Properties of logics of individual and group agency

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ABSTRACT. We provide proof-theoretic results about deliberative STIT logic. First we present STIT logic for individual agents without time, where the problem of satisfiability has recently been shown to be NEXPTIME-complete in the general case. Then we study STIT logic for groups of agents. We prove that satisfiability of STIT formulas involving groups of agents is undecidable by reducing the problem of satisfiability of a formula of the product logic $S5^n$ to group STIT satisfiability problem. We also prove that group STIT is not finitely axiomatizable.

Keywords: logics of agency, deliberative STIT, joint action, decidability, axiomatizability, complexity

1 Introduction

While logics of programs and actions such as PDL allow to reason about the relation between an action and its effects, so-called logics of agency are about the relation between an agent and the effects of his actions. The latter are relevant in game theory, theoretical computer science and philosophy of action.

In game theory, Pauly's coalition logic (CL) allows to reason about the capabilities of coalitions [14]. It provides for expressions of the kind "coalition J can make ϕ true at the next time point".

In theoretical computer science, Alternating-time Temporal Logics ATL and ATL* were introduced by Henzinger et al. in order to reason about distributed processes [2]. The formula $\langle\langle J\rangle\rangle\phi$ reads "coalition J has a strategy such that ϕ holds", where ϕ is a formula of linear temporal logic (that has to satisfy some restrictions in the case of ATL). Goranko showed that CL is nothing but a fragment of ATL (which in turn is a fragment of ATL*), by identifying the CL formula $[J]\phi$ with the ATL formula $\langle\langle J\rangle\rangle X\phi$ [9], where X is the temporal 'next' operator.

In philosophy of action constructions of the form $[i\ stit : \phi]$ were introduced by Belnap et col. [4], read "agent i sees to it that ϕ" or "i brings it about that ϕ". In this paper, we focus on the basic version that is called Chellas STIT [6] (thus baptized by [10]), noted $[i\ cstit : \phi]$ in the literature. (The original operator defined by Chellas is nevertheless notably different since it does not come with the principle of independence of agents that plays a central role in STIT theory.) The Chellas STIT was extended to group agency in [4, Section 10.C] and [11, Section 2.4]. For a set of agents J, the formula $[J\ cstit : \phi]$ reads "group J sees to it that ϕ". We here write $[J]\phi$ instead of $[J\ stit : \phi]$. These logics moreover have a modal operator of historical necessity $\Box$. Recently Broersen et al. showed that ATL can be embedded into the logic of the Chellas STIT, by identifying $\langle\langle J\rangle\rangle X\phi$ with $\neg\Box\neg[J]X\phi$ [5]. This highlights that the modal operators of CL and ATL are nothing but fusions of three modal operators. STIT-logics are therefore the most general formal framework for agency, allowing not only to reason about what agents *can do*, but also about what they *do*.

While it is known that the satisfiability problem is PSPACE-complete for coalition logic CL [14], EXPTIME-complete for ATL [19], and 2EXPTIME-complete for ATL* [15], only little is known about the mathematical properties of STIT logic. Up to now the only known results were restricted to the individual case: Wölfl gave an axiomatization [21], and Xu established axiomatization and decidability in [22] and [4, Chapter 17]. Wansing gave a complete tableaux calculus, but didn't prove termination [20]. In previous work we showed NEXPTIME completeness of the satisfiability problem [3].

The present paper investigates decidability and axiomatizability results for STIT logic without temporal operators. It is organized as follows. In Section 2 we recall the known results about individual STIT. In Section 3 we recall the definition of group STIT, providing also for an alternative semantics and a normal form which is built by rewriting all groups that are different from the 'grand coalition' (alias the set of all agents), to what we call 'anti-individuals' (alias complements of singleton groups). Building on results about the product Logic $S5^n$ that we recall in Section 4, we show in Section 5 that group STIT is undecidable, and in Section 6 that there is no finite axiomatization for it.

2 Individual STIT

In this section, we present the logic of agentive sentences of the form 'individual i sees to it that ϕ'.

The language $\mathcal{L}_{\mathsf{STIT}_n}$ of STIT_n logic is built from a countably infinite set of atomic propositions ATM and a finite set of agents $AGT = \{1, \ldots, n\}$. It is defined by the following BNF:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid [i]\phi \mid \Box\phi$$

where p ranges over ATM and i ranges over AGT. $[i]\phi$ is read "agent i sees to it that ϕ (whatever the other agents do)", and $\Box\phi$ is read "ϕ is historically necessary", or "ϕ holds whatever all the agents choose to do".

We use the usual dual operators: $\Diamond\phi$ abbreviates $\neg\Box\neg\phi$, and $\langle i\rangle\phi$ abbreviates $[i]\phi$.

REMARK 1. In the STIT literature the formula $[i]\phi$ is usually written $[i\ cstit : \phi]$, where '*cstit*' stands for 'Chellas STIT'. The formula $\Box\phi$ is sometimes written $Sett\ \phi$, where '*Sett*' stands for 'settled'.

REMARK 2. Other STIT operators exist: the so-called deliberative STIT operator $[i\ dstit : \phi]$ can be defined as an abbreviation of $[i]\phi \wedge \neg\Box\phi$; the so-called achievement STIT operator $[i\ astit : \phi]$ is more complex and will not be considered here.

2.1 Semantics of STIT_n

We present two semantics for $\mathcal{L}_{\mathsf{STIT}_n}$. The first one is the original one in terms of Belnap's branching-time models [4], while the second one does not mention time and is closer to standard presentations of Kripke models.

BT+AC models

Semantics is given to formulas of $\mathcal{L}_{\mathsf{STIT}_n}$ in terms of a branching-time (BT) structure augmented by an agent choice (AC) function.

DEFINITION 3 (BT structure). A *BT structure* is of the form $\langle M, <\rangle$, where M is a nonempty set of moments, and $<$ is a partial order on M (transitive and anti-symmetric) that is tree-like: for any m_1, m_2 and m_3 in M, if $m_1 < m_3$ and $m_2 < m_3$, then either $m_1 = m_2$ or $m_1 < m_2$ or $m_2 < m_1$.

