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DOI
10.1061/AJRUA6.0001227

Publication date
2022

Document Version
Final published version

Published in
ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering

Citation (APA)
Raju, N., Patil, S., Arkatkar, S. S., & Easa, S. (2022). Proposed Empirical Approach to Measuring Traffic String Stability. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, *8*(2), [04022005]. https://doi.org/10.1061/AJRUA6.0001227

Proposed Empirical Approach to Measuring Traffic String Stability

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Abstract: This study originated with the intent of qualifying traffic string stability from empirical observations. A new responsiveness angle measure was developed to assess driver instincts under vehicle-following conditions. In this measure, the degree of the follower vehicle's attention towards its leader vehicle's actions is quantified. In understanding string stability in the traffic stream and assessing the propagation of disturbances, the newly conceptualized measure was used along with a discrete Fourier transform to measure the frequencies associated with responsiveness angle sequences. In this transform, a higher frequency of the angle depicts unstable conditions and vice versa. In assessing string stability from the empirical observations, vehicular trajectory data were developed from three study sections. Two study sections tended to have homogeneous lane-wise traffic, whereas the third section had mixed (heterogeneous) traffic. The results of the string stability analysis over the study sections showed that string stability varied with the change in traffic flow conditions, road geometries, and traffic flow type. In the case of free-flow conditions, the traffic streams were found to be stable with marginal disturbances in the responsiveness angle. From the analysis, it was observed that, in the case of study Section 3, around 26 instances of the stream were extremely unstable conditions (frequency equal to 10). For study Sections 1 and 2, the traffic stream was unsteady for 4 and 13 instances, respectively. However, as the traffic flow level rose, string stability deteriorated. This study demonstrated a novel approach to analyzing string stability based on actual traffic conditions that can be implemented in real time for traffic stream monitoring. **DOI: 10.1061/AJRUA6.0001227.**

Author keywords: Traffic string stability; Driver attention; Trajectory data; Real-time management.

Introduction

Traffic stability is one of the key performance indicators to understand traffic stream performance. Sensing this, researchers in the past have worked on numerous concepts to gauge and model traffic stability. Along these lines, Herman et al. (1959) relied on a car-following model to assess stability, derived criteria for measuring stability, and finally adopted acceleration sensitivity in uncovering traffic stream disturbances. Further, Herman's study revealed the concept of local and string stability.

Local stability (Sun et al. 2018; Zhang and Jarrett 1997) can be defined as the disordered traffic movement between a leader–follower pair under car-following conditions. At the same time, string stability (Montanino et al. 2021; Qin and Li 2020) is the propagation of disturbances from one following vehicle to another over the chain of vehicles. Further, with the development of numerous car-following models, researchers in the past attempted to study stability using those models, such as Bando et al. (1995) with the optimum velocity model, Yang et al. (2013) with the safety distance model, and Tordeux et al. (2010) with the Newell car-following model. The intelligent driver model (IDM) (Kesting and Treiber 2008) has been used for stability analysis. The other methodologies tended to play a significant role and were able to model stability in the traffic stream. Nevertheless, stability results highly depend on the calibration precision of the car-following model. Given the data constraints, those methodologies have heavily relied on numerical simulations to estimate stability.

Further, driving behavior under car-following conditions plays a significant role in affecting traffic stability. It is a measure of propagating the attention of drivers from one vehicle to other subsequent vehicles. Along these lines, to sense the attention of vehicles, researchers came up with numerous surrogate measures, including time to collision (TTC) (Van Der Horst and Hogema 1994), deceleration rate to avoid crash (DRAC) (Chevalier et al. 2017), time exposed in TTC (TET) (Meng and Qu 2012), and time integrated TTC (TIT) (Li et al. 2017). Numerous studies have developed different surrogate measures to assess the attention of drivers in the traffic stream. However, at the same time, those measures were found to play a significant role in assessing traffic stream safety. Even though those measures were able to gauge drivers' attention, they profoundly failed in sensing the propagation of vehicle instincts in a traffic stream. In this direction, Kuang et al. (2015) expressed the limitations of surrogate safety measures, their unsuitability for measuring conflicts, and their dependability on leader–follower vehicle interactions and assumptions.

On the other hand, sensing the progress of autonomous vehicles in the traffic stream, researchers worked on different simulation methodologies, such as numerical simulation, microscopic traffic simulation, and driving simulation experiments. These strategies

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Note. This manuscript was submitted on September 18, 2021; approved on December 16, 2021; published online on January 27, 2022. Discussion period open until June 27, 2022; separate discussions must be submitted for individual papers. This paper is part of the ***ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering***, © ASCE, ISSN 2376-7642.

aimed to sense the impacts on the traffic stream's stability and efficiency and tested various aspects. Along these lines, Talebpour and Mahmassani (2016) studied the impact of autonomous vehicles on safety and efficiency. Stern et al. (2018) associated the dissipation of shockwaves by damping propagation and related it to string stability. Numerous researchers worked in this direction, with autonomous vehicles as a critical component (Milanés and Shladover 2014; Papadoulis et al. 2019; Pereira and Rossetti 2012; Van Arem et al. 2006; Ye and Yamamoto 2018). From the literature on understanding traffic string stability, researchers focused on numerous concepts, including heterogeneous driving characteristics (Montanino et al. 2021; Yao et al. 2021), cooperative adaptive cruise settings (Li and Wang 2017), vehicle communications (Qin and Li 2020), mixed cooperative cruise control settings (Qin et al. 2017), stability frameworks (Montanino and Punzo 2021; Zhang et al. 2019), delay analysis (Zhang et al. 2020), empirical analysis (Makridis et al. 2020), platooning (Guo et al. 2020; Ruan et al. 2021; Zhao et al. 2020), nonlinear platooning of vehicles (Hao et al. 2020), and so on.

