

Protoneutron star dynamos: pulsars, magnetars, and radio-silent X-ray emitting neutron stars (Research Note)

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Received 4 November 2005 / Accepted 15 February 2006

ABSTRACT

We discuss the mean-field dynamo action in protoneutron stars that are subject to instabilities during the early evolutionary phase. The mean field is generated in the neutron-finger unstable region where the Rossby number is ~ 1 and mean-field dynamo is efficient. Depending on the rotation rate, the mean-field dynamo can lead to the formation of three different types of pulsars. If the initial period of the protoneutron star is short, then the generated large-scale field is very strong ($>3 \times 10^{13}$ G) and exceeds the small-scale field at the neutron star surface. If rotation is moderate, then the pulsars are formed with more or less standard dipole fields ($<3 \times 10^{13}$ G) but with surface small-scale magnetic fields stronger than the dipole field. If rotation is very slow, then the mean-field dynamo does not operate, and the neutron star has no global field. Nevertheless, strong small-scale fields are generated in such pulsars, and they can manifest themselves as objects with very low spin-down rate but with a strong magnetic field inferred from the spectral features.

Key words. magnetohydrodynamics (MHD) – pulsars: general – stars: neutron – X-ray: stars – stars: magnetic fields

1. Introduction

A strong magnetic field in the neutron stars can be generated by turbulent dynamo during the first ~ 30 – 40 s of their life when the star is subject to hydrodynamic instabilities (Thompson & Duncan 1993; Bonanno et al. 2003, 2005, Paper I). Instabilities in protoneutron stars (PNSs) are driven by either lepton or entropy gradients and, as a result, two different instabilities may occur (Bruenn & Dineva 1996; Miralles et al. 2000). Convection is presumably connected to the entropy gradient, whereas the neutron-finger instability is caused by a lepton gradient. The neutron-finger instability grows on a timescale ~ 30 – 100 ms, that is ~ 10 – 100 times longer than the growth time of convection (Miralles et al. 2000). Turbulent motions generated by instabilities in combination with rotation make turbulent dynamo one of the most plausible mechanism of the pulsar magnetism. The character of turbulent dynamo depends on the Rossby number, $Ro = P/\tau$, where P is the PNS period and τ the turnover time. If $Ro \gg 1$, the effect of rotation on turbulence is weak and the mean-field dynamo is inefficient. If $Ro \leq 1$ and turbulence is strongly modified by rotation, the PNS can be subject to the mean-field dynamo action. The Rossby number is ~ 10 – 100 in the convective zone where the mean-field dynamo is not to work. On the contrary, except very slowly rotating PNSs, $Ro \sim 1$ in the neutron-finger unstable region (Bonanno et al. 2003) that favors the efficiency of the mean-field dynamo. Note that strong small-scale magnetic fields is generated by turbulence in the both unstable regions.

The conclusion regarding the mean-field dynamo action is at variance with the results by Thompson & Duncan (1993) who argued that only small-scale dynamos operates in most PNSs.

Their reasoning is based on disregarding the neutron-finger instability and the assumption that the whole PNS is convective with the turbulent velocity $v_T \sim 10^8$ cm/s. Since $Ro \gg 1$ for convection, Duncan & Thompson (1992) and Thompson & Duncan (1993) concluded that the mean-field dynamo does not operate in PNSs, except those with $P \sim 1$ ms. Turbulent dynamo in PNSs has also been considered by Rheinhardt & Geppert (2005) who modeled turbulent motions in the convective zone by a relatively complex but *stationary* velocity field. In reality, however, a velocity pattern changes on a timescale comparable to τ which is shorter than the growth time of a mean field. Therefore, the approach used is incorrect in principle because the stochastic nature of turbulent dynamo is entirely lost. In fact, the authors considered laminar rather than turbulent dynamo and, as a result, obtained a confusing result that a large-scale field can be generated in the convective zone where $Ro \gg 1$ in contradiction to the results by many authors for different astrophysical bodies (see, e.g., Thompson & Duncan 1993; Chabrier & Küker 2005).