A maximal set of linearly ordered moments from M is a *history*. When $m \in h$ we say that moment m is *on* the history h. $Hist$ is the set of all histories.

$$H_m = \{h \mid h \in Hist, m \in h\}$$

is the set of histories passing through m. A *moment-history pair* is a couple m/h, consisting of a moment m and a history h from H_m (i.e., a history and a moment in that history).

BT+AC models are BT structures augmented by agents' choices (AC) and a valuation.

DEFINITION 4 (BT+AC model). A *BT+AC model* is a tuple $\mathcal{M} = \langle M, <, Choice, V\rangle$, where:

- $\langle M, <\rangle$ is a BT structure;
- $Choice : AGT \times M \rightarrow 2^{2^{Hist}}$ is a function mapping each agent and each moment m into a partition of H_m such that for all m and all mappings $s_m : AGT \longrightarrow 2^{H_m}$ such that $s_m(i) \in Choice(i, m)$, we have $\bigcap_{i \in AGT} s_m(i) \neq \emptyset$;
- V is valuation function $V : ATM \rightarrow 2^{M \times Hist}$.

In terms of game theory, each mapping $s_m : AGT \rightarrow 2^{H_m}$ such that $s_m(i) \in Choice(i, m)$ for all i is a *strategy profile* at m. We write $Choice_i^m$ instead of $Choice(i, m)$. Each equivalence class belonging to $Choice_i^m$ can be thought of as a choice that is available to agent i at m: when $h, h' \in Choice_i^m$ then agent i's current choice at the moment-history pair m/h

cannot distinguish between h and h'. Given a moment m, we can view $Choice_i^m$ as a mapping from H_m to 2^{H_m} by defining:

$$Choice_i^m(h) = \{h' \in H_m \mid \text{ there is } Q \in Choice_i^m \text{ and } h, h' \in Q\}$$

Thus $Choice_i^m(h)$ returns the particular choice from $Choice_i^m$ containing h, or in other words, the particular action performed by i at the moment-history pair m/h: i's current choice at m/h forces the possible histories to be among $Choice_i^m(h)$.

We call the constraint of nonempty intersection of all possible simultaneous choices of agents at m the *independence constraint*.[1]

A formula is evaluated with respect to a model and a moment-history pair:

$$\begin{array}{lll} \mathcal{M}, m/h \models p & \text{iff} & m/h \in V(p), p \in ATM \\ \mathcal{M}, m/h \models \neg\phi & \text{iff} & \mathcal{M}, m/h \not\models \phi \\ \mathcal{M}, m/h \models \phi \wedge \psi & \text{iff} & \mathcal{M}, m/h \models \phi \text{ and } \mathcal{M}, m/h \models \psi \\ \mathcal{M}, m/h \models \Box\phi & \text{iff} & \mathcal{M}, m/h' \models \phi \text{ for all } h' \in H_m \\ \mathcal{M}, m/h \models [i]\phi & \text{iff} & \mathcal{M}, m/h' \models \phi \text{ for all } h' \in Choice_i^m(h) \end{array}$$

Validity in BT+AC models is defined as truth at every moment-history pair of every BT+AC model. A formula ϕ is satisfiable in BT+AC models if $\neg\phi$ is not valid in BT+AC models.

Kripke models

We now present an alternative semantics for $\mathcal{L}_{\mathsf{STIT}_n}$-formulas that is closer to that of standard modal logics, and was proposed in [13].

DEFINITION 5 (Kripke model). A Kripke model for the logic STIT_n is a tuple $\mathcal{W} = \langle W, R, V\rangle$ where:

- W is a nonempty set;
- R is a mapping associating to every $i \in AGT$ an equivalence relation R_i on W such that for all $(w_1, \ldots, w_n) \in W^n$, $\bigcap_{i \in AGT} R_i(w_i) \neq \emptyset$;
- V is a valuation function $V : ATM \to 2^W$.

Intuitively, R_i is nothing more than the equivalence relation corresponding to the partition $Choice_i^m$. The condition on R corresponds to the independence constraint of BT+AC models.

REMARK 6. Our Kripke models here correspond to the class of point-generated models of the semantics proposed in [3]. There, the independence constraint is formulated in a slightly different way (just because the models might not be point-generated).

A formula is evaluated as usual with respect to a model and a point.

[1] There are other constraints relating the BT structure and the choice function, such as “no choice between undivided histories”. They are not relevant here because we do not have temporal operators in our language, and we therefore omit them.

$\mathcal{W}, w \models p$	iff	$w \in V(p)$, for p $\in ATM$
$\mathcal{W}, w \models \neg\phi$	iff	$\mathcal{W}, w \not\models \phi$
$\mathcal{W}, w \models \phi \wedge \psi$	iff	$\mathcal{W}, w \models \phi$ and $\mathcal{W}, w \models \psi$
$\mathcal{W}, w \models \Box\phi$	iff	$\mathcal{W}, w' \models \phi$ for all $w' \in W$
$\mathcal{W}, w \models [i]\phi$	iff	$\mathcal{W}, w' \models \phi$ for all $w' \in R_i(h)$

Validity and satisfiability in Kripke models are defined as usual.

THEOREM 7. *A STIT_n-formula is satisfiable in BT+AC models iff it is satisfiable in Kripke models.*

Proof. The proof is done by transforming a given BT+AC model into a Kripke model and vice versa. It is a particular case of the proof of Theorem 11 in Section 3.1. ■

2.2 Axiomatization, decidability and complexity of STIT_n

Xu gave the following axioms:

S5($\Box$) the axiom schemas of S5 for $\Box$;

S5(i) the axiom schemas of S5 for every $[i]$, for every $i \in AGT$;

($\Box\!\to\! i$) $\Box\phi \to [i]\phi$, for every $i \in AGT$;

(AIA$_n$) $(\Diamond[1]\phi_1 \wedge \ldots \wedge \Diamond[n]\phi_n) \to \Diamond([1]\phi_1 \wedge \ldots \wedge [n]\phi_n)$.

