass of the heavy fermions. Then the Higgs-particle couplings to the light fermions are as important as those to the heavy fermions. This type of coupling occurs naturally in  $SU(3) \times U(1)$  models.<sup>7</sup>

Some models may include examples of both cases.

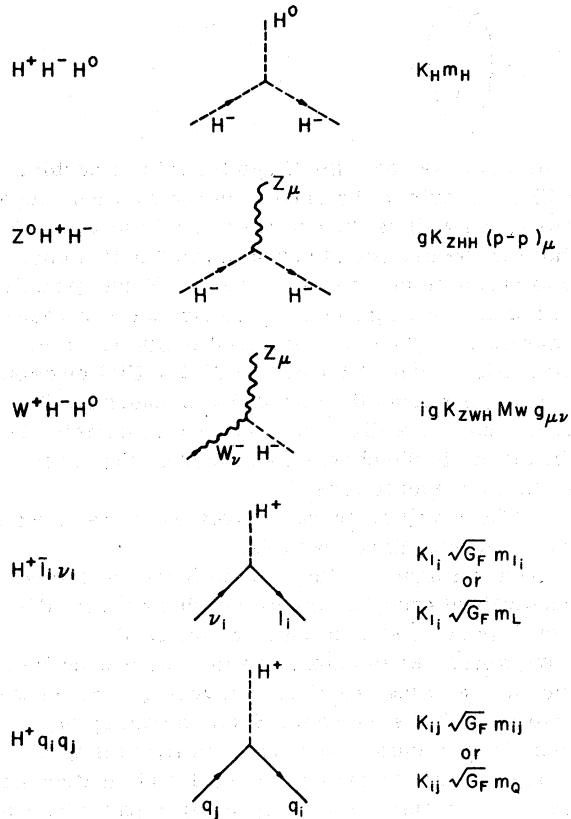


FIG. 1. Typical coupling strengths. The  $K$  factors stand for unknown mixing angles.  $m_L$  is the mass of some heavy lepton,  $m_{ij}$  is of the same order as the mass of  $q_i$  and  $q_j$ , and  $m_Q$  is the mass of some heavy quark.

As a guide to our phenomenology, we will write down "typical" Higgs-particle couplings. For example, there exists a trilinear  $H^0 H^+ H^-$  coupling that is governed by the same parameters which determine the masses of the Higgs particles, so we will write this vertex as  $K_H m_H$  with  $K_H$  summarizing the mixing angles. The remaining couplings are summarized in Fig. 1. Dependences on the gauge coupling constant  $g$  and masses are made explicit, while the unknown mixing angles are contained in the parameters labeled  $K$ .

### III. PHENOMENOLOGY

In this section we outline the physical signals for Higgs bosons. The Weinberg-Salam model's single Higgs particle has been discussed extensively<sup>3,8</sup> and we will be brief in that respect. Many of the properties of other neutrals can also be taken from previous work, so that most of the discussion centers on charged Higgs particles. Because of the uncertainties of any model with these extra parti-

cles, it is not possible to be precise in the estimates. However, we discuss the possible effects as a guide in the search for Higgs particles.

The range in masses that is possible for the Higgs particles is another obstacle in any such discussion. If their masses were large (i.e.,  $m_H \geq m_W$ ) it is clear that none of the methods described below will be fruitful in the near future.  $W$  bosons will be founded first, as their coupling is stronger than that of Higgs particles. However, if there are Higgs particles with mass  $m_H < 10$  GeV, the suggestions below are immediately applicable. There is a reasonable lower bound on the mass of any charged Higgs particle. Since a charged Higgs particle would couple  $\tau$  to  $\nu$  semiweakly,  $\tau$  decay would proceed dominantly through  $\tau^+ \rightarrow H^+ \nu$  if the process were kinematically allowed. This holds as long as the mixing parameters (called  $\sin\theta$  here) are such that  $\sin^2\theta > 10^{-5}$ . With this, we will restrict ourselves to  $m_{H^+} > m_\tau$ .

This section is subdivided into subsections on Higgs-particle decay,  $e^+e^-$  reactions, Neutrino production, hadronic production, virtual effects, and a summary.

#### A. Higgs-particle decay

As we have seen in the discussions of models, there are two general types of Higgs-particle coupling to quarks and leptons, (1) those whose coupling to fermion is  $(m_f G_F^{1/2})$  and (2) those where it is  $(m_F G_F^{1/2})$  where  $m_F$  refers to a heavy fermion  $F$ . These two cases will have different patterns of decay, and we will treat them separately.