Finally, from the literature, it can be concluded that, even in the present context, most studies relied on car-following models to assess stability. On the other hand, surrogate measures were constrained to evaluating safety. As a result, limited studies have attempted to understand stability from an empirical perspective, which forms a considerable research gap. At the same time, there is no unique measure available in the literature to trace the propagation of disturbances in a traffic stream. Further in recent times, researchers (Zheng 2021) are strongly advocating the importance of producing reproducible research in transportation engineering. Considering all these aspects, the authors initially worked on a new measure to assess driver attention under car-following conditions. The new measure is incorporated in this paper to assess string stability in the traffic stream and test it using empirical trajectory data.

Methodology

To addresses the research gaps in the literature, the entire research work was conducted in four stages, as shown in Fig. 1. In Stage 1, vehicular trajectory data were collected from three study sections of varying geometry and flow characteristics. In Stage 2, to assess driver attention under following conditions, a new measure responsiveness angle was introduced. In Stage 3, to understand traffic string stability, the discrete Fourier transform was applied to gauge the frequency of responsiveness angles and sense stability. Finally, in Stage 4, based on the conceptualized methodology, stability over the study sections was analyzed by evaluating frequency.

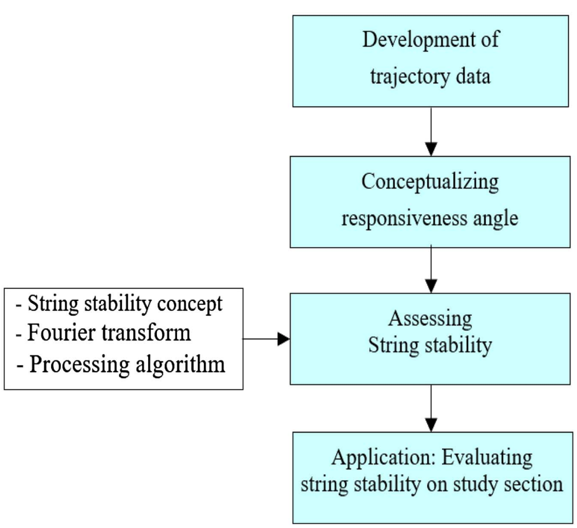

Fig. 1. Research methodology.

Development of Trajectory Data

In the present study, the analysis was carried out using vehicular trajectory data in which vehicular movements were recorded over the study sections with an update interval of 0.1 s. Three study sections in India were selected for the analysis. Section 1 was a four-lane divided highway with a shoulder, Section 2 was a six-lane divided highway with a shoulder, and Section 3 was a western expressway. The road segment was a 10-lane divided highway. In Section 1, traffic flow marginally varied, whereas in Section 2, traffic varied from free-flow conditions to near-capacity conditions. Thus, trajectory data were developed at two flow conditions for both study sections using the traffic data extractor (Vicraman et al. 2014). The traffic data extractor is a semiautomated image processing tool used in developing trajectory data. Initially, the road geometrics were marked as input, and then vehicles were tracked using computer mouse clicks for an update interval of 0.1 s, where vehicles were tracked by assigning the vehicle category.

On the other hand, for Section 3, trajectory data developed by the authors in previous studies were used. In Section 3, the traffic was mixed in nature, with five vehicle categories: motorized two wheelers (MTW), motorized three wheelers (MThW), buses, cars, and trucks. Unlike the other study sections, the traffic flow in Section 3 varied from free-flow conditions to near-capacity and stop-and-go conditions. Further, the details of the study sections' trajectory data are presented in Table 1, and the sample time-space plots are shown in Fig. 2.

Responsiveness Angle

In line with the aim of the work, to understand the attentiveness of the follower vehicle toward its leader, the time space plots of the vehicles were closely examined, as shown in Fig. 3(a). The leader traveled at speed V_{L1} and reduced its speed to V_{L2} at time T_1 (Point A). The follower traveled at speed V_{F1} and responded to the speed reduction of the leader at time T_2 (Point B) by reducing speed to V_{F2}. Time T_2 implicitly includes the perception-reaction time of the follower. The original distance gap between the two vehicles is D_o, and the distance gap at the follower response is D_r. Clearly, due to the lag in the follower's response, the distance gap decreased. To quantify the attentiveness of the follower, a new safety measure, responsiveness angle (α), was defined as the angle measured from the vertical line at A up to the line connecting the decision points of the leader and the follower. The angle is positive if it is anticlockwise and negative otherwise. The angle α depends, to a large extent, on both the distance gap and the level of attentiveness of the follower, reflected by the time lag t.