(AIA$_n$) is called the axiom schema for independence of agents. Xu's axiomatics has the standard inference rules of modus ponens and necessitation for $\Box$. From the latter necessitation rules for every $[i]$ follow by axiom ($\Box\!\to\! i$).

From Xu's completeness theorem [4, Chapter 4] and Theorem 7 we get:

THEOREM 8. *[4, Chapter 17] A formula ϕ of $\mathcal{L}_{\mathsf{STIT}_n}$ is valid in Kripke models iff ϕ is provable from the schemas S5($\Box$), S5(i), ($\Box\!\to\! i$), and (AIA$_n$) by the rules of modus ponens and $\Box$-necessitation.*

REMARK 9. An alternative axiomatization is given in [3], where (AIA$_n$) is replaced by the simpler axiom schema $\Diamond\phi \to \langle k\rangle \bigwedge\{\langle l\rangle\phi \mid 1 \leq l \leq n \text{ and } l \neq k\}$.

It is also shown there that $\Diamond\phi$ can be viewed as an abbreviation of $\langle i\rangle\langle j\rangle\phi$, for some arbitrary i and j.

The complexity of the satisfiability problem for STIT_n-formulas depends of the number of agents.

THEOREM 10. *[3] The problem of deciding satisfiability of a formula of $\mathcal{L}_{\mathsf{STIT}_n}$ is NP-complete if $n = 1$, and it is NEXPTIME-complete if $n \geq 2$.*

3 Group STIT

In this section, we extend the individual STIT to group STIT: we study the logic of agentive sentences of the form 'group J sees to it that ϕ'. Now modal operators have as arguments coalitions $J \subseteq AGT$.

Just as in the individual case, the language $\mathcal{L}_{\mathsf{STIT}_n^G}$ of STIT_n^G logic is built from a countable set of atomic propositions ATM and a finite set of agents $AGT = \{1, \ldots, n\}$. But now the modal operators have sets of agents as arguments, and $\mathcal{L}_{\mathsf{STIT}_n^G}$ is defined by the following BNF:

$$\phi \ ::= \ p \mid \neg\phi \mid (\phi \wedge \phi) \mid [J]\phi$$

where p ranges over ATM and J ranges over 2^{AGT}.

The language of individual STIT becomes a fragment of that of group STIT if we identify $[\{i\}]$ with $[i]$. The semantics of group STIT will guarantee that the $\mathcal{L}_{\mathsf{STIT}_n^G}$ formula $[\emptyset]\phi$ has the same interpretation as the $\mathcal{L}_{\mathsf{STIT}_n}$ formula $\Box\phi$.

3.1 Semantics of STIT_n^G

Again, we present the semantics both in terms of branching-time models as defined by Horty and Belnap [10] and Belnap and Perloff [4, Chapter 10], and in terms of Kripke models [13]. We prove that both classes of models have the same logic. The latter will be useful to establish the relationship with product logics.

BT+AC models

Models for STIT_n^G are the same those of the logic STIT_n, i.e. BT+AC models satisfying the independence constraint. The only thing we have to do is to extend the definition of *Choice* in order to interpret group agency.

Horty defines in [11] the notion of collective choice. He first introduces action selection functions s_m from AGT into 2^{H_m} such that for each $m \in M$ and $a \in AGT$, $s_m(a) \in Choice_a^m$. So a selection function s_m selects a particular action for each agent at m. Then for a given m,

$$Select_m = \{s_m : AGT \to 2^{H_m} \mid s_m(a) \in Choice_a^m, \text{for all } a \in AGT\}$$

is the set of all such selection functions. This allows to extend the definition of *Choice*. A collective choice for a nonempty group of agents $\emptyset \subsetneq J \subseteq AGT$ at moment m is defined as:

$$Choice_J^m = \{\bigcap_{j \in J} s_m(j) \mid s_m \in Select_m\}$$

For $J = \emptyset$ we define $Choice_\emptyset^m = \{H_m\}$. We can check that every $Choice_J^m$ is a partition of H_m.

As before,

$$Choice_J^m(h) = \{h' \in H_m \mid \text{ there is } Q \in Choice_J^m \text{ and } h, h' \in Q\}$$

is the particular choice from $Choice^m_J$ containing h, or in other words, the particular joint action performed by coalition J at the moment-history pair m/h. And as before, formulas are interpreted with respect to a model and a moment-history pair:

$$\mathcal{M}, m/h \models [J]\phi \quad \text{iff} \quad \mathcal{M}, m/h' \models \phi \text{ for all } h' \in Choice^m_J(h).$$

Observe that the $\mathcal{L}_{\mathsf{STIT}^G_n}$-formula $[\emptyset]\phi$ is true at m/h if and only if the $\mathcal{L}_{\mathsf{STIT}_n}$-formula $\Box\phi$ is true at m/h.

Validity and satisfiability are defined as before.

Kripke models

Kripke models for STIT^G_n are the same as for STIT_n. Just as we defined $Choice^m_J$ from $Choice^m_i$ in the last section, we here define R_J from the R_is.

Let $\mathcal{W} = \langle W, R, V\rangle$ be a Kripke model for STIT_n. For all nonempty $J \subseteq AGT$, we define

$$R_J = \bigcap_{i\in J} R_i$$

and $R_\emptyset = W \times W$.

A formula is evaluated as usual with respect to a model and a world:

$$\mathcal{W}, w \models [J]\phi \quad \text{iff} \quad \mathcal{W}, w' \models \phi \text{ for all } w' \in R_J(w)$$

THEOREM 11. *A STIT^G_n-formula is satisfiable in BT+AC models iff it is satisfiable in Kripke models.*

Proof. $\boxed{\Rightarrow}$ Let $\mathcal{M}' = \langle M', <, Choice, V'\rangle$ be a BT+AC model such that $\mathcal{M}', m_0/h_0 \models \phi$ for some a moment-history pair m_0/h_0. We define the tuple $\mathcal{W} = \langle W, R, V\rangle$ as follows:

- $W = H_{m_0}$;
- $R_i = \{(h, h') \mid \text{there exists } Q \in Choice^{m_0}_i \text{ such that } h, h' \in Q\}$;
- $V(p)$ is the set of histories $h \in H_{m_0}$ such that $m_0/h \in V'(p)$.