In case (1) the Higgs boson will decay into the heaviest particles available to it. The patterns for the neutral Higgs particles are well treated elsewhere. For charged Higgs particles in the mass range 2–5 GeV the primary modes will be  $\tau\bar{\nu}$  and  $c\bar{s}$ . The decay mode  $c\bar{s}$  will contain  $D$  or  $F$  mesons in the final state. In general, mixing parameters enter differently into these two couplings, so the relative strengths cannot be predicted. The direct decay to muons or electrons is suppressed, but since  $\tau$  has a leptonic branching ratio of 40% and  $D$  decays semileptonically roughly 20% of the time,  $\mu$  and  $e$  will often appear in the final state. However, they will not be as energetic as if the Higgs particles had decayed directly to  $e\nu$  or  $\tau\nu$ . The  $c\bar{s}$  mode has the obvious signal of two strange particles in the final state, which may prove to be a useful handle. In addition any future study of  $D$  mesons could combine observed  $K$ 's in the event to look for a mass bump. For masses greater than the  $T$ , the decay modes are even more model dependent. In the  $SU(2) \times U(1)$  model that we have studied, the charged-Higgs-boson coupling was proportional to

the weak-current coupling, which is known to be small between light ( $u, d, s, c$ ) and heavy ( $t, b$ ) quarks. In this case  $\tau\bar{\nu}$  and  $c\bar{s}$  still dominate. Other models may have new quarks in the final state, with the specific signature depending on the decay patterns of the new quarks.

In the second case, there is no preference for heavy masses, and all modes will occur at roughly the same rate, depending only on mixing parameters. If these parameters were the same for quarks and leptons, we could estimate  $e\bar{\nu}$ :  $\mu\bar{\nu}$ :  $\tau\bar{\nu}$ :  $u\bar{d}$ :  $c\bar{s} = 1:1:1:3:3$ . Then  $\frac{2}{9}$  would go directly to  $\mu$  or  $e$  and about 10% would produce  $\mu$  or  $e$  indirectly. We can see that regardless of the mixing parameters or choice of the two cases, 5% or more of the decays will involve a muon or an electron, with a more realistic value being twice this. In the hadronic decay modes the second case is less distinctive, but the  $c\bar{s}$  mode should not be negligible, and would still be a reasonable way to look for Higgs.

#### B. $e^+e^-$ reactions

The most obvious advantage for the detection of charged Higgs particles is that they can be produced electromagnetically. In  $e^+e^-$  colliding beams the cross section for  $e^+e^- \rightarrow H^+H^-$  is

$$\sigma(e^+e^- \rightarrow H^+H^-) = \frac{\pi\alpha^2}{3s} \left( \frac{s - 4m_H^2}{s} \right)^{3/2}, \quad (18)$$

and far above threshold they contribute a quarter of the strength of  $e^+e^- \rightarrow \mu^+\mu^-$ . It appears that present experiments do not have the sensitivity to detect the opening up of this threshold when so much other new physics is occurring at the same time. Slightly more promising is the possibility of looking at specific final states. One of the dominant decay modes of  $H^+$  may be to  $c\bar{s}$ , so that  $H^+H^-$  would often appear with four kaons in the final state. A threshold in the multikaon events could be looked for. Alternatively, if  $D$  mesons are studied at higher center-of-mass energies,  $H^+$  could show up as a bump in the  $DK$  invariant mass.

Another interesting possibility occurs when PEP and PETRA start operating. They will be able to produce the weakly decaying partners of the  $\Upsilon$ , which we will label by the generic name  $B$  and which are expected to have a mass of about 5 GeV. The charged-Higgs boson coupling is semiweak and changes flavor, so that as long as  $m_H < m_B$ , the decay mode  $B \rightarrow H + X$  will dominate over any of the standard weak processes. As the  $b$  quark's weak coupling is not known, it is hard to spell out the characteristics of this process with certainty. One interesting possible decay (if kinematically allowed) is  $B \rightarrow DH$  which may be reconstructable.<sup>9</sup>

Likewise, if new leptons are produced and a light Higgs particle exists,  $L^+ \rightarrow H^+\nu$  will dominate the

decay modes. This may confuse the lepton interpretation of  $L^+$  when it is first found, since one would be observing the decays of  $H^+$ . However, a unit contribution to  $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  would indicate that it is not a fractionally charged quark or a unit charged boson. The production of a single neutral  $H^0$  in  $e^+e^- \rightarrow H^0$  will be impossible to see if the coupling of  $e^+e^-H^0$  is proportional to  $m_e$ . If, however, a heavy lepton mass enters into the coupling, then the production cross section may be sizeable. This is discussed in more detail in Ref. 7.

