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Properties of conformal supergravity

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We complete our program of constructing the gauge theory of the superconformal group, and show that the previously proposed action is completely invariant under both local supersymmetries. The gauge algebra closes off-shell as well as on-shell. A flat-space model with a local supersymmetry is also presented.

I. INTRODUCTION

Supersymmetry unifies spacetime with internal symmetries by adding new, fermionic symmetries. The result is a graded group. Gauge theories based on these graded groups are called supergravity theories. Since there are two spacetime symmetry groups of interest, the Poincaré (or de Sitter) and the conformal group, there are two corresponding classes of supergravity theories. Poincaré-supergravity theories, which can accommodate $O(N)$ internal symmetries, have been studied extensively in the past two years and their advantages and shortcomings are by now fairly well understood.[1] In a recent series of articles[2,3] we have started to construct conformal supergravity theories which can accommodate $U(N)$ internal symmetries.[4] In this article we complete our previous work on conformal supergravity with U(1) internal symmetry.[3] In particular, we establish invariance of the action under both local supersymmetries (Q and S) corresponding to the square roots of the translation P_μ and conformal boosts K_μ ($Q=\sqrt{P}$ and $S=\sqrt{K}$). Previously we established S supersymmetry exactly and Q supersymmetry of the complete interacting theory up to terms in the varied action linear in the Q-gauge field. We also demonstrate that the algebra of conformal supergravity closes off-shell, unlike the gauge algebra of Poincaré supergravity.[5] Hence the new Feynman rules[6] for Poincaré supergravity are not needed for conformal supergravity. Finally, we show that a truncation of our theory leads to a flat-space model with a *local* supersymmetry, thus demonstrating that gravity is not strictly necessary for local supersymmetry.

Our method of construction[7,8] uses the group curvatures $R^A_{\mu\nu}$ given by

$$R^A_{\mu\nu}=\partial_\nu h^A_\mu-\partial_\mu h^A_\nu+f_{BC}{}^{A}h^B_\nu h^C_\mu\,, \tag{1.1}$$

where h^A_μ are the gauge fields corresponding to the generators X_A of the graded group with the (anti)commutation relations

$$[X_A,X_B\}=f_{BA}{}^{C}X_C\,. \tag{1.2}$$

We consider an action bilinear in these curvatures,

$$I=\int d^4x\{R^A_{\mu\nu}R^B_{\rho\sigma}Q^{\mu\nu\rho\sigma}_{AB}\}\,. \tag{1.3}$$

Proper choice of the coefficients Q can only lead to invariance under some of the local symmetries. Invariance under the remaining symmetries requires constraints on the curvatures. As in Poincaré gravity and supergravity[7] and in conformal gravity,[2] also in conformal supergravity,[3] one needs the constraint

$$R^a_{\mu\nu}(P)=0\,. \tag{1.4}$$

In addition, one also needs the following two constraints:

$$R_{\mu\nu}(Q)+\tfrac{1}{2}\tilde{R}_{\mu\nu}(Q)\gamma_5=0\,, \tag{1.5}$$

$$R_{\mu\nu}(Q)\sigma^{\mu\nu}=0\,, \tag{1.6}$$

which imply

$$R_{\mu\nu}(Q)\gamma^\mu=\tilde{R}_{\mu\nu}(Q)\gamma^\mu=0 \tag{1.7}$$

as well. The self-duality constraint on $R(Q)$ was previously found to be necessary for K and S invariance. The new constraint (1.6) is necessary for Q invariance as we shall show. In the noninteracting theory, the linearization of this constraint was found as an identity in Ref. 9.

The constraints on curvatures, which are the pivot of our work, follow naturally from the requirement of complete invariance under *all* local symmetries. Fields which are expressed in terms of other fields as a solution of the constraints in general no longer transform according to the gauge prescription. We present a useful theorem which enables one to obtain the modified transformation rules immediately. Use of this theorem leads to a simple proof of Q supersymmetry. These modified transformation rules also allow us to give a geometrical interpretation of the constraints.

The physical fields in our theory are the spin-2 vierbein field $e_{a\mu}$, the spin-$\frac{3}{2}$ Q-supersymmetry field ψ_μ, and the axial-vector field A_μ. The gauge field $\omega_{\mu ab}$ of local Lorentz invariance is eliminated by (1.4) while the S-supersymmetry gauge field is completely eliminated by (1.5) and (1.6). The constraint (1.5) alone allows still an independent spin-$\frac{1}{2}$ field χ, which we previously gauged away by a local S-supersymmetry transformation[3] but which we now eliminate by (1.6). The gauge field of conformal boosts is eliminated through its nonpropagating field equation while the dilatation gauge field drops out of the action altogether. The kinetic terms of the action are

$$L^{\text{kinetic}} = (R_{\mu\nu}{}^2 - \tfrac{1}{3}R^2) - \tfrac{3}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \tfrac{2}{3}\bar{\psi}_\mu[\not{\partial}(\Box\,\delta_{\mu\nu} - \partial_\mu\partial_\nu) - \tfrac{1}{2}\gamma_5\gamma_\rho\partial_\sigma\Box\,\epsilon^{\mu\nu\rho\sigma}]\psi_\nu\,, \qquad (1.8)$$

that is, a sum of the spin-2 Weyl, spin-1 Maxwell, and conformally invariant spin-$\frac{3}{2}$ action, and contain no lower-spin (gauge) components, as discussed in Ref. 4, since Q, S, D, and general coordinate invariance eliminate them. The particle content of this higher-derivative theory has been shown[9] to be two spin-2 (since $\Box^2$ describes two particles with each one $\Box$), three spin-$\frac{3}{2}$ (since $\not{\partial}\Box$ describes three particles with each one $\not{\partial}$), and one spin-1 particle. Thus there are an equal number of bosons and fermions in the theory.

In this article, we present mostly new results. A pedagogical review with many more details and explanations of this and previous articles on conformal supergravity is in preparation. In Sec. II we present the action and transformation laws. In Sec. III we determine the modification of the transformation laws of the nonphysical fields due to the presence of constraints. In Sec. IV we demonstrate invariance of the action under all 24 local symmetries, in particular complete Q-supersymmetry invariance. In Sec. V we obtain the gauge algebra and show that it closes even off-shell. Section VI contains the flat-space model with local supersymmetry. In Sec. VII we interpret our results.