We can check that $\mathcal{W}$ is a Kripke model. We can prove by induction on ψ that for all formulas ψ and for all $h \in H_{m_0}$, $\mathcal{W}, h \models \psi$ iff $\mathcal{M}', m_0/h \models \psi$. Hence, $\mathcal{W}, h_0 \models \phi$.

$\boxed{\Leftarrow}$ Let $\mathcal{W} = \langle W, R, V\rangle$ be a Kripke model such that $\mathcal{W}, w_0 \models \phi$ for some world $w_0 \in W$. We define the BT+AC model $\mathcal{M}' = \langle M', <, Choice, V'\rangle$ as follows:

- $M' = \{m_0\} \cup W$ for some $m_0 \notin W$;
- $< = \{m_0\} \times W$ (and thus $Hist = H_{m_0} = \{\{m_0, w\} \mid w \in W\}$);
- $Choice^{m_0}_i = \{\{\{m_0\} \times R_i(w)\} \mid w \in W\}$, and $Choice^w_i = \{\{h\}, h \in H_w\}$ for every $w \in W$;

- $V'(p)$ is the set of moment-history pairs $m_0/\{m_0, w\}$ such that $w \in V(p)$.

We can check that $\mathcal{M}'$ is a BT+AC model. We can also prove by induction on ψ that for all STIT_n formulas ψ and worlds $w \in W$, $\mathcal{M}', m_0/\{m_0, w\} \models \psi$ iff $\mathcal{W}, w \models \psi$. Hence $\mathcal{M}', m_0/\{m_0, w_0\} \models \phi$. ■

3.2 Normal form for STIT_n^G formulas

We now show that every formula of $\mathcal{L}_{\mathsf{STIT}_n^G}$ is equivalent to a formula where only the 'grand coalition' AGT and 'anti-individuals' occur, where the latter are complements of singleton groups.

LEMMA 12. *Let $\mathcal{W} = \langle W, R, V\rangle$ be a Kripke model. For all $(w_1, \ldots, w_k) \in W^k$ and $J_1, \ldots, J_k \in 2^{AGT}$ such that $j \neq l$ implies $J_j \cap J_l = \emptyset$ we have: $\bigcap_{j\in\{1\ldots k\}} R_{J_j}(w_j) \neq \emptyset$.*

Proof. This follows from the independence constraint, which says that

$$\bigcap_{i\in AGT} R_i(w_i) \neq \emptyset \text{ for all } (w_1, \ldots, w_n) \in W^n.$$

■

The following theorem holds for any J_1 and J_2 (that are not necessarily disjoint).

THEOREM 13. *Let $J_1, J_2 \subseteq AGT$. We have:*

$$\models_{STIT_n^G} [J_1 \cap J_2]\phi \leftrightarrow [J_1][J_2]\phi$$

Proof. Let $\mathcal{M} = \langle W, R, V\rangle$ be a Kripke model. We are going to prove that $R_{J_1\cap J_2} = R_{J_1} \circ R_{J_2}$.

$\boxed{\supseteq}$ As $J_1 \cap J_2 \subseteq J_1$, we have $R_{J_1} \subseteq R_{J_1\cap J_2}$ by definition of the relation R_J. Likewise, $R_{J_2} \subseteq R_{J_1\cap J_2}$. As $R_{J_1\cap J_2}$ is transitive, we have $R_{J_1} \circ R_{J_2} \subseteq R_{J_1\cap J_2}$.

$\boxed{\subseteq}$ Let $w, w' \in W$ such that $(w, w') \in R_{J_1\cap J_2}$. We are going to prove that $R_{J_1}(w) \cap R_{J_1}(w') \neq \emptyset$, i.e. that W contains a point u such that $(w, u) \in R_{J_1}$ and $(u, w') \in R_{J_2}$ (from which it immediately follows that $R_{J_1\cap J_2} \subseteq R_{J_1} \circ R_{J_2}$).

First, we have

$$R_{J_1}(w) \cap R_{J_1}(w') = R_{J_1\cap J_2}(w) \cap R_{J_1\setminus J_1\cap J_2}(w) \cap R_{J_1\cap J_2}(w') \cap R_{J_2\setminus J_1\cap J_2}(w')$$

by the above Lemma 12. Then, as $R_{J_1\cap J_2}(w) = R_{J_1\cap J_2}(w')$, we have

$$R_{J_1}(w) \cap R_{J_1}(w') = R_{J_1\cap J_2}(w) \cap R_{J_1\setminus J_1\cap J_2}(w) \cap R_{J_2\setminus J_1\cap J_2}(w').$$

As $J_1 \cap J_2$, $J_1 \setminus J_1 \cap J_2$, and $J_2 \setminus J_1 \cap J_2$ are pairwise disjoint, we have $R_{J_1}(w) \cap R_{J_1}(w') \neq \emptyset$ again by the above Lemma 12. ■

THEOREM 14. *Let $J \subsetneq AGT$ such that $AGT \setminus J = \{j_1, \ldots, j_r\}$, and let $\bar{j_i} = AGT \setminus \{j_i\}$. Then*

$$\models_{STIT_n^G} [J]\phi \leftrightarrow [\bar{j_1}] \ldots [\bar{j_r}]\phi$$

Proof. By induction on r, with base case $r = 1$ and using Theorem 13 for the induction step. ■

It follows that every STIT_n^G-formula can be written only using the grand coalition $[AGT]$ and anti-individuals $[\bar{i}]$s. From now on, we consider that a STIT_n^G-formula contains only such operators.

