

Propositional Logic of Supposition and Assertion

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Abstract This presentation of a system of propositional logic is a foundational paper for systems of illocutionary logic. The language $\mathcal{L}_{.75}$ contains the illocutionary force operators ‘ \vdash ’ for assertion and ‘ \ulcorner ’ for supposition. Sentences occurring in proofs of the deductive system $\mathcal{S}_{.75}$ must be prefixed with one of these operators, and rules of $\mathcal{S}_{.75}$ take account of the forces of the sentences. Two kinds of semantic conditions are investigated; familiar truth conditions and commitment conditions. Accepting a statement A or rejecting A commits a person to accepting some statements (the symbol ‘+’ marks this value), to rejecting some statements (–), and will leave the person uncommitted with respect to others (n). Commitment valuations assign the values +, –, n to sentences of $\mathcal{L}_{.75}$; such a valuation is conceived as reflecting the beliefs/knowledge of a particular person. This paper explores the relations between truth conditions and commitment conditions, and between semantic concepts defined in terms of these conditions.

1 Language and speech acts In this paper I develop a system of what I understand to be *illocutionary logic*. In order to motivate this system and make it intelligible to the reader, I briefly sketch a philosophical basis for the system. This sketchy foundation will not be argued for, because there is no room for that in the present paper. However, it should be clear how this foundation “gives rise to” the logical system that is developed.

On my view, the fundamental linguistic “reality” is constituted by speech acts, or linguistic acts. These are meaningful acts performed with expressions. The word ‘speech’ suggests someone talking out loud. But I use the phrase ‘speech act’ (and ‘linguistic act’) for acts of speaking out loud, or writing, or thinking with words. An audience that listens or reads with understanding also performs speech acts.

Words don’t have, or express, meanings; words are not meaningful. Linguistic acts are the primary meaningful items. Although it is linguistic acts that are meaningful, various expressions are conventionally used to perform different kinds of

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meaningful acts. The meanings commonly assigned to expressions belong to the acts which the expressions are conventionally used to perform. But it is not the conventions associating meanings with acts which make the acts meaningful. The meaning of a linguistic act depends primarily on the language user's intentions. However, it is normal to intend to use expressions to perform acts with which they are conventionally associated. Still, a person can by misspeaking produce the wrong word to perform a linguistic act. I might have been looking for the word 'Michelle' to refer to my daughter Michelle, but by mistake have uttered the word 'Megan'. I did refer to Michelle, because I intended Michelle when I uttered 'Megan'. I *directed my attention* to Michelle. However, my utterance will undoubtedly mislead my addressee into directing his attention to Megan.

The speech-act understanding of language drives a wedge between syntax and semantics, for it is expressions, not acts, that have syntactic features. Indeed, we can regard expressions as syntactic objects. But linguistic acts are the "owners" of semantic features. Linguistic acts are meaningful; some linguistic acts have truth conditions, and are true or false. The connection between syntactic features and semantic features is conventional and contingent.

The fundamental semantic feature of a linguistic act is its *semantic structure*. This is determined by the semantic characters of component acts and their organization. I will illustrate this with an example. If, considering the wall to my right, I say "That wall is white," I have made a statement. This statement has a syntactic character supplied by the expressions used. A semantic analysis of the statement can be given as follows.

- (1) The speaker (myself) referred to the wall.
- (2) This referring act identified the wall, and so provided a "target" for the act acknowledging the wall to be white (characterizing the wall as white).

The semantic structure is constituted by the referring act, the acknowledging (characterizing) act, and the enabling relation linking the two component acts. It is possible to characterize the semantic structure without mentioning the expressions used or the order in which they occurred. Such a characterization is language-independent. However, for a given semantic structure, there may be languages which (presently) lack the resources to instantiate the structure.

Speech acts performed with a single word or phrase can be distinguished from those acts (or activities) performed with a whole sentence. Some *sentential* acts can appropriately be evaluated in terms of truth and falsity; these are *propositional* acts. A language user can perform a propositional act without accepting it as true. A speaker might perform a propositional act and wonder if it is true, or doubt that it is true. A disjunctive speech act, which is accepted, might contain two propositional components, neither of which is accepted.

A propositional act can be performed and accepted all at once. I will characterize such an act as an *assertion*. This is at odds with the terminology of Austin and Searle and also with ordinary usage. As commonly understood, an assertion is performed by a speaker or writer who is addressing an audience that understands her. On the common understanding, we can distinguish sincere from insincere assertions. However, accepting the propositional act is what makes an assertion, as commonly understood,

sincere. On my usage, an assertion doesn't need an audience and it can be performed out loud, in writing (or signing), or in a person's head. All assertions (in my sense) are sincere. Assertions as I understand them are little different than judgments (i.e., judgments performed with sentences). On one usage, the word 'statement' serves to pick out assertions, but I shall use this word for propositional acts of all kinds. On my usage, then, some statements are assertions and some are not.

As well as accepting a propositional act, a person can reject one; a person can also *decline* to accept a propositional act, which is different from rejecting the act as false. Accepting, rejecting, and declining are all illocutionary forces which characterize some propositional acts. Someone can also *suppose* a propositional act to be true, or suppose it to be false, in order to derive the consequences of this supposition. If someone supposes *A* (to be true) and supposes *B*, and then infers a consequence like '[*A* & *B*]', this consequence also has the force of a supposition.

I understand an *inference* to be a speech act which begins with propositional premises having some illocutionary forces and moves to a conclusion having some illocutionary force, where the conclusion is thought to be supported by the premises. This is a *simple* inference, for simple inferences can be combined in various ways to constitute complex inferences. The word 'inference' is in order when a person carries out reasoning to discover something for herself. An *argument* is a speech act whose point is to support a conclusion selected in advance (or to show that certain suppositions lead to a certain suppositional conclusion). Arguments are also either simple or complex. A sequence of *n* premise statements and one conclusion statement can be considered apart from a context in which the premises are actually part of an inference or argument. Such a sequence will be called both an *inference* sequence and an *argument* sequence; argument sequences are not speech acts, though their components are. We consider an argument sequence in order to focus on the relation between the premises and the conclusion, when we wish to determine whether the premises support the conclusion.

The present speech-act approach to language employs the adjective 'propositional', but it provides no place for propositions as classically conceived. The only propositional acts that exist are acts performed by some person on some occasion. But we can represent kinds of propositional acts that no one has performed, just as we can draw pictures of kinds of events that never took place. Whatever status such propositional acts have must be conferred by whoever represents them. (They are not "there" ahead of time.)

2 *Logic and speech acts* A logical theory, or logical system, has three parts: (1) an artificial language, (2) an account of the truth conditions of sentences in the artificial language—a semantic account, (3) a deductive system which codifies the logically true sentences or the logically valid argument sequences of the language.

From my perspective, an artificial logical language isn't really a language, although I will continue to speak of logical languages. For no one speaks, writes, or thinks the sentences of these artificial languages. Sentences of artificial languages are not used to perform speech acts; these sentences are representations of propositional acts that might be performed using natural-language expressions. Sentences of most logical languages are primarily concerned with semantic structure, and provide

almost no syntactic information. For example, a sentence ‘ $[A \vee B]$ ’ can be used to represent statements in virtually any natural language, no matter how these languages differ from one another.

Although logical-language sentences represent statements that might actually be made, these sentences often fail to represent statements of kinds commonly made. In elementary logic courses, a statement made with

Some student in this class is a boy

might be translated like this:

$$(1) \quad (\exists x)[S(x) \ \& \ B(x)]$$

or

$$(2) \quad (\exists x)[S(x, a) \ \& \ B(x)]$$

The first-order sentences do not actually represent the semantic structure of the statement they translate. A statement which has the structure represented by the logical-language “translation” needs a sentence like this:

For something, it is a student in this class and it is a boy.

The first-order sentences represent a statement which is at best an approximation to the original statement. Dealing with such approximations is sufficient for many logical analyses, but it can be enlightening to devise artificial languages which perspicuously represent statements of kinds we commonly make.

An account of the syntax of an artificial logical language is a collection of rules or principles for constructing representations. There is no reason why such an account should shed light on syntactic principles of natural languages. The semantics of an artificial logical language is not directly concerned with the sentences of that language. For it isn’t sentences that are true or false, in an artificial language or a natural one. The truth-conditions of an artificial logical language are for statements represented by logical-language sentences.

A *concrete interpretation* of a logical language determines what statements (statement kinds) are represented by logical-language sentences. To provide a concrete interpretation of a first-order language, we might say something like the following.

Let ‘ $F(x)$ ’ mean *x is a fish*

Let ‘ $M(x)$ ’ mean *x is a mammal*.

Let ‘ a ’ mean Alaska, etc.