From the geometry of Fig. 3(a), the responsiveness angle can be derived as follows. The lag time is given by

$$t = T_2 - T_1 \tag{1}$$

where t = lag time (s); T_1 = time at which the leader reduces speed (s); T_2 = time at which the follower reduces speed (s); and D_o is original distance gap (m). Because $\tan \alpha = t/d_2$, then the responsiveness angle α can be written as

$$\alpha = \tan^{-1}\left(\frac{t}{d_2}\right) \tag{2}$$

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Table 1. Details of the trajectory data from the study sections

Study section	Trap length (m)	No. of lanes	Road width (m)	Traffic flow classification (V/C ratio)	Traffic composition (%)[a]	No. of vehicles tracked	Duration of trajectory data (min)
Section 1	150	2-lane	9.5	Flow 1 (0.19)	0, 0, 3, 92, 5, 0	159	10
				Flow 2 (0.25)	0, 0, 8, 82, 10, 0	211	10
Section 2	150	3-lane	11.2	Flow 1 (0.45)	0, 0, 2, 87, 11, 0	547	10
				Flow 2 (0.89)	0, 0, 7, 84, 9, 0	1,068	10
Section 3	120	5-lane	17.5	Flow 1 (0.35)	15, 35, 5, 40, 2, 3	1,080	15
				Flow 2 (0.71)	20, 29, 2, 45, 1, 3	1,715	15
				Flow 3[b]	17, 25, 5, 45, 3, 4	660	10

[a]Traffic composition: MThW, MTW, buses, cars, trucks, LCV.
[b]Stop and go conditions.

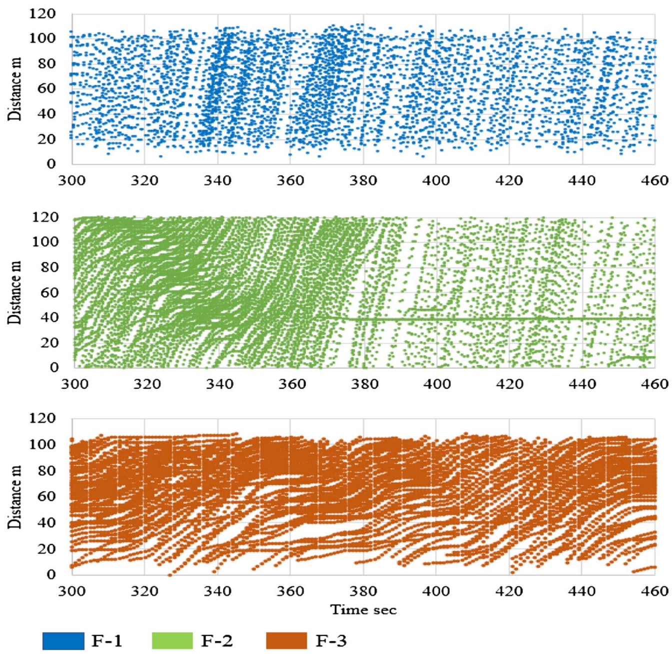

Fig. 2. Sample time–space plots from study Section 3.

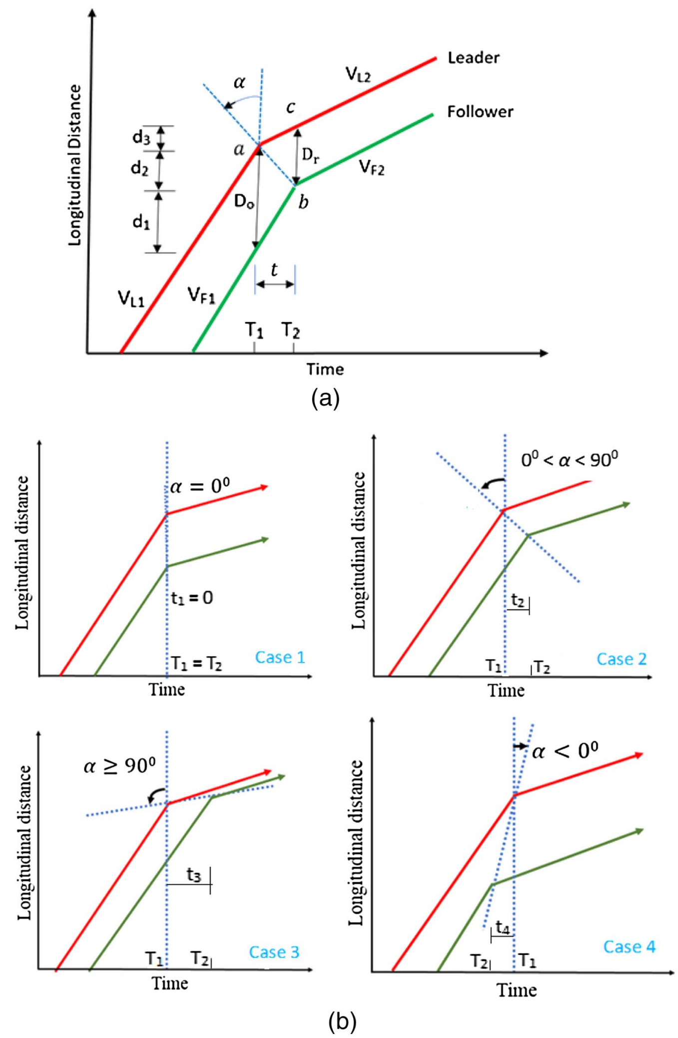

Fig. 3. Time–space diagram and different cases of the responsiveness angle: (a) time–space diagram; and (b) boundary conditions of responsiveness angle.

