Protecting Entanglement via the Quantum Zeno Effect

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We study the exact entanglement dynamics of two atoms in a lossy resonator. Besides discussing the steady-state entanglement, we show that in the strong coupling regime the system-reservoir correlations induce entanglement revivals and oscillations and propose a strategy to fight against the deterioration of the entanglement using the quantum Zeno effect.

The description of decoherence for bipartite entangled systems has recently reached notable theoretical [1] and experimental [2] results due to the introduction of the concept of entanglement sudden death. This describes the finite-time destruction of quantum correlations due to the detrimental action of independent environments coupled to the two subsystems. On the other hand, it is well known [3,4] that the interaction with a *common* environment leads to the existence of a highly entangled long-living decoherence-free (or subradiant) state. At the same time, another entangled state exists (orthogonal to the previous one, and called superradiant) that loses its coherence faster. In this Letter, we present a variant of the quantum Zeno effect that allows us to preserve the superradiant state without affecting the subradiant one, thus achieving a complete entanglement survival.

Specifically, we address an exactly solvable model describing two two-level atoms (qubits) resonantly coupled to a lossy resonator, which we treat through the pseudo-mode approach [5]. This model describes both atoms or ions trapped in an electromagnetic cavity [6] and circuit-QED setups [7–9], so that our results are directly verifiable in both systems.

We obtain the exact entanglement dynamics as a function of the environment correlation time and discuss its stationary value, which turns out to be maximal for a factorized initial state of the two atoms. In the past, the environment induced entanglement generation has been discussed in the Born-Markov limit [10] or for a pure dephasing case [11]. Here we extend these results to a dissipative coupling with the environment outside the Markovian regime, both for weak and strong couplings, corresponding to the bad and good cavity limits. In particular, in the latter regime, the long memory of the reservoir induces entanglement revivals and oscillations.

Furthermore, we describe a measurement induced quantum Zeno effect [12] for the entanglement, showing that the simple procedure of monitoring the population of the cavity mode leads to a protection of the entanglement well beyond its natural decay time. This effect too can be tested with slight modifications of already existing experimental

setups, both in the realm of cavity QED and with superconducting Josephson circuits.

We consider a two-qubits system interacting with a common zero-temperature bosonic reservoir. The microscopic Hamiltonian of the system plus reservoir is given by $H = H_0 + H_{\text{int}}$, with

$$H_0 = \omega_1 \sigma_+^{(1)} \sigma_-^{(1)} + \omega_2 \sigma_+^{(2)} \sigma_-^{(2)} + \sum_k \omega_k b_k^\dagger b_k, \quad (1)$$

$$H_{\text{int}} = (\alpha_1 \sigma_+^{(1)} + \alpha_2 \sigma_+^{(2)}) \sum_k g_k b_k + \text{H.c.} \quad (2)$$

Here, b_k is the annihilation operator of quanta of the environment, while $\sigma_\pm^{(j)}$ and ω_j are the inversion operator and transition frequency of the jth qubit, $j = 1, 2$, whose interaction with the reservoir is measured by the dimensionless constant α_j. This depends on the value of the cavity field at the qubit position and can be effectively manipulated, e.g., by means of dc Stark shifts tuning the atomic transition in and out of resonance. For the following discussion, it will prove useful to introduce a collective coupling constant $\alpha_T = (\alpha_1^2 + \alpha_2^2)^{1/2}$ and the relative strengths $r_j = \alpha_j/\alpha_T$ (as $r_1^2 + r_2^2 = 1$, we take only r_1 as independent). By varying α_T, we will explore both the weak and the strong coupling regimes.

For an initial state of the form

$$|\psi(0)\rangle = [c_{01}|1\rangle_1|0\rangle_2 + c_{02}|0\rangle_1|1\rangle_2]\bigotimes_k |0_k\rangle, \quad (3)$$

the time evolution of the total system is given by

$$|\Psi(t)\rangle = c_1(t)|1\rangle_1|0\rangle_2|0\rangle_E + c_2(t)|0\rangle_1|1\rangle_2|0\rangle_E$$
$$+ \sum_k c_k(t)|0\rangle_1|0\rangle_2|1_k\rangle_E, \quad (4)$$

