Prospect Theory, Mental Accounting, and Differences in Aggregated and Segregated Evaluation of Lottery Portfolios

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If individuals have to evaluate a sequence of lotteries, their judgment is influenced by the presentation mode. Experimental studies have found significantly higher acceptance rates for a sequence of lotteries if the overall distribution was displayed instead of the set of lotteries itself. Mental accounting and loss aversion provide an easy and intuitive explanation for this phenomenon. In this paper we offer an explanation that incorporates further evaluation concepts of Prospect Theory. Our formal analysis of the difference in aggregated and segregated portfolio evaluation demonstrates that the higher attractiveness of the aggregated presentation mode is not a general phenomenon (as suggested in the literature) but depends on specific parameters of the lotteries. The theoretical findings are supported by an experimental study. In contrast to the existing evidence and in line with our theoretical results, we find for specific types of lotteries an even lower acceptance rate if the overall distribution is displayed.
(*Prospect Theory; Mental Accounting; Evaluation Procedures*)

1. Introduction

Assume you are asked to participate in two independent draws of a simple lottery

$$\begin{cases} 0.5 & \$200 \\ 0.5 & -\$100. \end{cases}$$

Would you accept playing such a sequence of gambles? What about the alternative offer to play the lottery

$$\begin{cases} 0.25 & \$400 \\ 0.5 & \$100 \\ 0.25 & -\$200 \end{cases}$$

just once? Needless to say, both outcome distributions are identical, so the offers should be either rejected or accepted.

But do individuals in fact perceive both choice situations to be equivalent and thus make identical decisions? Kahneman and Lovallo (1993) argue that people show a strong tendency to consider problems as unique, neglecting the portfolio context of the overall choice situation. By this "narrow framing" the two draws in the repeated trial offer might be evaluated in isolation, neglecting the overall outcome distribution.

In this paper we address the question of whether such a perception of the portfolio context would make the repeated trial offer look more or less attractive. Stated in other words: Does the attractiveness of a portfolio of lotteries increase (or decrease) by explicit provision of aggregated outcome information? The literature on this point has provided a consistent answer. In a recent paper Benartzi and Thaler (1999, p. 366) conclude from their experimental data, "When subjects are shown the distributional facts about repeated plays of a positive expected value gamble they like them more rather than less." Benartzi and Thaler (1995) provide a theoretical expla-

0025-1909/01/4705/0716$5.00
1526-5501 electronic ISSN

nation for this phenomenon, which is based on mental accounting and loss aversion.

In this paper we argue that, despite its intuitive appeal, the explanation is incomplete and the phenomenon is not as general as it might seem. We present a theoretical analysis of the evaluation problem based on mental accounting and Prospect Theory (Tversky and Kahneman 1992). This analysis predicts the aggregated presentation format to be even less attractive if lotteries with specific risk profiles (low probability for high loss) are involved. The predictions are confirmed by an experimental study.

The general phenomenon has much practical relevance. New financial products might be considered highly attractive by individual investors, although they are just combinations of existing investment alternatives, which are less appreciated in isolated evaluation.[1] On the other hand, the attractiveness of a product might increase by splitting it and offering its components separately. In general, the question to be answered is how a portfolio of risky options should be presented to make it look more attractive.

The remainder of the paper is structured as follows. In §2 we give an overview of the literature. In §3 we formalize the evaluation problem, extend the consideration to a more general space of lotteries, and provide a theoretical analysis based on mental accounting and Prospect Theory. In §4 we derive hypotheses and report the results of an experimental study testing these hypotheses. We conclude in §5 with a discussion of the results and suggestions for further research.

2. Related Literature

Most of the early literature on repeated gambling focused on the different risk-taking behavior in single and multiple plays of a lottery, i.e., risk in the short and the long run (Samuelson 1963; Lopes 1981, 1996; Tversky and Bar-Hillel 1983; Keren and Wagenaar 1987; Keren 1991). Recently, however, attention has been drawn to the question of different evaluation modes for repeated gambles by Benartzi and Thaler (1995). They gave an explanation for the Equity Premium Puzzle[2] based on the argument that stocks look much less attractive to investors if repeatedly evaluated in short intervals than if considered within the overall investment horizon. In a more general formulation this argument reads as follows: A portfolio or sequence of gambles (stock price movements) is less attractive compared to the overall portfolio distribution if each gamble in the set is evaluated separately.

Benartzi and Thaler (1995) provided a simple example to demonstrate how an isolated evaluation and loss aversion might lead to the proposed difference in attractiveness. They consider a decision maker with a value function of the form

$$v(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 2.5 \cdot x, & \text{if } x < 0, \end{cases}$$

reflecting loss aversion through the 2.5 times stronger impact of losses compared to equal-sized gains. This decision maker would accept the aggregated gamble

$$\begin{cases} 0.25 & \$400 \\ 0.5 & \$100 \\ 0.25 & -\$200, \end{cases}$$

because the overall evaluation (+25) is positive. However, he assigns the negative value −25 to each of the two lotteries

$$\begin{cases} 0.5 & \$200 \\ 0.5 & -\$100. \end{cases}$$

Thus, for segregated evaluation the gamble is rejected, although it is accepted in aggregated evaluation.

Gneezy and Potters (1997) explicitly tested the interdependence of evaluation period and risk-taking behavior in an experimental study.[3] In their setting, subjects were repeatedly asked to invest part of their endowment in sequences of mixed gambles. The willingness to take the risk was found to depend on the frequency of previous outcome feedback. The sequence of risky gambles was considered

[1] Consider as an example the very popular *guarantee funds*, which can be constructed as a simple portfolio of zero-coupon bonds and stock options.

[2] The puzzle, as introduced by Mehra and Prescott (1985), states that the historical difference in returns of stocks and bonds is much too high to be explained by any plausible degree of risk aversion.

[3] Thaler et al. (1997) provided a similar test in parallel research.

more attractive if just the combined return of three consecutive draws was reported, but not each single outcome.

A direct experimental test of the influence of presentation mode on the attractiveness of a lottery portfolio was reported by Redelmeier and Tversky (1992). They asked subjects to state their willingness to take part in five independent draws of a lottery of the form

$$\begin{cases} 0.5 & \$2{,}000 \\ 0.5 & -\$500. \end{cases}$$

If the problem was presented as a repeated trial (i.e., segregated), 63% of the subjects accepted the offer. If the aggregated distribution was displayed instead, the acceptance rate increased to 83%. This result might also confirm that the higher attractiveness of an aggregated evaluation is a general phenomenon. Note, however, that the risk profile used by Redelmeier and Tversky (1992) is very similar to the lottery

$$\begin{cases} 1/3 & \$2.50 \\ 2/3 & -\$1.00 \end{cases}$$

used by Gneezy and Potters (1997) and to the one used in Benartzi and Thaler's (1995) illustrative theoretical argument.

3. Formal Analysis

Our formal analysis extends the previous literature in two ways. First, we incorporate a more general value function, expressing loss aversion and diminishing sensitivity as proposed by Prospect Theory. Second, we extend the space of lotteries to cover all kinds of gain/loss probabilities and gain/loss sizes. In §3.1 we define the general lottery space and the value function. In §3.2 we demonstrate how the evaluation difference for a two-lottery portfolio can be separated, and formally analyze the case of pure loss aversion. In §3.3 the analysis is extended to incorporate both loss aversion and diminishing sensitivity. In §3.4 we discuss the additional impact of probability weighting and increasing portfolio size.

3.1. The Lottery Space $\mathcal{L}_{2,500}$ and the Value Function

For the analysis we exclusively consider mixed two-outcome lotteries

$$\begin{cases} 1-p & g \\ p & \ell. \end{cases}$$

In the following it will turn out that our qualitative results are not influenced by the absolute size of gains and losses but rather by the relative size of gains compared to losses.[4] Hence we simplify the analysis without loss of generality by assuming a fixed difference Δ between the two lottery outcomes. Because we want the lottery

$$\begin{cases} 0.5 & \$2{,}000 \\ 0.5 & -\$500 \end{cases}$$

used in the Redelmeier and Tversky (1992) study to be included in our analysis, we choose Δ to be 2,500, i.e., $g = \ell + 2{,}500$.

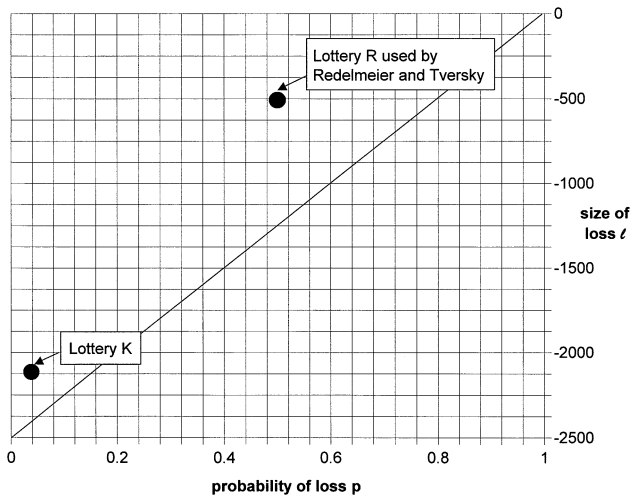
Given the Δ restriction, each mixed lottery can be described as a pair (p, ℓ) of loss probability and loss size. Our general lottery space is defined as $\mathcal{L}_{2,500} := \{(p, \ell) | \ell \in [-2{,}500, 0], p \in [0, 1]\}$. Note that the lotteries at the frontier of $\mathcal{L}_{2,500}$ are not mixed anymore but are included for limit considerations. The lotteries in $\mathcal{L}_{2,500}$ can be displayed in a (p, ℓ) coordinate system (see Figure 3.1). Each point within the rectangle corresponds exactly to one lottery in $\mathcal{L}_{2,500}$. At the left and the right boundary there are lotteries with sure gains and losses, respectively. All the lotteries on the diagonal have an expected value (EV) of 0. The expected value increases by moving up and to the left. Two specific lotteries are marked in the space. The point $R = (0.5; -500)$ corresponds to the lottery used by Redelmeier and Tversky in their study. The point $K = (0.04; -2{,}100)$, representing the lottery

$$\begin{cases} 0.96 & \$400 \\ 0.04 & -\$2{,}100 \end{cases}$$

is an example for a risk profile with low loss probability and high loss size, which will turn out to be particularly interesting in the further analysis. In the following we will refer to this type of lottery as *loan*

[4] This is because we assume throughout a specific type of power value function.