To give a concrete interpretation of a language of propositional logic, we would assign entire statements to atomic sentences. Although we understand what a concrete interpretation of an artificial language would accomplish, we don’t usually bother to provide such interpretations. When using an artificial language to analyze statements or arguments in a natural language, we commonly provide interpretations for only a small number of artificial-language expressions.

Interpreting functions (valuations) which assign values to expressions of artificial logical languages provide *abstract interpretations*. Different concrete interpretations can correspond to a single abstract interpretation. This is especially clear for

a propositional language for which interpreting functions assign truth and falsity to the atomic sentences, but it also holds for first-order and higher-order languages. An abstract interpreting function gives logically salient features of the concrete interpretations with which it is associated.

An artificial logical language makes it convenient to characterize certain classes of statements and argument sequences. The logical form of an artificial-language sentence is a visible feature of that sentence. An understanding of the truth conditions associated with logical form allows us to devise efficient procedures for determining that statement forms represent analytic truths or argument-sequence forms represent valid argument sequences. But the logical forms of logical-language sentences do not correspond to visible or perceptible features of the statements they represent. The logical form of an artificial-language sentence represents an abstract level of semantic structure. This logical form is an artifact of logical analysis; it is a mistake to look for such forms in natural-language sentences or statements.

The deductive system which constitutes part of a logical theory is primarily a system for codifying representations, and only indirectly a means for codifying natural-language statements or argument sequences. Logically true sentences of artificial languages represent statements which constitute a subclass of analytic statements. Logically valid (sentential) argument sequences represent argument sequences which constitute a subclass of (simply) valid argument sequences. However, some deductive systems are good for more than codifying representations. A real argument proceeds from asserted or supposed premises to an asserted or supposed conclusion. A natural deduction system sanctions the construction of deductions/proofs from hypotheses in which the structures of the deductions represent the structures of natural-language arguments. These proofs from hypotheses directly establish that a sentence is logically true or that a (sentential) argument sequence is logically valid; indirectly, they show that a statement is analytic or an argument sequence is valid. They also provide understanding of the structures of actual arguments.

3 *Some important semantic concepts* Approaching language and logic from the perspective of speech acts makes clear that many logically important semantic concepts have been largely overlooked by current research. The semantic concepts of entailment and consequence that have received most attention are *truth-conditional*. Statements/propositional acts A_1, \dots, A_n *truth-conditionally entail* statement/propositional act B if and only if any way of satisfying the truth conditions of A_1, \dots, A_n also satisfies those of B . B is a *truth-conditional consequence* of A_1, \dots, A_n in the same circumstances. A statement B is *truth-conditionally analytic* if and only if its truth conditions cannot fail to be satisfied.

A propositional act must be performed by someone at some time if it is to exist, though we can represent kinds of acts that are never performed. But the truth conditions of a propositional act are independent of the question of whether that act is accepted. However, some logically important semantic concepts “take account” of whether a propositional act is accepted or rejected.

Commitment is a fundamental feature of intentional acts. Its concept is too basic to be explained in terms of more fundamental notions. Everyone is familiar with commitment, but not everyone calls it ‘commitment’. A person can be committed to

perform an act. And performing one act can commit a person to performing another. This is *rational* commitment, not moral commitment. Someone is not immoral or sinful if she fails to honor rational commitment.

Deciding to do X commits a person to doing X . Deciding to catch the 8 o'clock train to New York City also commits a person to going to the station before 8 o'clock. And judging Peter to be in pain commits me to judging him to be uncomfortable and to not judging him to be enjoying himself. Some commitments are "come what may" commitments like the commitment to catch the train. Some are commitments that only come up in certain situations, like the commitment to close the upstairs windows if it rains while I am at home.

Statements/propositional acts A_1, \dots, A_n *basically entail* statement/propositional act B if and only if accepting A_1, \dots, A_n commits a person to accepting B . B is a *basic consequence* of A_1, \dots, A_n under the same circumstances. The commitment to accept B is not a come what may commitment. The person who accepts A_1, \dots, A_n is committed to accept B if the matter comes up. A statement is *basically analytic* if and only if simply making/performing the statement commits a person to accepting it.

Truth-conditional entailment and basic entailment coincide to a large extent, but not entirely. Statements made with these sentences (on the same day)

Today is Tuesday or Wednesday.

It isn't Wednesday.

both truth-conditionally and basically entail (on that day).

Today is Tuesday.

But if a given person accepts a statement A , this commits her to accepting the statement made with 'I believe that A '; however, satisfying the truth conditions for A won't satisfy the truth conditions of a statement that some particular person believes A . Some cases of truth-conditional entailment also fail to be cases of basic entailment. For example, we can represent a collection X of infinitely many propositional acts such that if all their truth conditions were satisfied, then B 's would be satisfied. There is a truth-conditional entailment from the statements in X to B . But there is no basic entailment, for no one can accept infinitely many statements.

A statement/propositional act is *basically analytic* if and only if performing the act commits a person to accepting it. The statement made with 'This statement is in English' will be basically analytic. But it won't be truth-conditionally analytic, for it isn't inevitable that a given speaker's current statement is in English. (There need not even be a current statement.) A more interesting example of a basically analytic statement is one made with: I think, I exist.

The two kinds of entailment (and analyticity) are different, and it is important to study each kind. With basic entailment, there is a distinction between simple, immediate entailments and mediate entailments. The immediate entailments are simply "grasped" once a person understands the propositional acts. Mediate basic entailments are constituted by chains of immediate entailments. There is no immediate/mediate distinction for truth-conditional entailment. We can see that basic entailment is like an "absolute" deducibility, a deducibility not tied to a particular deductive

system or other deductive apparatus. Basic entailment depends on the semantic structures of a language user's acts, and on the consequences of accepting certain propositional acts (the "premise" acts).

Although the focus of twentieth-century research in logic, since Tarski anyway, has been on truth-conditional entailment, logic need not and should not be so restricted. Both concepts of entailment have been important historically. One role of illocutionary logic as I understand this is to explore the two concepts and their interrelations.

4 A standard system of propositional logic The language \mathcal{L}_5 contains the connectives ' \sim ', ' \vee ', ' $\&$ ', and denumerably many atomic sentences. Brackets ($[,]$) are used for punctuation. Although the horseshoe ' \supset ' of material implication is not a primitive symbol of \mathcal{L}_5 , we can use ' $[A \supset B]$ ' to abbreviate ' $[\sim A \vee B]$ '. I have not made the horseshoe a primitive symbol, because it is conventionally used for material implication, and this is not a concept expressed by any ordinary expression. Sentences made with the horseshoe do not provide good translations of ordinary conditional statements. In a subsequent paper I will introduce a different expression for forming conditional sentences. A (*truth-conditional*) *interpreting function* of \mathcal{L}_5 assigns one of truth (T), falsity (F) to each atomic sentence of \mathcal{L}_5 . Given an interpreting function f , a valuation of \mathcal{L}_5 determined by f assigns to atomic sentences the values assigned by f , and assigns values to compound sentences on the basis of standard truth-tables. If A is a sentence of \mathcal{L}_5 which has T in the valuation determined by interpreting function f , I will indicate this by writing: $f(A) = T$.

A sentence A of \mathcal{L}_5 is *logically true* if and only if it is true for the valuation determined by every (truth-conditional) interpreting function of \mathcal{L}_5 . If A_1, \dots, A_n, B are sentences of \mathcal{L}_5 , then A_1, \dots, A_n *truth-conditionally imply* B if and only if there is no interpreting function f of \mathcal{L}_5 for which each of A_1, \dots, A_n has value T for the valuation determined by f , but B has value F for this valuation. Implication is the special case of entailment that is linked to the logical forms of artificial-language sentences.

If A_1, \dots, A_n, B are sentences of \mathcal{L}_5 , then $A_1, \dots, A_n/B$ is a (*sentential*) *argument sequence* of \mathcal{L}_5 . The sentences A_1, \dots, A_n are the *premises* and B is the *conclusion*. An argument sequence $A_1, \dots, A_n/B$ is *truth-conditionally logically valid* if and only if A_1, \dots, A_n imply B . Logical validity is the special case of validity that is linked to the logical forms of artificial-language sentences.

The deductive system \mathcal{S}_5 is a natural deduction system which employs tree proofs. The theorems of \mathcal{S}_5 are argument sequences $A_1, \dots, A_n/B$ such that $n \geq 0$. If $n = 0$, we have an argument sequence $/B$. A theorem $/B$ of \mathcal{L}_5 can also be written without the slant line: B . The *elementary* rules of inference for \mathcal{S}_5 are these:

<i>& Introduction</i>	<i>& Elimination</i>	\vee <i>Introduction</i>
$A \quad B$	$[A \ \& \ B] \quad [A \ \& \ B]$	$A \quad B$
$[A \ \& \ B]$	$A \quad B$	$[A \ \vee \ B] \quad [A \ \vee \ B]$

If we consider the defined symbol ' \supset ', we have the elementary principle Modus Ponens:

$$\frac{A \quad [A \supset B]}{B}$$

An instance of a rule in a tree proof is an *inference figure*. If all inference figures are elementary in a tree proof from hypotheses A_1, \dots, A_n to conclusion B , this tree proof establishes that the argument sequence $A_1, \dots, A_n/B$ is a theorem.