Future machines hope to discover a neutral  $Z^0$ . If this occurs, one can study a good deal of the weak interaction, including possibly Higgs particles, in its decay. If we use the  $ZH^+H^-$  coupling,

$$L_{ZH^+H^-} = K_{ZH^+H^-} g \epsilon_\mu (p - p')^\mu, \quad (19)$$

the decay rate is

$$\Gamma(Z^0 \rightarrow H^+H^-) = \frac{K_{ZH^+H^-}^2 g^2 M_Z}{48\pi} \left[ 1 - \frac{4M_H^2}{M_Z^2} \right]^{3/2}, \quad (20)$$

compared to

$$\Gamma(Z^0 \rightarrow \mu^+\mu^-) = \frac{(g_L^2 + g_R^2)}{24\pi} M_Z \quad (21)$$

for coupling

$$L_{Z\mu^+\mu^-} = \epsilon_\mu \bar{u}(p') \gamma^\mu \left( g_L \frac{(1 + \gamma_5)}{2} + g_R \frac{(1 - \gamma_5)}{2} \right) v(p). \quad (22)$$

If one counts each known quark (in each color) and lepton equally, this says that the branching rates for  $Z^0 \rightarrow H^+H^-$  is about 1%. This clearly makes it difficult to observe this mode, although, we hope, not impossible.

Finally we would like to mention a possible way to produce  $W^+$  mesons in association with Higgs at energies below  $2M_W$ . One of the couplings that is predicted by gauge theories is between  $Z^0$ ,  $W^+$ , and  $H^-$  of the form

$$K_{ZW^+H^-} g M_W (Z_\mu^0 W^{+\mu} H^- + \text{H.c.}); \quad (23)$$

this allows the process  $e^+e^- \rightarrow Z^0 \rightarrow W^+H^-$ , as depicted in Fig. 2. The full matrix element has the form

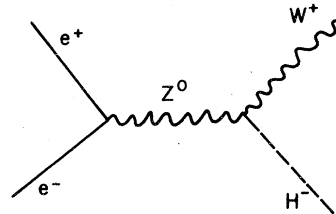


FIG. 2. A production mechanism for  $e^+e^- \rightarrow W^+H^-$ .

$$\frac{K_{ZWH}g^2M_W}{M_Z^2-s}\bar{\nu}(p')\gamma_\mu(g_V+g_A\gamma_5)u(p)\epsilon^\mu. \quad (24)$$

A virtual Higgs intermediate state is down by factors of lepton masses. The cross section for this process is

$$\begin{aligned} \sigma(e^+e^- \rightarrow W^+H^-) &= \frac{K_{ZWH}^2\alpha\pi(g_V^2+g_A^2)}{M_W^2\sin^4\theta_W}\left(\frac{M_W^2}{M_Z^2-s}\right)^2 \\ &\times \left(1 - \frac{2(M_W^2+M_H^2)}{S}\right. \\ &\quad \left. + \frac{(M_W^2-M_H^2)^2}{S}\right)^{1/2}. \end{aligned} \quad (25)$$

Above threshold this quickly becomes sizeable. The observation of this process would be a spectacular confirmation of gauge theories, as three basic ingredients ( $W$ ,  $H$ , and the gauge coupling) are involved.

Before we proceed, some comment should be made about the search for neutral Higgs mesons. All work published so far has assumed that the  $H^0$  decays only into quarks and leptons. However, if  $M_H^* < \frac{1}{2}M_{H^0}$  the decay  $H^0 \rightarrow H^+H^-$  is allowed. This is governed by the trilinear coupling in the (shifted) Higgs potential, which is of order of

$$L_{HHH} = K_H M_{H^0} H^0 H^+ H^-. \quad (26)$$

Using this, we find a decay width

$$\Gamma = \frac{K_H^2 M_{H^0}}{16\pi} \left(1 - \frac{4M_{H^0}^2}{M_{H^+}^2}\right)^{1/2}. \quad (27)$$

In words, the width is comparable to the Higgs mass. If this channel is open it will dominate and none of the other signals for  $H^0$  production are valid. Instead, one must look at the signature for  $H^+H^-$ . In addition if  $M_{H^+} > M_{H^0}/2$  the decays  $H^0 \rightarrow H^+ + (\text{hadrons})^-$  may take place as in Fig. 3. Numerical factors from the three-body phase-space integral suppresses it, but there is an enhancement from the  $H^-$  propagator if  $M_{H^+} \approx M_{H^0}/2$ . Using Eq. (26) and Fig. 1 we find [expanding in  $(2M_H^+ - M_{H^0})$ ]

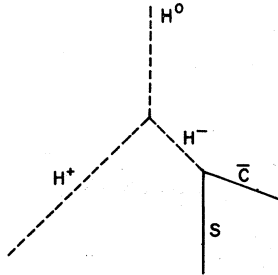


FIG. 3. Decay mechanism for  $H^0 \rightarrow H^+c\bar{s}$ .