Our conventions are as follows:

$$\begin{aligned}
&\delta_{\mu\nu} = (+,+,+,+)\,, \quad \mu,\nu = 1,2,3,4\,,\\
&\epsilon^{1234} = \epsilon_{1234} = +1\,, \quad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4\,, \quad \gamma_\mu = \gamma_\mu^\dagger\,, \quad \gamma_5 = \gamma_5^\dagger\,,\\
&\gamma_\mu{}^2 = \gamma_5{}^2 = 1\,, \quad \sigma_{\mu\nu} = \tfrac{1}{4}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\,, \quad e = \det e_{a\mu}\\
&\tilde{R}^{\rho\sigma} = e^{-1}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu}\,, \quad R_{\mu b} = R_{\mu\nu ab}\,e^{a\nu}\,, \quad \chi = C\,\bar{\chi}^T\\
&C = -C^T = -C^{-1}\,, \quad C\gamma_a C^{-1} = -\gamma_a^T\,, \quad \gamma_\mu = e_\mu^a\gamma_a\\
&g_{\mu\nu} = e_{a\mu}e_\nu^a\,,
\end{aligned} \qquad (1.9)$$

where $R_{\mu b}$ is the Ricci tensor, χ is a Majorana spinor, and C is the charge conjugation matrix. The tensor $R_{\mu\nu}(Q)$ starts with $(\partial_\nu\bar{\psi}_\mu + \frac{1}{2}\bar{\psi}_\mu\sigma^{ab}\,\omega_{\nu ab}) - (\mu \leftrightarrow \nu)$. We define $\bar{R}_{\mu\nu}(Q)$ to be minus the charge conjugate of $R^T_{\mu\nu}(Q)$ so that it starts with $(\partial_\nu - \frac{1}{2}\omega_{\nu\,ab}\,\sigma^{ab})\,\psi_\mu - (\mu \leftrightarrow \nu)$. The vierbien field $e_{a\mu}$ and its inverse $e^{a\mu}$ are used to convert local Lorentz to world indices and vice versa. $g_{\mu\nu}$ (and its inverse $g^{\mu\nu}$) is used to lower (and raise) world indices and the flat-space metric δ_{ab} (and δ^{ab}) is used to lower (raise) local Lorentz indices. $\epsilon^{\mu\nu\rho\sigma}$ is a tensor density, so that $e^{-1}\epsilon^{\mu\nu\rho\sigma}$ is a tensor (as is $e\epsilon_{\mu\nu\rho\sigma}$).

II. THE ACTION, CONSTRAINTS, AND TRANSFORMATION RULES

The superconformal group[10] has the following 24 generators X_A, gauge fields h_μ^A, and gauge parameters ϵ^A:

$$\begin{aligned}
&X_A = P_\mu, K_\mu, M_{\mu\nu}, D, A, Q_\alpha, S_\alpha\,,\\
&h_\mu^A = e_{a\mu}, f_{a\mu}, \omega_{\mu ab}, b_\mu, A_\mu, \bar{\psi}_\mu^\alpha, \bar{\phi}_\mu^{\,\alpha}\,,\\
&\epsilon^A = \epsilon^a, \epsilon_K^a, \lambda^{ab}, \lambda_D, \lambda_A, \bar{\epsilon}_Q^{\,\alpha}, \bar{\epsilon}_S^{\,\alpha}\,.
\end{aligned} \qquad (2.1)$$

The (anti)commutation relations of the generators are given in Table I.

Parity conservation and the absence of dimensional constants led to the following action[3]:

$$I = \int d^4x\{\epsilon^{\mu\nu\rho\sigma}[\alpha R_{\mu\nu}^{ab}(M)R_{\rho\sigma}^{cd}(M)\,\epsilon_{abcd} + \beta R_{\mu\nu}(Q)\gamma_5\bar{R}_{\rho\sigma}(S) + \gamma R_{\mu\nu}(A)R_{\rho\sigma}(D)] + \delta e R_{\mu\nu}(A)R^{\mu\nu}(A)\}\,,$$
$$\beta = \delta = 2i\gamma = -8\alpha\,. \qquad (2.2)$$

TABLE I. (Anti) commutation relations of the superconformal group.

$[M_{ab}, M_{cd}] = \eta_{bc} M_{ad} + \eta_{ad} M_{bc} - \eta_{ac} M_{bd} - \eta_{bd} M_{ac}$
$[M_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b$
$[M_{ab}, K_c] = \eta_{bc} K_a - \eta_{ac} K_b$
$[P_a, D] = P_a$
$[K_a, D] = -K_a$
$[K_a, P_b] = -2(\eta_{ab} D + M_{ab})$
$[S, P_a] = \gamma_a Q, \quad [S, A] = \frac{3}{4} i\gamma_5 S, \quad [S, D] = -\frac{1}{2} S,$
$[Q, K_a] = -\gamma_a S, \quad [Q, A] = -\frac{3}{4} i\gamma_5 Q, \quad [Q, D] = \frac{1}{2} Q,$
$[Q, M_{ab}] = \sigma_{ab} Q, \quad \{Q_\alpha, Q_\beta\} = -\frac{1}{2}(\gamma^a C)_{\alpha\beta} P_a$
$[S, M_{ab}] = \sigma_{ab} S, \quad \{S_\alpha, S_\beta\} = \frac{1}{2}(\gamma^a C)_{\alpha\beta} K_a$
$\{Q_\alpha, S_\beta\} = -\frac{1}{2} C_{\alpha\beta} D + (\sigma^{ab} C)_{\alpha\beta} M_{ab} + (i\gamma_5 C)_{\alpha\beta} A \quad (a > b).$

TABLE II. Curvatures of the superconformal group. All curvatures are to be antisymmetrized in (μ, ν).

$R_{\mu\nu ab}(M) = R^{(0)}_{\mu\nu ab} - 4(e_{a\mu} f_{b\nu} - e_{b\mu} f_{a\nu}) - 2\bar\psi_\mu \sigma_{ab} \phi_\nu$
$R_{\mu\nu}(D) = -2\partial_\mu b_\nu + 4 e_{a\mu} f^a_\nu + \bar\psi_\mu \phi_\nu$
$R_{\mu\nu}(A) = -2\partial_\mu A_\nu - 2i\bar\psi_\mu \gamma_5 \phi_\nu$
$R^\alpha_{\mu\nu}(Q) = (2D_\nu \bar\psi_\mu + 2\bar\phi_\mu \gamma_\nu + b_\nu \bar\psi_\mu - \frac{3}{2} i A_\nu \bar\psi_\mu \gamma_5)^\alpha$
$R^\alpha_{\mu\nu}(S) = (2D_\nu \bar\phi_\mu - 2\bar\psi_\mu \gamma_a f^a_\nu - b_\nu \bar\phi_\mu + \frac{3}{2} i A_\nu \bar\phi_\mu \gamma_5)^\alpha$
$R_{\mu\nu a}(P) = -2\partial_\mu e_{a\nu} + 2\omega^{\ b}_{\mu a} e_{b\nu} + \frac{1}{2}\bar\psi_\mu \gamma_a \psi_\nu + 2 e_{a\mu} b_\nu$
$R_{\mu\nu a}(K) = -2\partial_\mu f_{a\nu} + 2\omega^{\ b}_{\mu a} f_{b\nu} - \frac{1}{2}\bar\phi_\mu \gamma_a \phi_\nu - 2 f_{a\mu} b_\nu .$

The curvatures are obtained from (1.1) and are given in Table II. The sums over (a,b), (c,d), and $\mu\nu$ are unrestricted.

The constraint $R_{\mu\nu a}(P) = 0$ can be solved algebraically to give

$$\begin{aligned} \omega_{\mu ab} &= -\omega_{\mu ab}(e) + (e_{b\mu} b_a - e_{a\mu} b_b) \\ &\quad + \tfrac{1}{4}(\bar\psi_\mu \gamma_b \psi_a - \bar\psi_\mu \gamma_a \psi_b - \bar\psi_a \gamma_\mu \psi_b), \\ \omega_{\mu ab}(e) &= \tfrac{1}{2}[e^\nu_a (e_{b\nu,\mu} - e_{b\mu,\nu}) + e^\lambda_a e^\rho_b e_{c\lambda,\rho} e^c_\mu] \\ &\quad - (a \leftrightarrow b). \end{aligned} \tag{2.3}$$

Thus there is torsion if b_μ or ψ_μ are nonzero.