A *nonelementary* rule is for a move which “cancels” or “discharges” hypotheses in a subproof. For example, this tree proof

$$\frac{\frac{A \quad B}{[A \& B]} \quad \& I}{B} \quad \& E$$

establishes the theorem: $A, B/B$. The inference principle \supset *Introduction* allows us to *cancel* the hypothesis A .

$$\frac{\frac{\frac{x}{A \quad B}}{[A \& B]} \quad \& I}{B} \quad \& E}{[A \supset B]} \quad \supset I, \text{ cancel } A$$

This is a proof from uncanceled hypothesis B to conclusion $[A \supset B]$, establishing this result: $B/[A \supset B]$. An ‘ x ’ is placed above canceled hypotheses.

One primitive nonelementary rule is: \vee *Elimination*.

$$\frac{\frac{\{A\} \quad \{B\}}{[A \vee B] \quad C \quad C}}{C}$$

The sentences in braces are the hypotheses that are canceled by the rules. The second nonelementary rule is: \sim *Elimination*.

$$\frac{\frac{\{\sim A\} \quad \{\sim A\}}{B \quad \sim B}}{A}$$

It is to be understood that the sentence $\sim A$ is a hypothesis of one or both of the subproofs leading to the conclusions B and $\sim B$. All occurrences of $\sim A$ as hypothesis in either subproof are canceled by the application of this rule. Alternatively, the rule \sim *Elimination* may be regarded as having three forms:

$$\frac{\frac{\{\sim A\}}{B \quad \sim B}}{A} \quad \frac{\frac{\{\sim A\}}{B \quad \sim B}}{A} \quad \frac{\frac{\{\sim A\} \quad \{\sim A\}}{B \quad \sim B}}{A}$$

For the defined symbol ‘ \supset ’, the inference principle \supset *Introduction* is nonelementary:

$$\frac{\{A\} \\ B}{[A \supset B]}$$

A proof from uncanceled hypotheses A_1, \dots, A_n to conclusion B establishes that $A_1, \dots, A_n/B$ is a theorem of \mathcal{S}_5 .

It is easy to show that \mathcal{S}_5 is *truth-conditionally sound*, which means that every proof whose uncanceled hypotheses are true also has a true conclusion. This can be proved by induction on the rank of a tree proof. The rank of a proof is the number of distinct inference figures it contains. (0 is the lowest rank; a sentence standing alone is a proof having rank 0.) \mathcal{S}_5 is also complete in the sense of having all truth-conditionally valid inference sequences among its theorems. Since every logically true sentence A corresponds to a truth-conditionally valid sequence $\vdash A$, the system \mathcal{S}_5 is complete with respect to logical truth. The system is also complete with respect to the truth-conditional logical consequences of a collection X of sentences of \mathcal{L}_5 .

Proofs in \mathcal{S}_5 represent natural-language arguments. We can also use tree structures which are not proofs in \mathcal{S}_5 to represent arguments. For example, the premises of the following argument

$$\frac{\sim A \quad [A \vee B]}{B}$$

imply the conclusion, while the premise of this argument

$$\frac{\sim [A \& B]}{\sim B}$$

does not imply the conclusion.

It is common in elementary logic texts to characterize arguments as valid or invalid. Although I have (unfortunately) done this in the logic texts I have written, I have not done this in the present paper. Validity has been defined for argument sequences, not for arguments. If we understand an *abstract premise-conclusion argument* to be a pair X, B where X is a collection of sentences (statements) and B is a sentence (statement), we can easily adapt our earlier definition to cover abstract premise-conclusion arguments. However, validity is not properly applied to arguments conceived as speech acts.

An argument can be either simple or complex. The concept of validity which applies to argument sequences or abstract premise-conclusion arguments can be “extended” to apply to simple arguments. For example, we might choose to characterize this argument (the argument this represents)

$$\frac{\sim A \quad [A \vee B]}{B}$$

as valid. But the ordinary concept of validity has no application to complex arguments. In this complex argument

$$\begin{array}{c}
 \sim A \quad [A \vee B] \\
 \hline
 B \qquad [B \supset C] \\
 \hline
 C \qquad \sim A \\
 \hline
 [\sim A \ \& \ C]
 \end{array}$$

the premises of each component argument entail their conclusion, and the hypotheses of the overall argument entail the overall conclusion. But the overall argument is neither valid nor invalid.

Someone might think we should take a complex argument to be valid if its hypotheses entail the ultimate conclusion. Such a decision would lead us to characterize the argument above as valid. But then the following argument would also come out to be valid,

$$\begin{array}{c}
 A \\
 \hline
 [A \ \& \ B] \quad [C \supset [A \ \& \ B]] \\
 \hline
 C \\
 \hline
 [C \supset A]
 \end{array}$$

even though none of its components is.¹

It is certainly possible to redefine ‘valid’ and ‘invalid’ so that we can characterize complex arguments as valid or invalid. But as customarily defined, the concepts don’t apply to complex arguments. I think we should limit these concepts to argument sequences or abstract premise-conclusion arguments. Those arguments which are speech acts are either *deductively correct* or *deductively incorrect*. As a first approximation, we shall understand a simple argument to be deductively correct if its premises entail its conclusion. A complex argument is deductively correct if its component arguments are deductively correct, and the uncanceled hypotheses (the premises) of the complex argument entail the overall conclusion.

5 Representing illocutionary force Proofs in \mathcal{S}_5 are satisfactory for identifying logical truths and truth-conditionally logically valid argument sequences. And proofs in \mathcal{S}_5 represent (some) deductively correct arguments carried out with statements of natural languages. But tree-structure arguments constructed from sentences of \mathcal{L}_5 are not entirely perspicuous representations of natural-language arguments. When some person makes an inference or argument from premises to a conclusion, if the premises provide deductive support to the conclusion, that person should “see” this. When a person recognizes that the conclusion follows from the premises, she is recognizing that she is *committed* to accept the conclusion, once she has accepted the premises. Commitment and its recognition provide the “motive power” taking an arguer from premises to conclusion. Commitment is not generated by force-neutral propositional acts. *Accepting/asserting* some statements commits a person to accepting or rejecting other statements. *Supposing* statements also commits a person to supposing others.

A perspicuous representation of an argument should include symbols for representing illocutionary force. If A is a sentence of the artificial language, I will write

$$\vdash A$$

to indicate that A is accepted (asserted). To reject A , I will write: $\neg A$. To decline to accept A , I would like to use the symbol ‘ \vdash ’ with a line through it, but this is not convenient with my word processor. So I will use

$$x \vdash A$$

to decline to assert A . I can decline in this fashion if I judge A to be false or if I simply don’t know whether A is true.

A sentence A is a *plain* sentence. And $\vdash A$ (or $\neg A$, etc.) is a *completed* sentence (of the artificial language). The plain sentence A represents a propositional act, and ‘ $\vdash A$ ’ represents the act of performing-and-accepting A ’s propositional act. It makes no sense (it is not allowed) to iterate force indicators: $\vdash \vdash A$. And a completed sentence $\vdash A$ cannot be a component of a larger sentence, as in $[\vdash A \vee \neg B]$. It might seem that the prohibition on including one illocutionary force operator within the scope of another is a departure from ordinary usage, for as well as making a statement:

- (1) I assert that Richard has resigned.

It is also possible to say:

- (2) I assert that I assert that Richard has resigned.

However, in (2) only the first ‘I assert that’ can serve as an illocutionary-force indicating device. The inner ‘I assert that’ merely predicates asserting of me (of the speaker). In $\mathcal{L}_{.75}$, the illocutionary force operators have no predicative use.

To represent an act of supposing a statement true, I use the top half of the assertion sign: \sqsubset . And for supposing a statement (propositional act) false, I use the bottom half of the sign of rejection: \neg . Rotating the positive signs 180° yields the negative force indicators, and vice versa.

If we expand $\mathcal{L}_{.5}$ with the force indicators ‘ \vdash ’ and ‘ \sqsubset ’, we get the language $\mathcal{L}_{.75}$. A sentence of $\mathcal{L}_{.5}$ is a *plain* sentence of $\mathcal{L}_{.75}$. There are no other plain sentences. If A is a plain sentence of $\mathcal{L}_{.75}$, then $\vdash A$ and $\sqsubset A$ are *completed* sentences of $\mathcal{L}_{.75}$. There are no other completed sentences. (To keep $\mathcal{L}_{.75}$ relatively simple, I am not introducing the force operators ‘ \neg ’, ‘ \neg ’, and ‘ $x \vdash$ ’.)