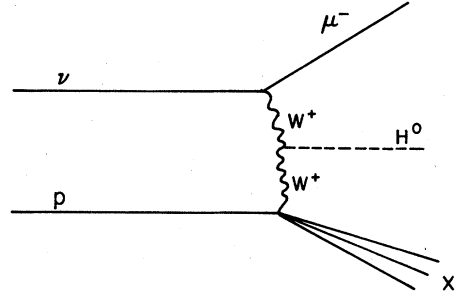


FIG. 4. Neutrino production of  $H^0$ .

$$\begin{aligned} \Gamma(H^0 \rightarrow H^+c\bar{s}) &= \frac{G_F m_c^2 K_q^2 K_H^2 M_{H^0}}{128\pi^2} \left(\frac{M_{H^+}}{2M_{H^+} - M_{H^0}}\right)^{1/2} \\ &\quad + O((2M_{H^+} - M_{H^0})^{1/2}). \end{aligned} \quad (28)$$

This may be an important mode.

### C. Neutrino production

Higgs mesons may also be produced in neutrino reactions. There has been considerable effort made in the study of neutral Higgs-particle production.<sup>3,10</sup> The dominant diagram is that illustrated in Fig. 4. Unfortunately the cross section turns out to be extremely small. As quoted in Ref. 3 the ratio of Higgs-particle production to the standard neutrino cross section approaches

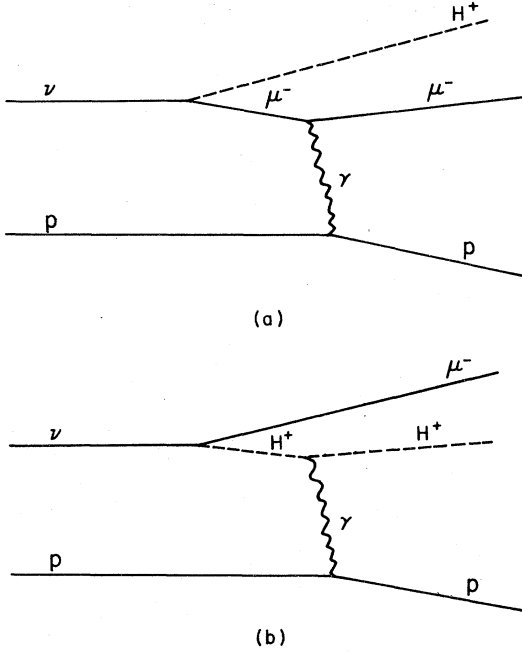
$$\frac{\sigma(\nu + T \rightarrow H^0 + \mu^- + X)}{\sigma(\nu + T \rightarrow \mu^- + X)} = 3 \times 10^{-8} [E_\nu(\text{GeV})] \quad (29)$$

far above threshold. It appears very unlikely that Higgs mesons will be discovered this way. This conclusion is not modified for neutral mesons in other gauge theories.

Charged Higgs particles have a considerably more advantageous production mechanism. Since their coupling is formally semiweak, they can be produced in the Coulomb field of the nucleus by the diagrams of Fig. 5. As we will see, this turns out to be a reasonable way to search for light Higgs particles if their coupling is strong enough.

This process is similar to that considered for the production of  $W$  mesons by neutrinos.<sup>11,12</sup> There the dominant diagram was found to be that of Fig. 5(a) because the muon can get closer to its mass shell before emitting a soft virtual photon, thereby resulting in an enhancement due to the propagator. This makes it easy to convert the work on  $W$  production to our process. To do this we neglect Fig. 5(b), which introduces a (10–20)% error, but is acceptable for our purposes. We must multiply by  $\frac{1}{2}$  to remove the sum over  $W$  polarizations, since this sum has the form



FIG. 5. Neutrino production of  $H^+$ .

$$\gamma_\mu u(p')\bar{u}(p')\gamma_\nu g^{\mu\nu} = -2u(p')\bar{u}(p'), \quad (30)$$

if we neglect the mass of the muon. The work on  $W$  production uses the coupling

$$L_{W\mu\nu} = \frac{g'}{\sqrt{2}}\bar{\mu}\gamma_\mu \frac{(1+\gamma_5)}{2}\nu W^\mu, \quad (31)$$

with

$$\frac{g'^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}. \quad (32)$$

In the above equation  $M_W$  is taken to be the  $W$  meson produced, not the gauge theories  $W$ . For the Higgs-boson process we use the standard coupling

$$L_{H\mu\nu} = \frac{g}{\sqrt{2}}\frac{K_\mu}{M_W}\bar{\mu}\frac{(1+\gamma_5)}{2}\nu H^* \quad (33)$$