$|1_k\rangle_E$ being the state of the reservoir with only one excitation in the kth mode. Setting $\delta_k^{(j)} = \omega_j - \omega_k$, the equations for the probability amplitudes take the form

$$\dot{c}_j(t) = -i\alpha_j \sum_k g_k e^{i\delta_k^{(j)} t} c_k(t), \qquad j = 1, 2, \quad (5)$$

$$\dot{c}_k(t) = -ig_k^*[\alpha_1 e^{-i\delta_k^{(1)} t} c_1(t) + \alpha_2 e^{-i\delta_k^{(2)} t} c_2(t)]. \quad (6)$$

Integrating Eq. (6) and inserting its solution into Eqs. (5), one gets two integro-differential equations for the amplitudes $c_{1,2}(t)$. In the continuum limit for the environment, and by introducing the correlation function $f(\tau)$, and its Fourier transform $J(\omega)$ (which is the reservoir spectral density), these two equations become

$$\dot{c}_1 = -\int_0^t dt_1 f(t-t_1)[\alpha_1^2 c_1(t_1) + \alpha_1\alpha_2 c_2(t_1)], \quad (7)$$

$$\dot{c}_2 = -\int_0^t dt_1 f(t-t_1)[\alpha_2^2 c_2(t_1) + \alpha_2\alpha_1 c_1(t_1)]. \quad (8)$$

Before discussing the general time evolution, we notice that a constant solution can be found, independently of the form of the spectral density. Namely, a subradiant, decoherence-free state exists, that does not decay in time

$$|\psi_-\rangle = r_2|1\rangle_1|0\rangle_2 - r_1|0\rangle_1|1\rangle_2. \quad (9)$$

In the following, we consider the case in which the two atoms have the same Bohr frequency, i.e., $\omega_1 = \omega_2 = \omega_0$, and interact resonantly with a reservoir with Lorentzian spectral density $J(\omega) = (W^2\lambda/\pi)[(\omega - \omega_0)^2 + \lambda^2]$. This is, e.g., the case of two atoms interacting with a cavity field in presence of cavity losses. Because of the nonperfect reflectivity of the cavity mirrors, the spectrum of the cavity field displays a Lorentzian broadening. In this case the correlation function decays exponentially $f(\tau) = W^2 e^{-\lambda\tau}$, the quantity $1/\lambda$ being the reservoir correlation time. The ideal cavity limit is obtained for $\lambda \to 0$; in this limit $J(\omega) = W^2\delta(\omega - \omega_0)$, corresponding to a constant correlation function $f(\tau) = W^2$. The system, then, reduces to a two-atom Jaynes-Cummings model [13] with a vacuum Rabi frequency $\mathcal{R} = \alpha_T W$. On the other hand, for small correlation times (with λ much larger than any other frequency scale), we obtain the Markovian regime characterized by a decay rate $\gamma = 2\mathcal{R}^2/\lambda$. For generic parameter values, the model interpolates between these two limits.

As $|\psi_-\rangle$ does not evolve in time, the only relevant time evolution is that of its orthogonal, superradiant, state

$$|\psi_+\rangle = r_1|1\rangle_1|0\rangle_2 + r_2|0\rangle_1|1\rangle_2. \quad (10)$$

Its survival amplitude $\mathcal{E}(t) = \langle\psi_+|\psi_+(t)\rangle$ is given by

$$\mathcal{E}(t) = e^{-\lambda t/2}[\cosh(\Omega t/2) + \frac{\lambda}{\Omega}\sinh(\Omega t/2)], \quad (11)$$

where $\Omega = \sqrt{\lambda^2 - 4\mathcal{R}^2}$.

In the $\{|1\rangle, |0\rangle\}$ basis, the reduced density operator for the two qubits is given by

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |c_1(t)|^2 & c_1(t)c_2^*(t) & 0 \\ 0 & c_1^*(t)c_2(t) & |c_2(t)|^2 & 0 \\ 0 & 0 & 0 & 1 - |c_1|^2 - |c_2|^2 \end{pmatrix}, \quad (12)$$

where, letting $\beta_\pm = \langle\psi_\pm|\psi(0)\rangle$, one has

$$c_1(t) = r_2\beta_- + r_1\mathcal{E}(t)\beta_+, \quad (13)$$

$$c_2(t) = -r_1\beta_- + r_2\mathcal{E}(t)\beta_+. \quad (14)$$

The solution is exact as we have not performed neither the Born nor the Markov approximation. We now use this result to obtain the dynamics of the qubit entanglement as measured by the concurrence $C(t)$ [14], ranging from 0 (for separable states) to 1 (for maximally entangled ones). For the density matrix given by Eq. (12), the concurrence is