The deductive system $\mathcal{S}_{.75}$ is obtained from $\mathcal{S}_{.5}$. But in $\mathcal{S}_{.75}$, only completed sentences can occur as steps in a proof. All of the rules of $\mathcal{S}_{.5}$ are “transformed” to constitute rules of $\mathcal{S}_{.75}$. But the rules of $\mathcal{S}_{.75}$ take account of illocutionary force. For an elementary rule, if at least one premise is a supposition, so is the conclusion. If all premises are asserted/accepted, then so is the conclusion. The following are all examples of *& Introduction*:

$$\begin{array}{ccc} \frac{\sqsubset A \quad \sqsubset B}{\sqsubset [A \& B]} & \frac{\sqsubset A \quad \vdash B}{\sqsubset [A \& B]} & \frac{\vdash A \quad \vdash B}{\vdash [A \& B]} \end{array}$$

So, if we believe (or know) B , we can suppose A and reason to the conclusion ‘ $[A \& B]$ ’. We are not entitled to *accept* this conclusion, because it depends (in part)

on a supposition. Given that we accept B , supposing A commits us to *supposing* ' $A \& B$ '.

We shall understand an argument like this

$$\frac{\neg A \quad \vdash B}{\neg[A \& B]}$$

to establish a result about *relative*, or enthymematic, logical validity. The argument shows that ' $A/[A \& B]$ ' is *logically valid with respect to a background of belief/knowledge that includes B* . If an argument sequence ' $A_1, \dots, A_n/B$ ' is valid with respect to our current beliefs, then supposing A_1, \dots, A_n will commit us to supposing B , and accepting A_1, \dots, A_n will commit us to accepting B .

Tree proofs can now begin either with an assertion $\vdash A$ or a hypothesis $\neg B$. An assertion at the top of a branch is an *initial assertion* of the tree proof. A hypothesis is an *initial supposition*. For the nonelementary rules

$$\begin{array}{c} \vee\text{Elimination} \\ \frac{? [A \vee B] \quad \{\neg A\} \quad \{\neg B\} \quad \neg C \quad \neg C}{?C} \end{array} \qquad \begin{array}{c} \sim\text{Elimination} \\ \frac{\{\neg A\} \quad \{\neg A\} \quad ?B \quad ?\sim B}{?A} \end{array}$$

if the only uncanceled hypotheses in the subproofs leading to the sentences on the line are those in braces, then the conclusion is an assertion. Otherwise it is a supposition.

The following proof

$$\frac{\frac{\frac{x}{\neg A} \quad \vdash [A \supset C]}{\neg C} \text{MP} \quad \frac{\neg C}{\neg[C \vee D]} \vee I}{\vdash [A \vee B] \quad \neg[C \vee D]} \quad \frac{\frac{x}{\neg B} \quad \frac{\frac{\neg[\sim D \supset \sim B] \quad \frac{x}{\neg \sim D}}{\neg \sim B} \text{MP}}{\neg D} \sim E, \text{ drop } \neg \sim D}{\neg[C \vee D]} \vee I}{\neg[C \vee D]} \vee E, \text{ drop } \neg A, \neg B$$

establishes that ' $[\sim D \supset B]/[C \vee D]$ ' is logically valid with respect to knowledge/belief that includes ' $[A \vee B]$ ' and ' $[A \supset C]$ '. This proof

$$\frac{\frac{x}{\neg \sim \sim A} \quad \neg \sim A}{\neg A} \sim E, \text{ drop } \neg \sim A$$

establishes that ' $\sim \sim A/A$ ' is logically valid. Since this proof contains no initial assertions, it establishes an "absolute" result. And this proof

$$\frac{\neg A \quad \neg B}{\vdash [A \& B]}$$

The premise acts, with the forces indicated, do not support the conclusion act.

We can now make a second attempt to characterize deductive correctness. A simple argument is *deductively correct* if and only if performing its premise acts commits a person to performing the conclusion act. A complex argument is *deductively correct* if and only if its component arguments are deductively correct, and accepting/making the initial assertions and supposing the hypotheses of the complex argument commits a person to performing the conclusion act (with its indicated force).

6 Commitment semantics We have characterized truth-conditional entailment and basic entailment, as well as truth-conditional validity and basic validity. There are other important commitment-based concepts. Statements A_1, \dots, A_n *suppositionally entail* statement B , and $A_1, \dots, A_n/B$ is *suppositionally valid* if and only if supposing A_1, \dots, A_n commits a person to supposing B . Suppositional entailment doesn't coincide with basic entailment. A statement A basically entails 'I believe that A ', but there is no suppositional entailment in this case. For supposing A is not the same thing as supposing that A is believed; to suppose A is to suppose that A is true. So while this principle

$$\frac{\vdash A}{\vdash \text{I believe that } A}$$

is deductively correct, this one is not:

$$\frac{\neg A}{\neg \text{I believe that } A}$$

The truth-conditional semantic account for $\mathcal{L}_{.5}$ also applies to $\mathcal{L}_{.75}$ (to the plain sentences of $\mathcal{L}_{.75}$). In addition to the truth conditions of sentences of $\mathcal{L}_{.75}$ (and the statements these represent), it is appropriate to provide an account of *commitment conditions* for sentences of $\mathcal{L}_{.75}$. These conditions are relative to a given person or community. To indicate that the relevant person is committed to accepting A , I will use '+'. To show that she is committed to rejecting A , I will use '-'. If she is committed in neither "direction," I will use ' n '. So a commitment matrix for ' \sim ' looks like this:

$$\begin{array}{cc} A & \sim A \\ \hline + & - \\ - & + \\ n & n \end{array}$$

The matrices for '&', ' \vee ', and ' \supset ' are as follows:

A	B	$[A \& B]$	$[A \vee B]$	$[A \supset B]$
+	+	+	+	+
+	-	-	+	-
+	n	n	+	n
-	+	-	+	+
-	-	-	-	+
-	n	-	n	+
n	+	n	+	+
n	-	-	n	n
n	n	$-, n$	$+, n$	$+, n$

The values of component sentences are not in every case sufficient to uniquely determine the values of the compound sentence. If A and B each have n , and are irrelevant to one another, then ‘ $[A \vee B]$ ’ should have value n . But if A has value n , then so does $\sim A$; still, ‘ $[A \vee \sim A]$ ’ should have $+$. And if $A =$ ‘We will have spaghetti for dinner’ while $B =$ ‘We will have tuna fish salad for dinner’, then ‘ $[A \vee B]$ ’ might have $+$ even though each disjunct has n . Because of the last row in the matrices above, the matrices are not sufficient for determining an acceptable *commitment valuation* for $\mathcal{L}_{.75}$. However, the failure of functionality for the three values is not a defect of the commitment semantics. The three values, and the matrices, are important for capturing our intuitions about commitment. The matrices will be supplemented to provide an adequate commitment semantics; the present treatment is similar to the accounts found in Kearns [1], [2], and [3].

To complete the semantic account based on commitment, we need to link it to the truth conditional account. While not all of the statements a person accepts are true, no one believes a statement which she thinks to be false. In developing commitment semantics, we will idealize somewhat, and adopt the perspective of a person whose beliefs might all be true—her beliefs don’t conflict with one another. This is appropriate for uncovering deductive standards for arguments. We want to know what arguments will preserve truth on the presumption that our beliefs so far are true. A *commitment valuation* of $\mathcal{L}_{.75}$ is a function which assigns (exactly) one of $+$, $-$, n to each sentence of $\mathcal{L}_{.75}$.

Let $\mathcal{E}_1, \mathcal{E}_2$ be commitment valuations of $\mathcal{L}_{.75}$. Then \mathcal{E}_2 is an extension of \mathcal{E}_1 if and only if both

- (1) If $\mathcal{E}_1(A) = +$, then $\mathcal{E}_2(A) = +$;
- (2) If $\mathcal{E}_1(A) = -$, then $\mathcal{E}_2(A) = -$.

So an extension of a commitment valuation disagrees with the original valuation only for sentences assigned n by the original valuation.

Let f be a truth-conditional interpreting function of the plain atomic sentences of $\mathcal{L}_{.75}$. A commitment valuation \mathcal{E} is *based on* f if and only if \mathcal{E} assigns $+$ only to sentences true for the valuation determined by f , and assigns $-$ only to sentences false for that valuation. A commitment valuation \mathcal{E} is *coherent* if and only if it is based on a truth-conditional interpreting function of $\mathcal{L}_{.75}$. A person whose beliefs and disbeliefs are picked out by a coherent commitment valuation is a person whose beliefs might all be true and whose disbeliefs might all be false. (Here the ‘might’

indicates what I have elsewhere called *absolute epistemic possibility*.)