Let D_o be the original distance gap (m) and D_r the clear distance during the lag time. Then, the leader travels a distance d_3, and therefore the clear distance D_r between the leader and the follower at time T_2 is $(d_2 + d_3)$. Thus

$$D_0 = d_1 + d_2 \tag{3}$$

$$D_r = d_2 + d_3 \tag{4}$$

Adding Eqs. (3) and (4) yields

$$2d_2 + d_1 + d_3 = D_0 + D_r \tag{5}$$

$$2d_2 + 0.278\ V_{F1}t + 0.278\ V_{L2}t = D_0 + D_r \tag{6}$$

Rearranging the terms, then d_2 is given by

$$d_2 = \frac{D_0 + D_r - 0.278(V_{F1} + V_{L2})t}{2} \tag{7}$$

Substituting for d_2 from Eq. (2) into Eq. (7), then

$$\alpha = \tan^{-1}\left(\frac{2t}{D_0 + D_r - 0.278(V_{F1} + V_{L2})t}\right) \tag{8}$$

where α = responsiveness angle (degrees); D_0 = clear distance between the leader and the follower at time T_1 (m); D_r = clear distance between the leader and the follower at time T_2 (m); V_{F1} = original speed of the follower at time T_1 (km/h), V_{L2} = reduced speed of the leader at time T_2 (km/h); and t = lag time of the follower (s).

Various possible cases of the responsiveness angle (degree) are shown in Fig. 3(b). In Case 1 ($\alpha = 0$), the follower made the decision immediately when the leader decreased its speed with zero lag time $t_1 = 0$, indicating that the follower was fully attentive to the leader (full attention). In Case 2 ($0 < \alpha < 90$), the follower responded with some time lag $t_2 < D_o/0.278\,V_F$, indicating that the follower was displaying partial attention to the reader (partial attention). In Case 3 ($\alpha \geq 90$), the follower responded with a time lag corresponding to the original distance gap, $t_3 \geq D_o/0.278\,V_F$, indicating that the follower was not attentive at all to the leader (no attention). In Case 4 ($\alpha < 90$), the follower reduced the speed before the leader's action, indicating also that the follower was fully attentive. The distance gap in Case 1 did not change, whereas it decreased in Cases 2 and 3 and increased in Case 4 (opening process). Clearly, an increase in the lag time of the follower depicts the inattentiveness of the follower. Thus, the boundary conditions of α are defined as follows:

$$\alpha = \begin{cases} 0, & \text{(full attention)} \\ 0 < \alpha < 90^0, & \text{(partial attention)} \\ \geq 90^0, & \text{(no attention)} \\ < 0, & \text{(opening process)} \end{cases} \quad (9)$$

Fig. 3(b) shows Case 3 for $\alpha \geq 90$, which is the boundary for this case because $\alpha > 90$ represents a worse situation. Clearly, the responsiveness angle represents the rate of shift in the vehicle-following process.

Assessing String Stability

String Stability Concept

In the traffic stream, there is a series of vehicles moving one after another. The response of a following vehicle in the traffic stream can be transferred to the next consecutive vehicles, depending on their sensitivities. To better explain this, hypothetical time–space plots of vehicles are presented, as shown in Fig. 4(a). From the time–space plots, it can be seen that the first vehicle in the traffic movement is varied due to some random discrepancy. After a certain lag with sensitivity, its follower vehicle is adjusted by altering its movement.

Further, the phenomenon propagates through consecutive vehicles as a medium. Based on the magnitude of the movement variation, the propagation over the vehicles varies and depends on traffic conditions and vehicular behavior. Thus, the entire phenomenon ascertains the string stability of a traffic stream. Let $\alpha_{t1}, \alpha_{t2}, \ldots, \alpha_{tn}$ be the responsiveness angles of vehicles at time $t_1, t_2, \ldots, t_n$, as shown in Fig. 4(b).

In the case of regular traffic, there can be numerous fluctuations in the traffic stream. Due to this, at each time instant, a vehicle can have a responsiveness angle associated with it. The angle will be true zero at perfect following conditions. Let N be the number of vehicles in the traffic stream (in a single lane) whose responsiveness angles at time instant t are $1_t, \alpha 2_t, \cdot, \cdot, \cdot, \cdot, \cdot, \alpha N_t$. Thus

$$f(t1) = \alpha 1_{t1}, \alpha 2_{t1}, \cdot, \cdot, \cdot, \cdot, \cdot, \alpha N_{t1}$$
$$f(t2) = \alpha 1_{t2}, \alpha 2_{t3}, \cdot, \cdot, \cdot, \cdot, \cdot, \alpha N_{tn}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$f(tp) = \alpha 1_{tp}, \alpha 2_{tp}, \cdot, \cdot, \cdot, \cdot, \alpha N_{tp}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$f(tn) = \alpha 1_{tn}, \alpha 2_{tn}, \cdot, \cdot, \cdot, \cdot, \alpha N_{tn}$$

The variation among the responsiveness angles defines traffic disorders. For example, if this variation is minimum and near zero, the flow is consistent and leads to ideal behavior among the vehicles. On the other hand, if the variation among the angles is high, this signifies that the traffic movement is disordered. Therefore, based on this, traffic string stability can be estimated. Thus

$$\text{var}(\alpha_{tp}) = \begin{cases} \sim 0 & \text{Stable flow conditions} \\ \neq 0 & \text{Unstable flow condtions} \end{cases} \quad (10)$$

where var(α_{tp}) = variation among the responsiveness angles α_{tp}at time tp.