The duality constraint $R_{\mu\nu}(Q) + \frac{1}{2}\tilde R_{\mu\nu}(Q)\gamma_5 = 0$ can be solved algebraically for ϕ_μ,

$$\begin{aligned} \phi_\mu &= \tfrac{1}{4}\gamma^\nu (S_{\mu\nu} + \tfrac{1}{2}\gamma_5 \tilde S_{\mu\nu}) + \gamma_\mu \chi, \\ S_{\mu\nu} &= (D_\nu \psi_\mu + \tfrac{1}{2} b_\nu \psi_\mu - \tfrac{3}{4} i A_\nu \gamma_5 \psi_\mu) - (\mu \leftrightarrow \nu), \\ D_\nu \psi_\mu &= \partial_\nu \psi_\mu - \tfrac{1}{2}\omega_{\nu ab}\sigma^{ab}\psi_\mu . \end{aligned} \tag{2.4}$$

The field χ is an arbitrary spin-$\frac{1}{2}$ field related to ϕ_μ by $\chi = \frac{1}{4}\gamma\cdot\phi$. The new constraint $R_{\mu\nu}(Q)\sigma^{\mu\nu} = 0$ eliminates χ,

$$\chi = \tfrac{1}{2}\sigma^{\mu\nu} S_{\mu\nu} . \tag{2.5}$$

Substituting (2.5) into (2.4) yields

$$\phi_\mu = \tfrac{1}{3}\gamma^\nu (S_{\mu\nu} + \tfrac{1}{4}\gamma_5 \tilde S_{\mu\nu}), \tag{2.6}$$

which can also be obtained directly from solving $R_{\mu\nu}(Q)\gamma^\nu = 0$.

The field equation for the proper conformal gauge field $f_{a\mu}$ is algebraic so that $f_{a\mu}$ can be eliminated,

$$f_{\mu\nu} = -\tfrac{1}{4}(\hat R_{\nu\mu} - \tfrac{1}{6} g_{\mu\nu}\hat R) + \tfrac{1}{8} R_{\lambda\mu}(Q)\gamma_\nu \psi^\lambda - \frac{i}{16}\tilde R_{\mu\nu}(A) \tag{2.7}$$

where $\hat R_{\nu\mu}$ denotes $R_{\nu\rho a\mu}(M) e^{a\rho}$ with $f_{a\mu}$ put equal to zero, and $\hat R = g^{\mu\nu}\hat R_{\mu\nu}$. Note that $R_{\mu\nu}$ is not symmetric due to torsion and the $\bar\psi\sigma\phi$ term. This equation holds whether or not $\omega_{\mu ab}$ and ϕ_μ have been eliminated since they are f independent. Note also that $f = g^{\mu\nu} f_{\mu\nu} = -\frac{1}{12}\hat R$.

Inserting (2.7) into (2.2) yields the action,

$$\begin{aligned} I = 8\alpha \int d^4x \{ & e R_{\mu\nu}(M) R^{\nu\mu}(M) - (e/3) R^2(M) + 4\epsilon^{\mu\nu\rho\sigma}\bar\phi_\rho \gamma_5 \gamma_\sigma D_\nu \phi_\mu - (3e/4) R_{\mu\nu}(A) R^{\mu\nu}(A) \\ & + e(i/2)\bar\psi_\mu \phi_\nu R^{\mu\nu}(A) + (3i)\partial_\sigma A_\nu \bar\psi_\rho \phi_\mu \epsilon^{\mu\nu\rho\sigma} + e(i/2) R_{\mu\nu}(M) R^{\mu\nu}(A) - 3i\epsilon^{\mu\nu\rho\sigma}\bar\phi_\rho \gamma_\sigma \phi_\mu A_\nu \\ & - e R^{\mu\nu}(M) R_{\lambda\mu}(Q)\gamma_\nu \psi^\lambda + e(i/4) R^{\mu\nu}(A) R_{\lambda\mu}(Q)\gamma_\nu \psi^\lambda + e\tfrac{1}{4}[R_{\rho\nu}(A)\gamma^\mu \psi^\sigma][R_{\lambda\mu}(Q)\gamma^\nu \psi^\lambda] \\ & - \bar\psi_\mu \sigma_{ab}\phi_\nu \bar\psi_\rho \gamma_5 \sigma^{ab}\phi_\sigma \epsilon^{\mu\nu\rho\sigma} \}, \end{aligned} \tag{2.8}$$

where ϕ_μ is given by (2.4). The first four terms yield (1.8). As we shall show shortly, the dilaton field b_μ drops from the action, so that b_μ can be put to zero in each of the terms in (2.8).

This action is invariant under gauge transformations on the remaining physical fields, $e_{a\mu}$, ψ_μ, and A_μ:

$$\delta^{\text{gauge}} h^A_\mu = (D_\mu \epsilon)^A = \partial_\mu \epsilon^A + f_{BC}{}^A h^B_\mu \epsilon^C , \tag{2.9}$$

for M, D, A, S, and Q symmetries. Invariance under K is trivial since the physical fields are K inert, while the action is not invariant under P-gauge transformations but rather (by construction) under general coordinate transformations. For example, under Q supersymmetry, the physical fields transform as

$$\begin{aligned}
&\delta_Q e_{a\mu} = \tfrac{1}{2}\bar{\epsilon}_Q \gamma_a \psi_\mu \,,\\
&\delta_Q \psi_\mu = \partial_\mu \epsilon_Q - \tfrac{1}{2}\omega_{\mu ab}(e,\psi)\sigma^{ab}\epsilon_Q - (3i/4)A_\mu \gamma_5 \epsilon_Q \,,\\
&\delta_Q A_\mu = (-i/3)\bar{\epsilon}_Q \gamma_5 \gamma^\lambda (\Sigma_{\mu\lambda} + \tfrac{1}{4}\gamma_5 \tilde{\Sigma}_{\mu\lambda}) \,,\\
&\Sigma_{\mu\nu} = [\partial_\nu - \tfrac{1}{2}\omega_{\nu\, ab}(e,\psi)\sigma^{ab} - (3i/4)A_\nu \gamma_5]\psi_\mu \\
&\qquad - (\mu \leftrightarrow \nu) \,.
\end{aligned} \tag{2.10}$$

The field b_μ may be omitted from the transformation laws since it is not present in the action.

III. MODIFICATION OF TRANSFORMATION LAWS IN THE PRESENCE OF CONSTRAINTS

As mentioned in the Introduction, the transformations of $\omega_{\mu ab}$ and ϕ_μ as given in (2.3) and (2.6) differ from the group transformations in (2.9). One can of course obtain the former by tedious application of the chain rule; instead, one may apply to advantage the following obvious theorem:

Theorem: The actual transformation laws of gauge fields in the presence of constraints are such as to maintain these constraints under variation.