If the magnetic field in PNSs is generated by turbulent dynamos one can then expect that the field of pulsars is complex with both small- and large-scale structures presented. After the unstable phase, small-scale fields decay rapidly due to ohmic dissipation but fields with the lengthscale $\gtrsim 1$ km can survive during the lifetime of radiopulsars (Urpin & Gil 2004).

In this note, we show that the PNS dynamo can consistently account for the origin of both “standard” radiopulsar magnetic fields and ultra-strong fields in “magnetar”. Our model predicts also the existence of X-ray emitting isolated neutron stars which possess a strong small-scale field at their surfaces but exhibit no radio-pulsations because of a weak large-scale field.

Table 1. The critical periods P_0 for different PNS models.

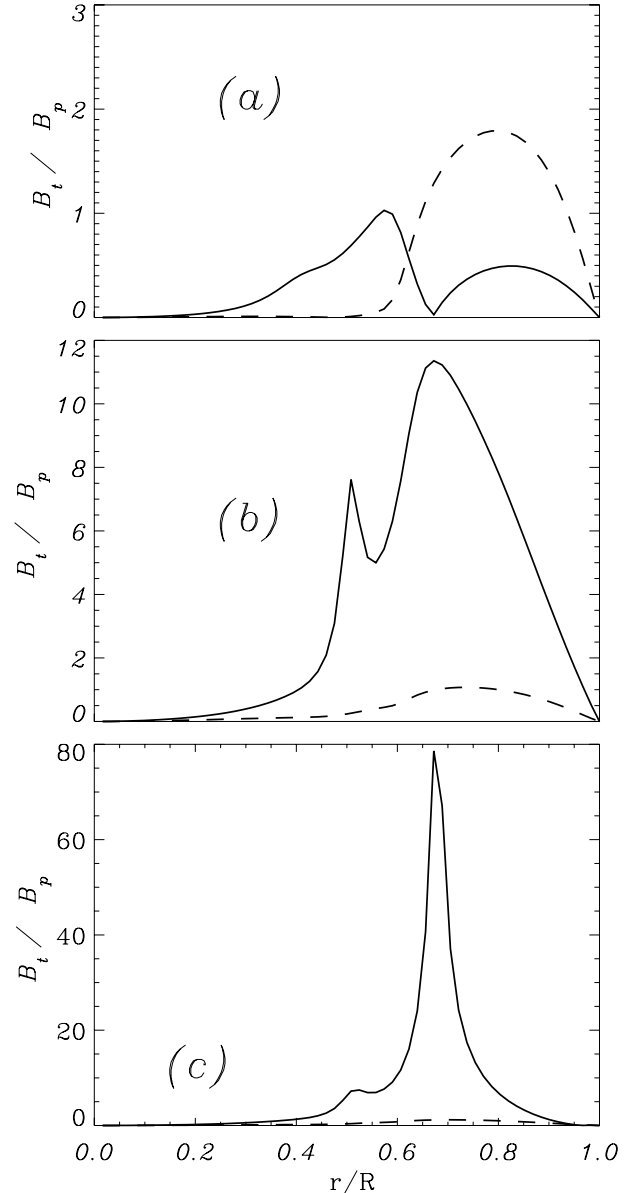
P_0 [s]	η_{nc}/η_c	model	parity
0.96	0.1	a	A0
1.13	0.1	b	A0
1.22	0.1	c	A0
0.94	0.1	a	A1
0.93	0.1	b	A1
0.89	0.1	c	A1
0.84	0.02	a	A0
0.97	0.02	b	A0
1.15	0.02	c	A0
0.81	0.02	a	A1
0.83	0.02	b	A1
0.81	0.02	c	A1