with

$$\frac{g^2}{8(72 \text{ GeV})^2} = \frac{G_F}{\sqrt{2}}. \quad (34)$$

We must rescale the coupling constant ( $g' \rightarrow g$ ) by multiplying the cross section by  $72 \text{ GeV}/M_H$ , and of course must multiply everything by  $K^2 m_\mu^2/M_W^2$ . With these changes we can extract the cross section from Ref. 12. Due to the exchanged photon the coherent scattering off of a proton will dominate over the deep-inelastic process. For the process  $\nu + p \rightarrow \mu + H^* + p$ , a very good parametrization of the theory turns out to be

$$\sigma(\nu + p \rightarrow \mu^- + H^* + p) = \frac{K^2 A}{M_H^{3/2}} e^{-a+b/E_\nu} M_H, \quad (35)$$

with

$$\begin{aligned} A &= 1.4 \times 10^{-38} \text{ cm}^2 \text{ GeV}^{3/2}, \\ a &= 0.16 \text{ GeV}^{-1} \\ b &= 75 \end{aligned} \quad (36)$$

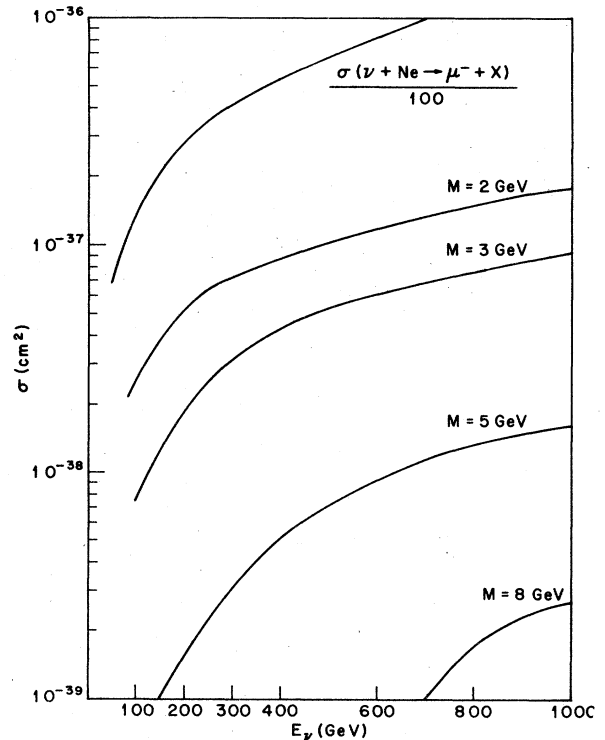
This form is reasonable for  $2 \text{ GeV} \leq M_H \leq 25 \text{ GeV}$  and for  $50 \leq E_\nu < 1000 \text{ GeV}$ . The cross section is identical for neutrinos and antineutrinos.

For  $M_H = 2 \text{ GeV}$  and  $K^2 = 1$  the ratio

$$R = \frac{\sigma(\nu + p \rightarrow \mu^- + H^* + p)}{\sigma(\nu + p \rightarrow \mu^- + X)} \quad (37)$$

peaks at  $10^{-3}$  at  $E = 140 \text{ GeV}$  and decreases slightly with increasing energy. For  $M_H = 3 \text{ GeV}$  the peak is about  $3 \times 10^{-4}$  and for  $M_H = 4 \text{ GeV}$  it is about  $10^{-4}$ . Clearly if  $K > 1$ , this increases the effect on this process.

On nuclear targets, coherent scattering off the nucleus dominates at sufficiently high energies.<sup>13</sup> Using neon, the nuclear and single-proton scatterings are equal when  $E_\nu = 80 \text{ GeV}$  for  $M_H = 3 \text{ GeV}$  ( $E = 250 \text{ GeV}$  for  $M_H = 5 \text{ GeV}$  or  $E = 1000 \text{ GeV}$  for  $M_H = 10 \text{ GeV}$ ). We have extracted the nuclear coherent cross sections from Ref. 12, and plotted in Fig. 6  $\sigma_{\text{tot}}$ , where

FIG. 6. Cross section for  $\nu + \text{Ne} \rightarrow \mu + H^* + X$  with the coupling given by Eq. (33) and  $K_\mu = 1$ .

$$\sigma_{\text{tot}} = Z\sigma(\nu p) + \sigma(\text{coherent})$$

for neon. There are some uncertainties due to nuclear corrections, which we have not included. For  $K^2=1$ ,  $M_H=2$  GeV, and  $E_\nu \approx 200$  GeV, we find  $2 \times 10^{-3}$  and  $R(\text{Ne})$  decreases roughly exponentially with mass [ $R(\text{Ne}) \approx 0.025e^{-1.2M_H}$ ]. For antineutrinos values of  $R$  would be about three times larger.

