Prospects for Geometric Complexity Theory

Peter Bürgisser
Institute of Mathematics
University of Paderborn
Germany
pbuerg@upb.de

***Abstract*—It is a remarkable fact that two prominent problems of algebraic complexity theory, the permanent versus determinant problem and the tensor rank problem (matrix multiplication), can be restated as explicit orbit closure problems. This offers the potential to prove lower complexity bounds by relying on methods from algebraic geometry and representation theory. While this basic idea for the tensor rank problem goes back to work by Volker Strassen from the mid eighties, the geometric complexity program has gained visibility and momentum in the past years. Some modest lower bounds for border rank have recently been proven by the construction of explicit obstructions. For further progress, a better understanding of irreducible representions of symmetric groups (tensor products and plethysms) is required. Interestingly, asymptotic versions of the the latter questions are of relevance in quantum information theory.**

***Index Terms*—permanent versus determinant, tensor rank, matrix multiplication, complexity lower bounds, orbit closure problem, representations, Kronecker coefficients**

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