$$C(t) = 2|c_1(t)c_2^*(t)|. \quad (15)$$

We begin by noticing that there exists a nonzero stationary value of C due to the entanglement of the decoherence-free state: $C_s = C(t \to \infty) \equiv C(|\psi_-\rangle)|\langle\psi_-|\psi(0)\rangle|^2 = 2|r_1 r_2||\beta_-|^2$. To better discuss the time evolution of the concurrence as a function of the initial amount of entanglement stored in the system, we consider initial states of the form (3) with

$$c_{01} = \sqrt{\frac{1-s}{2}}, \quad c_{02} = \sqrt{\frac{1+s}{2}}e^{i\phi}, \quad \text{with} \quad -1 \le s \le 1.$$

Here, the separability parameter s is related to the initial concurrence as $s^2 = 1 - C(0)^2$. Figure 1(a) displays the stationary concurrence versus r_1 and s. Because of the interaction with the cavity field, initial separable states ($s = \pm1$) become entangled. In fact, for $\phi = 0$, the maximum stationary entanglement $C_s^{\max} \simeq 0.65$ is obtained for initially factorized states, i.e., $s = \pm1$. While the details depend on the phase ϕ, the qualitative picture is generic and essentially independent of ϕ, apart from the isolated case of an initial state coinciding with $|\psi_-\rangle$. In such a situation all of the entanglement initially encoded in the qubits remains there for long times. For positive r_j, this occurs for $\phi = \pi$, see Fig. 1(b).

We now look at the entanglement dynamics in the good and bad cavity limits, i.e., for $R \gg 1$ and $R \ll 1$, respectively, with $R = \mathcal{R}/\lambda$. In Fig. 2 we show the concurrence as a function of $\tau = \lambda t$ in the bad (upper row) and good (lower row) cavity limits. We compare the dynamics of an initially factorized state ($s = 1$) with that of an initially maximal entangled state ($s = 0$) for four different values
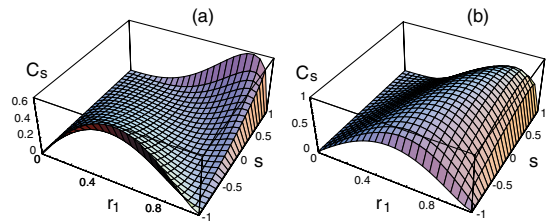

FIG. 1 (color online). Stationary concurrence as a function of the relative coupling constant r_1 and of the initial separability s of the state, for (a) $\phi = 0$, and (b) $\phi = \pi$. In the first case, the maximum of C_s is achieved for asymmetrical couplings: at $r_1 \simeq 0.87$ for $s = 1$, and at $r_1 \simeq 0.5$ for $s = -1$. In the second case the maximum is achieved for $|\psi(0)\rangle = |\psi_-\rangle$. We point out that these plots are independent of λ.

of the coupling parameter r_1, namely $r_1 = 0, 1/\sqrt{2}, 0.87$, 1. The plots for $r_1 = 0$ and $r_1 = 1$ overlap as they both describe a case in which one of the two atoms is effectively decoupled. The value $r_1 = 0.87$ corresponds to the optimal stationary entanglement for $s = 1$ and $\phi = 0$. Finally, for $r_1 = 1/\sqrt{2}$ the two atoms are equally coupled with the reservoir. Other values of r_1 show qualitatively similar behavior.

For weak couplings and/or bad cavity, $R = 0.1$, and for an initially separable state ($s = 1$), the concurrence increases monotonically up to its stationary value; whereas, for initially entangled states, the concurrence first goes to zero before increasing towards C_s. The strong coupling/ good cavity case $R = 10$ is more rich and presents entanglement oscillations and revival phenomena for every initial atomic states. One can prove that for maximally entangled initial states ($s = 0$) the revivals have maximum amplitude when only one of the two atoms is effectively coupled to the cavity field, i.e., for $r_1 = 0, 1$. In this case, indeed, the system performs damped oscillations between the states $|\psi_+\rangle$ and $|\psi_-\rangle$. On the other hand, for an initially factorized state, the interaction with the cavity field in the strong coupling regime generates a high degree of entanglement. Indeed, for $R = 10$, at $\tau = \bar{\tau} \simeq 0.31$, C attains the value $C(\bar{\tau}) \simeq 0.96$, at $r_1 \simeq 0.92$ (for $s = 1$) or $r_1 \simeq 0.4$ (for $s = -1$).