Fourier Transform

Further, in the present case, the series of angles at a time instant can be generalized as a wave function, with transfers of following instincts from one to vehicle to another longitudinally. Further quantifying this nature helps in understanding string stability. Along these lines, to understand the nature of the sequence of angles, Fourier transform is employed to decompose the sequence of angles to a frequency form. For example, a higher frequency represents higher instability and vice versa. In general, a Fourier transform (Plonka et al. 2018) discretizes a function into the basic frequency form. For example, with Fourier transform, a sound wave can be classified based on its frequency. At the same time, the Fourier transform can identify the frequencies, even when

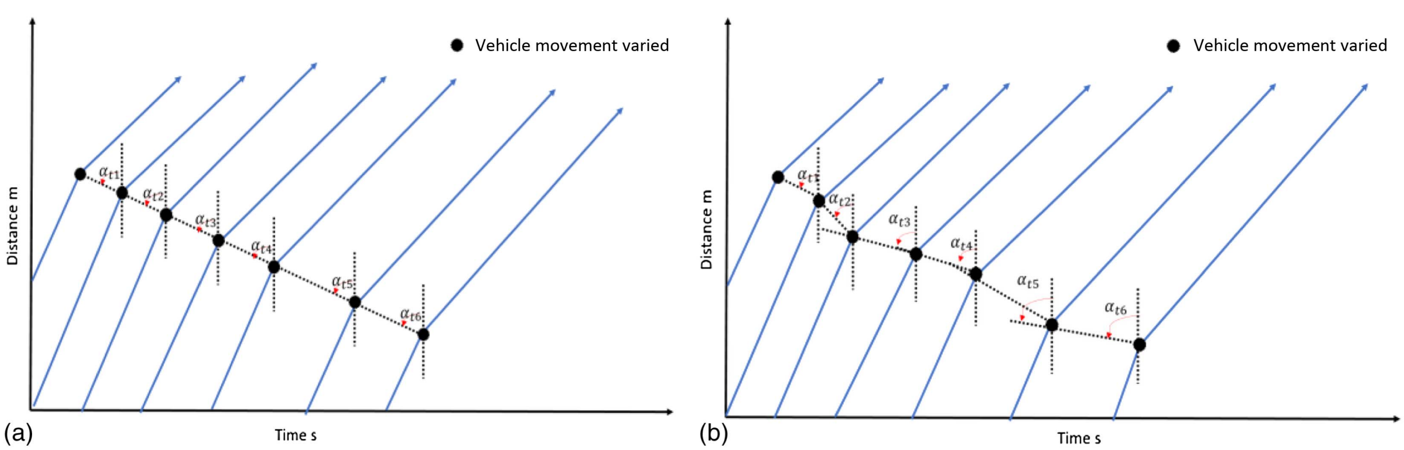

Fig. 4. Time–space plots of vehicles in a hypothetical scenario: (a) stable flow conditions; and (b) unstable flow conditions.

Input: vehicular trajectory data
For (Given subject vehicle)
    Identify the subsequent vehicles and compute lateral overlap
    If (subsequent vehicles having lateral overlap)
        Compute longitudinal distances
        Assemble them as leader-follower pairs
    End If
End For
For (Given leader-follower pair)
    Compute distance gap, relative speed for leader-follower pairs
    If (leader reduces speed) **#** *Computing responsiveness angle*
        Identify the time stamp as $\mathbf{T_1}$; measure the distance gap $\mathbf{D_0}$;
        Identify the speed of the follower $\mathbf{V_{F1}}$;
        If (the follower reduces speed after the leader's action)
            Identify the time stamp as $\mathbf{T_2}$
            Compute lag time **t** (Equation 1)
            Clear distance between leader and follower at T_2 is D_r
            Calculate distance d_2 (Equation 7)
            Calculate α (Equation 8)
        End If
    End If
    Report α, V_{F1}, and D_0 to carry out safety analysis.
End For
Output: Leader-follower pairs and their responsiveness angles

Fig. 5. Algorithm for calculating responsiveness angle (scripted in Python).