Let a constraint be given by $R^A_{\mu\nu}\Gamma^{\mu\nu}_A = 0$ where Γ may depend on h^A_μ. Defining

$$\delta' h^A_\mu = \delta^{\text{actual}} h^A_\mu - \delta^{\text{gauge}} h^A_\mu \,, \tag{3.1}$$

one finds by applying the above theorem

$$(\delta^{\text{gauge}} R^A_{\mu\nu})\Gamma^{\mu\nu}_A + R^A_{\mu\nu}(\delta^{\text{gauge}}\Gamma^{\mu\nu}_A) = -\delta' h^B_\rho \frac{\delta}{\delta h^B_\rho}(R^A_{\mu\nu}\Gamma^{\mu\nu}_A) \,. \tag{3.2}$$

From (2.9) and the corresponding homogeneous rotation of curvatures,

$$\delta^{\text{gauge}} R^A_{\mu\nu} = f_{BC}{}^A R^B_{\mu\nu}\epsilon^C \,, \tag{3.3}$$

one can directly obtain $\delta'\omega$ and $\delta'\phi$.

Under K, M, D, A, and S gauge transformations one finds that the constraints transform into themselves. Consequently, the left-hand side of (3.2) vanishes and $\delta' h^A_\mu = 0$ for all these symmetries. Under Q-supersymmetry gauge transformations, however, the constraints are not maintained so that one finds nonzero $\delta'\omega$ and $\delta'\phi$. Since $R^a_{\mu\nu}(P)$ rotates into $R_{\mu\nu}(Q)$ under Q, (3.2) yields

$$\tfrac{1}{2}[\bar{\epsilon}_Q \gamma^a \bar{R}_{\mu\nu}(Q)] + (\delta'\omega_{\mu ab})e^b_\nu - (\delta'\omega_{\nu ab})e^b_\mu \,. \tag{3.4}$$

This can be solved for $\delta'\omega$ in the same way as one solves for the Christoffel symbol in terms of $g_{\mu\nu,\lambda}$:

$$\delta'\omega_{\mu ab} = -\tfrac{1}{4}[\bar{\epsilon}_Q \gamma_\mu \bar{R}_{ab}(Q) + \bar{\epsilon}_Q \gamma_a \bar{R}_{\mu b}(Q) + \bar{\epsilon}_Q \gamma_b \bar{R}_{a\mu}(Q)] \,. \tag{3.5}$$

Using the cyclic identity of (1.7) one finally obtains

$$\delta'\omega_{\mu ab} = \tfrac{1}{2}[R_{ab}(Q)\gamma_\mu \epsilon_Q] \,. \tag{3.6}$$

A similar procedure applied to the constraint $\gamma^\nu \bar{R}_{\mu\nu}(Q) = 0$ [which is equivalent to (1.5) and (1.6)] yields

$$\gamma^\mu[(3i/4)\gamma_5\epsilon_Q R_{\mu\nu}(A) - \tfrac{1}{2}R_{\mu\nu}(D)\epsilon_Q + \tfrac{1}{2}\sigma_{ab}\epsilon_Q R^{ab}_{\mu\nu}(M) - \tfrac{1}{2}\delta'\omega_{\mu ab}\sigma^{ab}\psi_\nu + \tfrac{1}{2}\delta'\omega_{\nu ab}\sigma^{ab}\psi_\mu] + \tfrac{1}{2}(\bar{\epsilon}_Q\gamma_\mu\psi_a)\gamma^a\bar{R}_{\mu\nu}(Q) = 2\delta'\phi_\nu + \gamma_\nu(\gamma\cdot\delta'\phi) \,. \tag{3.7}$$

After multiplication by γ_ν one solves for $\gamma\cdot\delta'\phi$, and reinserting the result and using (3.6) one finds

$$\begin{aligned}
\delta'\phi_\nu = {}& \tfrac{1}{4}\gamma^\mu[\gamma_5 R_{\mu\nu}(A) + \tfrac{1}{4}\tilde{R}_{\mu\nu}(A)]\epsilon_Q - \tfrac{1}{6}\gamma_\mu[R_{\mu\nu}(D) + \tfrac{1}{4}\gamma_5\tilde{R}_{\mu\nu}(D)]\epsilon_Q \\
& + \tfrac{1}{6}\gamma_\mu\sigma^{ab}[R_{\mu\nu\, ab}(M) + (\gamma_5/4)\tilde{R}_{\mu\nu\, ab}(M)]\epsilon_Q + \tfrac{1}{4}(\bar{\epsilon}_Q\gamma^\mu\psi^\lambda)[\gamma_\lambda\bar{R}_{\mu\nu}(Q)] - \tfrac{1}{12}(\bar{\epsilon}_Q\gamma^\mu\psi^\lambda)[\gamma_\nu\bar{R}_{\mu\lambda}(Q)] \\
& - \tfrac{1}{8}[\bar{\epsilon}_Q\gamma_\nu\bar{R}_{ab}(Q)](\gamma^\mu\sigma^{ab}\psi_\mu) + \tfrac{1}{24}[\bar{\epsilon}_Q\gamma^\mu\bar{R}_{ab}(Q)](\gamma_\nu\sigma^{ab}\psi_\mu) - \tfrac{1}{8}(\gamma^\mu\epsilon_Q)[R_{\lambda\mu}(Q)\gamma_\nu\psi^\lambda] \,,
\end{aligned} \tag{3.8}$$

where $\tilde{R}_{\mu\nu ab} = e\epsilon_{\mu\nu\rho\sigma}R^{\rho\sigma}{}_{ab}$. Amazing simplifications lead to the final result,

$$\delta'\phi_\nu = (i/4)\gamma^\mu[\gamma_5 R_{\mu\nu}(A) + \tfrac{1}{2}\tilde{R}_{\mu\nu}(A)]\epsilon_Q \,, \tag{3.9}$$

which we now prove. We proceed in several steps:

(1) We extract the $f_{a\mu}$ dependence from (3.8). This is,

$$\delta'_f \phi_\nu = -\gamma^\mu f_{\mu\nu} \,, \tag{3.10}$$

and this can be expressed in terms of $R(M)$, $R(Q)$, and $R(A)$ by means of (2.7).

(2) We collect together all $R(M)$, $R(Q)$, and $R(A)$ terms. The total $R(A)$ dependence is just that of (3.9) while the $R(M)$ terms sum to

$$\tfrac{1}{12}\gamma^{\mu}(\hat{R}_{\nu\mu}-\hat{R}_{\mu\nu})\epsilon_Q+(e^{-1}/12)\gamma_5\gamma_\rho\epsilon_Q[\epsilon^{\mu\rho ab}\hat{R}_{\mu\nu ab}-\tfrac{1}{2}\epsilon^{\lambda\sigma\tau\eta}\tilde{R}_{\lambda\sigma\tau}{}^{\rho}g_{\eta\nu}]. \tag{3.11}$$

(3) Some $R(Q)\psi$ terms in (3.8) must be Fierz transformed so that they are all of the generic form $[\bar{\psi}R(Q)]\epsilon_Q$. All terms in (3.8) then have the spinor structure $O^A\epsilon_Q$ where O^A are the 16 Dirac matrices. The terms for which O^A is a scalar, pseudoscalar, or tensor come only from a Fierz transformation of $R(Q)\psi$ terms and must vanish separately. Using the $R(Q)$ constraints this can be shown to be the case. All remaining terms have O^A equal to a vector or an axial vector.