A coherent commitment valuation is *minimally acceptable* if and only if it satisfies the matrices above. If \mathcal{E} is a minimally acceptable commitment valuation based on interpreting function f , then $\langle f, \mathcal{E} \rangle$ is a *minimally acceptable pair*.

Let \mathcal{E}_0 be a coherent commitment valuation. The *commitment valuation determined by \mathcal{E}_0* is the valuation \mathcal{E} such that for every plain sentence A ,

1. if for every minimally acceptable pair $\langle f^*, \mathcal{E}^* \rangle$ such that \mathcal{E}^* is an extension of \mathcal{E}_0 , we have $f^*(A) = \text{T}$, then $\mathcal{E}(A) = +$;
2. if for every minimally acceptable pair $\langle f^*, \mathcal{E}^* \rangle$ such that \mathcal{E}^* is an extension of \mathcal{E}_0 , we have $f^*(A) = \text{F}$, then $\mathcal{E}(A) = -$;
3. Otherwise, $\mathcal{E}(A) = n$.

In the preceding definition, we can think of $\mathcal{E}_0, \mathcal{E}$ as follows. The person/community for whom the commitment semantics is developed is the *designated subject*. The valuation \mathcal{E}_0 assigns $+$ to those sentences (statements) that the designated subject has explicitly thought about and accepted (and which she still remembers). \mathcal{E}_0 assigns $-$ to those sentences she has thought about and rejected. So \mathcal{E}_0 characterizes the designated subject's explicit beliefs and disbeliefs at a given time. Then \mathcal{E} is intended to be the valuation which picks out the sentences that the designated subject is committed to accept and reject on the basis of her explicit beliefs. It is initially plausible that \mathcal{E} does this; the following results help to show the adequacy of our definitions. Results that can be established in a straightforward fashion will be stated without proof or with very sketchy proofs.

Lemma 6.1 *Let \mathcal{E}_0 be a coherent commitment valuation of $\mathcal{L}_{.75}$. Let \mathcal{E}_1 be a minimally acceptable extension of \mathcal{E}_0 , and \mathcal{E}_2 be a minimally acceptable extension of \mathcal{E}_1 . Then \mathcal{E}_2 is a minimally acceptable extension of \mathcal{E}_0 .*

Lemma 6.2 *Let f be a truth-conditional interpreting valuation of $\mathcal{L}_{.75}$, and let \mathcal{E}_0 be a commitment valuation based on f . Let \mathcal{E} be the commitment valuation determined by \mathcal{E}_0 . Then \mathcal{E} is based on f .*

Proof: Suppose $\mathcal{E}(A) = +$. There are two cases.

Case 1: $\mathcal{E}_0(A) = +$. Then $f(A) = \text{T}$.

Case 2: $\mathcal{E}_0(A) = n$. Then for every minimally acceptable pair $\langle f^*, \mathcal{E}^* \rangle$ such that \mathcal{E}^* is an extension of \mathcal{E}_0 , $f^*(A) = \text{T}$. Let \mathcal{E}^{**} be the commitment valuation such that $\mathcal{E}^{**}(A) = +$ if and only if $f(A) = \text{T}$ and $\mathcal{E}^{**}(A) = -$ if and only if $f(A) = \text{F}$. It is clear that $\langle f, \mathcal{E}^{**} \rangle$ is a minimally acceptable pair and \mathcal{E}^{**} extends \mathcal{E}_0 . Hence, $f(A) = \text{T}$.

Similarly, if $\mathcal{E}(A) = -$, then $f(A) = \text{F}$. □

Theorem 6.3 *Let \mathcal{E}_0 be a coherent commitment valuation of $\mathcal{L}_{.75}$, and let \mathcal{E} be the commitment valuation determined by \mathcal{E}_0 . Then \mathcal{E} is minimally acceptable and \mathcal{E} is the commitment valuation determined by \mathcal{E} .*

Proof: By Lemma 6.2, \mathcal{E} is coherent. Now we must show that \mathcal{E} satisfies the commitment matrices. Consider negation. Suppose A is a sentence of $\mathcal{L}_{.75}$ such that $\mathcal{E}(A) = +$. There are two cases.

Case 1: $\mathcal{E}_0(A) = +$. Then for every minimally acceptable pair $\langle f_1, \mathcal{E}_1 \rangle$ such that \mathcal{E}_1 extends \mathcal{E}_0 , $\mathcal{E}_1(\sim A) = -$ and $f_1(\sim A) = F$. So $\mathcal{E}(\sim A) = -$.

Case 2: $\mathcal{E}_0(A) = n$. Then for every minimally acceptable pair $\langle f_1, \mathcal{E}_1 \rangle$ such that \mathcal{E}_1 extends \mathcal{E}_0 , $f_1(A) = T$. But, for every such pair $f_1(\sim A) = F$. So $\mathcal{E}(\sim A) = -$.

If $\mathcal{E}(A) = -$, we can argue as above to show $\mathcal{E}(\sim A) = +$. Suppose $\mathcal{E}(A) = n$. Then $\mathcal{E}_0(A) = n$, and there is a minimally acceptable pair $\langle f_1, \mathcal{E}_1 \rangle$ such that \mathcal{E}_1 extends \mathcal{E}_0 , and $f_1(A) = T$, and another minimally acceptable pair $\langle f_2, \mathcal{E}_2 \rangle$ such that \mathcal{E}_2 extends \mathcal{E}_0 , and $f_2(A) = F$. Then $f_1(\sim A) = F$ and $f_2(\sim A) = T$. Hence, $\mathcal{E}(\sim A) = n$.

Consider ‘&’. Suppose $\mathcal{E}(A) = \mathcal{E}(B) = +$. Then for every minimally acceptable pair $\langle f_1, \mathcal{E}_1 \rangle$ such that \mathcal{E}_1 extends \mathcal{E}_0 , $f_1(A) = f_1(B) = T$. Hence $f_1[A \& B] = T$. So $\mathcal{E}[A \& B] = +$.

If $\mathcal{E}(A) = -$ or $\mathcal{E}(B) = -$, we can show that $\mathcal{E}[A \& B] = -$. Suppose $\mathcal{E}(A) = +$, $\mathcal{E}(B) = n$. Then there is a minimally acceptable pair $\langle f_1, \mathcal{E}_1 \rangle$ such that \mathcal{E}_1 extends \mathcal{E}_0 , and $f_1(A) = T$, $f_1(B) = T$, and $f_1[A \& B] = T$, while for another pair $\langle f_2, \mathcal{E}_2 \rangle$, \mathcal{E}_2 extends \mathcal{E}_0 , $f_2(A) = T$, $f_2(B) = F$, and $f_2[A \& B] = F$. Hence $\mathcal{E}[A \& B] = n$.

If $\mathcal{E}(A) = n$, $\mathcal{E}(B) = +$, the argument is similar. If $\mathcal{E}(A) = \mathcal{E}(B) = n$, then for some minimally acceptable pair $\langle f_1, \mathcal{E}_1 \rangle$ such that \mathcal{E}_1 extends \mathcal{E}_0 , $f_1(A) = F$, and $f_1[A \& B] = F$. Hence $\mathcal{E}[A \& B] \neq +$. We can similarly show that \mathcal{E} satisfies the matrix for ‘ \vee ’.

To show that \mathcal{E} is the commitment valuation determined by itself, we argue as follows. Suppose that A is a sentence such that for every minimally acceptable pair $\langle f^*, \mathcal{E}^* \rangle$ in which \mathcal{E}^* extends \mathcal{E} , $f^*(A) = T$. And suppose that $\mathcal{E}(A) \neq +$. Then there is a minimally acceptable pair $\langle f_1, \mathcal{E}_1 \rangle$ such that \mathcal{E}_1 extends \mathcal{E}_0 and $f_1(A) = F$. Clearly, \mathcal{E}_0 is based on f_1 . By Lemma 6.2, \mathcal{E} is based on f_1 . But then, $\langle f_1, \mathcal{E} \rangle$ is a minimally acceptable pair in which \mathcal{E} extends \mathcal{E} , and $f_1(A) \neq T$. This is *impossible!* Hence, $\mathcal{E}(A) = +$. Similarly, if A is a plain sentence such that for every minimally acceptable pair $\langle f^*, \mathcal{E}^* \rangle$ in which \mathcal{E}^* extends \mathcal{E} , $f^*(A) = F$, then $\mathcal{E}(A) = -$.