Figure 3.1 **The Lottery Space $\mathcal{L}_{2,500}$**



lottery because its risk profile is typical for the one faced by a creditor in a standard loan contract. The lottery K represents a \$2,100 loan, defaulting with a probability of 4% and an interest rate of 19%.[5]

Tversky and Kahneman (1992) proposed the following functional form for the value function:

$$v(x) = \begin{cases} x^{\alpha}, & \text{if } x \geq 0 \\ -k(-x)^{\beta}, & \text{if } x < 0 \end{cases}.$$

The parameter $k \geq 1$ describes the degree of loss aversion and $\alpha, \beta \leq 1$ measure the degree of diminishing sensitivity. $\alpha = \beta = 1$ represents the case of pure loss aversion discussed in the previous section. Tversky and Kahneman (1992) estimated the parameters α, β, and k and identified $\alpha = \beta = 0.88$ and $k = 2.25$ as median values. We will use these parameters for most of our calculations.

It should be mentioned that this specific functional form of the value function is not a necessary condition for our results. Similar results can be derived for other value functions reflecting loss aversion and diminishing sensitivity. However, the two-parameter form v_k^{α} (we will assume $\alpha = \beta$ throughout and omit the index β in the following) nicely demonstrates how the two concepts (loss aversion and diminishing sensitivity) influence the overall evaluation difference.

[5] This argument assumes that the loan engagement is evaluated ex ante using the status quo as a reference point and offsetting outputs against inputs. Cf. Casey (1994, 1995) for different ways of framing this kind of decision problem.

3.2. Separation of the Evaluation Difference and the Case of Pure Loss Aversion

We will use the notation $X + X$ to describe a portfolio consisting of two lotteries X (we call this segregated presentation). By $2X$ we refer to the lottery resulting as the overall distribution of the portfolio $X + X$ (we call this *aggregated presentation*). For each lottery $X = (p, \ell)$, the aggregated distribution $2X$ has the three outcomes 2ℓ, $\ell + g$, and $2g$. While the sign of 2ℓ and $2g$ is obvious, the mixed outcome $\ell + g$ is positive or negative depending on the size of ℓ and g. By A_L, A_G, and A_M we notate the values of these outcomes, i.e.,

$$A_L := v(2\ell), \quad A_G := v(2g) \quad \text{and} \quad A_M := v(\ell + g).$$

Similarly p_L, p_G, and p_M are used to refer to the probabilities for the pure loss, the pure gain, and the mixed case. The overall evaluation of the portfolio distribution $2X$ is then obtained as

$$A := p_L \cdot A_L + p_M \cdot A_M + p_G \cdot A_G.$$

Next we consider the portfolio $X + X$ if both lotteries are evaluated separately and the values are summed up to derive the overall evaluation of the portfolio. This segregated evaluation process results in a total value of $S = 2 \cdot [p \cdot v(\ell) + (1-p) \cdot v(g)]$.

Defining $S_L := 2 \cdot v(\ell)$, $S_G := 2 \cdot v(g)$, and $S_M := [v(\ell) + v(g)]$, we can write

$$\begin{aligned} S &= p^2 \cdot 2 \cdot v(\ell) + (1-p)^2 \cdot 2 \cdot v(g) \\ &\quad + 2 \cdot (1-p) \cdot p \cdot [v(\ell) + v(g)] \\ &= p_L \cdot S_L + p_G \cdot S_G + p_M \cdot S_M. \end{aligned}$$

It turns out that the total difference D between the two types of evaluation can be separated into three parts (a loss part, a gain part, and a mixed part),

$$\begin{aligned} D = A - S &= p_L \cdot [A_L - S_L] + p_G \\ &\quad \cdot [A_G - S_G] + p_M \cdot [A_M - S_M] \\ &= p_L \cdot D_L + p_G \cdot D_G + p_M \cdot D_M. \end{aligned}$$

The relevant expressions for the separation are summarized in Table 3.1. By definition a positive value of

Table 3.1 Separation of Evaluation Difference into *L*-, *G*-, and *M*-part

	L-part	*G*-part	*M*-part
Probability for the case	$p_L = p^2$	$p_G = (1-p)^2$	$p_M = 2\cdot(1-p)\cdot p$
Contribution to value if evaluated aggregated	$A_L = v(2\cdot\ell)$	$A_G = v(2\cdot g)$	$A_M = v(\ell+g)$
Contribution to value if evaluated segregated	$S_L = 2\cdot v(\ell)$	$S_G = 2\cdot v(g)$	$S_M = v(\ell)+v(g)$
Difference of value contributions	$D_L := A_L - S_L = v(2\cdot\ell) - 2\cdot v(\ell)$	$D_G := A_G - S_G = v(2\cdot g) - 2\cdot v(g)$	$D_M := A_M - S_M = v(\ell+g) - v(\ell) - v(g)$

D corresponds to a higher attractiveness of an aggregated evaluation. Using this notation we can formally state the main question as follows:

- For which lotteries $X = (p, \ell)$ in $\mathcal{L}_{2,500}$ do we get positive (negative) values of D, i.e., a higher (lower) attractiveness of an aggregated evaluation?[6]

We should further examine the question:

- How does the sign of D, i.e., the preference for one of the presentation modes, depend on the parameters α and k of the value function v?

To contrast with the later results, we start the analysis with the case of pure loss aversion ($\alpha = 1$). The linearity of the value function in the gain as well as in the loss domain immediately yields $D_G = v(2\cdot g) - 2\cdot v(g) = 0$ and $D_L = v(2\cdot\ell) - 2\cdot v(\ell) = 0$, s.t. neither the loss part L nor the gain part G contributes to the overall evaluation difference.

Further we have $D_M = [v(\ell+g) - v(\ell) - v(g)] = (k-1)\cdot\min\{g, |\ell|\}$, thus the mixed part is positive for all lotteries in the interior of $\mathcal{L}_{2,500}$. Proposition 1 follows immediately:

PROPOSITION 1. (The Case of Pure Loss Aversion). *Assuming a value function*

$$v(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -k(-x), & \text{if } x < 0 \end{cases}$$

with $k > 1$, the difference D between aggregated and segregated evaluation of a portfolio is positive for each mixed lottery X in the interior of $\mathcal{L}_{2,500}$.

Despite the simplicity of the argument, let us visualize some of the details to contrast with the further

[6] At this point it becomes clear that the restriction of the lottery space to a fixed gain/loss difference $\Delta = 2{,}500$ does not mean a qualitative loss of generality. Assuming a power value function v_k^α, a scaling of both outcomes g and ℓ by a factor c causes D to change by the factor c^α. This does not influence the sign of D.

Figure 3.2 Value Differences $D_L(\ell)$, $D_G(\ell)$, $D_M(\ell)$ and Probabilities $p_L(p)$, $p_G(p)$, $p_M(p)$ for $\alpha = 1$

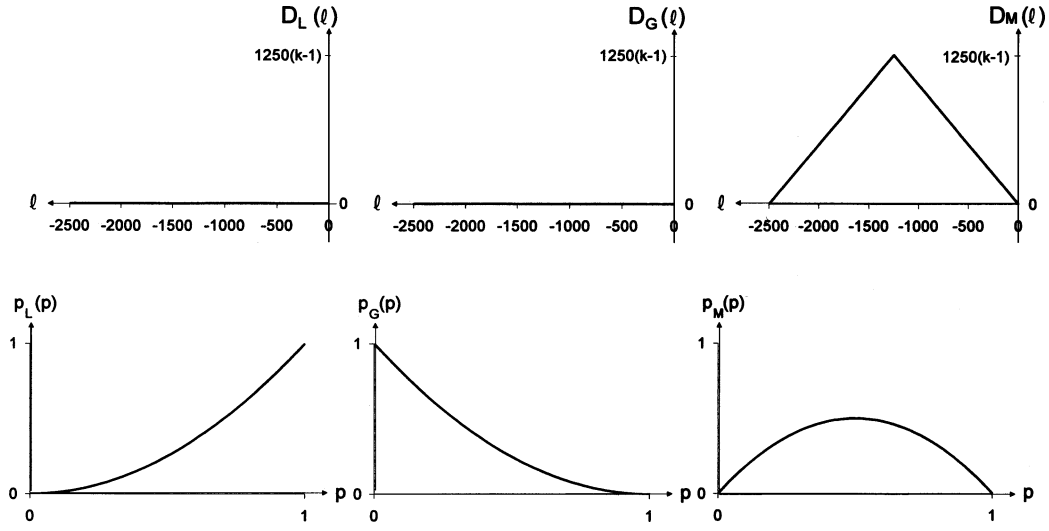
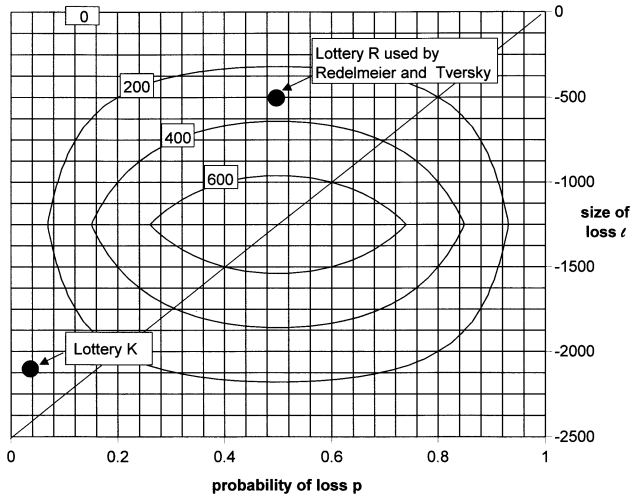