(4) We collect together all the "$\bar{\psi}\phi$" terms coming from $R(M)$ and $R(D)$, but not $R(A)$.

The result of these manipulations is

$$\begin{aligned}\delta'\phi_\nu={}&(i/4)\gamma^\lambda[\gamma_5R_{\lambda\nu}(A)+\tfrac{1}{4}\tilde{R}_{\lambda\nu}(A)]\epsilon_Q+\tfrac{1}{6}\gamma^\mu(b_{\mu\nu}+\gamma_5\tilde{b}_{\mu\nu}/4)\epsilon_Q-\tfrac{1}{24}[\bar{\psi}\cdot\gamma\bar{R}_{\lambda\nu}(Q)]\gamma^\lambda\epsilon_Q\\&-\tfrac{1}{12}[\bar{\psi}\cdot\gamma\gamma_5\bar{R}_{\lambda\nu}(Q)]\gamma_5\gamma^\lambda\epsilon_Q+\tfrac{1}{8}[\bar{\psi}^\mu\gamma_\lambda\gamma_5\bar{R}_{\mu\nu}(Q)]\gamma_5\gamma^\lambda\epsilon_Q+\tfrac{1}{12}\gamma^\mu\epsilon_Q(R^{(0)}_{\nu\mu}-R^{(0)}_{\mu\nu})\\&+(e^{-1}/12)\gamma_5\gamma_\lambda\epsilon_Q(\epsilon^{\mu\lambda ab}R^{(0)}_{\mu\nu\,ab}+\tfrac{1}{2}\epsilon_\nu{}^{\mu\alpha\beta}R^{(0)}_{\alpha\beta\mu}{}^{\lambda})+\tfrac{1}{12}\gamma^\mu\epsilon_Q(\bar{\psi}_\mu\sigma^\lambda{}_\nu\phi_\lambda-\bar{\psi}_\lambda\sigma^\lambda{}_\nu\phi_\mu-\bar{\psi}_\nu\sigma^\lambda{}_\mu\phi_\lambda+\bar{\psi}_\lambda\sigma^\lambda{}_\mu\phi_\nu)\\&-(e^{-1}/12)\gamma_5\gamma_\lambda\epsilon_Q\epsilon^{\mu\lambda ab}(\bar{\psi}_\mu\sigma_{ab}\phi_\nu-\bar{\psi}_\nu\sigma_{ab}\phi_\mu)+(e^{-1}/12)\gamma_5\gamma_\lambda\epsilon_Q\epsilon^{\mu\rho\alpha\beta}\bar{\psi}_\alpha\sigma^\lambda_\mu\phi_\beta g_{\nu\rho}\\&-\tfrac{1}{12}(\bar{\psi}_\mu\phi_\nu-\bar{\psi}_\nu\phi_\mu)\gamma^\mu\epsilon_Q+(e^{-1}/24)\gamma_5\gamma_\mu\epsilon^{\mu\rho\alpha\beta}(\bar{\psi}_\alpha\phi_\beta)\epsilon_Q g_{\rho\nu},\end{aligned} \tag{3.12}$$

where $b_{\mu\nu}=\partial_\mu b_\nu-\partial_\nu b_\mu$. In Fig. 1 we give a schematic representation of where all the various terms come from. $R^{(0)}_{\mu\nu\,ab}$ is $R_{\mu\nu\,ab}(M)$ without the "ef" and without the "$\bar{\psi}\phi$" terms. To simplify (3.12) still further, we use the results that

$$e^{-1}\epsilon^{\lambda\rho\mu\nu}R^{(0)}_{\lambda\rho\mu\sigma}=\tilde{b}_\sigma{}^\nu-\tfrac{1}{2}(\bar{\psi}_\mu\gamma_\sigma\tilde{S}^\mu{}_\nu), \tag{3.13}$$

$$\begin{aligned}e^{-1}\epsilon^{\mu\rho ab}R^{(0)}_{\mu\nu\,ab}={}&\tilde{b}^\rho{}_\nu-\tfrac{1}{2}(\bar{\psi}_\alpha\gamma_\nu\tilde{S}^{\rho\alpha})\\&+\tfrac{1}{4}(\bar{\psi}_\alpha\gamma_\beta\tilde{S}^{\beta\alpha})\delta^\rho_\nu,\end{aligned} \tag{3.14}$$

$$\begin{aligned}R^{(0)}_{\mu\nu}-R^{(0)}_{\nu\mu}={}&2b_{\mu\nu}-\tfrac{1}{2}(\bar{\psi}_\nu\gamma^\alpha S_{\alpha\mu}-\bar{\psi}_\mu\gamma^\alpha S_{\alpha\nu}\\&+\bar{\psi}\cdot\gamma S_{\mu\nu}),\end{aligned} \tag{3.15}$$

where $S_{\mu\nu}$ was defined in (2.4). The right-hand side of these equations comes from the nonzero torsion. This allows us to rewrite the $R(M)$ terms in the form "$\bar{\psi}S$" as the explicit $b_{\mu\nu}$ terms in (3.13)–(3.15) cancel the explicit $b_{\mu\nu}$ terms in (3.12). We may also write the $R(Q)$ terms in this form by means of the identity

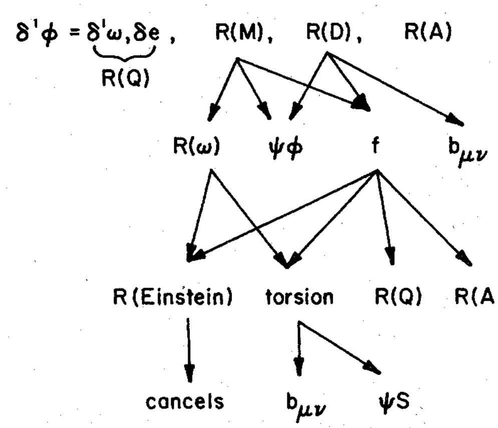

FIG. 1. Diagram of contributions to $\delta'\phi$.

$$\bar{R}_{\mu\nu}(Q)=\tfrac{1}{3}(S_{\mu\nu}-\tfrac{1}{2}\gamma_5\tilde{S}_{\mu\nu}+\sigma_\mu{}^\alpha S_{\nu\alpha}-\sigma^\alpha_\nu S_{\mu\alpha}). \tag{3.16}$$

Finally, the "$\bar{\psi}\phi$" terms in (3.12) may be cast into the "$\bar{\psi}S$" form by using the formulas

$$\begin{aligned}\sigma_{\lambda\mu}\phi_\nu=\tfrac{1}{6}(&\gamma_\lambda S_{\nu\mu}-\gamma_\mu S_{\nu\lambda}+\tfrac{1}{2}\gamma_\nu S_{\lambda\mu}+\tfrac{1}{2}\gamma^\alpha S_{\alpha\lambda}g_{\nu\mu}\\&-\tfrac{1}{2}\gamma^\alpha S_{\alpha\mu}g_{\nu\lambda}-\tfrac{1}{4}\gamma_5\gamma_\lambda\tilde{S}_{\nu\mu}+\tfrac{1}{4}\gamma_5\gamma_\mu\tilde{S}_{\nu\lambda}\\&-\tfrac{1}{2}\gamma_5\gamma_\nu\tilde{S}_{\lambda\mu}+\tfrac{1}{2}\gamma_5\gamma^\alpha\tilde{S}_{\alpha\mu}g_{\lambda\nu}\\&-\tfrac{1}{2}\gamma_5\gamma^\alpha\tilde{S}_{\alpha\lambda}g_{\mu\nu}),\end{aligned} \tag{3.17}$$

$$\sigma_{\lambda\mu}\phi^\mu=-\tfrac{1}{8}(\gamma_5\gamma^\mu\tilde{S}_{\mu\lambda}). \tag{3.18}$$