2. The model and results

The problem for dynamo modelling has been described in Paper I. We model the PNS as a sphere of radius R with two different turbulent zones separated at R_c . The inner part ($r < R_c$) corresponds to the convective zone, while the outer one ($R_c < r < R$) to the neutron-finger unstable zone. Generation is governed by the standard dynamo equation with the α -term and turbulent magnetic diffusivity η included. We consider three models of cylindrical rotation given by Eq. (3) of Paper I with $\Omega_{\text{cyl}}^{(1)} = 0, -1/2, -2/3$ (model a, b, c). Properties of turbulence are different at $0 \leq r \leq R_c$ and $R_c \leq r \leq R$. To model this, we assume that η in the convective and neutron-finger unstable regions is equal to η_c and η_{nf} , respectively. The α -parameter is small in the convective zone and equal to $\alpha_{nf} = \text{const.}$ in the neutron-finger unstable region. We recall that in turbulence with the length-scale ℓ_T and moderate Ro , $\alpha_{nf} \approx \Omega \ell_T^2 \nabla \ln(\rho v_T^2)$ (Rüdiger & Kitchatinov 1993). Denoting the density length-scale as L , we have $\alpha_{nf} \approx 2\pi \varepsilon L/P$ where $\varepsilon = \ell_T^2/L^2$. Since the maximal length-scale of instability is $\sim L$, we have $\varepsilon \sim 1$. By solving the dynamo equation, we determine the critical period $P_c = \varepsilon P_0$ corresponding to the marginal dynamo stability; P_0 is the critical period for $\varepsilon = 1$. Generation is possible if $P < P_c$ but PNSs with $P > P_c$ will not be subject to the mean-field dynamo action. The critical period is rather long ($P_0 \sim 1$ s) and, likely, the mean-field dynamo is effective in most PNSs.

In Table 1, we compare the value of P_0 for PNS models with different η_c and rotation laws. In calculations, we assume $R_c/R = 0.6$ and $\eta_{nf} = 10^{11} \text{ cm}^2 \text{ s}^{-1}$. The dependence of P_0 on the rotation law, η_c , and azimuthal wave number m is rather weak. Clearly, larger η_c enhances diffusion of the magnetic field from the neutron-finger unstable zone and, as a result, decreases the critical period. However, this decrease is not large, and $P_0 \sim 1$ s for all considered models.

The geometry of the generated field is rather complex. The field is basically concentrated in the neutron-finger unstable region where the mean-field dynamo operates. The field in the convective zone is substantially weaker. In Fig. 1, we plot the radial dependence of the ratio of toroidal B_t and poloidal B_p fields for the axisymmetric mode ($m = 0$) and different rotation laws. If rotation is almost rigid, then the α^2 -dynamo is more efficient, and $\xi \equiv |B_t/B_p| \sim 1-3$ in the neutron-finger unstable region. However, ξ is larger if differential rotation is strong. For instance, $\xi \sim 6-10$ and $\sim 20-60$ for our models (b) and (c). In these cases, the generation is determined by a combined effect of the α^2 - and $\alpha\Omega$ -dynamo, and the toroidal field is noticeably stronger. The field tends to concentrate near the polar region for the α^2 -dynamo and closer to the equator for the $\alpha\Omega$ -dynamo.

**Fig. 1.** The radial dependence of B_t/B_p for $\varepsilon = 1$, $\eta_{nf}/\eta_c = 0.1$, and for the polar angle $\theta = 60^\circ$ (solid lines) and 20° (dashed lines).

The values of ξ are approximately the same for the $m = 1$ mode. In Fig. 2, we show same as in Fig. 1 but for the model with $\eta_{nf}/\eta_c = 0.02$. More efficient diffusion into the convective region decreases the ratio ξ , particularly, for the models (b) and (c) but this decrease is not significant and does not change our main conclusions.

Generally, the steady-state Ω -profile in turbulent stars depends on P , as it emerges both from observations and numerical modelling. For instance, Reiners & Schmitt (2003) found that differential rotation in convective F-stars is much more common for stars with moderate or slow rotation than for rapid rotators. Simulations also show that the Ω -profile depends on Ro in convective stars (see, e.g., Küker & Rüdiger 2005). Rotation is almost rigid in fast rotators ($\text{Ro} \leq 1$) whereas slow rotators ($\text{Ro} \gg 1$) may exhibit a significant differential rotation. Therefore, we can expect that PNSs with $\text{Ro} \leq 1$ in the neutron-finger unstable zone (or with $P < 30-100$ ms) rotate approximately rigidly and are subject to the α^2 -dynamo. For such stars, $B_t \sim B_p$ and the internal and surface mean fields are comparable.