Clearly, if A is a plain sentence for which there is a minimally acceptable pair $\langle f^*, \mathcal{E}^* \rangle$ in which \mathcal{E}^* extends \mathcal{E} and $f^*(A) = T$, and there is another minimally acceptable pair $\langle f^{**}, \mathcal{E}^{**} \rangle$ in which \mathcal{E}^{**} extends \mathcal{E} and $f^{**}(A) = F$, then $\mathcal{E}(A) = n$. \square

Theorem 6.3 gives us reason to adopt the following definition.

Definition 6.4 A commitment valuation \mathcal{E} is *acceptable* if and only if there is a coherent commitment valuation \mathcal{E}_0 such that \mathcal{E} is the commitment valuation determined by \mathcal{E}_0 .

7 Relative truth-conditional concepts A proof from hypotheses in $\mathcal{S}_{.75}$ will have initial assertions $\vdash A_1, \dots, \vdash A_m$ ($m \geq 0$) and hypotheses (initial suppositions) $\lrcorner B_1, \dots, \lrcorner B_n$ ($n \geq 0$). If the conclusion of the proof is $\lrcorner C$, the proof establishes that argument sequence ‘ $B_1, \dots, B_n / C$ ’ is logically valid with respect to background knowledge/belief which includes A_1, \dots, A_m . If $m = 0$, then the argument sequence is logically valid without qualification. If the conclusion of the proof is $\vdash C$, then

there are no uncanceled hypotheses, and the proof establishes that C is true with respect to background knowledge that includes A_1, \dots, A_m . If the conclusion is $\vdash C$, and $m = 0$, then the proof establishes that C is a logical truth/law.

In order to characterize proofs in $\mathcal{S}_{.75}$ and the results these establish, we need to introduce semantic concepts that are relative to the designated subject's beliefs at a given time. We can think of a coherent commitment function \mathcal{E}_0 as characterizing the designated subject's beliefs and disbeliefs at some time. From the perspective of this cognitive state, different futures are epistemically possible. In such a future, the designated subject will not have given up any beliefs or disbeliefs, but she may have acquired additional beliefs/disbeliefs—and not simply on the basis of inferences from her present beliefs/disbeliefs. In an epistemically possible future, sentences will have truth values that cohere with her present beliefs and disbeliefs.

Let \mathcal{E}_0 be a coherent commitment valuation of $\mathcal{L}_{.75}$. Let f be a truth-conditional interpreting function of $\mathcal{L}_{.75}$, \mathcal{F}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$ which is an extension of \mathcal{E}_0 and is based on f , and \mathcal{F} be the commitment valuation determined by \mathcal{F}_0 . Then $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ is an *epistemically possible future* for \mathcal{E}_0 . The following results characterize epistemically possible futures.

Lemma 7.1 *Let \mathcal{E}_0 be a coherent commitment valuation of $\mathcal{L}_{.75}$, and let \mathcal{E} be the commitment valuation determined by \mathcal{E}_0 . Let \mathcal{F}_0 be a coherent commitment valuation which extends \mathcal{E}_0 , and let \mathcal{F} be the commitment valuation determined by \mathcal{F}_0 . Then \mathcal{F} is an extension of \mathcal{E} .*

Proof: Let A be a plain sentence of $\mathcal{L}_{.75}$ such that $\mathcal{E}(A) = +$. Suppose $\mathcal{F}(A) \neq +$. Then $\mathcal{E}_0(A) = n$. And in every minimally acceptable pair $\langle f, \mathcal{E}^* \rangle$ such that \mathcal{E}^* extends \mathcal{E}_0 , $f(A) = T$. But in some minimally acceptable pair $\langle f_0, \mathcal{F}^* \rangle$ in which \mathcal{F}^* extends \mathcal{F}_0 , $f_0(A) = F$. This is *impossible*, because \mathcal{F}^* extends \mathcal{E}_0 . Hence, $\mathcal{F}(A) = +$. Similarly, if $\mathcal{E}(A) = -$, then $\mathcal{F}(A) = -$. \square

Theorem 7.2 *Let A be a plain sentence of $\mathcal{L}_{.75}$. Let \mathcal{E}_0 be a coherent commitment valuation of $\mathcal{L}_{.75}$, and let \mathcal{E} be the commitment valuation determined by \mathcal{E}_0 . Then $\mathcal{E}(A) = +(-)$ if and only if in every epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 , $f(A) = T(F)$.*

Proof: Suppose $\mathcal{E}(A) = +$. Then in every minimally acceptable pair $\langle f^*, \mathcal{E}^* \rangle$ such that \mathcal{E}^* is an extension of \mathcal{E}_0 , $f^*(A) = T$. Suppose there is an epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 such that $f(A) = F$. By the lemma, $\langle f, \mathcal{F} \rangle$ is a minimally acceptable pair such that \mathcal{F} is an extension of \mathcal{E}_0 . This is *impossible*. So in every epistemically possible future $\langle f, \mathcal{E}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 , $f(A) = T$.

Suppose that in every epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 , $f(A) = T$. Suppose $\mathcal{E}(A) \neq +$. Then there is a minimally acceptable pair $\langle f^*, \mathcal{E}^* \rangle$ such that \mathcal{E}^* extends \mathcal{E}_0 and $f^*(A) = F$. But then $\langle f^*, \mathcal{E}^*, \mathcal{E}^* \rangle$ is an epistemically possible future for \mathcal{E}_0 . This is *impossible*, so $\mathcal{E}(A) = +$.

We can similarly show that $\mathcal{E}(A) = -$ if and only if in every epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 , $f(A) = F$. \square

We will define relative truth-conditional implication and validity in terms of epistemically possible futures. Let \mathcal{E}_0 be a coherent commitment valuation of $\mathcal{L}_{.75}$, and let

A_1, \dots, A_n, B be plain sentences of $\mathcal{L}_{.75}$. Then A_1, \dots, A_n *truth-conditionally imply* B with respect to \mathcal{E}_0 , and argument sequence $A_1, \dots, A_n/B$ is *truth-conditionally logically valid with respect to* \mathcal{E}_0 if and only if there is no epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 such that $f(A_1) = \dots = f(A_n) = T$, but $f(B) = F$. Sentence B is *logically true with respect to* \mathcal{E}_0 if and only if there is no epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 such that $f(B) = F$.

The following theorem, which relates the old absolute truth-conditional concepts to the new relative ones is entirely obvious.

Theorem 7.3 *Let $A_1, \dots, A_n/B$ be an argument sequence of $\mathcal{L}_{.75}$, and let C be a sentence of $\mathcal{L}_{.75}$. Then*

- (a) $A_1, \dots, A_n/B$ is *truth-conditionally logically valid* if and only if $A_1, \dots, A_n/B$ is *truth-conditionally logically valid with respect to every coherent commitment valuation* \mathcal{E}_0 ;
- (b) C is *logically true* if and only if C is *logically true with respect to every coherent commitment valuation* \mathcal{E}_0 .

A little care is needed to properly state soundness and completeness results for $\mathcal{S}_{.75}$ with respect to truth-conditional semantic concepts. But soundness and completeness proofs for \mathcal{S}_5 can be adapted to yield analogous results for $\mathcal{S}_{.75}$.

One difficulty in dealing with proofs in $\mathcal{S}_{.75}$ is that $\mathcal{L}_{.75}$ does not contain the force operator for rejection (the symbol ‘ \dashv ’). But a coherent commitment valuation \mathcal{E}_0 might assign $-$ to a sentence A and n to $\sim A$. The “idea” of our proofs is that initial assertions should be sentences assigned $+$ by \mathcal{E}_0 . But if $\mathcal{E}_0(A) = -$ while $\mathcal{E}_0(\sim A) = n$, then the fact that A is believed false will not play a role in such proofs. If we had introduced ‘ \dashv ’ and inference principles for rejection, we could have $\dashv A$ serve as an initial denial. Instead we will allow $\vdash \sim A$ to be an initial assertion if $\mathcal{E}_0(A) = -$, regardless of whether $\mathcal{E}_0(\sim A) = +$. The appropriate soundness result for $\mathcal{S}_{.75}$ is the following, which can be proved by induction on the rank of Γ .

Theorem 7.4 *Let Γ be a proof in $\mathcal{S}_{.75}$ from initial assertions $\vdash A_1, \dots, \vdash A_m$ and (uncanceled) hypotheses $\lrcorner B_1, \dots, \lrcorner B_n$. Let \mathcal{E}_0 be a coherent commitment valuation such that each A_i is either assigned $+$ by \mathcal{E}_0 or is the negation of a sentence assigned $-$ by \mathcal{E}_0 . Then*

- (a) if the conclusion of Γ is $\lrcorner C$, then B_1, \dots, B_n *truth-conditionally imply* C with respect to \mathcal{E}_0 ;
- (b) if the conclusion is $\vdash C$, then $n = 0$ and in every epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 , $f(C) = T$ (i.e., C is a logical truth with respect to \mathcal{E}_0).