This reduces all terms in (3.12), except the $R(A)$ terms, to the generic form

$$\begin{aligned}&\gamma\epsilon_Q(\bar{\psi}\gamma S),\ \ \gamma\epsilon_Q(\bar{\psi}\gamma_5\gamma\tilde{S}),\ \ \gamma_5\gamma\epsilon_Q(\bar{\psi}\gamma\tilde{S}),\\&\gamma_5\gamma\epsilon_Q(\bar{\psi}\gamma_5\gamma S).\end{aligned} \tag{3.19}$$

All four sets of terms vanish independently. Thus $\delta'\phi_\nu$ is given by (3.9).

IV. FULL LOCAL SUPERSYMMETRY AND OTHER INVARIANCES

The main result of this section is the proof that our action (2.8) is locally Q supersymmetric. Previously we established local S supersymmetry as well as invariance under all other bosonic local symmetries, but our investigation of Q supersymmetry was incomplete. In order to establish Q invariance, we need the new constraint (1.6). First we must show that this constraint does not invalidate the previously established local symmetries.

We begin by considering the action in (2.2) where we can use the rotation of curvatures in (3.3). Under local M, D, and A gauge transformations this action is invariant, but under K and S gauge transformations one finds

$$\delta_K I = \int d^4x\,[-4\alpha R^{cd}_{\rho\sigma}(M) R^{b}_{\mu\nu}(P)\epsilon_{abcd} - \beta R_{\mu\nu}(Q)\gamma_5\gamma_a \bar{R}_{\rho\sigma}(Q) - 2\gamma R_{\rho\sigma}(A) R_{\mu\nu a}(P)]\,\epsilon^{\mu\nu\rho\sigma}\xi^a_K\,, \tag{4.1}$$

$$\delta_S I = \int e\, d^4x\,\{\tilde{R}^{cd}_{\mu\nu}(M)\bar{\epsilon}_S\sigma_{cd}\gamma_5\bar{R}^{\mu\nu}(Q)(4\alpha + \beta/2) + R_{\mu\nu}(Q)\gamma_5\epsilon_S\tilde{R}^{\mu\nu}(D)(-\beta/2 + i\gamma) + R_{\mu\nu}(A)[2i\delta R^{\mu\nu}(Q)\gamma_5\epsilon_S + (3i\beta/4 - \gamma/2)\tilde{R}^{\mu\nu}(Q)\epsilon_S] - \beta\tilde{R}^{\mu\nu}(S)\gamma_5\gamma_a\epsilon_S R^{a}_{\mu\nu}(P)\}\,. \tag{4.2}$$

Local K invariance requires the constraints

$$R^{a}_{\mu\nu}(P) = 0 = R_{\mu\nu}(Q) + c\tilde{R}_{\mu\nu}(Q)\gamma_5\,, \tag{4.3}$$

where c is an arbitrary constant. If c is not equal to $\pm\frac{1}{2}$ then the duality constraint implies that $R_{\mu\nu}(Q) = 0$, which constitutes 24 equations for the 16 components ϕ_μ and is therefore too strong a constraint. If $c = \pm\frac{1}{2}$ there are only 12 equations; the remaining four components of ϕ_μ constitute the spin-$\frac{1}{2}$ field χ encountered in Sec. II. We choose $c = +\frac{1}{2}$ which amounts to a sign choice for γ_5.

Local S invariance now follows from these constraints if one puts $\beta = \delta = 2i\gamma = -8\alpha$. We will assume these values from now on.

In principle the transformations of the fields are modified after solving the constraints and extra terms proportional to $\delta'\omega$ and $\delta'\phi$ would appear in (4.1) and (4.2). However, as shown in Sec. III, the δ' terms vanish for M, D, A, K, and S transformations so that all these invariances remain after solving (4.3). We now need to impose the constraint $\sigma^{\mu\nu}\bar{R}_{\mu\nu}(Q) = 0$. The same arguments show that this constraint also respects the M, D, A, K, and S local symmetries.

The reader might object that there are also possible terms in the varied action proportional to $\delta' f_{a\mu}$ since this gauge field has also been expressed in terms of other fields by means of its field equation. However, any such variation is multiplied by the $f_{a\mu}$ field equation in the varied action, which vanishes by definition.[11,8]

We now prove invariance of the action under Q supersymmetry. The variation of the action (2.2) under Q-gauge transformations following from (2.9) is given by

$$\delta^{\text{gauge}}_Q I = 8\alpha \int e\,d^4x\,\{\tilde{R}^{\mu\nu}(Q)\gamma_5\gamma_a\epsilon_Q R^{a}_{\mu\nu}(K) + 2iR^{\mu\nu}(S)[\gamma_5 R_{\mu\nu}(A) + \tfrac{1}{2}\tilde{R}_{\mu\nu}(A)]\epsilon_Q - \tfrac{1}{2}(\bar{\epsilon}_Q\gamma\cdot\psi)R_{\mu\nu}(A)R^{\mu\nu}(A) + 2(\bar{\epsilon}_Q\gamma^{\nu}\psi_\sigma)R_{\mu\nu}(A)R^{\mu\sigma}(A)\}\,. \tag{4.4}$$

There are additional variations containing $\delta'\phi$ and $\delta'\omega$. These extra terms are

$$\delta'_Q I = 8\alpha \int e\,d^4x\,\{2\delta'\omega^{ab}_\mu \tilde{R}^{\mu\nu c}(K)\epsilon_{dabc}e^d{}_\nu + 4\tilde{R}^{\mu\nu}(S)\gamma_\mu\gamma_5\delta'\phi_\nu + 4i\bar{\psi}_\mu[\gamma_5 R^{\mu\nu}(A) + \tfrac{1}{2}\tilde{R}^{\mu\nu}(A)]\delta'\phi_\nu\}\,. \tag{4.5}$$

This result was obtained by using the following formula[7] for the variation of the action (1.3) for arbitrary variations δh^A_μ:

$$\delta I = 4\int d^4x\,[\delta h^A_\mu h^B_\nu R^C_{\rho\sigma}(f_{BC}{}^D Q^{\mu\nu\rho\sigma}_{AD} - f_{AB}{}^D Q^{\mu\nu\rho\sigma}_{DC})] + \int d^4x\,(R^A_{\mu\nu}R^B_{\rho\sigma}\delta Q^{\mu\nu\rho\sigma}_{AB}) - 4\int d^4x\,[\delta h^A_\mu (D_\nu R_{\rho\sigma})^B Q^{\mu\nu\rho\sigma}_{AB} + \delta h^A_\mu R^B_{\rho\sigma}\partial_\nu(Q^{\mu\nu\rho\sigma}_{AB})]\,. \tag{4.6}$$