Figure 3.3 **Iso *D* Lines for the Case $\alpha = 1$ and $k = 2.25$**



analysis. Figure 3.2 presents the value and probability components of the separation.

In Figure 3.3 we present Iso-D lines in a (p, ℓ) coordinate system representing the space $\mathcal{L}_{2,500}$ (for $\alpha = 1$ and $k = 2.5$). It can be seen that D is positive everywhere and converges to 0 only at the boundaries of the space $\mathcal{L}_{2,500}$, where we get pure gain (loss) lotteries.

3.3. The Case of Loss Aversion and Diminishing Sensitivity

If we generalize the value function to reflect diminishing sensitivity as well as loss aversion, we can still separate D into an L-, G-, and M-Part. Because of the concavity of v in the gain domain and the convexity in the loss domain, the components $D_L = v(2 \cdot \ell) - 2 \cdot v(\ell)$ and $D_G = v(2 \cdot g) - 2 \cdot v(g) = v(2 \cdot \ell + 5{,}000) - 2 \cdot v(\ell + 2{,}500)$ are decreasing functions in ℓ that are positive or negative, respectively (see Figure 3.4).

This is a formal description of a general rule first explicitly stated by Thaler (1985): It is more attractive to consider losses in aggregation ($D_L > 0$) but gains in segregation ($D_G < 0$).

The negativity of D_G allows us to state an immediate existence theorem:

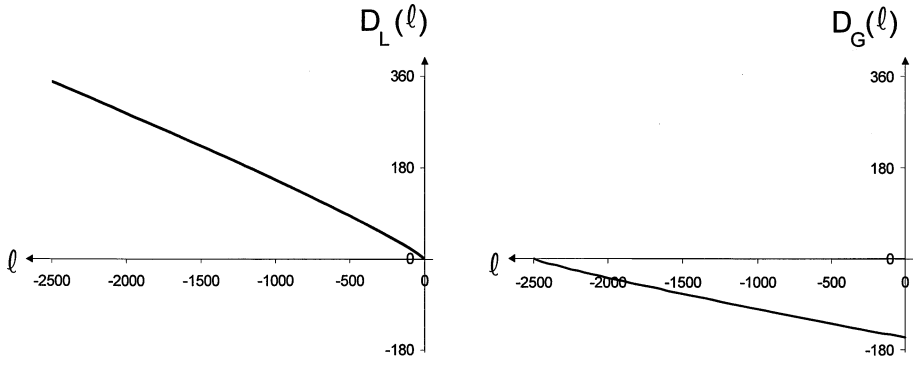
Proposition 2. (Existence of mixed lotteries with negative evaluation difference D). *For any value function $v_k^\alpha(x)$ with $\alpha < 1$, there exist mixed lotteries in $\mathcal{L}$ with a negative evaluation difference D, i.e., with a higher segregated than aggregated evaluation.*

The proposition can be proven by a simple continuity argument. If we have $\alpha < 1$ and pick an arbitrary $\ell \in (-2{,}500, 0]$, we get a negative value $D_G(\ell)$. If follows $D = D_G(\ell) < 0$ for the lottery $(0, \ell)$, as it holds $p_L = p_M = 0$ and $p_G = 1$ for $p = 0$. Because D is a continuous function in p on [0, 1], there must exist a small $p > 0$ such that D is still negative for a lottery (p, ℓ).

This is an intuitive result. Mixed lotteries that are almost sure gains are more attractive in segregated evaluation because the negative value difference in the G-part is dominant. With the same arguments it can be concluded that we will find positive D values if the lotteries have very high loss probabilities. In fact, it is true that for each loss size $\ell \in (-2{,}500, 0)$ there exist lotteries (p, ℓ) with positive and negative D values.

However, these existence theorems are not really helpful for practical considerations because they do

Figure 3.4 **$D_L(\ell)$ and $D_G(\ell)$ for $k = 2.5$ and $\alpha = 0.88$**



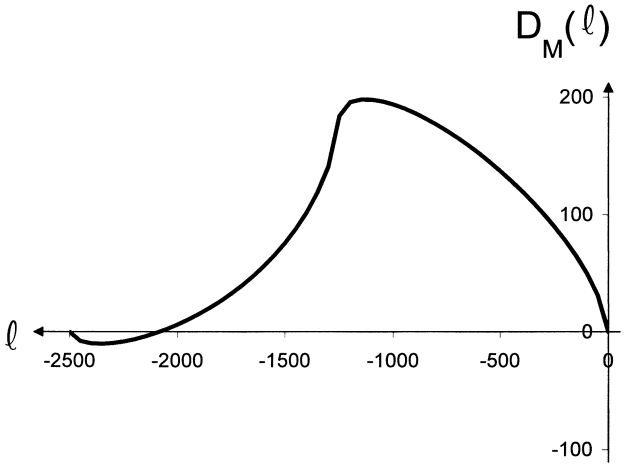
not provide any information about the size of the effect. In particular, we do not get clear predictions for the lotteries we consider to be most important for practical applications. These are lotteries with moderately positive expected values, placed slightly above the diagonal in the (p, ℓ) coordinate system. We distinguish three types of such lotteries: (a) lotteries with low probability for a high loss, (b) lotteries with rather moderate probabilities and outcome sizes, and (c) lotteries with high probability for a small loss.

For the first step of the further analysis, let us ignore the mixed M-part of the evaluation difference D and focus on the term $D_{LG} := [p_L \cdot D_L + p_G \cdot D_G]$. We can conclude from the monotonicity of p_L, p_G, D_L, and D_G that D_{LG} grows in ℓ as well as in p. Hence, we have low D_{LG} values for very attractive, and high D_{LG} values for very unattractive, lotteries. But what about different risk profiles with similar expected values as in (a), (b), and (c) above? An examination of the lotteries on the diagonal ($EV = 0$) can provide some basic insight for the situation of simultaneous ℓ-increase and p-decrease. It can be shown that there exists a unique $p_0 \in (0, 1)$ s.t. we have for each lottery $(p, \ell) \in \mathcal{L}_{2,500}$ with $EV = 0: D_{LG} < 0$ if and only if $p < p_0$.[7]

Hence, concerning the impact of the L- and the G-part we should rather expect to find negative D values for the risk profile (a) above, i.e., for lotteries with low probabilities for high losses. However, so far we did not consider the contribution of the mixed M-part, which proved to be solely responsible for the positive D value in the pure loss-aversion case.

Recalling the definition of the value difference $D_M = v(\ell + g) - v(\ell) - v(g)$, we have to examine in which cases a sure gain and a sure loss should be combined to provide a higher overall evaluation. This question was also addressed by Thaler (1985). He stated that small losses and high gains should be aggregated and small gains and high losses should be segregated. A formal analysis leads to the result that there is indeed

Figure 3.5 $D_M(\ell)$ **for** $\alpha = 0.7$ **and** $k = 2.25$



a unique critical loss size ℓ°, s.t. D_M is negative on $(-2{,}500, \ell^\circ)$ and positive on $(\ell^\circ, 0)$ (see Figure 3.5).[8]

Because of loss aversion the positive interval dominates.[9] Hence we find a negative contribution of the M-part to the evaluation difference D if and only if we have lotteries with sufficiently high losses. From the three interesting risk profiles (a), (b), and (c) we should thus expect profile (a)—low probability for high loss—to be best suited to result in a negative D value. While these thoughts are informal, an Iso-D diagram can clarify the point (see Figure 3.6). The isoline for $D = 0$ is the boundary between the dark and the light area. All lotteries in the dark portion of the $\mathcal{L}$ space have negative D values and should thus look more attractive in segregated evaluation. Note in particular the dark bulge in the lower left corner where the lotteries with low probability for a high loss are located.

It can be seen that there is still a high positive D value assigned to the lottery R used by Redelmeier and Tversky. In contrast, the D value for the loan-type lottery K is negative. Figure 3.7 demonstrates the impact of diminishing sensitivity by reducing α from

[7] p_0 is decreasing in k as well as in α. For $k = 1$ it holds $p_0 = 1/2$. See Result 1 in the appendix for a formal proof.

[8] We have chosen the parameter $\alpha = 0.7$ to better visualize the general shape of D_M. For higher values of α, the negative part of D_M is hard to recognize.