4 The product logic $\mathsf{S5}^n$

In this part, we briefly recall the product logic $\mathsf{S5}^n$. The reader is referred to [7] for more details.

Just as $\mathcal{L}_{\mathsf{STIT}_n}$, the language of $\mathsf{S5}^n$ logic is built from a countably infinite set of atomic propositions ATM and a set of parameters $\{1, \ldots, n\}$. It is defined by the following BNF:

$$\phi \ ::= \ p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i \phi$$

where p ranges over ATM and i ranges over $\{1, \ldots, n\}$.

4.1 Semantics of $\mathsf{S5}^n$

A Kripke model for $\mathsf{S5}^n$ is a cartesian product.

DEFINITION 15 ($\mathsf{S5}^n$ model). A $\mathsf{S5}^n$ model is a tuple $\mathcal{X} = (X, R, V)$ where:

- $X = X_1 \times \ldots \times X_n$ for some nonempty sets $X_1, \ldots, X_n$;
- R is a mapping associating to every $i \in \{1, \ldots, n\}$ the equivalence relation

$$R_i = \{\langle (x_1, \ldots, x_n), (y_1, \ldots, y_n) \rangle \in X^2 \mid x_j = y_j \text{ for all } j \neq i\}$$

- $V : ATM \to 2^X$.

Note that the usual $\mathsf{S5}^n$ models are more generally products of equivalence relations. Our $\mathsf{S5}^n$ models here are the subclass of point-generated models (that suffice for the characterization of $\mathsf{S5}^n$).

Definitions of truth conditions, validity and satisfiability are as usual.

4.2 A nonstandard axiomatics for $\mathsf{S5}^n$

We first recall the definition of finite axiomatizability of [7, Chapter 1].

DEFINITION 16 (finite axiomatizability). A logic L is finitely axiomatizable if there is a finite set Ax of formula schemas such that $\phi \in L$ iff there is a sequence $(\phi_1, \ldots, \phi_k)$ of formulas such that for $1 \leq i \leq k$, one of the following holds:

- ϕ_i is a tautology of classical proposition logic or an instance of an axiom in Ax;

- ϕ_i is obtained by necessitation from ϕ_j, where $j < i$;
- ϕ_i is obtained by modus ponens from ϕ_j and ϕ_k, where $j, k < i$;
- $\phi_k = \phi$.

THEOREM 17. *[7, Theorem 8.2] The logic $\mathsf{S5}^n$ is not finitely axiomatizable for $n \geq 3$.*

While $\mathsf{S5}^n$ can thus not be axiomatized in the standard way, there exists an axiomatization by means of a nonstandard rule.

THEOREM 18. *[18] $\mathsf{S5}^n$ is axiomatized by the following axiom schemas:*

- *$S5(\Box_i)$: the axiom schemas for S5, for every modal operator $\Box_i$*
- $\vdash \Box_i \Box_j \phi \leftrightarrow \Box_j \Box_i \phi$
- *Modus Ponens rule:*
$$\frac{\vdash \phi \qquad \vdash \phi \to \psi}{\vdash \psi}$$
- *Necessitation rule:*
$$\frac{\vdash \phi}{\vdash \Box_i \phi}$$
- *Rectangle Rule:*
$$\frac{\vdash (p \land \tau(\neg\phi \land p)) \to \phi}{\vdash \phi} \text{ if } p \text{ does not occur in } \phi$$
where $\tau(\chi) = \Box_1 \dots \Box_n[(\bigwedge_{i \in \{1,\dots,n\}} \Diamond_1 \dots \Diamond_{i-1} \Diamond_{i+1} \dots \Diamond_n \chi) \to \chi]$.

It is the Rectangle Rule which is nonstandard.

REMARK 19. The axiomatics in [18] does not have all the axioms of $S5(\Box_i)$. These are nevertheless valid in $\mathsf{S5}^n$ models and we have chosen to add them explicitly.

4.3 Undecidability of the satisfiability problem for $\mathsf{S5}^n$-formulas

While satisfiability of a formula of $\mathsf{S5}^n$ is decidable for $n = 2$, things get worse beyond.

THEOREM 20. *[18, Theorem 8.6] The problem of satisfiability of a formula of $\mathsf{S5}^n$ is undecidable for $n \geq 3$.*

5 Group STIT satisfiability is undecidable

We are going to map the problem of satisfiability in $\mathsf{S5}^n$ to the problem of satisfiability in STIT_n^G. The range of our mapping is the fragment of $\mathcal{L}_{\mathsf{STIT}_n^G}$ formulas where only the 'grand coalition' and 'anti-individuals' occur, i.e. the set of groups J such that either $J = AGT$, or $J = AGT \setminus \{i\}$ for some $i \in AGT$. We note $\bar{i}$ such sets. As satisfiability is undecidable for $\mathsf{S5}^n$, satisfiability in STIT_n^G cannot be decidable either.

The $\mathcal{L}_{\mathsf{S5}^n}$ formula $\Box_i\phi$ will be mapped to the $\mathcal{L}_{\mathsf{STIT}_n^G}$ formula $[\bar{i}]\phi$. For the ease of exposition, we identify these two kinds of formulas from now on, and suppose that formulas $\Box_i\phi$ are part of the STIT_n^G language.

Let $atm(\phi)$ be the set of all atomic propositions occurring in ϕ.

THEOREM 21. *For any $\phi \in \mathcal{L}_{\mathsf{S5}^n}$, the following are equivalent:*

1. *ϕ is $\mathsf{S5}^n$-satisfiable;*
2. *ϕ is satisfiable in a STIT_n^G model where $R_{AGT} = id_W$;*
3. *$[\emptyset](\bigwedge_{p\in atm(\phi)}[AGT]p \leftrightarrow p) \wedge \phi$ is STIT_n^G-satisfiable.*[2]

Proof. $\boxed{1. \Rightarrow 2.}$ Let $\mathcal{X} = \langle X, R, V\rangle$ be an $\mathsf{S5}^n$ model and let $x_0 \in X$ be a point such that $\mathcal{X}, x_0 \models \phi$. We define a triple $\mathcal{W}' = \langle W', R', V'\rangle$ as follows:

- $W' = X$;
- R' is a mapping associating to every $i \in AGT$ the equivalence relation

$$R'_i = \{\langle (x_1,\ldots,x_n),(y_1,\ldots,y_n)\rangle \in W'^2 \mid x_i = y_i\};$$

- $V' = V$.