The two terms in the last set of square brackets in (4.6) only contribute if $Q^{\mu\nu\rho\sigma}_{AB}$ is not proportional to $\epsilon^{\mu\nu\rho\sigma}$. This follows from the fact that $\partial_\nu\epsilon^{\mu\nu\rho\sigma} = 0$ and the Bianchi identity $\epsilon^{\mu\nu\rho\sigma}(D_\nu R_{\rho\sigma})^B = 0$. But we use $\delta h^A_\mu = \delta' h^A_\mu$ and only $\delta'\phi_\mu$ and $\delta'\omega_{\mu ab}$ are nonzero. For these variations $Q^{\mu\nu\rho\sigma}_{AB}$ is proportional to $\epsilon^{\mu\nu\rho\sigma}$ in these terms which therefore do not contribute to (4.5). Note that when $A = (a,b)$, one considers only $a > b$. Since we sum always over unrestricted indices, care is required to obtain the correct factor 2 in (4.5).

We now consider the cancellation of independent terms in (4.4) and (4.5) separately. The $R(Q)R(K)$ term in (4.4) cancels with the $\delta'\omega R(K)$ term in (4.5) provided $\delta'\omega$ is given by (3.6). If one would only have the two constraints in (4.3), $\delta'\omega$ would be given by (3.5), and no cancellation would occur. At this point we discovered the need for the last constraint,

$$\sigma^{\mu\nu}\bar{R}_{\mu\nu} = 0\,, \tag{4.7}$$

which we adopt from this point on. Anticipating this constraint we were able to derive in Sec. III the simple form for $\delta'\phi$ of (3.9). Substituting (3.9) into (4.5) one finds immediately cancellations of the $R(A)R(S)$ and the $R(A)R(A)$ terms. This completes the proof of Q invariance.

Finally, we demonstrate that the dilaton field b_μ drops from the action as mentioned previously. The simplest proof of this fact is that, of the fields $e_{a\mu}$, ψ_μ, A_μ, and b_μ, only b_μ transforms under K. Its infinitesimal transformation is

$$\delta^K b_\mu = -2\xi^K_\mu . \tag{4.8}$$

Clearly, a finite K transformation can eliminate b_μ. On the other hand, the action is K invariant, hence it is b_μ independent. This elegant argument was checked explicitly.

V. CLOSURE OF THE GAUGE ALGEBRA OFF SHELL

In Poincaré supergravity the gauge algebra does not close off shell, i.e., the commutator of two local symmetry operations is only then again a local symmetry if one uses the equation of motion of some of the gauge fields. Recently it was shown that this lack of closure leads to modified Feynman rules for the quantized theory.[6] In particular a four-ghost coupling was needed to restore unitarity. It is therefore of importance for the quantization of conformal supergravity to investigate its gauge algebra. Surprisingly, the gauge algebra on the physical fields e^a_μ, ψ_μ, and A_μ does close without the need to use their field equations. This reinforces a previous conjecture[12,4] that in Poincaré supergravity the gauge algebra might close (and a simpler structure arise) with the addition of at least one auxiliary axial-vector field.

The nonphysical fields ϕ_μ, $\omega_{\mu ab}$, and $f_{a\mu}$ were eliminated by constraints and nonpropagating field equations. Hence b_μ will be the only nonphysical field contained in the gauge transformations of physical fields. In the action b_μ was absent, hence putting b_μ equal to zero is a consistent truncation. In the gauge algebra, however, it makes a difference whether one carries b_μ and its variations along or whether one puts it equal to zero right from the beginning. We will consider both cases separately and start with the b_μ-dependent gauge transformations.

The general commutator is given by

$$[\delta_2, \delta_1] h^A_\mu = \delta_3 h^A_\mu + [f_{BC}{}^A \delta'_2 h^B_\mu \epsilon^C_1 - (1 \leftrightarrow 2)] , \tag{5.1}$$

where A is restricted to the physical group indices belonging to $e_{a\mu}$, ψ_μ, and A_μ, and where δ_3 is again a gauge transformation with parameter $\epsilon^A_3 = f_{BC}{}^A \epsilon^B_2 \epsilon^C_1$. We note the following points:

(1) Only for Q-supersymmetry transformations are the $\delta' h^A_\mu$ nonzero, and only for $h^A_\mu = \omega_{\mu ab}$, ϕ_μ, and $f_{a\mu}$.

(2) $\delta' f_{a\mu}$ never contributes.

(3) $\delta'\phi_\mu$ and $\delta'\omega_{\mu ab}$ contribute only to the commutator of two local Q-supersymmetry transformations or to a $[P, Q]$ commutator.

(4) However, as in Poincaré supergravity, P is not a symmetry of the action but one must consider instead general coordinate transformations. One may check that the commutator of a general coordinate transformation and any of the local symmetries except P is again a local symmetry.

(5) $\delta'\phi_\mu$ contributes only to $[\delta_Q, \delta_Q] A_\mu$ and $\delta'\omega_{\mu ab}$ only to $[\delta_Q, \delta_Q]\psi_\mu$.

Using the previously obtained results for $\delta'\omega$ and $\delta'\phi$ in (3.6) and (3.9) one obtains the result

$$[\delta^Q_2, \delta^Q_1]\bar\psi_\mu = \delta^P_3 \bar\psi_\mu + \tfrac{1}{2}(\bar\epsilon_1 \gamma^\lambda \epsilon_2) R_{\mu\lambda}(Q) , \tag{5.2}$$

$$[\delta^Q_2, \delta^Q_1] A_\mu = \delta^P_3 A_\mu + \tfrac{1}{2}(\bar\epsilon_1 \gamma^\lambda \epsilon_2) R_{\mu\lambda}(A) , \tag{5.3}$$

$$[\delta^Q_2, \delta^Q_1] e_{a\mu} = \delta^P_3 e_{a\mu} , \tag{5.4}$$

where δ^P_3 is a P-gauge transformation with parameter $\frac{1}{2}(\bar\epsilon_1 \gamma_a \epsilon_2)$. However, a general coordinate transformation with parameter ρ^λ can be written as[13]

$$\delta_{\text{g.c.}} h^A_\mu = (D_\mu \epsilon)^A + \rho^\lambda R^A_{\mu\lambda} , \quad \epsilon^A = \rho^\lambda h^A_\lambda , \tag{5.5}$$

where $(D_\mu \epsilon)^A$ is a gauge transformation with parameters ϵ^A. Clearly, (5.2)–(5.4) are a sum of a general coordinate transformation plus local gauge transformations other than P. This establishes the closure of the gauge algebra even off shell in the presence of the nonphysical field b_μ.