The following theorems state completeness results.

Theorem 7.5 *Let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$. Let X be a set of plain sentences of $\mathcal{L}_{.75}$ and let C be a plain sentence of $\mathcal{L}_{.75}$ such that in every epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 in which each member of X has value T for f , $f(C) = T$. Then there is a proof in $\mathcal{S}_{.75}$ from initial assertions which are either assigned $+$ by \mathcal{E}_0 or are the negations of sentences assigned $-$ by \mathcal{E}_0 and from initial hypotheses which are sentences in X to the conclusion $\lrcorner C$.*

Lemma 7.6 *Let Γ be a proof in $S_{.75}$ from initial assertions $\vdash A_1, \dots, \vdash A_m$ and hypotheses $\lrcorner B_1, \dots, \lrcorner B_n$ to conclusion $\lrcorner C$. Then there is a proof in $S_{.75}$ from initial assertions $\vdash A_1, \dots, \vdash A_m, \vdash B_1, \dots, \vdash B_n$ to conclusion $\vdash C$.*

Theorem 7.7 *Let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$.*

- (a) *Let ‘ $B_1, \dots, B_n/C$ ’ be an argument sequence which is truth-conditionally logically valid with respect to \mathcal{E}_0 . Then there is a proof whose initial assertions are either assigned $+$ by \mathcal{E}_0 or are the negations of sentences assigned $-$ by \mathcal{E}_0 and whose hypotheses are $\lrcorner B_1, \dots, \lrcorner B_n$ which has conclusion $\lrcorner C$.*
- (b) *Let C be a sentence which is logically true with respect to \mathcal{E}_0 . Then there is a proof from initial assertions which are either assigned $+$ by \mathcal{E}_0 or are the negations of sentences assigned $-$ by \mathcal{E}_0 to the conclusion $\vdash C$.*

8 Commitment-based concepts The commitment semantics above provides the resources to define basic implication and basic logical validity. Let A_1, \dots, A_n, B be plain sentences of $\mathcal{L}_{.75}$, and let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$. Then A_1, \dots, A_n *basically imply* B with respect to \mathcal{E}_0 and ‘ $A_1, \dots, A_n/B$ ’ is *basically logically valid with respect to \mathcal{E}_0* if and only if for every epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 such that each A_i is either assigned $+$ by \mathcal{F}_0 or is the negation of a sentence assigned $-$ by \mathcal{F}_0 , $\mathcal{F}(B) = +$.

It isn’t appropriate to use ‘true’ for a concept based on commitment, so there are no basic logical truths. Instead, a sentence C is *basically logically analytic with respect to \mathcal{E}_0* if and only if in every epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 , $\mathcal{F}(C) = +$.

With respect to $\mathcal{L}_{.75}$, basic implication coincides with truth-conditional implication, and basic logical analyticity coincides with logical truth, as the following theorems show.

Theorem 8.1 *Let A_1, \dots, A_n, B be plain sentences of $\mathcal{L}_{.75}$, and let \mathcal{E}_0 be a coherent commitment valuation. Let A_1, \dots, A_n truth-conditionally imply B with respect to \mathcal{E}_0 . Then these sentences basically imply B with respect to \mathcal{E}_0 .*

Proof: The sentence ‘ $[[A_1 \& \dots \& A_n] \supset B]$ ’ is true in every epistemically possible future for \mathcal{E}_0 . By Theorem 7.2, $\mathcal{E}[[A_1 \& \dots \& A_n] \supset B] = +$. Now suppose that $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ is an epistemically possible future for \mathcal{E}_0 in which each A_i is either assigned $+$ by \mathcal{F}_0 or is the negation of a sentence assigned $-$ by \mathcal{F}_0 . By the lemma to Theorem 7.2, $\mathcal{F}[[A_1 \& \dots \& A_n] \supset B] = +$. But $\mathcal{F}[A_1 \& \dots \& A_n] = +$. By the matrices, $\mathcal{F}(B) = +$. \square

Corollary 8.2 *Let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$, and let A be a plain sentence of $\mathcal{L}_{.75}$ which is logically true with respect to \mathcal{E}_0 . Then A is basically logically analytic with respect to \mathcal{E}_0 .*

Lemma 8.3 *Let A, B be plain sentences of $\mathcal{L}_{.75}$. Let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$, and let \mathcal{E} be the commitment valuation determined by \mathcal{E}_0 . And in every epistemically possible future $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ for \mathcal{E}_0 in which $\mathcal{F}(A) = +$, let $\mathcal{F}(B) = +$. Then $\mathcal{E}[A \supset B] = +$.*

Theorem 8.4 *Let A_1, \dots, A_n, B be plain sentences of $\mathcal{L}_{.75}$. Let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$ such that A_1, \dots, A_n basically imply B with respect to \mathcal{E}_0 . Then A_1, \dots, A_n truth-conditionally imply B with respect to \mathcal{E}_0 .*

Proof: By the lemma, $\mathcal{E}[[A_1 \& \dots \& A_n] \supset B] = +$. Suppose $\langle f, \mathcal{F}_0, \mathcal{F} \rangle$ is an epistemically possible future for \mathcal{E}_0 such that $f(A_1) = \dots = f(A_n) = \text{T}$. By the lemma to Theorem 7.2, $\mathcal{F}[[A_1 \& \dots \& A_n] \supset B] = +$. Hence $f[[A_1 \& \dots \& A_n] \supset B] = \text{T}$. Clearly, $f(B) = \text{T}$. \square

Corollary 8.5 *Let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$, and let A be a plain sentence of $\mathcal{L}_{.75}$ that is basically logically analytic with respect to \mathcal{E}_0 . Then A is logically true with respect to \mathcal{E}_0 .*

The fact that basic implication coincides with truth-conditional implication for $\mathcal{L}_{.75}$ is not a deep result about the relation between these two semantic features. Instead the coincidence is due to the poverty of expressive power of $\mathcal{L}_{.75}$. (It is due to the extreme simplicity of the semantic features represented by sentences of $\mathcal{L}_{.75}$.) If $\mathcal{L}_{.75}$ were enriched with expressions that capture natural-language conditionals or with epistemic modal operators, basic implication would diverge from truth-conditional implication. However, for $\mathcal{L}_{.75}$ and $\mathcal{S}_{.75}$ we have the following as easy consequences of the truth-conditional soundness and completeness of $\mathcal{S}_{.75}$.

Theorem 8.6 *Let Γ be a proof in $\mathcal{S}_{.75}$ from initial assertions $\vdash A_1, \dots, \vdash A_m$ and hypotheses $\lrcorner B_1, \dots, \lrcorner B_n$. Let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$ such that each A_i is either assigned $+$ by \mathcal{E}_0 or is the negation of a sentence assigned $-$ by \mathcal{E}_0 . Then*

- (a) *if the conclusion of Γ is $\lrcorner C$, then B_1, \dots, B_n basically imply C with respect to \mathcal{E}_0 ;*
- (b) *if the conclusion of Γ is $\vdash C$, then $n = 0$, and C is basically logically analytic with respect to \mathcal{E}_0 .*

Corollary 8.7 *If $\Gamma, A_1, \dots, A_m, B_1, \dots, B_n, C, \mathcal{E}_0$ are as in the statement of Theorem 8.6, \mathcal{E} is the commitment valuation determined by \mathcal{E}_0 and the conclusion of Γ is $\vdash C$, then $\mathcal{E}(C) = +$.*

Theorem 8.8 *Let B_1, \dots, B_n, C be plain sentences of $\mathcal{L}_{.75}$, and let \mathcal{E}_0 be a coherent commitment valuation for $\mathcal{L}_{.75}$. Then*

- (a) *if B_1, \dots, B_n basically imply C with respect to \mathcal{E}_0 , there is a proof in $\mathcal{S}_{.75}$ from initial assertions of sentences which are either assigned $+$ by \mathcal{E}_0 or are the negations of sentences assigned $-$ by \mathcal{E}_0 and from hypotheses which are among $\lrcorner B_1, \dots, \lrcorner B_n$ to conclusion $\lrcorner C$;*
- (b) *if C is basically logically analytic with respect to \mathcal{E}_0 , then there is a proof from initial assertions of sentences which are either assigned $+$ by \mathcal{E}_0 or are the negations of sentences assigned $-$ by \mathcal{E}_0 to the conclusion $\vdash C$.*

Corollary 8.9 *If \mathcal{E}_0 is a coherent commitment valuation for $\mathcal{L}_{.75}$, \mathcal{E} is the commitment valuation determined by \mathcal{E}_0 , and C is a plain sentence of $\mathcal{L}_{.75}$ such that $\mathcal{E}(C) = +$, then there is a proof from initial assertions of sentences which are either*

assigned + by \mathcal{E}_0 or are the negations of sentences assigned – by \mathcal{E}_0 to conclusion $\vdash C$.