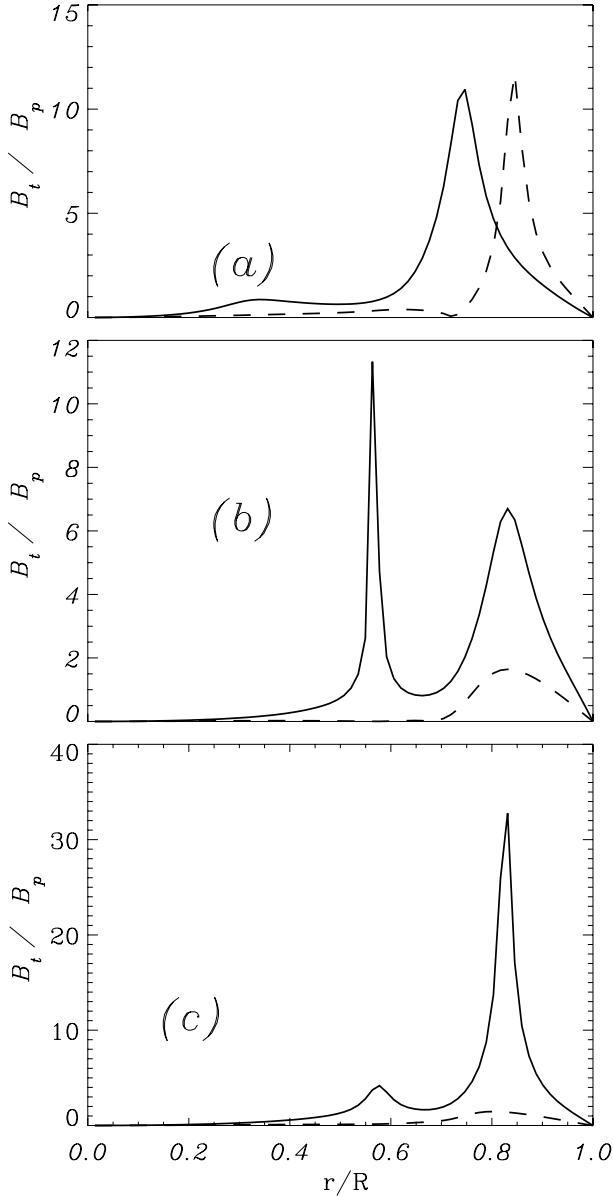


Fig. 2. Same as in Fig. 1 but for $\eta_{\text{nf}}/\eta_c = 0.02$.

Slowly rotating PNSs ($P > 30$ – 100 ms) can be differential rotators. Likely, the α^2 -dynamo is accompanied by the $\alpha\Omega$ -dynamo in such stars, and the internal toroidal field can be substantially stronger than the surface field.

3. Discussion

The critical period that determines the onset of mean-field dynamo is rather long, and dynamos should be effective in most PNSs. The unstable stage lasts ~ 40 s, and this is sufficient for dynamo to reach saturation. We can estimate a saturation field assuming the simplest α -quenching with non-linear α given by $\alpha(\bar{B}) = \alpha_{\text{nf}}(1 + \bar{B}^2/B_{\text{eq}}^2)^{-1}$, where \bar{B} is a characteristic strength of the generated field, and B_{eq} is the equipartition small-scale field. The saturation is reached when α becomes equal to α_c corresponding to the marginal stability. This yields

$$B_s \approx B_{\text{eq}} \sqrt{P_c/P - 1}. \quad (1)$$

A further activity of the PNS as a radiopulsar is determined by the poloidal component of this field, B_{ps} . Using the estimate $B_s \sim B_{\text{ps}}(1 + \xi)$, we obtain

$$B_{\text{ps}} \approx B_{\text{eq}}(1 + \xi)^{-1} \sqrt{P_c/P - 1}. \quad (2)$$