[9] For $k = 1$ we have symmetry, for increasing k the ℓ° decreases, approaching $-2{,}500$ for $k \to \infty$. See Result 2 in the appendix for a formal proof of these facts. More general results can be found in Langer (1999).

Figure 3.6 $\mathcal{L}$ **space with Iso-D Lines for** $\alpha = 0.88$ **and** $k = 2.25$

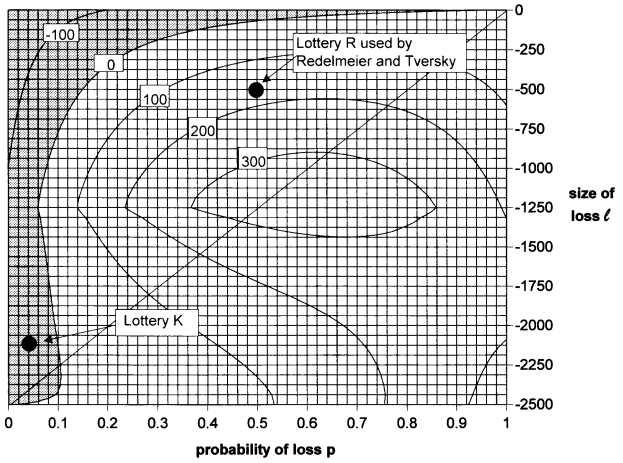
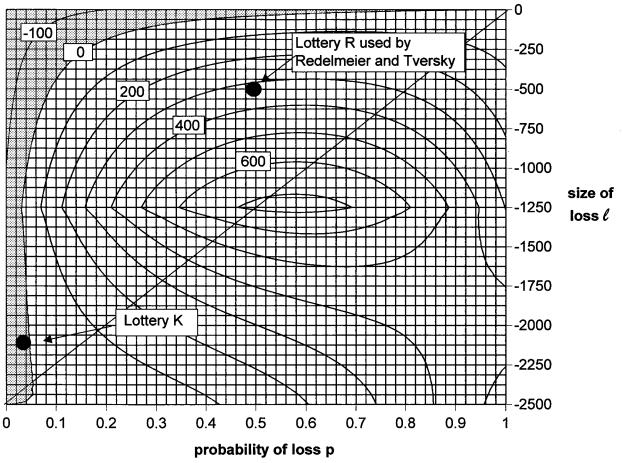


Figure 3.8 **Iso-D Lines for** $\alpha = 0.88$ **and** $k = 3.5$



0.88 to 0.7. As could be expected from our analysis, the area of negative D values, and especially the size of the bulge, grows.

The consequence of increasing loss aversion is displayed in Figure 3.8. For $k = 3.5$ the bulge almost disappears. From the known properties of D_{LG} and D_M we can conclude that each mixed lottery in $\mathcal{L}$ has a positive D value for a sufficiently high degree of loss-aversion k.

3.4. Extensions

Next we present some results for larger portfolios and probability weighting. For an increased portfolio size it is still possible to separate the evaluation difference D as in §3.2. However, because of the increasing number of parts and the complexity of each value component, a detailed formal analysis does not provide any additional insights. Hence, we just visualize the effect of increasing portfolio size in Iso-D diagrams. Figures 3.9 and 3.10 display the evaluation difference D for portfolios consisting of 5 and 10 identical lotteries. The larger the portfolio, the less the bulge is pronounced in the lower left corner. The proportion of the lottery space $\mathcal{L}$ with a negative evaluation difference D increases with growing portfolio size.

Figure 3.11 demonstrates the evaluation difference for even larger portfolios. It presents the relative evaluation difference D/n for a portfolio of n lotteries R

Figure 3.7 **Iso-D Lines for** $\alpha = 0.7$ **and** $k = 2.25$

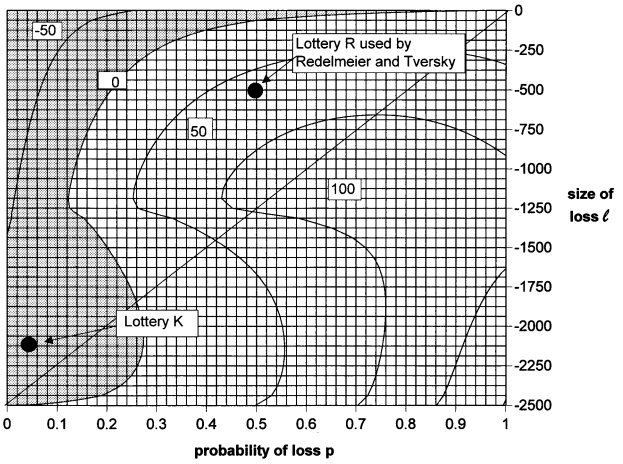


Figure 3.9 **Portfolio of Size 5 for** $\alpha = 0.88$ **and** $k = 2.25$

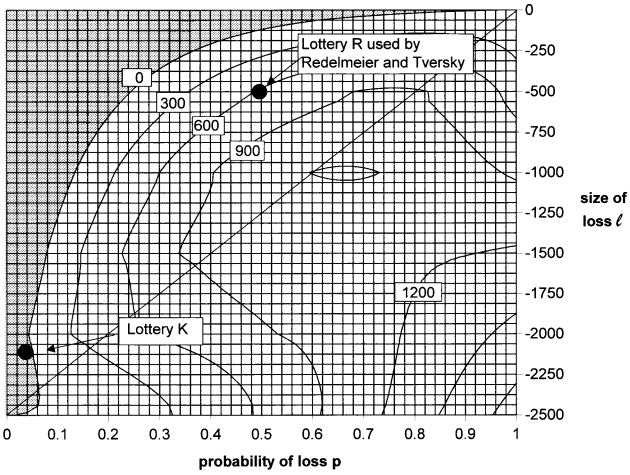
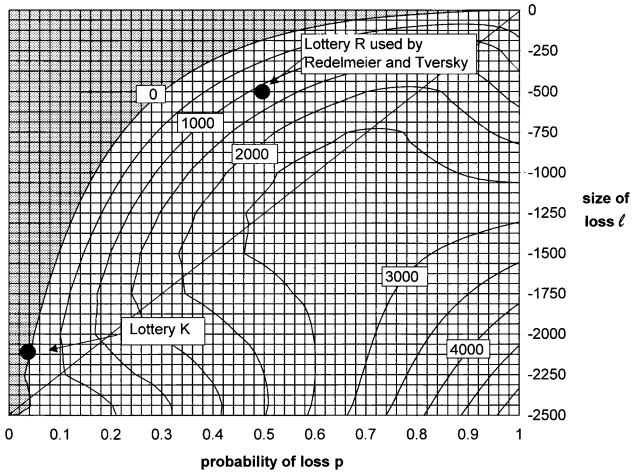


Figure 3.10 **Portfolio of Size 10 for $\alpha = 0.88$ and $k = 2.25$**



and K. For sufficiently high n, not only the relative difference D/n, but also the absolute difference D, decreases in n. Result 3 in the appendix shows that any gamble in $\mathcal{L}$, which is accepted in single play, has a higher segregated than aggregated evaluation (i.e., $D < 0$) for a sufficiently large portfolio size. Thus, even for the lottery R we would eventually (i.e., for higher n) find negative D/n values in Figure 3.11. It should further be noted that the function D/n is neither monotonic nor required to have a single peak. In fact, for some lotteries the D/n values change their signs several times for increasing n.

In the light of the experimental results provided by Benartzi and Thaler (1999), we do not consider our results for the large portfolios to have much practical relevance. For large portfolios (Benartzi and Thaler used sizes between 100 and 150), the effect caused by the value transformation seems to be superimposed by a severe bias in probability judgment. The subjects in the Benartzi and Thaler study strongly overestimated the very small probability for an overall loss in the repeated trial format.

Prospect Theory does not only predict value transformation but also the transformation of probabilities into decision weights. These weights are used in the evaluation procedure instead of the actual probabilities. Unfortunately, the probability weighting rules out the simple separation and general analysis as presented in §3.2. We cannot define "weights" of the G-, L-, and M-part any longer, because the decision weights depend on the type of portfolio segmentation, i.e., they are different in aggregated and segregated evaluation. Informally, one can conclude that the preference for a segregated evaluation of a loan lottery portfolio is weakened if probability

Figure 3.11 **D/n as a Function of n for $\alpha = 0.88$ and $k = 2.25$ (the Lines Represent Scale Changes)**