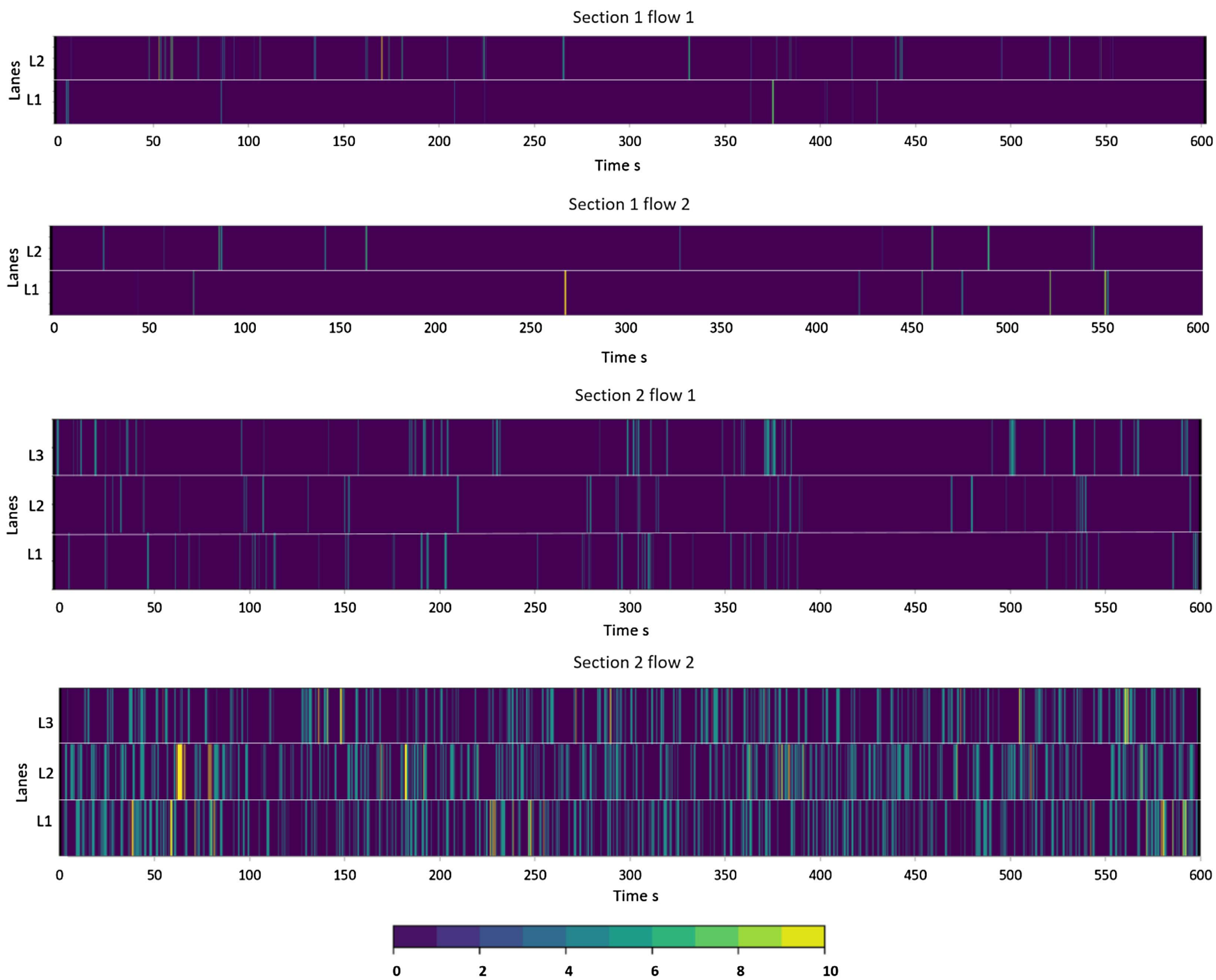

Fig. 6. Variation of string stability of lanes over time in study Sections 1 and 2.

multiple functions are combined. Further, the inverse Fourier transform can synthesize mathematically and revert the original function with its frequency. In general, for the data sequences, the Fourier transform of the function $f(t)$ is given by

$$F(\alpha) = \int_0^{N-1} f(t) e^{-j\alpha t} dt \quad (11)$$

where F = function of responsiveness angles α. The function $F(\alpha)$ is a complex number given by

$$F(\alpha) = \int_0^{N-1} f(t)\cos(\alpha t)dt - j\int_0^{N-1} f(t)\sin(\alpha t)dt \quad (12)$$

where $|F(\alpha)|$ defines the amplitude of the function $f(t)$, and $\angle F(\alpha)$ backs out the phase of the function $f(t)$.

In the present case, the frequency of the function $f(t)$ depicts the variation of the responsiveness angle sequence. Thus, the higher frequency of the data sequence represents more considerable variation in the responsiveness angle sequence. The value of $F(\alpha)$ tends to be maximum, and the order of that maximum value is taken as the frequency of the responsiveness angle sequence. The frequency of the function depicts the string stability, as follows:

$$k = \max\{F(\alpha){:}x = 1, 2, \ldots, n\} \quad (13)$$

$$\vartheta = \text{Order of } k \quad (14)$$

where ϑ = frequency of the function $F(\alpha)$.

Responsiveness Angle Algorithm

Further, based on the developed trajectory data over both study sections, the responsiveness angles were evaluated using real field conditions. Along these lines, the discussed safety framework was applied to sense the stability over the study sections. However, at the same time, it becomes incredibly tricky and complicated to capture the responsiveness angle directly from such substantial trajectory data. Thus, to compute the responsiveness angle, an algorithm was developed and scripted in Python 3.7.2, as shown in Fig. 5. The lateral overlap between the ego vehicle and its tentative leader and follower vehicles was checked to consider the lane-changing process. If the leader and follower vehicle tended to have the lateral overlap with the ego vehicle, the ego vehicle was immediately added to the changed lane and the angle of responsiveness was checked. Further, with the help of the Fourier transform, the frequency was checked.