The equipartition field, $B_{\text{eq}} \approx 4\pi\rho v_T^2$, varies during the unstable stage, rising rapidly soon after collapse and then going down when the temperature and lepton gradients are smoothed. We can estimate B_{eq} at a peak as $\sim 10^{16}$ G in the convective zone and $\sim (1-3) \times 10^{14}$ G in the neutron-finger unstable zone (Urpin & Gil 2004). However, v_T and B_{eq} decrease whereas the turnover time τ increases as the PNS cools down. The mean-field dynamo description applies as the quasi-steady condition $\tau_{\text{cool}} \gg \tau$ is fulfilled with τ_{cool} being the cooling timescale. We assume that the final strength of the generated magnetic field is determined by B_{eq} at the moment when the quasi-steady condition breaks down and $\tau \sim \tau_{\text{cool}}$. This is in a contrast to the assumption made by Thompson & Duncan (1993) that the final field strength is approximately equal to the maximal one. If $\tau \sim \tau_{\text{cool}}$, then we have $v_T \sim \pi\ell_T/\tau_{\text{cool}}$ and

$$B_{\text{eq}} \sim \sqrt{4\pi\rho}v_T \sim \pi\sqrt{4\pi\rho}\ell_T\tau_{\text{cool}}^{-1}. \quad (3)$$

The final equipartition field is same for the both unstable zones. Estimate (3) yields $B_{\text{eq}} \sim (1-3) \times 10^{13}$ G for $\ell_T \sim 1-3$ km if $\tau_{\text{cool}} \sim$ few seconds. We can distinguish three types of neutron stars which exhibit different magnetic characteristics.

Strongly magnetized neutron stars. If the period satisfies the condition $P < P_m \equiv P_c[1 + (1 + \xi)^2]^{-1}$ then dynamo leads to the formation of a strongly magnetized PNS with $B_{\text{ps}} > B_{\text{eq}} \sim 3 \times 10^{13}$ G. Since $\text{Ro} \leq 1$ for fast rotators, we can expect that such stars rotate almost rigidly, and the α^2 -dynamo is operative. Therefore, $\xi \sim 1-3$ in the neutron-finger unstable region, and such strongly magnetized stars can be formed if P is shorter than $\sim 0.1P_c = 0.1\epsilon P_0$. Then, Eq. (2) yields

$$B_{\text{ps}} \sim 0.3B_{\text{eq}}\sqrt{P_c/P - 1} \sim 10^{13}\sqrt{P_c/P - 1} \text{ G}. \quad (4)$$

The shortest possible period is ~ 1 ms, hence, the strongest magnetic field generated by dynamo is $\sim 3 \times 10^{14}$ G. This is a bit higher than the maximum field inferred from the spin-down data in radiopulsars, $\sim 9.4 \times 10^{13}$ G (McLaughlin et al. 2003). Likely, the field of all five high-magnetic-field radiopulsars known at the moment such as PSR J1847-0130 (9.4×10^{13} G), PSR J1718-3718 (7.4×10^{13} G), PSR J1814-1744 (5.5×10^{13} G), PSR J1119-6127 (4.4×10^{13} G), and PSR B0154+61 (2.1×10^{13} G) has been generated in this regime, and these pulsars had $P \sim$ few ms at their birth. Note that some PNSs which were strongly magnetized just after the generation stage, now may possess a dipole field $< 3 \times 10^{13}$ G since a fraction of the electric current decays during the early evolution when the conductivity is relatively low (Urpin & Gil 2004). Since the small-scale field is weaker than the large-scale field, one can expect that radiopulsations from such stars may have a more regular structure than those from low-field pulsars.

The maximum field generated is comparable to estimates of the surface field in some SGRs and AXPs which are believed to be the candidates in magnetars. However, estimates of B inferred from spin-down are not reliable for AXPs and SGRs because \dot{P} can vary significantly on a short timescale that seems to be unrealistic for the magneto-dipole braking (Kaspi & McLaughlin 2004).