These cross sections are somewhat small unless  $K^2 > 1$ , but the signal for this process is striking. Because of the muon propagator, the  $\mu^-$  picks up relatively little of the neutrinos energy. For example, for  $E=200$  GeV and  $M_H=5$  GeV 80% of the events have  $y = E_\nu - E_\mu/E_\nu > 0.9$ . The outgoing proton also has very little energy, and is produced at large angles ( $\langle \theta_{\text{proton}} \rangle \approx 45^\circ$ ). The Higgs particle carries away almost all ( $\approx 90\%$  on the average) of the neutrinos energy and is peaked very far forward (85% have  $\theta \leq 0.8^\circ$  for  $E_\nu = 200$  GeV and  $M_H = 5$  GeV). The decay products of  $H^+$  are isotropic in its rest frame, and have a maximum momentum perpendicular to the beam direction of about  $M_H/2$ . In the laboratory frame, the distribution of the decay products peak at  $\theta = 0$  and most events are confined to angles less than a few degrees. If the Higgs particle decays leptonically then we have the outstanding signal of a fast positive lepton, a slow  $\mu^-$ , a slow recoil proton, and no extra hadrons. If  $H^+$  decays to hadrons, the events are characterized by very fast forward hadrons. Classified as deep inelastic they are high  $y$  and very low  $x$ . In Ref. 12 it was estimated that 30% of the events would have both  $x < 0.03$  and  $y > 0.85$ . If the Higgs particles are coupled most strongly to high mass states,  $H^+$  will decay mainly to  $\tau^+\nu_\tau$  and/or  $\bar{s}c$ . Since  $\tau^+$  has a branching ratio to  $\mu^+$  of about 20%, and about 10% of  $D$  meson contain a muon, we expect dimuon events with the above unusual properties. These characteristics make it possible that light-mass Higgs mesons may be visible in neutrino scattering.

#### D. Hadronic production

Much work has recently been done on the hadronic production of a neutral Higgs particle in the Weinberg-Salam model. The most promising process appears to be gluon annihilation as in Fig. 7. This process has been discussed in Ref. 8.

Charged Higgs particles again have an advantage over neutral particles due to their electromagnetic coupling. If the energy is great enough  $H^+H^-$  pairs may be produced by the Drell-Yan processes in Fig. 8. The ratio of this to the standard  $\mu$  pair production at the same  $q^2$  is

$$\frac{\sigma(AB \rightarrow H^+H^-X)}{\sigma(AB \rightarrow \mu^+\mu^-X)} = \frac{1}{4} \left(1 - \frac{4m_H^2}{q^2}\right)^{3/2}. \quad (38)$$

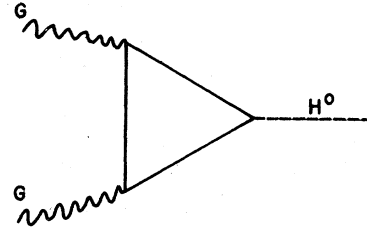


FIG. 7. Dominant production mechanism for a neutral Higgs particle.

Detection of  $H^+H^-$  through its decay products may be difficult. If  $H^+H^-$  decays (either directly or by cascading through heavier states) to  $\mu^+\mu^-X$  the muon pair will appear to have lower invariant mass than the real (but unobservable)  $q^2$ , and will thus be swamped in a much larger background than Eq. (38) suggests. Looking for a threshold this way is futile. Perhaps more reasonable is to look for muon pairs with high transverse momentum. In the standard Drell-Yan process the muon pair have a known limited transverse momentum. However, the Higgs-particle decay can give a single muon a transverse momentum of up to  $M_H/2$  and both muons may appear in the same direction. Thus a way to look for the Higgs-particle process is to look for a threshold in the production of muons with anomalously large transverse momentum for the pair. Detection through the hadronic decays seems less promising for existing experiment.

If an experiment does not have the energy to produce a pair of Higgs particles, it is still possible to produce a single one. In this regard charged Higgs particles are inferior to neutral ones, because the diagram of Fig. 7 is not allowed. Instead we are left with the diagram of Fig. 9. The diagram of Fig. 9(a) has been calculated in Ref. 8 where a cross section of

$$\frac{d\sigma}{dy} = \frac{1}{6} \pi \frac{G_F m_d^2}{12 M_H^2} \tau F_u(\tau^{1/2} e^y) F_d(\tau^{1/2} e^{-y}) \quad (39)$$