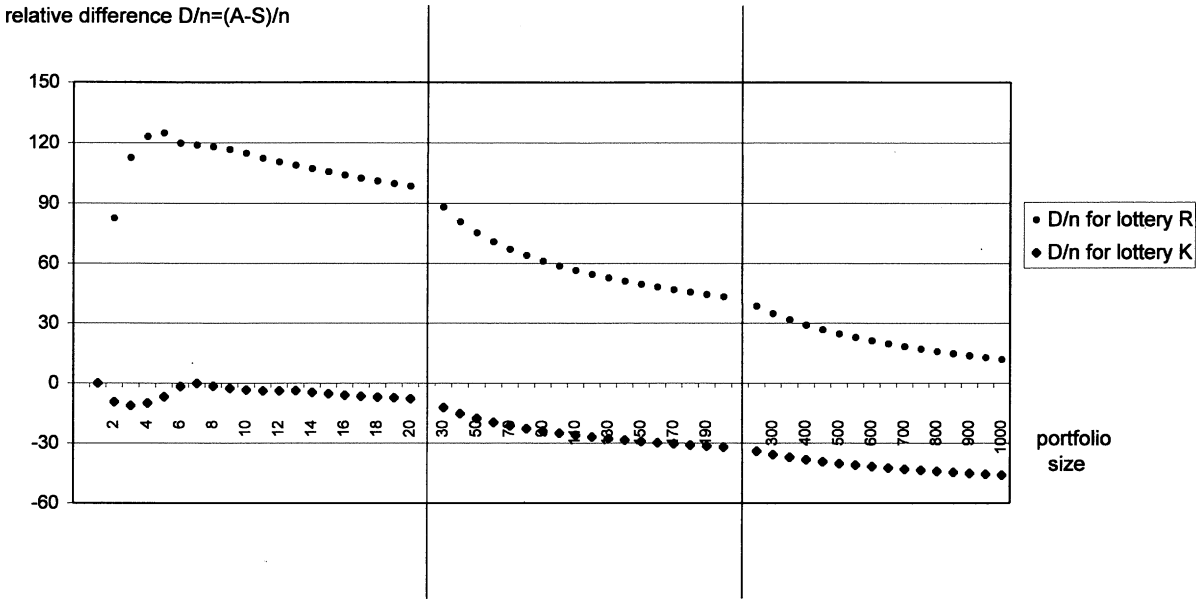


Figure 3.12 **Iso-*D* Lines for $\alpha = 0.88$, $k = 2.25$, $\gamma^+ = 0.69$, and $\gamma^- = 0.61$**

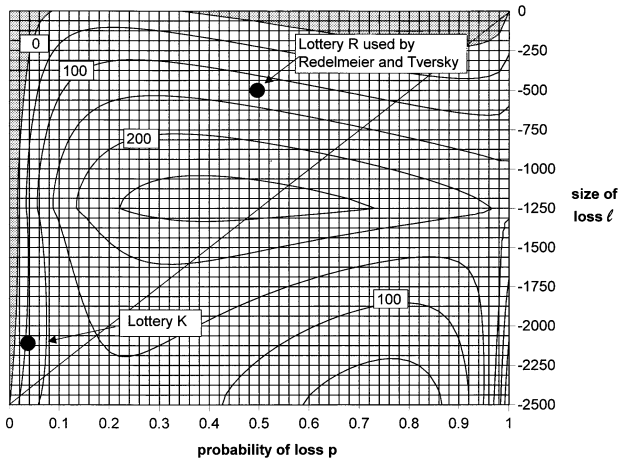
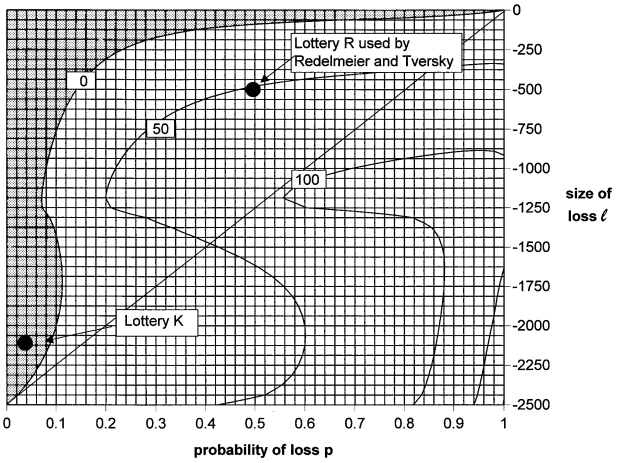


Figure 3.13 **Iso-*D* Lines for $\alpha = 0.7$, $k = 2.25$, $\gamma^+ = \gamma^- = 0.8$**



weighting is incorporated.[10] This effect is illustrated in Figure 3.12, where Iso-*D* lines are presented in the $\mathscr{L}$ space for an evaluation that takes into account loss aversion, diminishing sensitivity, and probability weighting. The computation is based on Cumulative Prospect Theory and incorporates probability-weighting functions

$$w_\gamma(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

with $\gamma < 1$ as introduced by Tversky and Kahneman (1992). Using their parameter estimates $\gamma^+ = 0.69$ and $\gamma^- = 0.61$ for gains and losses, the dark bulge in the lower left corner completely disappeared. Figure 3.13 presents Iso-*D* lines for γ values closer to 1, e.g., $\gamma^+ = \gamma^- = 0.8$ (see Figure 3.7 for $\gamma = 1$).[11]

4. Experimental Study

In this section the design and the results of two experimental studies are reported. We begin with a few general comments regarding the experimental design and the choice of risk profiles used in these experiments.

A comparison of acceptance rates as used by Redelmeier and Tversky (1992) provides a convenient and simple test for our theoretical predictions. We can easily hypothesize how the sign of the difference in acceptance rates should depend on the risk profiles of the involved lotteries. It is a major strength of this method that subjects are confronted with very simple choice situations and are not unduly influenced by coding or other framing effects. A weakness of this method is the fact that its use is restricted to a specific class of lotteries. For very attractive or very unattractive lotteries we must expect to find acceptance rates close to 100% or 0% for both presentation modes. As the acceptance decision does not capture the strength of a preference, the method is restricted to lotteries with moderately positive expected values for which we can hope to find switches in acceptance.

Although these specific lotteries are surely most important for practical applications, an extension of the empirical tests to other regions of the lottery space would be interesting to examine the general correctness of our predictions. There are two possible designs for such an extension. First, subjects could be asked to directly compare the two presentations of the same lottery portfolio. Second, subjects could provide certainty equivalents for both presentations. A com-

[10] This is because the small probability of a loss (with the positive D_L contribution) is overweighted and the high gain probability (with the negative D_G contribution) is underweighted. However, it should be clear that the specific effect is restricted to loan lotteries. In the upper right corner of the lottery space a reverse impact of probability weighting should be expected by the same argument (see Figure 3.12).

[11] Cf. Wu and Gonzalez (1996) for empirical evidence of less curved probability-weighting functions.

parison of these certainty equivalents could serve as a direct test of our theoretical results.

Both methods have severe drawbacks. At this point we want to point out once again that our research does not address the question of how the problem presentation influences an individual's coding. Instead, our theoretical predictions are based on specific assumptions about the coding (with the central assumption that a segregated presentation causes a segregated evaluation). We can only expect to find support for our predictions if the experimental design induces the participants to code the decision problems in the assumed way. The more lotteries are involved in each decision and the more complex the decision situation is, the more ambiguous the coding procedure will be. We do not know how individuals code a direct comparison of a three-outcome lottery with a sequence of two-outcome lotteries. In our opinion it is not very likely, however, that independent evaluations are derived and compared for the final decision. We thus do not consider the direct comparison method to represent an appropriate design.

For the certainty equivalent method and for all other methods in which the lottery (sequence) is compared to a sure amount, there is another severe coding problem. If we assume subjects to neglect the overall decision context given a segregated presentation, it is straightforward not only to assume a segregated evaluation of the lotteries but to further assume an isolated assignment of certainty equivalents to the lotteries. Hence an elicitated certainty equivalent must be interpreted as a sum of single-play certainty equivalents. Because of the nonlinearity of the value function (which not only transforms the lottery outcomes but also the certainty equivalents), a comparison of elicitated certainty equivalents thus would not provide an appropriate test for our theory. To illustrate this point, assume an individual assigns a value of $+100$ to each of the two lotteries in a sequence and a lower total value of $+190$ to the aggregated distribution. The resulting negative D value is not reflected in the certainty equivalents. Assuming a parameter $\alpha = 0.88$ of the value function, the individual assigns a certainty equivalent of $\sim$374 DM ($= 187 + 187$ DM) to the repeated trial presentation, but a higher certainty equivalent of $\sim$388 DM to the aggregated version.

This problem turns out to be particularly severe for the very attractive lotteries in the upper left corner of our lottery space. While these lotteries (with their high negative D values) might seem well suited to generate easy experimental support for our main theoretical result (that specific lotteries are preferred in segregated presentation), their appropriateness is undermined by the coding problems of the elicitation procedure. It turns out that by the concavity of the gain function the negative D values are transformed into positive differences of certainty equivalents for all such lotteries.[12]

We thus restricted our attention to risk profiles with moderately positive expected values, enabling us to use an acceptance rate comparison as experimental procedure. From our analysis in §3, we know that the restricted set should still be sufficiently rich (D values with different signs) to allow an empirical test of our main theoretical results.

Design of Study 1. This introductory study was set up to directly test the robustness of the Redelmeier/Tversky results and to verify the basic predictions of our theoretical analysis. For that reason we chose a 2×2 design as presented in Figure 4.1, varying the lottery type as well as the portfolio size.

The entry $R/5$ is a replication of Redelmeier and Tversky (1992): a portfolio consisting of 5 lotteries of type $R = (0.5; -500)$ using DM instead of U.S. dollars. The entries $R/2$ and $K/2$ allow us to test the predictions of our theoretical analysis for portfolios of Size 2. Assuming that a presentation in repeated trial format results in a segregated evaluation S, while the aggregated value A is assigned to the overall distribution, we predict

Hypothesis 1: (Portfolios of Size 2)

a) *The acceptance rate for playing the gamble* $R = (0.5; -500$ *DM*) *twice will be higher if the overall distribution of the sequence is explicitly mentioned.*

[12] We did actually run some studies with very attractive lotteries using the certainty equivalent method and recalculated values from the elicited certainty equivalents. We do not report these results, as it turned out that their interpretation strongly depends on assumptions about the specific parameters of the value function.