We can check that for all $(w_1,\ldots,w_n) \in W'^n, \bigcap_{i\in AGT} R_i(w_i) \neq \emptyset$. Thus, $\mathcal{W}'$ is a STIT_n^G-Kripke model as defined in Section 3.1. We can see that

$$\begin{aligned} R'_{\bar{i}} &= \textstyle\bigcap_{j\in\bar{i}} R'_j \text{ (by definition, cf. Section 3.1)}\\ &= \textstyle\bigcap_{j\in\bar{i}}\{\langle (x_1,\ldots,x_n),(y_1,\ldots,y_n)\rangle \in W'^2 \mid x_j = y_j\}\\ &= \{\langle (x_1,\ldots,x_n),(y_1,\ldots,y_n)\rangle \in W'^2 \mid x_j = y_j \text{ for all } j \neq i\}\\ &= R_i \end{aligned}$$

and that

$$\begin{aligned} R'_{AGT} &= \textstyle\bigcap_{j\in AGT} R'_j \text{ (by definition, cf. Section 3.1)}\\ &= \textstyle\bigcap_{j\in AGT}\{\langle (x_1,\ldots,x_n),(y_1,\ldots,y_n)\rangle \in W'^2 \mid x_j = y_j\}\\ &= \{\langle (x_1,\ldots,x_n),(x_1,\ldots,x_n)\rangle \in W'^2\}\\ &= id_{W'} \end{aligned}$$

We can check by induction on ϕ that $\mathcal{X}, z \models \phi$ iff $\mathcal{W}', z \models \phi$ for all $z \in W$.

$\boxed{2. \Rightarrow 3.}$ Let $\mathcal{W} = \langle W, R, V\rangle$ be a STIT_n^G Kripke model such that $R_{AGT} = id_W$, and let $w_0 \in W$ be a world s.t. $\mathcal{W}, w_0 \models \phi$. As $R_{AGT} = id_W$, we have $\mathcal{W}, w_0 \models [\emptyset](\bigwedge_{p\in atm(\phi)}[AGT]p \leftrightarrow p)$. Thus $\mathcal{W}, w_0 \models [\emptyset](\bigwedge_{p\in atm(\phi)}[AGT]p \leftrightarrow p) \wedge \phi$.

[2] Remember that $[\emptyset]$ abbreviates $[\bar{1}]\ldots[\bar{n}]$.

3. $\Rightarrow$ 2. Let $\mathcal{W}' = \langle W', R', V' \rangle$ be a STIT_n^G Kripke model and let $w_0' \in W'$ be a world such that $\mathcal{W}', w_0' \models [\emptyset](\bigwedge_{p \in atm(\phi)} [AGT]p \leftrightarrow p) \wedge \phi$. We prove that there exists a STIT_n^G Kripke model $\mathcal{W} = \langle W, R, V \rangle$ with $R_{AGT} = id_W$ and a point $w_0 \in W$ such that $\mathcal{W}, w_0 \models \phi$. Let $\mathcal{W} = \langle W, R, V \rangle$ where:

- $W = \{R'_{AGT}(x) \mid x \in W'\}$;
- $R_i = \{(R'_{AGT}(x), R'_{AGT}(y)) \mid (x, y) \in R'_i\}$;
- $V(p) = \{U \in W \mid U \subseteq V'(p)\}$.

Notice that $R_J = \{(R'_{AGT}(x), R'_{AGT}(y)) \mid (x, y) \in R'_J\}$, and that $R_{AGT} = id_W$. We can prove by structural induction that for all $w \in W'$ and for all subformulas ψ of ϕ:

$$\mathcal{W}', w \models \psi \text{ iff } \mathcal{W}, R'_{AGT}(w) \models \psi$$

Indeed:

$$\begin{array}{lll}
\mathcal{W}', w \models p & \text{iff} & \mathcal{W}', w \models [AGT]p \\
 & & \text{(because } \mathcal{W}', w \models [\emptyset](\bigwedge_{p \in atm(\phi)} [AGT]p \leftrightarrow p)) \\
 & \text{iff} & \mathcal{W}', w' \models p \text{ for all } y \in R'_{AGT}(z) \\
 & \text{iff} & w' \in V'(p) \text{ for all } w' \in R'_{AGT}(w) \\
 & \text{iff} & R'_{AGT}(w) \subseteq V'(p) \\
 & \text{iff} & R'_{AGT}(w) \in V(p) \\
 & \text{iff} & \mathcal{W}, R'_{AGT}(w) \models p
\end{array}$$

$$\begin{array}{lll}
\mathcal{W}', w \models [\bar{i}]\psi & \text{iff} & \mathcal{W}', w' \models \psi \text{ for all } w' \in R'_{\bar{i}}(w) \\
 & \text{iff} & \mathcal{W}, R'_{AGT}(w') \models \psi \text{ for all } w' \in R'_{\bar{i}}(w) \\
 & \text{iff} & \mathcal{W}, R'_{AGT}(w') \models \psi \\
 & & \text{for all } R'_{AGT}(w') \in R_{\bar{i}}(R'_{AGT}(w)) \\
 & \text{iff} & \mathcal{W}, R'_{AGT}(w) \models [\bar{i}]\psi
\end{array}$$

2. $\Rightarrow$ 1. Let $\mathcal{W} = \langle W, R, V \rangle$ be a STIT_n^G Kripke model with $R_{AGT} = id_W$, and let $w_0 \in W$ be a world such that $\mathcal{W}, w_0 \models \phi$. From $\mathcal{W}$ we define a $\mathsf{S5}^n$ model $\mathcal{X}' = \langle X', R', V' \rangle$ as follows:

- $X' = X_1 \times \ldots \times X_n$ where for all $i \in AGT$, $X_i = \{R_i(w) \mid w \in W\}$;
- R' is a mapping associating to every $i \in AGT$ the equivalence relation

  $$R'_i = \{((x_1, \ldots, x_n), (y_1, \ldots, y_n)) \in X'^2 \mid x_j = y_j \text{ for all } j \neq i\};$$

- $V'(p) = \{(x_1, \ldots, x_n) \mid \bigcap_{i \in AGT} x_i \in V(p)\}$ (identifying $\bigcap_{i \in AGT} x_i = \{y\}$ and y).