was obtained with  $\tau = M_H^2/s$ , for interaction

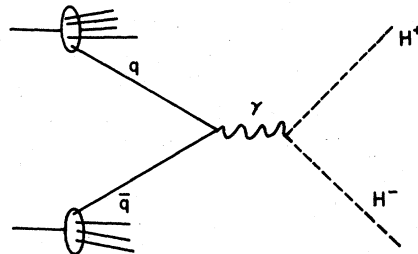
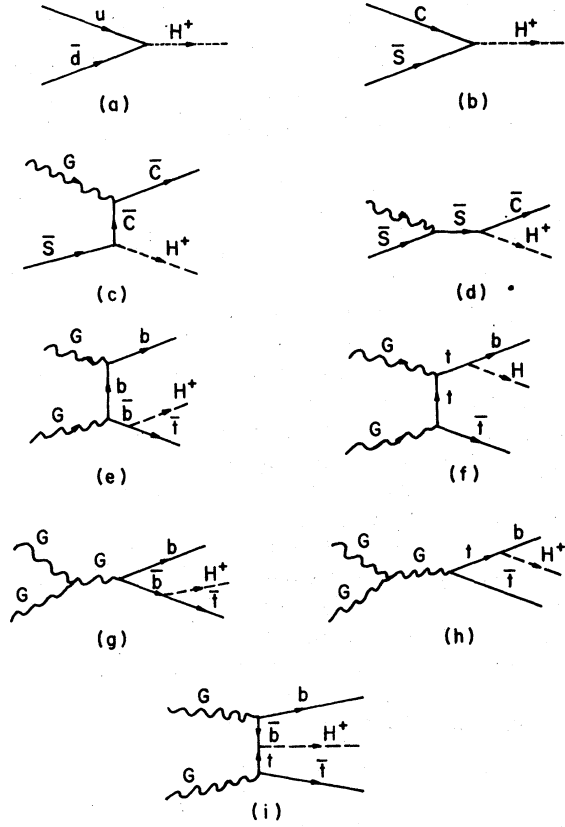


FIG. 8. Drell-Yan production of  $H^+H^-$ .

FIG. 9. Hadronic production of a single  $H^+$ .

$$L_{H\bar{u}d} = K_d G_F^{1/2} m_d \bar{u} H^+ d. \quad (40)$$

We see that this is a poor production mechanism when  $K$  is  $O(1)$  because of the small masses of  $m_d$  and  $m_u$ . If  $K$  were  $O(m_Q/m_d)$  with  $Q$  being a heavy quark, this would presumably be the dominant process. With  $K \sim O(1)$  we must search for processes with higher masses involved. Figure 9(b) satisfies this, but if involves two sea distributions, and the charmed sea especially is expected to be strongly suppressed. The diagrams 9(c)–9(i) are other ways to obtain strong coupling. We have included diagrams 9(e)–9(i), but we will not consider them further, because they contain an energy threshold due to the production of two heavy quarks in addition to  $H^+$ . For  $H^+$  masses to be probed by currently conceived experiments, this threshold is as bad or worse than that in  $H^+H^-$  production, so that the Drell-Yan process would dominate. This leaves Figs. 9(c) and 9(d) as potentially dominant contributions. These are of the form

$$\frac{d\sigma}{d\Omega} = \int dx_G F_G(x_G) \int dx_s F_s(x_s) \frac{d\sigma}{d\Omega}(\hat{s}, \hat{t}, \hat{u}),$$

where  $x_G$  and  $x_s$  are the momentum fractions of the

gluon and the strange quark within their respective hadrons, and  $(d\sigma/d\Omega)(\hat{s}, \hat{t}, \hat{u})$  is the cross section for the subprocess  $G + s \rightarrow H + c$ . The expression for the full cross section is lengthy and we will not display it here. However, at high  $\hat{s}$  it is dominated by the  $t$  channel pole in Fig. 9(c). Neglecting terms which vanish as  $\hat{s}^{-1}$  or faster, and using

$$L_{H\bar{c}s} = K G_F^{1/2} m_c \bar{c} \left( \frac{1 + \gamma_5}{2} \right) s H^+, \quad (41)$$

we find

$$\frac{d\sigma}{d\Omega}(\hat{s}, \hat{t}, \hat{u}) \approx \frac{K^2 G_F^2 \alpha_s m_c^2}{3} \frac{\hat{t} - m_H^2}{(\hat{t} - m_c^2)^2}. \quad (42)$$

Most of the cross sections are concentrated at low transverse momentum where most of the hadronic debris is found, so observing such a small signal would be extremely difficult.

#### E. Virtual effects

In addition to direct production it may be possible to observe Higgs particles the way we have learned about  $W$  bosons, i.e., through their virtual effects. Below we will list some of these ways. However, the successes of the standard picture already tell us that no such effects have yet been seen. These processes then are rather limits on the parameters in a given theory. Observing Higgs particles this way will only be possible if experiments are pushed beyond their present limits and deviations from the standard picture are found.