The flow of the algorithm works as follows. Initially, trajectory data were loaded as a data frame. In the next step, for the subsequent vehicles over the road space, the lateral overlap among vehicles was computed. If the vehicular pairs were found to have any lateral overlap, those pairs were considered leader–follower pairs. Further, to evaluate the responsiveness angle, the extracted leader–follower pairs were thoroughly investigated. The responsiveness angle is nothing but the angle between the lines joining the actions of the leader and follower vehicles with the vertical axis. In computing the responsiveness angle, initially, the trajectory data of the leader vehicle were scrutinized, and the instants at which the leader vehicle dropped its speeds were identified. Later, the actions of the follower vehicle, particularly speed drops, were examined. On this basis, the follower responses were organized with the leader instincts. Then, the angle between the lines joining the action of the leader and its subsequent follower action with the vertical axis was computed as the responsiveness angle. By adopting this framework, the responsiveness angle was computed for all pairs over the study sections.

Application

In line with the study objectives, the proposed methodology was used to evaluate traffic string stability. In the case of regular traffic, lane-based traffic movement was observed. As a result, the vehicles tended to move one after another in each lane. Due to this, the generalized stability varied based on the traffic lanes. In the present work, stability over the traffic lanes in the study section was evaluated. Initially, using the algorithm, the responsiveness angles were estimated for all leader–follower interactions, and later they were segregated based on the lane. This helped in ascertaining the leader–follower sequences, like the vehicles in Fig. 5. Given this, every time instant, there was a series of responsiveness angles at every lane. In understanding the string stability on each lane, the responsiveness angle's frequency was evaluated as an indirect measure for string stability.

Further, the string stability was evaluated over a given time interval for each of the lanes in the study sections, as shown in Fig. 6, and details of unstable instances are presented in Table 2. From the analysis, the frequency was observed in the range of 0 to 10. In Section 1, both traffic flow conditions were near free flow; most of the time, the traffic was found to be in a stable regime, with less frequency in the responsiveness angles. In Section 2 at Flow 1 (near free flow), a similar inference to that of Section 1 was observed. Interestingly, for Flow 2, traffic string stability was found to be affected at the higher volume level, resulting in discrepancies in the frequencies over the lanes with time. From this, it can be seen that as traffic volume increased, string stability deteriorated. In all three lanes, the frequencies were found to be independent over the lanes, whereas for Section 3, the traffic was observed to be mixed in nature, with five different vehicle categories. Earlier works on Section 3 trajectory data showed great lateral movements of vehicles and smaller vehicles' dominance in disturbing the traffic medium.

Table 2. Unstable instances of the study sections over the lanes

Study section	Flow	Lane	No. of instances of frequency is equal to 10
Section 1	Flow 1	Lane 1	0
		Lane 2	0
	Flow 2	Lane 1	0
		Lane 2	2
Section 2	Flow 1	Lane 1	0
		Lane 2	0
		Lane 3	0
	Flow 2	Lane 1	3
		Lane 2	4
		Lane 3	6
Section 3	Flow 1	Lane 1	0
		Lane 2	2
		Lane 3	3
		Lane 4	2
		Lane 5	1
	Flow 2	Lane 1	0
		Lane 2	0
		Lane 3	0
		Lane 4	4
		Lane 5	5
	Flow 3	Lane 1	0
		Lane 2	0
		Lane 3	4
		Lane 4	2
		Lane 5	9