We can check that

$$\mathcal{X}', (R_1(w), \ldots, R_n(w)) \models \phi \text{ iff } \mathcal{W}, w \models \phi$$

for all $w \in W$. Indeed:

$$\begin{array}{lll} \mathcal{X}', (R_1(w), \ldots, R_n(w)) \models p & \text{iff} & (R_1(w), \ldots, R_n(w)) \in V'(p) \\ & \text{iff} & \bigcap_{i \in AGT} R_i(w) \in V(p) \\ & \text{iff} & w \in V(p) \\ & & (\text{notice that } \bigcap_{i \in AGT} R_i(w) = \{w\}) \\ & \text{iff} & \mathcal{W}, w \models p \end{array}$$

and

$\mathcal{X}', (R_1(w), \ldots, R_n(w)) \models [\bar{i}]\psi$

iff $\mathcal{X}', (R_1(w), \ldots, R_{i-1}(w), R_i(w'), R_{i+1}(w), \ldots R_n(w)) \models \psi$ for all $w' \in W$

iff $\mathcal{X}', (R_1(w''), \ldots, R_i(w''), \ldots R_n(w'')) \models \psi$ where $R_1(w) \cap \ldots \cap R_i(w') \cap \ldots \cap R_n(w) = \{w''\}$ for all $w' \in W$

iff $\mathcal{X}', (R_1(w''), \ldots, R_i(w''), \ldots R_n(w'')) \models \psi$ for all $w'' \in R_{\bar{i}}(w)$

iff $\mathcal{W}, w'' \models \psi$ for all $w'' \in R_{\bar{i}}(w)$

iff $\mathcal{W}, w \models [\bar{i}]\psi$

■

THEOREM 22. *The problem of satisfiability of a formula of STIT_n^G is undecidable for $n \geq 3$.*

Proof. By Theorem 20 and 21. ■

6 Group STIT is not finitely axiomatizable

THEOREM 23. *There is no finite axiomatization of logic STIT_n^G if $n \geq 3$.*

Proof. Suppose for a contradiction that STIT_n^G is finitely axiomatizable, i.e. that there exists a finite set of axioms Ax such that for every STIT_n^G-formula ϕ, we have $\models_{\mathsf{STIT}_n^G} \phi$iff there is a deduction of ϕ from (instances of) Ax using Modus Ponens and Necessitation. Let us define an axiomatics Ax' obtained from Ax by removing all $[AGT]$ operators. We are going to prove that for all formulas $\phi \in \mathcal{L}_{\mathsf{S5}^n}$, $\models_{\mathsf{S5}}^n \phi$ iff there is a deduction of ϕ from (instances of) Ax using Modus Ponens and Necessitation. Hence, $\mathsf{S5}^n$ would be finitely axiomatizable and there is a contradiction.

Let us prove first that $\vdash_{Ax'} \phi$ implies $\models_{\mathsf{S5}}^n \phi$. We do so by proving that each instance of Ax' is valid in $\mathsf{S5}^n$. Let us consider an instance ψ' of an axiom of Ax'. ψ' is obtained from an instance ψ of Ax by removing all $[AGT]$ operators. We have $\models_{\mathsf{STIT}_n^G} \psi$. Therefore, ψ is valid in the class of STIT_n^G-models where $R_{AGT} = id_W$. Hence, ψ' is valid in the class of STIT_n^G-models where $R_{AGT} = id_W$. It follows that $\models^n_{\mathsf{S5}} \phi$.

Here is an outline of the $\boxed{\Leftarrow}$-sense of the proof. First, for all $\mathsf{S5}^n$-formulas ϕ,

$$\begin{array}{lll} \models^n_{\mathsf{S5}} \phi & \text{iff} & \models_{\mathsf{STIT}_n^G} [\emptyset](\bigwedge_{p\in atm(\phi)}[AGT]p \leftrightarrow p) \to \phi \\ & \text{iff} & \vdash_{Ax} [\emptyset](\bigwedge_{p\in atm(\phi)}[AGT]p \leftrightarrow p) \to \phi \\ & \text{implies (1)} & \vdash_{Ax,[AGT]\psi\leftrightarrow\psi} \phi \\ & \text{implies (2)} & \vdash_{Ax'} \phi \end{array}$$

It remains to prove (1) and (2).

As to (1), it suffices to prove the following:

$$\vdash_{Ax} [\emptyset](\bigwedge_{p\in atm(\phi)} [AGT]p \leftrightarrow p) \to \phi \text{ implies } \vdash_{Ax,[AGT]\psi\leftrightarrow\psi} \phi$$

This can be established using necessitation and principles of classical propositional logic. Basically the proof goes as follows:

$$\dfrac{\dfrac{\ldots\text{(necessitation and principles of classical propositional logic)}}{\vdash_{Ax,[AGT]\psi\leftrightarrow\psi} [\emptyset](\bigwedge_{p\in atm(\phi)}[AGT]p \leftrightarrow p)} \qquad \dfrac{\text{(by hypothesis)}}{\vdash_{Ax,[AGT]\psi\leftrightarrow\psi} [\emptyset](\bigwedge_{p\in atm(\phi)}[AGT]p \leftrightarrow p) \to \phi}}{\vdash_{Ax,[AGT]\psi\leftrightarrow\psi} \phi}$$

As to (2), suppose $Ax + \psi \leftrightarrow [AGT]\psi$ is the axiom system obtained from Ax by adding the schema $\psi \leftrightarrow [AGT]\psi$. Then we can prove that $\vdash_{Ax + \psi\leftrightarrow[AGT]\psi} \phi$ implies $\vdash_{Ax'} \phi$.