These entanglement revivals are a truly new result due to the memory depth of the reservoir. Small revivals can occur in the Markovian regime [15], and in the non-Markovian regime with independent reservoirs [16]. In our case, however, the amount of revived entanglement is huge, being comparable to the previous maximum. This feature only appears in the strong coupling regime and with a nonzero environmental correlation time. The surprising

aspect here is that an irreversible process such as the spontaneous emission is so strongly counteracted by the memory effect of the environment, which not only creates entanglement, but also lets it oscillate quite a few times before a stationary value is reached.

If we express the initial state of the qubits as a superposition of $|\psi_\pm\rangle$, that is $|\psi(0)\rangle = \beta_-|\psi_-\rangle + \beta_+|\psi_+\rangle$, we see that, while part of the initial state will be trapped in the subradiant state $|\psi_-\rangle$, another part will decay following Eq. (11). Thus, as discussed above, the amount of entanglement that survives depends on the specific state (and on the value of the r_j). In the following, we present a quantum Zeno effect for the entanglement and show that this effect can be used to preserve the initial entanglement *independently* of the state in which it is stored.

We consider the action of a series of nonselective measurements on the collective atomic system, performed at time intervals T, which have the two following properties: (i) one of the possible measurement outcomes is the projection onto the collective ground state $|\psi_g\rangle = |0\rangle_1|0\rangle_2$, and (ii) the measurement cannot distinguish between the states $|1\rangle_1|0\rangle_2$ and $|0\rangle_1|1\rangle_2$. Any procedure fulfilling these two conditions will do the job of preserving the entanglement. In particular, one could measure the collective atomic energy [condition (ii) holds in this case since the transition frequencies are equal] or, more simply, one could monitor the state of the cavity: if a photon is found, then the qubits have necessarily decayed into $|\psi_g\rangle$, while if no photon is found, then the excitation still resides on the atoms. This can be done both in cavity QED setups (by sending a probe atom through the cavity that can absorb the photon) and with superconducting circuits (by sending a short measuring voltage pulse to the resonator, similarly to Ref. [9]).
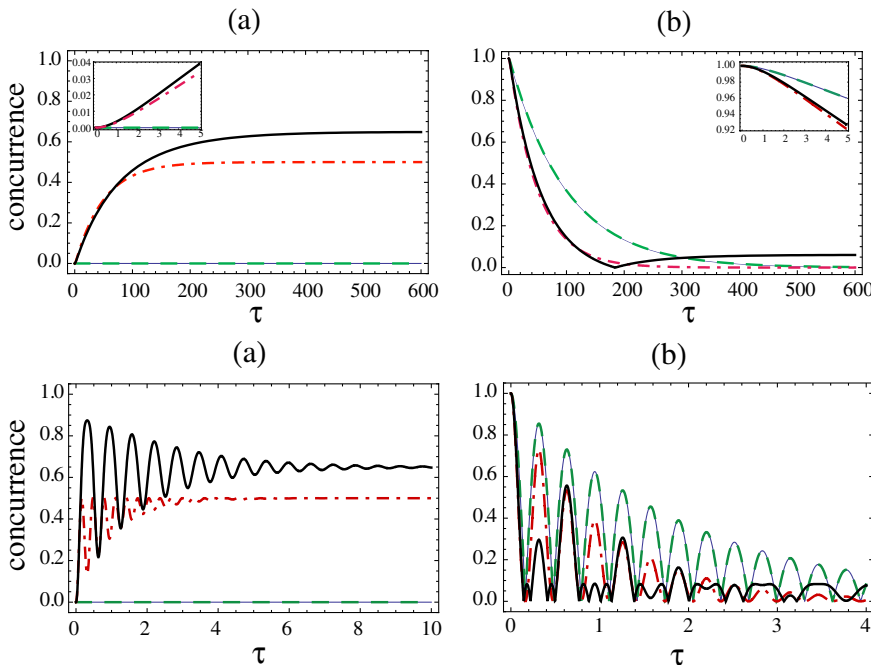