Figure 4.1 **The 2×2 Design of Study 1**

	Portfolio size / Type of lottery	2	5
Lottery used by Redelm./Tversky: 50% +2000 DM, 50% - 500 DM	R	✖	✖
Loan type lottery: 96% +400 DM, 4% - 2100 DM	K	✖	✖

b) *The acceptance rate for playing the gamble* $K = (0.04; -2{,}100\ DM)$ *twice will be lower if the overall distribution of the sequence is explicitly mentioned.*

Finally, the case $K/5$ was included to show the acceptance rate increase/decrease to depend on the type of lottery even for a portfolio size as in the Redelmeier and Tversky study.

Ninety-five subjects (79 advanced business students and 16 other students from the University of Mannheim) took part in this study. They filled out questionnaires either in combination with another lab experiment or within a classroom lecture. Typical questions were of the form, "Are you willing to accept a gamble where you can win 400 DM with a probability of 96% and lose 2,100 DM with a probability of 4%?" Additionally, the gambles were displayed in tree form

$$\begin{cases} 0.96 & 400\ \text{DM} \\ 0.04 & -2{,}100\text{DM}. \end{cases}$$

We did not provide any monetary incentives.

For each lottery type (K and R), the portfolios of different size (2 and 5) had to be evaluated in two modes (aggregated and segregated). In addition, the willingness to play the gambles K and R just once had to be stated. Overall, 10 different questions were relevant within the 2×2 design. We did not present all 10 questions to each subject, however. Instead, two types of questionnaires were used to cover the full set. Within-subject information concerning the different presentation modes was just collected for the $K/2$ case, which we considered most interesting from a theoretical point of view. Six subjects were completely eliminated from the sample because they left open answers in all four cells of the 2×2 design. Other questionnaires that lacked answers in just some of the tasks were not excluded from the analysis.

Table 4.1 **Results of Study 1**

Lottery	Accept. Rate (%)	*N*	Lottery	Accept. Rate (%)	*N*
		Portfolio Size 2			
N	59	86	*K*	20	87
$R+R$	59 } **	41	$K+K$	24 } ***	87
$2R$	76 } **	41	$2K$	10 } ***	87
		Portfolio Size 5			
R	59	86	*K*	20	87
$R+R+\cdots+R$	67 } ***	45	$K+K+\cdots+K$	41	41
$5R$	88 } ***	41	$5K$	30	46

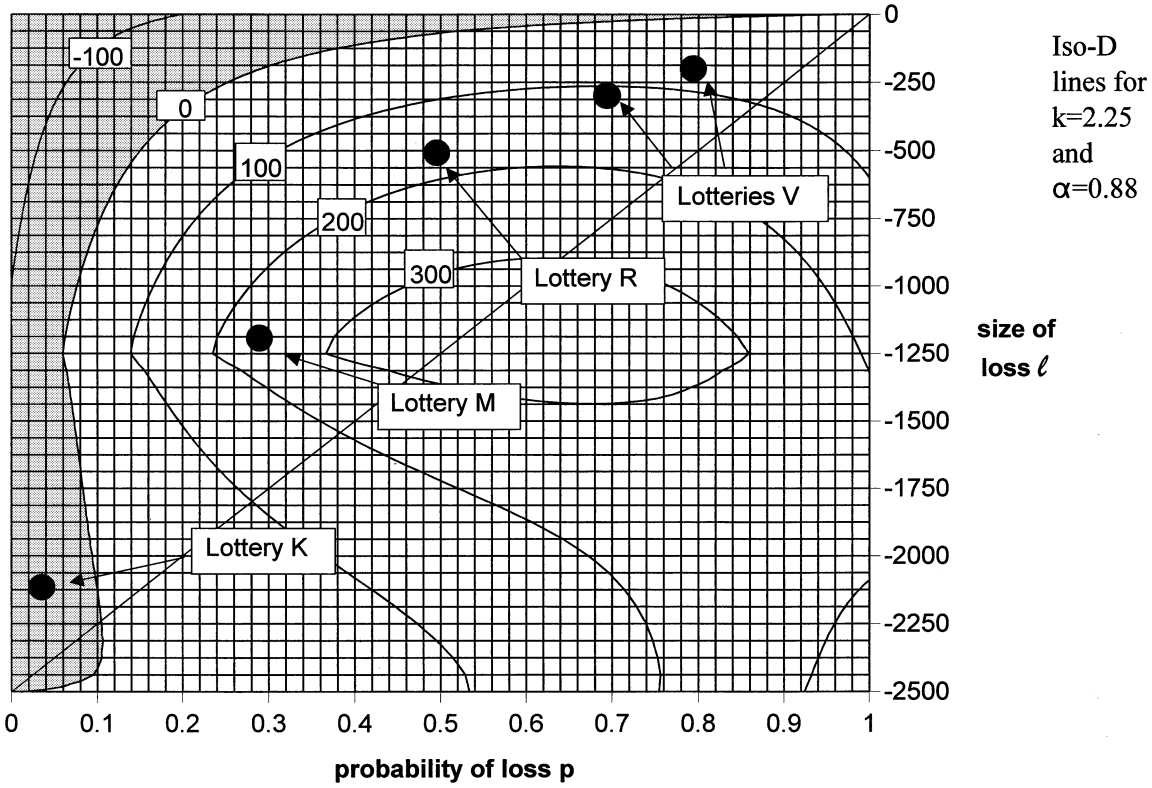
**Significant at the 0.05 level. ***Significant at the 0.01 level.

Results of Study 1. The results for the designs $R/2$ and $K/2$, supporting our hypotheses, are summarized in Table 4.1. Fifty-one of 86 subjects (59%) were willing to accept a single gamble R. Two independent draws of R were accepted by 24 out of 41 students (59%) when presented as repeated trial, and by 31 out of 41 students (76%) when displayed as aggregated distribution. Hence, we find a higher attractiveness of aggregated evaluation for the lottery type R as predicted by our formal analysis. A t test shows the difference of the acceptance rates to be significant at the 5% level. It is further interesting to note that we have the same acceptance rate for a single draw of the lottery R and the segregated presentation of the portfolio. This is well in line with our assumption about the coding of the repeated trial presentation.

A single play of the lottery K was accepted by 20% of the subjects (17 out of 87). With 24%, the acceptance rate was slightly higher (21 out of 87) if subjects were offered to play the gamble twice. For the aggregated distribution $2K$, however, it decreased to 10% (9 out of 87). Here we can provide a within-subject analysis. Sixty-five students made consistent decisions (4 accepted both, 61 rejected both offers). Five students accepted $2K$ but rejected $K+K$, while 17 moved in the other direction. A paired t test shows the decrease of the acceptance rate to be significant at the 1% level. Again, this result is in line with our formal analysis and Hypothesis 1.

The results for Portfolio Size 5 are also presented in Table 4.1. Our replication of the study by Redelmeier

Figure 4.2 **The Lottery Types K, M, R, and V in $\mathcal{L}$ space**



and Tversky ($R/5$) found a strong qualitative similarity of the results, although our acceptance rates were a little higher in general. Fifty-one of 86 students (59%) were willing to play the gamble R once. Thirty of 45 (67%) agreed to play it five times in a row, while 36 of 41 (88%) accepted the aggregated distribution $5R$. The increase of the acceptance rate (67% to 88%) is significant at the 1% level. Concerning the lottery K, we found the effect to be somewhat weaker for Portfolio Size 5 than in the $K/2$ scenario but still pointing in the same direction. Seventeen of 41 students (41%) agreed to play the gamble K five times. If the aggregated outcome distribution was presented, the acceptance rate decreased to 30% (14 of 46).[13] This decrease is not significant. Nevertheless the results are in line with our formal analysis and show that the direction of the effects reported by Redelmeier and Tversky (1992) is not invariant under variations of the lottery parameters.

Design of Study 2. While Study 1 focused exclusively on differences concerning the lotteries R and K, Study 2 was run to analyze the phenomenon more generally. To fully cover the spectrum of possible risk profiles, we added two new types of lotteries to the existing set $\{R, K\}$. A lottery type M ("*M*oderate outcomes") was included with gains and losses of approximately the same size. Further, we introduced two lotteries of type V with rather small probabilities for high gains (as typical for *v*enture investments). Figure 4.2 displays the location of the lotteries K, M, R, V_1, and V_2 within the space $\mathcal{L}$. For the V-lotteries we had the following hypothesis:

Hypothesis 2: (Lotteries of Type V, Portfolio Sizes 2 and 5). *The acceptance rate for playing lotter-*

[13] In the questionnaire the exact distribution of $5K$ was replaced by a simplified lottery. This lottery was constructed in a way, it stochastically dominated the real distribution of $5K$. Hence, the real distribution of $5K$ should be even less attractive.

ies of type V twice (*five times*) *will be higher if the overall distribution of the sequence is explicitly mentioned*.

The lottery M was only examined for Portfolio Size 2. From our theoretical analysis of the two-lottery case we predicted the effect of increasing acceptance rate to be especially strong for the risk profile of the lottery M.

Hypothesis 3: (Lottery of Type M, Portfolio Size 2). *The acceptance rate for playing the lottery M twice will be higher if the overall distribution of the sequence is explicitly mentioned*.