9 Coherence Statements have both truth conditions and commitment conditions, and we have used abstract interpretations of $\mathcal{L}_{.75}$ to give both the truth conditions and the commitment conditions of statements represented by the sentences of the artificial language. These abstract interpretations have enabled us to define truth-conditional and basic concepts.

The commitment conditions that figure in commitment valuations involve commitments to accept and reject statements. A person can also be committed to suppose a statement true or to suppose it false, but we have not introduced symbols to represent these “values.” It seems undesirable to have a special symbol to mark the statements which the designated subject is committed to suppose, for there is considerable variation about this from one moment to the next. However, commitments to suppose statements are fundamental to proofs from hypotheses. If we suppose A_1 and A_2 , and then infer supposition B , our inference is deductively correct only if A_1, A_2 suppositionally entail B .

Although we need to understand and characterize suppositional implication if we are to properly assess our proofs from hypotheses, in $\mathcal{L}_{.75}$ suppositional implication is defined as coinciding with truth-conditional implication. To suppose A is to suppose A true, and this commits us to further suppose whatever statements we can determine to be true once we are given that A 's truth conditions are satisfied. The coincidence of suppositional and truth-conditional implication for systems of classical logic is one reason why many have thought that the all logically important semantic concepts are truth-conditional.

Statements are consistent or compatible with one another independently of their illocutionary force. (It is propositional acts that are mutually consistent/compatible.) But if A is incompatible with B , then accepting $A(\vdash A)$ and accepting $B(\vdash B)$ are acts which are incoherent with each other. Accepting $A(\vdash A)$ and supposing $B(\perp B)$, or supposing both $(\perp A, \perp B)$ are also incoherent. The fundamental kind of incoherence is between both performing and declining to perform a single act (a single specific kind of act): $\vdash A$ and $x \vdash A$ are incoherent with each other, and so are $\perp A$ and $x \perp A$. Acts which lead to this fundamental incoherence are also incoherent. This new usage of ‘coherent’ and ‘incoherent’ “fits” our earlier definition of ‘coherent commitment valuation’. If \mathcal{E}_0 is a coherent commitment valuation, then it is coherent to accept the sentences (statements) assigned + by \mathcal{E}_0 and to reject the sentences assigned –.

It is never correct or appropriate to make incoherent assertions. If $\vdash A$ is incoherent with $\vdash B$, and a person finds herself committed to both $\vdash A, \vdash B$, then she needs to modify her beliefs so that she is no longer committed to accept both A and B . However, it is legitimate to make incoherent suppositions. That is the very “idea” behind the inference principle \sim Elimination. (It is the idea behind indirect proofs, or *reductio ad absurdum*.)

If A_1, \dots, A_n are incompatible, then they truth-conditionally entail every statement B . If accepting all of A_1, \dots, A_n is incoherent, then A_1, \dots, A_n basically entail every statement B . And if supposing A_1, \dots, A_n is incoherent, then these statements suppositionally entail every statement B . Tracing the truth conditions or the commit-

ment conditions of the individual statements leads us to see that the conditions of an arbitrary statement must also be satisfied.

The commitments generated by accepting or supposing individual statements are not sufficient to explain our deductive practice. There is in addition a general commitment to coherence (and, by extension, to compatibility/consistency). Consider the argument represented by this proof from hypotheses:

$$\begin{array}{c}
 \begin{array}{cc}
 x & x \\
 \underline{\neg \sim B} & \underline{\neg A} \\
 \hline
 & \&I \\
 \underline{\neg[\sim B \& A]} & \\
 \hline
 & \&E \\
 \underline{\neg A} & \underline{\neg \sim A} \\
 \hline
 \underline{\neg[A \vee B]} & \underline{\neg B} & \sim E, \text{ drop } \underline{\neg \sim B} & x \\
 & & & \underline{\neg B} \\
 \hline
 & & \vee E, \text{ drop } \underline{\neg A}, \underline{\neg B} \\
 & & \underline{\neg B}
 \end{array}
 \end{array}$$

This establishes that the argument sequence ‘ $[A \vee B], \sim A/B$ ’ is logically valid (in all three senses). The inference which exemplifies *~Elimination* is not motivated simply by truth conditions, because when considering only truth conditions, a person is simply led to further suppositions. The inference principle *~Elimination* yields a further conclusion (possibly a supposition), but it also cancels a supposition. In making a “move” according to *~Elimination*, a person is not simply tracing out the commitments of her initial assertions and her hypotheses. There is an independent (come what may) commitment to act coherently. Once the contradictory suppositions are reached in the argument above, the arguer is committed to remove the incoherence/inconsistency. The rule *~Elimination* prescribes a certain format for doing this, but other formats would be equally legitimate. Another format could sanction dropping a different hypothesis.

Even in our deductive system the commitment to coherence can call for an arbitrary one of two moves. In the proof

$$\begin{array}{c}
 \begin{array}{cc}
 & x \\
 \underline{\neg \sim \sim A} & \underline{\neg \sim A} \\
 \hline
 & \sim E, \text{ drop } \underline{\neg \sim A} \\
 \underline{\neg A}
 \end{array}
 \end{array}$$

which establishes that the argument sequence ‘ $\sim \sim A/A$ ’ is logically valid, the rules allow us to reach either the conclusion $\neg A$ or $\neg \sim A$. Either hypothesis might be canceled/discharged. The commitment here is to give up one of the inconsistent hypotheses.

Our deductive practice depends for its correctness on the commitments generated by accepting or supposing statements and on the general commitment to coherence. The commitment to coherence doesn’t always require that we accept or suppose some particular statement or statements—it simply requires that we take steps to eliminate incoherence.

10 Correctness Our earlier account of deductive correctness still stands: a simple argument is deductively correct if and only if performing the premise acts with their forces commits a person to performing the conclusion act with its force. A complex argument is deductively correct if and only if its component arguments are deductively correct and performing the (uncanceled) initial acts of the complex argument commits a person to performing the conclusion act with its force.

It isn't enough for an argument to be deductively correct. The following argument

$$\frac{\vdash A \quad \vdash [A \supset B]}{\vdash B}$$

is deductively correct, but this argument is *inappropriate* for a person who doesn't believe both A and ' $[A \supset B]$ '. The following deductively correct argument

$$\frac{\vdash A \quad \vdash \sim A}{\vdash B}$$

is inappropriate for everyone. A really satisfactory argument needs to meet certain epistemic requirements. Let us say that an argument is *epistemically appropriate* if and only if (1) it is deductively correct, (2) the arguer accepts the initial assertions, and (3) the initial assertions are mutually coherent.

Even an epistemically appropriate argument can fall short of what we ideally want. A person might construct an epistemically appropriate argument using rules that he had been taught in his logic class, but whose correctness he had failed to grasp. (He is like many of the successful students in our own logic classes.) An *epistemically adequate* argument is an epistemically appropriate argument in which the arguer recognizes/grasps the correctness of each component argument. Although many introductory logic texts claim that deductive perfection in an argument consists in a soundness which amounts to validity together with true premises, what is really wanted is epistemic adequacy in an argument for which the arguer has good grounds for accepting the premises.

11 Future developments The present paper provides the foundation and the framework for the future development of illocutionary logic. This is a quite different sort of contribution than found in Vanderveken [4] and [5]. In those works, the focus is on illocutionary acts in general, and logical features of all such acts. The present paper investigates specific kinds of illocutionary acts, not illocutionary acts in general. We have considered only suppositions and assertions. Natural next steps will extend the present treatment to accommodate acts of declining to accept/assert and declining to suppose, as well as acts of rejecting and supposing to be false.

Apart from laying the foundations, the present paper has provided a thorough treatment of deductive standards for arguments. The limited application of validity and the completely unimportant character of soundness understood as validity plus true premises have been made apparent. Deductively, the best kind of argument is not a sound argument, it is an epistemically adequate argument for which the arguer has good grounds for accepting the premises.

Because of the limited resources available in $\mathcal{L}_{.75}$, what certain statements of the language truth-conditionally imply is the same as what they basically imply and what they suppositionally imply. However, these three features will not always coincide. We are now in a position to extend the present treatment to deal with a variety of interesting and important cases for which the concepts diverge. These include a natural treatment of natural-language conditionals, and the exploration of various epistemic modal concepts such as those treated in Kearns [1], [2], and [3].

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NOTE

1. That the ordinary concept of validity does not apply to complex arguments was first pointed out to me in a talk that Gwen Burda delivered to the Buffalo Logic Colloquium in the spring semester of 1995.

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