The proof of that goes as follows. Assume that $\vdash_{Ax,[AGT]\psi\leftrightarrow\psi} \phi$. There exists a proof of ϕ, that is to say a sequence $(\phi_1, \ldots, \phi_k)$ such that for $1 \leq i \leq k$, one of the following holds:

- ϕ_i is a tautology, an instance of an axiom in Ax or an instance of $[AGT]\psi \leftrightarrow \psi$;
- ϕ_i is obtained by necessitation from ϕ_j where $j < i$;
- ϕ_i is obtained by modus ponens from ϕ_j and ϕ_k where $j, k < i$;
- $\phi_k = \phi$.

Now, we construct $(\phi'_1, \ldots, \phi'_n)$ where ϕ'_i is ϕ_i in which we have removed all $[AGT]$ operators. The reader can check that $(\phi'_1, \ldots, \phi'_n)$ is a proof of ϕ.

This concludes the proof. ■

7 Discussion

Now, we are going to propose a generalization of these results, and try to classify some more fragments of STIT_n^G. First, for a given a family $\mathcal{C} \subseteq 2^{AGT}$ of subsets of 2^{AGT} we define the language $\mathcal{L}_{\mathsf{STIT}[\mathcal{C}]}$ by the following BNF:

$$\phi \ ::= \ p \mid \neg\phi \mid (\phi \wedge \phi) \mid [J]\phi$$

where p ranges over ATM and J ranges over the set of coalitions $\mathcal{C}$.

Let us call $\mathsf{STIT}[\mathcal{C}]$ the fragment of STIT_n^G where formulas are in $\mathcal{L}_{\mathsf{STIT}[\mathcal{C}]}$. Thus, $\mathsf{STIT}_n^G = \mathsf{STIT}[2^{2^{AGT}}]$ and $\mathsf{STIT}_n = \mathsf{STIT}[\{\emptyset, \{1\}, \ldots, \{n\}\}]$.

We have the following result:

PROPOSITION 24. *Let $\mathcal{C} \subseteq 2^{AGT}$. If $\mathcal{C}$ has a linear structure, then the problem of satisfiability of a formula in $\mathsf{STIT}[\mathcal{C}]$ is NP-complete.*

Proof. We can prove that if a formula is satisfiable, then it is so in a polynomial-sized model. The proof is based on a selection-of-points argument. More details can be found in [16]. ■

We conjecture the following result (which would cover Theorems 10 and 22).

CONJECTURE 25. Given $\mathcal{C} \subseteq 2^{AGT}$, the problem of satisfiability of a formula in $\mathsf{STIT}[\mathcal{C}]$ is:

1. undecidable if there are $J_1, J_2, J_3 \in \mathcal{C}$ such that J_1, J_2, J_3, $J_1 \cap J_2$ and $J_2 \cap J_3, J_1 \cap J_3$ are distinct;

2. NEXPTIME-complete if there is no $J_1, J_2, J_3 \in \mathcal{C}$ such that J_1, J_2, J_3, $J_1 \cap J_2$, $J_2 \cap J_3$ and $J_1 \cap J_3$ are distinct, but there exists $J_1, J_2 \in \mathcal{C}$ such that J_1, J_2, $J_1 \cap J_2$ are distinct;

We therefore conjecture, e.g., that: the problem of satisfiability of a formula in $\mathsf{STIT}[\mathcal{C}]$ is undecidable if $\mathcal{C} = \{\{1,3,4\}, \{1,3,5\}, \{4,5\}\}$, that it is NEXPTIME-complete if $\mathcal{C} = \{\{1,3,4\}, \{1,3,5\}, \{1\}\}$, and that it is NP-complete if $\mathcal{C} = \{\emptyset, \{1\}, \{1,2\}, \ldots, \{1,2,\ldots,n\}\}$.

Finally, we also conjecture that a nonstandard axiomatization of STIT_n^G can be obtained from that of $\mathsf{S5}^n$.

8 Conclusion

The paper contains mathematical results for deliberative STIT logic, both in its individual and group version: while the fragment STIT_n allowing to reason only about individual agency is decidable in nondeterministic exponential time (NEXPTIME), the entire logic STIT_n^G (allowing for joint agency) is undecidable and cannot be finitely axiomatized. The result for STIT_n was established in [3], while the general result for STIT_n^G is new.

The results for STIT_n^G apply a fortiori to extensions of STIT_n^G with the temporal 'next' operator. Given these rather negative results, it is interesting to look for decidable fragments of STIT_n^G and its temporal extensions. One of these fragments is Pauly's coalition logic, whose satisfiability problem is decidable in polynomial space (PSPACE-complete). As we said in the introduction, the CL and ATL formula $\langle\langle J\rangle\rangle X\phi$ corresponds to STIT_n^G's $\neg\Box\neg[J]X\phi$: in CL, the three modal operators $\Box$, $[J]$ and X are fused into a single operator. The latter is non-normal: it does not satisfy the K-axiom of standard modal logics. In recent work we have investigated a non-normal modal logic between coalition logic and STIT_n^G where $\Box$ and $[J]$ are fused, while X is the standard temporal 'next' [8]. We called the resulting logic CL^* because it extends CL in the same way as ATL^* extends ATL. We have shown that contrarily to ATL^*, the extension CL^* provides is for free: CL^* has the same complexity as CL. We have also argued that the epistemic extension of CL^* is more powerful than that of CL: contrarily to the latter, CL^* allows to reason about the agents' power, i.e. about agents' knowledge of the right action to choose in order to achieve something. In other words, in the epistemic extension of CL^* we can say that an agent 'knows how to play'. Logics having such expressive power have attracted a lot of attention recently [17, 12, 1].

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