Eighty-three students from the University of Mannheim took part in the computerized experiment. They were recruited through an experimental mailing list and majored in different disciplines. For various combinations of lottery type and portfolio size they had to state their willingness to play the (sequence of) gambles. The questions were formulated as in Study 1. Each portfolio in the set was presented in segregated as well as in aggregated form to all subjects. Thus, within-subject information is available for all type/size combinations. We presented the questions in four different orders to control for order effects. Identical (but differently framed) questions never appeared in close temporal vicinity. As in Study 1, there were no monetary incentives given.

Results of Study 2. Two types of lotteries (V) with rather high loss-probabilities, but comparably small losses, were used. The lottery V_1 was defined as (0.7; −300 DM), the lottery V_2 was given as (0.8; −200 DM). The risk profiles of these two lotteries look similar. Nevertheless, we included both types in our study because we expected the difference in loss probability to be important in the case of Portfolio Size 2. Note that for the aggregated distribution $2V_1$ the loss probability is lower than 50%, while it is above 50% for $2V_2$. Although this kind of effect cannot be explained by Prospect Theory, it is known from the literature that people pay special attention to the overall probabilities of gains and losses and less to the outcome sizes (cf. Lopes 1996). Because 50% is especially salient, we expected the attractiveness of the distributions $2V_1$ and $2V_2$ to be strongly influenced by the small difference in loss probability of V_1 and V_2. Table 4.2 summarizes the results for the V lotteries.

Table 4.2 Results for the Lotteries of Type V (Venture Lotteries)

Type V_1		Type V_2	
Lottery	Accept. Rate (%)	Lottery	Accept. Rate (%)
$V_1 =$ 0.30 2,200 DM; 0.70 −300 DM	43	$V_2 =$ 0.20 2,300 DM; 0.80 −200 DM	30
$V_1 + V_1$	35	$V_2 + V_2$	36
0.09 4,400 DM		0.04 4,600 DM	
$2V_1 =$ 0.42 1,900 DM	60 ***	$2V_2 =$ 0.32 2,100 DM	35
0.49 −600 DM		0.64 −400 DM	
$V_1 + V_1 + V_1 + V_1 + V_1$	63***	$V_2 + V_2 + V_2 + V_2 + V_2$	59*
$5V_1$	82***	$5V_2$	69*

***Significant at the 0.10 level.

For the lottery V_1 we found a strong effect in line with Hypothesis 2. While only 29 of 83 subjects were willing to play the lottery V_1 twice, the acceptance rate increased to 50 of 83 for an aggregated presentation of the portfolio distribution. This increase (35% to 60%) is significant at the 1% level. For the portfolio consisting of five lotteries V_1, the effect was of similar strength. The number of subjects accepting the portfolio increased from 52 to 68 (63% to 82%). This difference is significant at the 1% level. Hypothesis 2 is thus confirmed for V_1. However, a similarly strong effect could not be found for the lottery V_2. In case of the Portfolio Size 5, the increase from 59% to 69% is just weakly significant at the 10% level. For the Portfolio Size 2 the acceptance rate did not even move in the hypothesized direction.

Comparing the acceptance rates for the distributions $2V_1$ and $2V_2$, we come to the conclusion that the observed difference between V_1 and V_2 is driven by the fact that subjects pay special attention to the overall probability of losses. For the lottery M, defined as (0.30; −1,200 DM), Hypothesis 3 is confirmed (see Table 4.3). Of 83 subjects, 38 accepted a single play of M, 46 were willing to play two independent draws, and 56 stated their willingness to participate if the portfolio distribution $2M$ was displayed. The increase of the acceptance rate (55% to 67%) from segregated to aggregated presentation is significant at the 2% level.

Table 4.3 Results for Lottery *M*

Lottery			Accept. Rate (%)	
$M =$	0.70	1,300 DM	46	
	0.30	−1,200 DM		
$M + M$			55	**
	0.49	2,600 DM		
$2M =$	0.42	100 DM	67	**
	0.09	−2,400 DM		

**Significant at the 0.05 level.

5. Conclusion

If individuals have to evaluate a portfolio or sequence of lotteries, their judgment is influenced by the portfolio presentation mode. In an experimental study, Redelmeier and Tversky (1992) found a significantly higher acceptance rate for a sequence of lotteries if the overall distribution was displayed instead of the set of lotteries itself. There seems to be an easy explanation for this response pattern. A tendency for a narrow framing (Kahneman and Lovallo 1993) of the decision problem causes individuals to neglect the portfolio context if the overall outcome distribution is not explicitly mentioned. Because of loss aversion, this type of mental accounting leads to a lower overall evaluation of a portfolio of mixed lotteries as exemplarily demonstrated by Benartzi and Thaler (1995). In the "segregated evaluation" of the mental accounts (i.e., the isolated evaluation of each lottery), the asymmetric impact of losses and gains is fully taken into account, while some of the gains and losses would cancel in an "aggregated evaluation" of the overall distribution. The difference in aggregated and segregated evaluation of portfolios has obvious implications for economic problems, particularly in the financial sector. The basic question is whether a bank or an investment fund (e.g., venture capital fund) can make its loan or investment portfolio look more attractive to customers by (not) providing isolated details about the single risks within the portfolio.

In the theoretical part of our paper we extended the loss-aversion explanation and assumed the value function to reflect diminishing sensitivity too. We formally analyzed the sign and the size of the evaluation difference dependent on the gain/loss probabilities and the relative gain/loss sizes of the lotteries in the portfolio. It turned out that the simple lottery (0.5, \$2,000; 0.5, −\$500) used by Redelmeier and Tversky in their experimental design (and frequently used in similar form in the literature for related questions) is especially suited to result in a higher evaluation of the aggregated distribution.

However, there also exist portfolios of mixed lotteries with a higher segregated evaluation. A portfolio of lotteries with moderate to low probabilities for rather high losses should appear less attractive to decision makers if an aggregated evaluation is performed. This type of lottery can be found in a bank loan setting. In case of a typical loan the investor has to bear a high loss (compared to the possible interest gain) in the rather unlikely event of default.

Because we could derive analytical results for only small portfolios of simple lotteries, we ensured the robustness of our findings by computer-supported simulations. The full range of lottery types (low to high loss probability and low to high loss size) in a given lottery space was examined and graphically displayed for portfolios with up to 10 identical lotteries and for different parameter settings. The general findings of our formal analysis translate to bigger portfolio sizes. While for the loan-type lotteries segregated evaluation remains favored (even for bigger portfolios), for most other lotteries with moderately positive expected values the aggregated evaluation is much higher. This holds in particular for the Redelmeier/Tversky type of lotteries (median probability for moderate loss) and a probability/size combination we called "venture lotteries." These lotteries offer a high probability of loss but extremly high gain chances in case of success (typical profile of venture capital investments).

The different theoretical predictions for loan, venture, and other lotteries were finally investigated in two experimental studies. The experimental results were in line with our theoretical predictions. It should be mentioned, however, that the strength of the effect seems to be very sensitive to small changes of the lottery parameters. For two similar risk profiles within the venture category we found the difference in acceptance rates for the two presentation modes to be surprisingly distinct. These patterns can

hardly be explained within a Prospect Theory framework. Obviously, simple heuristics (e.g., a maximum loss-probability rule) might be used by some subjects to decide about the acceptance of outcome distributions and lottery portfolios. These rules of thumb will surely gain even more importance if the evaluation tasks get more complex. It is not clear how well a Prospect Theory–based explanation will predict individual decisions if more complicated (and not identical) lotteries within larger portfolios are considered. First experimental evidence for large portfolios was provided by Benartzi and Thaler (1999). In their study, the effect of nonlinear value transformation is superimposed by a strong bias in probability judgment. Hence, they do not observe the dependence of the evaluation difference on the risk profiles of the lotteries, which we identify for smaller portfolios. Further experimental research is needed to be able to make reliable predictions of individual behavior in more complex real-world decision situations.

We see another challenging area for further research in the experimental examination of evaluation differences for lotteries that do not allow a simple acceptance rate comparison. For these extremely (un)attractive lotteries the complex coding problems have to be analyzed, then an appropriate experimental method must be developed to provide a more general test for the empirical correctness of our theoretical predictions.

Acknowledgments

This research was financially supported by the SFB 504 "Rationalitätskonzepte, Entscheidungsverhalten und ökonomische Modellierung" at the University of Mannheim. We thank participants of the 1998 Mannheim ESA conference, the 1998 Miami BDRM conference, the 1999 Marrakech FUR conference, the 1999 Philadelphia INFORMS conference, and participants of the SFB 504 colloquium for helpful discussion.