In most semileptonic processes, the contribution of Higgs-particle exchange relative to  $W$  exchange is of order  $(K_e m_e K_q m_u / M_W^2)$  (or  $m_e \rightarrow m_u$ ) which is  $\leq 0$  ( $10^{-7}$ ) and impossible to see. An exception is in pion decay, where the helicity structure of the vector and axial-vector currents suppresses the normal rate dramatically. Scalar interactions are not helicity suppressed. If we write the Higgs-particle coupling as

$$L_{H\bar{u}d} = G_F^{1/2} 2^{-1/4} H^+ [K_q (m_d + m_u) \bar{u} \gamma_5 d + K_e m_e \bar{e} (1 + \gamma_5) \nu], \quad (43)$$

the matrix element to  $\pi \rightarrow e \nu$  is

$$\begin{aligned} \mathfrak{M} = & \frac{f_\pi G_F m_e}{\sqrt{2}} \left[ 1 + \frac{K_q K_e}{M_H^2} \frac{(m_u + m_d)}{f_\pi} \langle \pi | \bar{u} \gamma_5 d | 0 \rangle \right] \\ & \times \bar{e} (1 + \gamma_5) \nu, \end{aligned} \quad (44)$$

where the first term is the usual  $W$ -exchange piece. An estimate of  $\langle \pi | \bar{u} \gamma_5 d | 0 \rangle$  using PCAC (partial conservation of axial-vector current) [ $(m_u + m_d) \bar{u} \gamma_5 d = \partial_\mu \bar{u} \gamma_\mu \gamma_5 d = f_\pi m_\pi^2 \phi_\pi$ ] yields

$$\mathfrak{M} = \frac{f_\pi G_F m_e}{\sqrt{2}} \left( 1 + \frac{K_q K_e m_\pi^2}{M_H^2} \right) \bar{e} (1 + \gamma_5) \nu. \quad (45)$$

If  $K_e \neq K_\mu$  this could easily destroy  $\mu e$  universality in

$$R = \frac{\Gamma(\pi \rightarrow e\nu)}{\Gamma(\pi \rightarrow \mu\nu)} = R_0 \left( \frac{1 + K_e K_e m_\tau^2 / M_H^2}{1 + K_e K_\mu m_\tau^2 / M_H^2} \right)^2, \quad (46)$$

where  $R_0$  is the calculated ratio without Higgs-particle exchange. In particular if the Higgs-particle couplings to leptons were governed by heavy leptons  $K_e \approx m_L / m_e$  and  $K_\mu \approx m_L / m_\mu$ . With  $K_q \approx 1$  and  $m_L = m_\tau$ , a Higgs particle of  $m_H = 80$  GeV would produce a 1% shift in  $R$ . In such theories this process provides a strong limit on the product of quark and lepton couplings of any Higgs particle. The present experimental value is approximately 2 standard deviations from the best theoretical value including radiative corrections. If higher-precision experiments are performed and a discrepancy is found, Higgs particles may be the cause.

In purely hadronic weak interactions, charged Higgs particles may play a role. Branco and Holstein have both suggested that the  $\Delta I = \frac{1}{2}$  rule may result from Higgs exchange.<sup>14</sup> The charged Higgs particles couple to both  $\bar{c}s$  and  $\bar{c}d$  and the Lagrangian

$$L = K_c^2 \frac{G_F}{\sqrt{2}} \frac{m_q^2}{m_H^2} \cos\theta_c \sin\theta_c \bar{d}(1 - \gamma_5)c \bar{c}(1 + \gamma_5)s \quad (47)$$

is purely  $\Delta I = \frac{1}{2}$ . This involves charmed quarks which are not common in hadrons, but we may have  $K_q > 1$  to compensate for this. In any case this needs a light Higgs particle, which would soon be directly observable.

Another way in which Higgs particles may have reasonable effects is in radiative corrections such as  $(g-2)$  for the muon and electron. Using the notation of Eq. (43) the contribution to the magnetic moment of the electron is

$$\mu_e = \frac{e}{48\pi^2} \frac{G_F}{\sqrt{2}} m_e \frac{K_e^2 m_e^2}{m_H^2}, \quad (48)$$

with an obvious change for the muon case. This may be significant if  $K_\mu \sim m_L / m_e$  and  $m_H$  is light.

#### F. Summary

Since it is the scalar particles that govern the pattern of spontaneous symmetry breaking in a gauge theory, there are many situations where additional Higgs particles are theoretically desirable. We have outlined the consequences of such a situation, and have looked for signature for light enough Higgs particles. Charged Higgs particles have an advantage that they can be produced in pairs by a virtual photon, making  $e^+e^-$  reactions a good place to look for them. They also mediate semiweak flavor-changing interactions in quarks and leptons, allowing them to be produced in decays of heavy leptons and new quarks and in neutrino scattering. While the detection of Higgs particles is well known to be difficult, their discovery would provide the exciting evidence that the weak and electromagnetic interactions are described by a spontaneously-broken gauge theory.

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