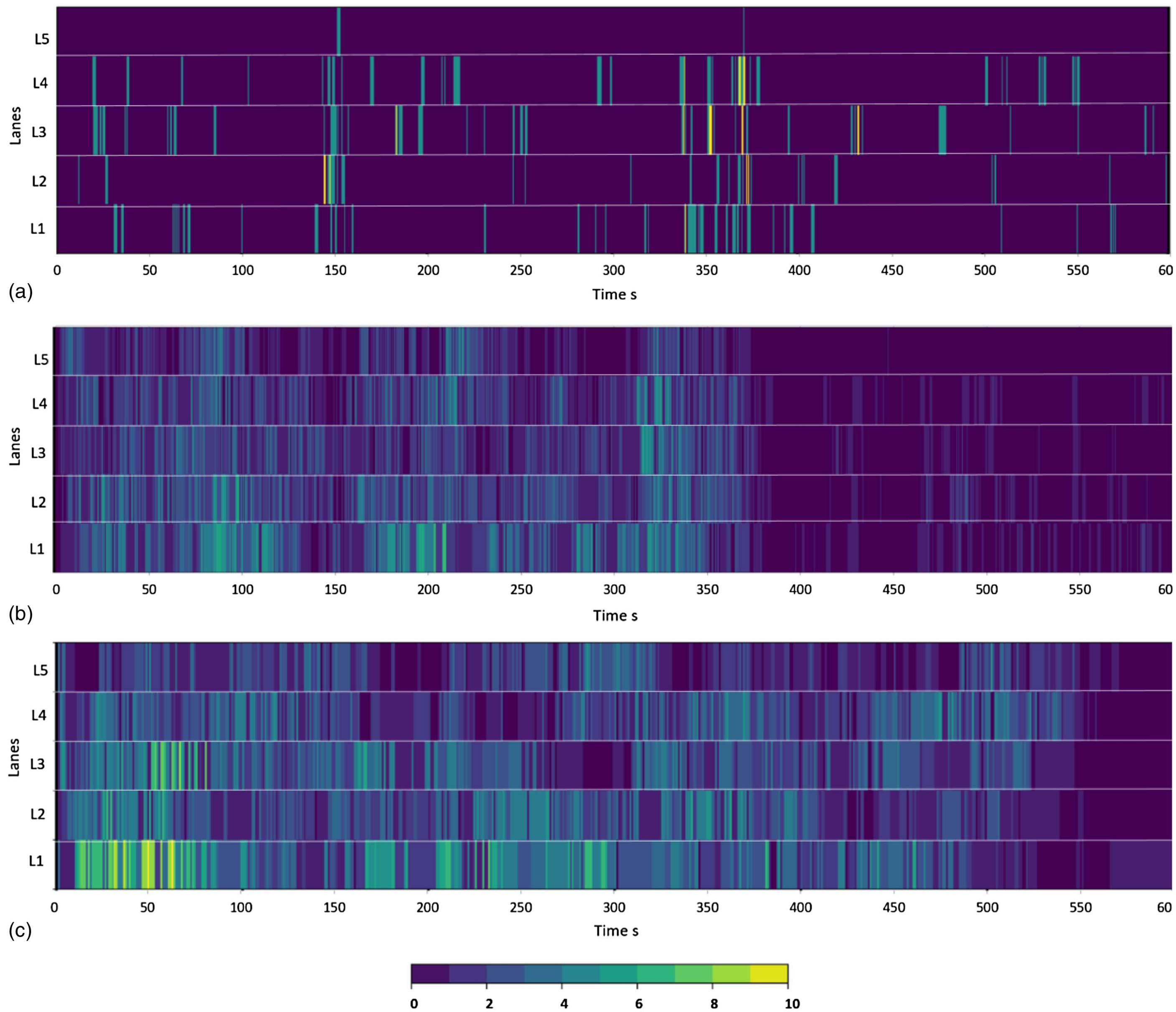

Fig. 7. Variation of string stability of lanes over time in study Section 3: (a) Flow 1; (b) Flow 2; and (c) Flow 3.

The stability was analyzed over Section 3 at all three flow conditions, as shown in Fig. 7. For Flow 1, near free-flow conditions, the traffic over the lanes tended to be stable most of the time. On the other hand, for Flow 2, traffic flow was transferred at around 370 s; even stability was varied from that time instant.

Similarly, in the case of stop and go conditions at Flow 3, stability was found to deteriorate more, depicting high frequencies over time. From the results, it is concluded that, with an increase in volume conditions, traffic stability was depreciated, which can be attributed to numerous factors. For example, in the case of free-flow conditions, drivers can maintain a necessary gap from their leaders, resulting in consistent vehicle movements, resulting in less variation of the responsiveness angles among the vehicles, whereas as traffic volume increases, the driver's freedom is affected, and drivers are forced to follow one another, which leads to variations of the responsiveness angle while following one another and finally impacts traffic string stability.

Conclusions

This study has presented a methodology for evaluating traffic string stability using the responsiveness angle, capturing the attention of follower vehicles. In understanding string stability and assessing the propagation of disturbances, Fourier transform was applied to assess the frequency of the responsiveness angle data sequences. Finally, the presented methodology was applied to the study sections to assess string stability. Based on this study, the following conclusions are offered:

1. It is noted from the literature that most studies depended on car-following models to assess stability. Those methodologies were highly confined to numerical simulation, and as a result, very few studies have attempted to estimate stability from empirical observations using trajectory data. This study can be considered the first of its kind in this direction. The developed methodology can be implemented with real-time trajectory data and can be useful in monitoring traffic stability on a real-time basis.

2. The analysis of the study sections showed that traffic was more stable at free-flow conditions, given there were fewer interactions among the vehicles. On the other hand, as the flow increases, car-following interactions among the vehicles tend to increase. Given the human element and associated lag in car-following conditions, traffic string stability deteriorated in those flow regimes. This was observed in both homogeneous and mixed traffic conditions.
3. In the present work, string stability was analyzed over the lanes, even though the methodology tends to be valid in lane-wise traffic. On the other hand, in the case of mixed traffic, given the numerous vehicle categories and weak lane discipline still in this direction, a few more data sets are required to develop a context for applying the developed stability framework in mixed traffic conditions. This can act as a limitation in the present work.
4. Given the real-time trajectory data developed over the study sections, the conceptualized methodology can be embedded in traffic systems for monitoring traffic stability. Based on this, traffic streams can be monitored for traffic stability on a real-time basis. By thresholding the frequency, stable and unstable instances in traffic streams can be quantified far in advance. Further, the concerned traffic authority can make the decision well in advance before occurring any catastrophic events. Simultaneously, the methodology can be instrumental in regulating variable speed limits with no damage to traffic efficiency levels.
5. Researchers heavily focused on autonomous vehicles and their impacts on the traffic stream in the present context. As a result, various surrogate safety measures are employed to assess the safety and trace the traffic efficiencies over the study sections. However, on the other hand, given the combination of autonomous and human-driven vehicles, string stability plays a balancing force in regulating safety and efficiency. Given this, studies of the present nature play a critical factor in grading traffic stability.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request. This includes:

- Trajectory data.
- Python codes for computing the angle of responsiveness and Fourier transforms.

Acknowledgments

This research work is an outcome of MITACS Global Research Award. Application Ref IT16143.

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