Appendix

Result 1. There exists a $p_0 \in (0,1)$, s.t. for all mixed lotteries (p, ℓ) with $EV(p, \ell) = 0$:

$$D_{LG}(p, \ell) < 0 \Leftrightarrow p < p_0. \quad p_0 \text{ decreases in } k \text{ and in } \alpha.$$

Proof. For a mixed lottery $(p, \ell) \in \mathcal{L}_{2,500}$ with $EV(p, \ell) = 0$ we have $p\ell + (1-p)g = 0$, thus:

$$\frac{-\ell}{g} = \frac{1-p}{p}.$$

Further it holds for a value function v_k^α:

$$D_L(\ell) = v(2\ell) - 2v(\ell) = k(2-2^\alpha)(-\ell)^\alpha \text{ and}$$
$$D_G(g) = v(2g) - 2v(g) = -(2-2^\alpha)(g)^\alpha.$$

It follows:

$$\frac{D_L(\ell)}{-D_G(g)} = k \cdot \left(\frac{-\ell}{g}\right)^\alpha = k \cdot \left(\frac{1-p}{p}\right)^\alpha$$

and we can conclude:

$$D_{LG}(p, \ell) < 0 \Leftrightarrow p_L(p)D_L(\ell) + p_G(g)D_G(g) < 0 \Leftrightarrow \frac{D_L(\ell)}{-D_G(g)} < \frac{p_G(p)}{p_L(p)}$$
$$\Leftrightarrow k \cdot \left(\frac{1-p}{p}\right)^\alpha < \left(\frac{1-p}{p}\right)^2 \Leftrightarrow p \cdot k^{1/(2-\alpha)} < 1-p$$
$$\Leftrightarrow p < \left(1 + k^{1/(2-\alpha)}\right)^{-1} =: p_0.$$

The monotonicity of p_0 in k and in α is obvious. □

Result 2. For $k > 1$ there exists a unique loss size $\ell^\circ \in (-2{,}500, -1{,}250)$ s.t.

$$D_M(\ell) < 0 \Leftrightarrow \ell \in (-2{,}500, \ell^\circ). \quad \ell^\circ \text{ decreases in } k \text{ and in } \alpha.$$

Proof. We notate the gain part of the value function v by $v_\oplus$ and the loss part by $v_\ominus$. The existence and uniqueness of ℓ° can be proven for all continuous value functions with the properties:

(i) Loss aversion: $v_\oplus(x) < -v_\ominus(-x)$ for all $x > 0$.

(ii) Concavity [convexity] of the continuously differentiable function $v_\oplus[v_\ominus]$.

(iii) A sufficiently high $v'_\oplus(x)$ for $x \searrow 0$ (explicitly: $v'_\ominus(-2{,}500) < \lim_{x\to 0} v'_\oplus(x)$).

Note that these properties are given in particular for value functions v_k^α. From (iii) and the continuity of v' in the gain and in the loss domain we derive the existence of some small g with: $v'_\ominus(-2{,}500 + 2g) < v'_\oplus(2g)$. Using (ii), it follows

$$v_\oplus(g) > g \cdot v'_\oplus(g) > g \cdot v'_\oplus(2 \cdot g) > g \cdot v'_\ominus(-2{,}500 + 2 \cdot g)$$
$$\geq v_\ominus(-2{,}500 + 2 \cdot g) - v_\ominus(-2{,}500 + g).$$

Defining $\ell = g - 2{,}500$, this proves the existence of some $\ell \in (-2{,}500, 0)$ with $D_M(\ell) < 0$. From (i) and (ii), we further derive for all $\ell \in [-1{,}250, 0)$

$$D_M(\ell) = v_\oplus(\ell + g) - v_\oplus(g) - v_\ominus(\ell)$$
$$\geq -v_\oplus(-\ell) - v_\ominus(\ell) > -v_\oplus(-\ell) + v_\oplus(-\ell) = 0.$$

By the continuity of $D_M(\ell)$, this suffices to conclude the existence of some $\ell^\circ \in (-2{,}500, -1{,}250)$ with $D_M(\ell^\circ) = 0$.

To prove the uniqueness of ℓ°, we show for all $\ell^\circ \in (-2{,}500, -1{,}250)$ with $D_M(\ell^\circ) = 0$

$$D_M < 0 \text{ on } (-2{,}500, \ell^\circ).$$

For each ℓ° with $D_M(\ell^\circ) = 0$ and $\ell \in (-2{,}500, \ell^\circ)$, we define $g^\circ := \ell^\circ + 2{,}500$ and $g := \ell + 2{,}500$. Because we have $\frac{g}{g^\circ} \in (0, 1)$, the convexity of $v_\ominus$ yields

$$\begin{aligned}\frac{g}{g^\circ} \cdot v_\ominus(\ell^\circ + g^\circ) + \left(1 - \frac{g}{g^\circ}\right) \cdot v_\ominus(\ell^\circ) \\ > v_\ominus\left(\frac{g}{g^\circ} \cdot (\ell^\circ + g^\circ) + \left(1 - \frac{g}{g^\circ}\right) \cdot \ell^\circ\right) = v_\ominus(\ell^\circ + g).\end{aligned}$$

Subtracting $v_\ominus(\ell^\circ)$ on both sides gives

$$\frac{g}{g^\circ} \cdot [v_\ominus(\ell^\circ + g^\circ) - v_\ominus(\ell^\circ)] > v_\ominus(\ell^\circ + g) - v_\ominus(\ell^\circ).$$

Using the property $D_M(\ell^\circ) = 0$, i.e., $v_\oplus(g^\circ) = v_\ominus(\ell^\circ + g^\circ) - v_\ominus(\ell^\circ)$ we finally conclude

$$\begin{aligned}v_\oplus(g) &> \frac{g}{g^\circ} \cdot v_\oplus(g^\circ) = \frac{g}{g^\circ} \cdot [v_\ominus(\ell^\circ + g^\circ) - v_\ominus(\ell^\circ)] \\ &> v_\ominus(\ell^\circ + g) - v_\ominus(\ell^\circ) > v_\ominus(\ell + g) - v_\ominus(\ell).\end{aligned}$$

This proves $D_M(\ell) < 0$.

The proof of monotonicity of ℓ° in k and in α relies on the specific functional form v_k^α of the value function. D_M is a function of ℓ, α, and k. For $g := \ell + 2{,}500$ we get

$$D_M(\alpha, k, \ell) = v_k^\alpha(\ell + g) - v_k^\alpha(\ell) - v_k^\alpha(g).$$

Substituting v_k^α, we easily derive the positive monotonicity of $D_M(\alpha, k, \ell)$ in k for all ℓ and α. Using the above insights about the general shape of $D_M(\ell)$, this suffices to prove the monotonicity of ℓ° in k.

The same argument cannot be used to show the monotonicity of ℓ° in α, because D_M does not decrease in α for all k and ℓ. Instead we show the weaker, but sufficient property that D_M has a negative partial derivative in α at each point $(\hat{\alpha}, k, \hat{\ell})$ with $\hat{\alpha} < 1$ and $\hat{\ell} = \ell_k^\circ(\hat{\alpha})$. Using the known property $\frac{\partial}{\partial \ell} D_M(\hat{\alpha}, k, \hat{\ell}) > 0$, we then conclude by implicit differentiation that $\frac{\partial}{\partial \alpha} \ell_k^\circ(\hat{\alpha}) < 0$.

Fix k and choose $\hat{\alpha} < 1$. Then $\ell^\circ(\hat{\alpha}) < -1{,}250$ and for $\hat{\ell} = \ell_k^\circ(\hat{\alpha})$ and $\hat{g} := \hat{\ell} + 2{,}500$ we have

$$D_M(\hat{\alpha}, k, \hat{\ell}) = k\left[(-\hat{\ell})^{\hat{\alpha}} - (\hat{\ell} - \hat{g})^{\hat{\alpha}}\right] - \hat{g}^{\hat{\alpha}} = 0. \qquad (*)$$

It follows

$$\begin{aligned}\frac{\partial}{\partial \alpha} D_M(\hat{\alpha}, k, \hat{\ell}) &= k\left[\ln(-\hat{\ell}) \cdot (-\hat{\ell})^{\hat{\alpha}} - \ln(-\hat{\ell} - \hat{g}) \cdot (-\hat{\ell} - \hat{g})^{\hat{\alpha}}\right] - \ln(\hat{g}) \cdot \hat{g}^{\hat{\alpha}} \\ &> k \cdot \ln(-\hat{\ell})\left[(-\hat{\ell})^{\hat{\alpha}} - (-\hat{\ell} - \hat{g})^{\hat{\alpha}}\right] - \ln(\hat{g}) \cdot \hat{g}^{\hat{\alpha}} \\ &\overset{(*)}{=} \ln(-\hat{\ell}) \cdot \hat{g}^{\hat{\alpha}} - \ln(\hat{g}) \cdot \hat{g}^{\hat{\alpha}} > 0.\end{aligned}$$

□

Result 3. For $\alpha < 1$ there exists for each lottery $(p, \ell) \in \mathcal{L}_{2{,}500}$ with $S(p, \ell) > 0$ a portfolio size n_0 s.t. the evaluation difference $D_n(p, \ell) = A_n(p, \ell) - S_n(p, \ell)$ is negative for all $n > n_0$.

Proof. Let $(p, \ell) \in \mathcal{L}_{2{,}500}$ with $s := S(p, \ell) > 0$. Choose $n_0 > [(\ell + 2{,}500)^\alpha / s]^{1/(1-\alpha)}$, and it follows for all $n > n_0$

$$n \cdot s > n^\alpha \cdot (\ell + 2{,}500)^\alpha.$$

Now note that the total outcome of n draws of a lottery (p, ℓ) cannot exceed $n \cdot (\ell + 2{,}500)$. Thus, the sure amount $n \cdot (\ell + 2{,}500)$ cannot have a lower evaluation than the aggregated distribution of a portfolio of n lotteries (p, ℓ), i.e.,

$$A_n(p, \ell) \le v(n \cdot (\ell + 2{,}500)) = n^\alpha \cdot (\ell + 2{,}500)^\alpha.$$

With $S_n(p, \ell) = n \cdot S(p, \ell) = n \cdot s$ the proposition is immediate. □

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Accepted by Mako Bockholt; received July 13, 1999. This paper was with the authors 7 months for 2 revisions.