FIG. 2 (color online). Time evolution of the concurrence in the bad cavity limit ($R = 0.1$, top plots) and good cavity limit ($R = 10$, bottom plots), with (a) $s = 1$, and (b) $s = 0$, both with $\phi = 0$, for the cases of (i) maximal stationary value, $r_1 = 0.87$ (solid line), (ii) symmetrical coupling $r_1 = 1/\sqrt{2}$ (dot-dashed line), and iii) only one coupled atom $r_1 = 0, 1$ (dotted line). The insets show the initial quadratic behavior of the concurrence for $R = 0.1$.
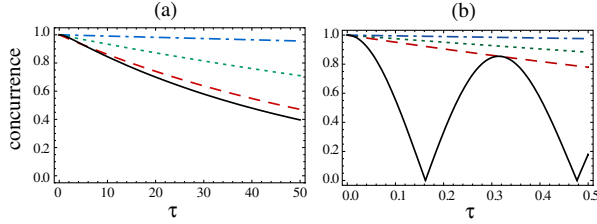
FIG. 3 (color online). Time evolution of the concurrence, for $s = 0$ and $r = 1/\sqrt{2}$, in the absence of measurements (solid line) and in the presence of measurements performed at intervals (a) $\lambda T = 5, 1, 0.1$ (dashed, dotted, and dot-dashed lines, respectively) with $R = 0.1$ (weak coupling) and (b) $\lambda T = 0.01, 0.005, 0.001$ (dashed, dotted, and dot-dashed lines, respectively) with $R = 10$ (strong coupling).

The measurements described above disentangle the qubits from the reservoir after each period T. Choosing T such that $\langle \Psi_g | \rho(T) | \Psi_g \rangle \ll 1$, it is straightforward to prove that the state $|\psi_-\rangle$ is unaffected and that, at the same time, the decay of $|\psi_+\rangle$ is slowed down. Its survival probability $P_+^{(N)}(t) = \langle \psi_+ | \rho(t) | \psi_+ \rangle$ in the presence of N measurements is given by $P_+^{(N)}(t) = |\beta_+(0)|^2 \exp[-\gamma_z(T)t]$, where $t = NT$ and with an effective decay rate

$$\gamma_z(T) = -\frac{\log[\mathcal{E}(T)^2]}{T}. \tag{16}$$

In the limit $T \to 0$ and $N \to \infty$, with a finite $t = NT$, $\gamma_z(T) \to 0$ and the decay is completely suppressed.

Besides affecting the probability $P_+(t)$, the projective measurements also modify the time evolution of the entanglement, whose effective dynamics now depends on T. Explicitly, the concurrence at time $t = NT$, after performing N measurements, is given by

$$C^{(N)}(t) = 2|(\beta_+ r_1 e^{-\gamma_z t/2} + \beta_- r_2)$$
$$\times (\beta_+ r_2 e^{-\gamma_z t/2} - \beta_- r_1)|. \tag{17}$$

In Fig. 3 we compare the dynamics of $C(\tau)$ in the absence and in the presence of measurements performed at various intervals T for an initially maximal entangled state. Both in the weak and in the strong coupling regimes (left and right plots, respectively) the presence of measurements quenches the decay of the concurrence. Thus, we have achieved a quantum Zeno protection of entanglement from the effect of decoherence. Again, decreasing the interval between the measurements, $C^{(N)}(t)$ remains closer and closer to its initial value. It is worth stressing that this quantum Zeno effect for the entanglement is not straightforwardly predictable, since it crucially depends on the environmental spectral density and on the resonance condition. In fact an inverse-Zeno effect could be obtained in some cases, giving rise to an enhanced decay of the entanglement (see, e.g., Ref. [16]).

To sum up, we discussed an exactly solvable model describing two qubits interacting with a nonideal resonator. We analyzed in detail the stationary value of the entanglement and its dynamics both in the weak and strong coupling limits, showing that entanglement revivals can appear due to the finite memory of such an environment. We also investigated the quantum Zeno effect for this system, showing that the entanglement can be preserved independently of the state in which it is encoded, with the help of repeated projective measurements. As anticipated above, our results apply both to cavity QED experiments with trapped atoms and to the case of superconducting circuits, with on-chip qubits and resonator. In the first case, it has already been demonstrated that both atoms and ions can be confined inside high finesse optical cavities and their quantum states can be fully controlled [6]. In the second case, quantum communication between two Josephson qubits has been achieved using a transmission line as a cavity [8,9].

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