The more realistic case in which one puts b_μ equal to zero in the transformation laws right from the beginning now follows easily. We return to (5.1) and derive expressions for $\delta'\phi_\mu$ and $\delta'\omega_{\mu ab}$ valid when b_μ and its variations are absent. These are obtained from the explicit b dependence of ϕ_μ and $\omega_{\mu ab}$,

$$\delta'_Q \omega_{\mu ab}(b=0) = \delta'_Q \omega_{\mu ab}(b \neq 0) - e_{b\mu}(\delta_Q b_a) + e_{a\mu}(\delta_Q b_b) , \tag{5.6}$$

$$\delta'_Q \phi_\mu(b=0) = \delta'\phi_\mu(b \neq 0) - \tfrac{1}{2}(\delta_Q b_a)\gamma^a \psi_\mu , \tag{5.7}$$

where $\delta_Q b_\mu = \frac{1}{2}\bar\epsilon_Q \phi_\mu$. Inserting these modified expressions into the $[\delta_Q, \delta_Q]$ commutator leads to an extra S-gauge supersymmetry transformation with field-dependent parameter. The final result is given by

$$[\delta_Q,\delta_Q]h_\mu^A(b=0)=\delta_{\text{g.c.}}[\tfrac{1}{2}(\bar{\epsilon}_1\gamma_\lambda\epsilon_2)]+\delta_{\text{loc. Lorentz}}[-\tfrac{1}{2}(\bar{\epsilon}_1\gamma^\lambda\epsilon_2)\omega_{\lambda ab}]$$
$$+\delta_{\text{chiral}}[-\tfrac{1}{2}(\bar{\epsilon}_1\gamma^\lambda\epsilon_2)A_\lambda]+\delta_{\text{Q-sym}}[-\tfrac{1}{2}(\bar{\epsilon}_1\gamma^\lambda\epsilon_2)\bar{\psi}_\lambda]$$
$$+\delta_{\text{S-sym}}[-\tfrac{1}{2}(\bar{\epsilon}_1\gamma^\lambda\epsilon_2)\bar{\phi}_\lambda]+\delta_{\text{S-sym}}[\tfrac{1}{4}(\bar{\phi}_\mu\epsilon_1)\bar{\epsilon}_2\gamma^\mu-(1\leftrightarrow 2)]\,, \tag{5.8}$$

where the last term is not present in (5.2) and (5.4). Again, the algebra closes off shell.

VI. FLAT-SPACE LOCAL SUPERSYMMETRY

Conformal supergravity differs from Poincaré supergravity in that it has two rather than one local supersymmetry, denoted by Q and S. Because in the graded Poincaré group $\{Q_\alpha, Q_\beta\}=(\gamma^\mu C)_{\alpha\beta}P_\mu$, it was suspected in the process of constructing supergravity that local supersymmetry is a deeper symmetry, underlying and necessarily implying gravity. Although this argument is correct for Poincaré supergravity, it does not apply to S supersymmetry in conformal supergravity. From the anticommutator

$$\{S_\alpha, S_\beta\}=\tfrac{1}{2}(\gamma^\mu C)_{\alpha\beta}K_\mu\,, \tag{6.1}$$

one might expect at first that an S-supersymmetric theory necessarily leads to f-curved spacetime, since in the algebra (P,Q) and (K,S) appear symmetrically up to signs. However, the constraints destroy this symmetry since they involve only the P and Q curvatures, and, as we have seen, one can eliminate the gauge field $f_{a\mu}$ but not $e_{a\mu}$. Since under S supersymmetry

$$\delta_S e_{a\mu}=0\,,\quad \delta_S\psi_\mu=-\gamma_\mu\epsilon_S\,,\quad \delta_S A_\mu=i(\bar{\epsilon}_S\gamma_5\psi_\mu) \tag{6.2}$$

one can consistently put $e_\mu^a=\delta_\mu^a$ in our action. This reduction of the U(1) conformal supergravity action is thus a flat-space locally supersymmetric field theory containing one spin-$\frac{3}{2}$ and one spin-1 field (or three spin-$\frac{3}{2}$ particles and one spin-1 particle).

Thus this model demonstrates that local supersymmetry can exist in flat spacetime, and that Fermi-Bose symmetry and spacetime symmetries are independent concepts.

One should investigate whether this model is also free from the higher-spin inconsistencies.

VII. CONCLUSIONS

We have shown that the previously constructed theory of conformal supergravity with U(1) internal symmetry[3] is invariant under both local supersymmetries, Q and S, as well as under all other bosonic symmetries. The action contains the spin-2 vierbein field, a spin-$\frac{3}{2}$ field, and an axial-vector field, and is a supersymmetric extension of Weyl's $R_{\mu\nu}{}^2-\frac{1}{3}R^2$ theory of conformal-invariant gravity.

We have developed techniques for handling gauge theories with constraints on the group curvatures. The requirements of invariance of the action under K (or S) and Q local symmetry led directly to the three constraints

$$R_{\mu\nu}^a(P)=0\,,\quad R_{\mu\nu}(Q)+\tfrac{1}{2}\tilde{R}_{\mu\nu}(Q)\gamma_5=0\,,$$
$$R_{\mu\nu}(Q)\sigma^{\mu\nu}=0\,. \tag{7.1}$$

In hindsight one can understand these constraints geometrically. In the absence of constraints, the $\{Q,Q\}=P$ relation tells us that the commutator of two Q-gauge transformations is a P-gauge transformation. However, the action is invariant by construction under general coordinate and not under P-gauge transformations. The relation between the two is given in (5.5). Hence, not all fields can transform according to the gauge prescription under Q transformations. Therefore there must be modified transformation laws denoted in (3.1) by $\delta_{\text{actual}}h_\mu^A=(D_\mu\epsilon)^A+\delta' h_\mu^A$. The relations $\delta_Q e_\mu^a=\frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu$ and $\delta_Q\psi_\mu=D_\mu\epsilon_Q$ lead to the same commutators as in Poincaré supergravity. Hence the same relation $\omega_\mu^{ab}=\omega_\mu^{ab}(e,\psi)$ as there is needed in order to lead to a general coordinate transformation. This is equivalent to requiring $R_{\mu\nu}^a(P)=0$.

The same argument for the commutator of two Q-supersymmetry transformations on A yields

$$[\delta_Q^2,\delta_Q^1]A_\mu=-i\bar{\epsilon}_Q^{(1)}\gamma_5\delta_Q'^{(2)}\phi_\mu-(1\leftrightarrow 2)\,, \tag{7.2}$$

which shows that $\delta_Q'\phi_\mu$ must be nonzero. The constraints on the Q curvatures lead indeed to the expression for $\delta'\phi_\nu$ in (3.9), which leads to the desired result in (5.3). Thus the role of the constraints is to convert the P-gauge transformations of the group to the general coordinate transformations of the spacetime manifold.

It would be interesting to see whether a superspace approach to conformal supergravity[14,9] would lead to the same constraints. In that case another geometrical interpretation might be possible.

We have shown that the gauge algebra of conformal supergravity closes without the need to use field equations. Thus the four-ghost coupling needed for unitarity in Poincaré supergravity[6] is absent in conformal supergravity. It is instructive to understand in this language why the Poincaré algebra does not close on ψ_μ [it does on e^a_μ since $R^a_{\mu\nu}(P)=0$]. The reason is that the cyclic constraint $\epsilon^{\mu\nu\rho\sigma}\gamma_\nu \overline{R}_{\rho\sigma}(Q)=0$ is not available to cast $\delta'\omega_{\mu ab}$ into the simple form of (3.6). In fact, this relation is in Poincaré supergravity just the spin-$\frac{3}{2}$ field equation.

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