Neutron stars with moderate magnetic fields. If $P_c > P > P_m$ (or approximately, $\epsilon P_0 > P > 0.1\epsilon P_0$) then the generated dipolar

field is weaker than the small-scale field, $B_{ps} < B_{eq} \sim 3 \times 10^{13}$ G. Since $Ro \sim 1$ or slightly larger for such PNSs, their rotation can be differential. Departures from rigid rotation are weak if $P \sim P_m$ but can be noticeable for $P \sim P_c$. As a result, ξ can vary within a wide range for these PNSs, $\xi \sim 5-60$, and $B_t \gg B_p$. The toroidal field is stronger than the small-scale field if $P_c/2 > P > P_m$, and weaker if $P_c > P > P_c/2$. Note that both differential rotation and strong toroidal field can influence the thermal evolution of such stars. Heating caused by dissipation of differential rotation is important during the early cooling phase since viscosity operates on a timescale $\sim 10^2-10^3$ yr. On the contrary, ohmic dissipation of the toroidal field is a slow process and can maintain the surface temperature $\sim (1-5) \times 10^5$ K during $\sim 10^8$ yr (Miralles et al. 1998).

The magnetic field of this group of stars should be very irregular with a number of sunspot-like magnetic structures on their surface. Particularly, this concerns slowly rotating PNSs with $P \sim P_c$ where only a weak dipole field can be generated. Urpin & Gil (2004) argued that magnetic spots with the lengthscale $\gtrsim 1$ km can survive during the lifetime of radiopulsars. Therefore, one can expect that radiopulsations from these pulsars may have a complex behaviour. For example, small-scale magnetic structures can be responsible for the phenomenon of drifting subpulses observed in many pulsars (Gill & Sendyk 2003). Note also that features in the X-ray spectra of these pulsars may indicate the magnetic field which differs essentially from that inferred from the spin-down data. This can happen because spectral features provide information about the strength of small-scale fields at the surface rather than about the mean field associated to the magneto-dipole braking.

Neutron stars with no large-scale field. If P is longer than $P_c = \varepsilon P_0$ then the mean-field dynamo does not operate but the small-scale dynamo can still be efficient. We expect that such neutron stars have only small-scale fields with the strength $B_{eq} \sim 3 \times 10^{13}$ G and no dipole field. If $\varepsilon \sim 0.3$ then such object can be formed if the initial PNS period is > 0.3 s. Likely, such slow rotation is rather difficult to achieve, and the number of such exotic PNSs is small.

The characteristics of pulsars originated from such PNSs should be rather unexpected. Since the dipole field is negligible,

the spin-down rate should be low for these objects (if spin-down is determined by the magneto-dipole braking alone). Therefore, periods of such pulsars do not increase much in the course of evolution. These neutron stars cannot manifest themselves as bright radiopulsars because of a negligible dipole field. Most likely, they are almost radio-silent. Nevertheless, they can be observed due to periodicity in X-rays caused by the presence of small-scale magnetic structures at their surface. Pulsations must be weak because inhomogeneities of the surface temperature caused by a nonuniform magnetic field $\sim 3 \times 10^{13}$ G are rather small (e.g., Potekhin & Yakovlev 2001). Note that the thermal evolution of these objects should be very similar to that of pulsars with a moderate magnetic field, and they can be X-ray emitting during a rather long time ~ 1 Myr. Likely, the most remarkable property of these neutron stars is a discrepancy between the magnetic field that can be inferred from spin-down measurements and the field strength obtained from spectral observations. Features in X-ray spectra may indicate the presence of a rather strong magnetic field $\sim 3 \times 10^{13}$ G (or a bit weaker because of ohmic decay during the early evolution) associated to sunspot-like magnetic structures at the surface. The field inferred from spin-down data should be essentially lower.

Acknowledgements. V.U. thanks *Dipartimento di Fisica ed Astronomia*, University of Catania, for financial support.

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