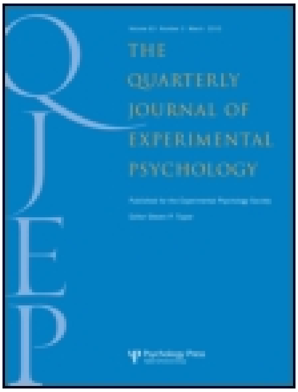

Quarterly Journal of Experimental Psychology
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/pqje19

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Version of record first published: 11 Jun 2007.

To cite this article: P. C. Wason & P. N. Johnson-Laird (1969): Proving a disjunctive rule, Quarterly Journal of Experimental Psychology, 21:1, 14-20

To link to this article: http://dx.doi.org/10.1080/14640746908400189

Q. Jl exp. Psychol. (1969) **21**, 14–20

PROVING A DISJUNCTIVE RULE

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This experiment was designed to determine whether individuals reason correctly about disjunctive rules. The task consisted in the selection of appropriate instances either to prove, or to disprove, a given disjunctive rule. When the first component of the rule was negated, i.e. when the rule was logically equivalent to implication ($\bar{p}$ v q), the selection of appropriate instances was significantly more difficult than when the first component was not negated. The majority of subjects, however, revealed patterns of reasoning which were unstable and labile. The results are discussed in relation to those of previous experiments in which subjects had to reason about conditional sentences.

Introduction

Previous experiments (Wason, 1968) have shown that very few intelligent adults reason adequately about conditional sentences of the form, "if p then q". For example, given a sentence like, "if there is a D on one side of a card, then there is a 3 on the other side", together with four cards bearing respectively D, 3, B, and 7, very few subjects recognized that the 7 was relevant to finding out whether the sentence was true or false. The form of the words, "if p then q," however, is only one way of expressing the concept of implication ($p \supset q$). It can be expressed in different words using different logical constants, e.g. "there isn't p without q"—($p\bar{q}$), or in disjunctive form, "either there isn't p or there is q, or both" ($\bar{p}$ v q). Is the difficulty of "if p then q" inherent in the concept of implication, or is it due to the words used to express the concept? The present investigation attempts to answer this question by testing the disjunctive form. It should be noted that the appropriate disjunctive form is *inclusive* rather then *exclusive*, i.e. the sentence is true when $\bar{p}$ is true, when q is true, and when both $\bar{p}$ and q are true; it is false only when p and $\bar{q}$ are true. In the present investigation, however, the inclusive inter-pretation was assumed and the words, "or both," were omitted from the material.

Since the propositional calculus counts the following combinations of the conditional as true, pq, $\bar{p}$q, $\bar{p}\bar{q}$, and only the one combination, $p\bar{q}$, as false, it can readily be seen that ($p \supset q$) and ($\bar{p}$ v q) have the same truth table. But although the conditional and the disjunctive form of implication have the same truth table, they have a different meaning. Previous research (Wason, 1968; Johnson-Laird and Tagart, unpublished) has shown that the expression, "if p then q," is construed as true only when both p and q are true. Thus the expression, "either there isn't p or there is q," would appear to be a more accurate description of the truth table.

It is known that the errors made in evaluating negative sentences are considerably greater than those made in evaluating affirmative sentences (Wason, 1959, 1961). Hence in the present investigation the disjunctive form of implication ($\bar{p} \vee q$) was compared with the unnegated disjunction ($p \vee q$). ($p \vee q$) provides a base line against which the difficulty of ($\bar{p} \vee q$) may be measured. Even though the present task is effectively binary, i.e. $\bar{p}$ takes only one value, it was predicted that more errors would occur under the ($\bar{p} \vee q$) condition than under the ($p \vee q$) condition.

In the previous experiments (Wason, 1968), the subjects' task was to select all those values necessary to determine whether a given conditional sentence was true or false. In the present investigation the subject was either told that the sentence was true, or that it was false, and then instructed to select values in order to prove this. Knowledge that the sentence is true or false does not logically affect the correct response—it is the same in both cases. When the sentence is in the form ($\bar{p} \vee q$) the correct response is to select p and $\bar{q}$; when it is in the form ($p \vee q$) the correct response is to select $\bar{p}$ and $\bar{q}$. However, when the sentence is known to be true it was considered more likely that the subject would make an erroneous "matching response," i.e. he would select just those values mentioned in the sentence. Similar results have been noted in the attainment of disjunctive concepts by Bruner *et al.* (1956), and in a classification task involving disjunctive sentences by Johnson-Laird and Tagart (unpublished). It was considered, in addition, that this effect would be more pronounced with ($\bar{p} \vee q$) than with ($p \vee q$) since it was anticipated that the latter would be too easy to reflect the effect. Hence the following interaction was predicted: the difference between performance in the "true" and "false" conditions would be larger for ($\bar{p} \vee q$) than for ($p \vee q$).

The test sentences which were used in this experiment referred to all the stimuli which were presented simultaneously to the subject, i.e. they were in the form, "Every card hasn't a P on one side, or it has a Q on the other side," or "Every card has a P on one side, or it has a Q on the other side." This allowed another factor in reasoning to be examined. There is an asymmetry in proving the rule true and proving it false. Since under the true condition every correct stimulus was consistent with the rule, and under the false condition every correct stimulus was inconsistent with the rule, every correct stimulus has to be inspected to prove the rule true but a single correct stimulus is sufficient to prove the rule false.

Method

Design

There were four basic experimental conditions: ($\bar{p} \vee q$)T, i.e. true, ($\bar{p} \vee q$)F, i.e. false, ($p \vee q$)T and ($p \vee q$)F. The subjects acted as their own controls and were presented with these conditions once, ordered in a different one of each of the 4! permutations.

Subjects

Twenty-four volunteer undergraduate subjects of University College London were individually tested.

Materials

The following four types of lexical material were constructed:

(A) Every card has a number which is Roman on one side, or it has a letter which is capital on the other side.

(B) Every card has a rectangle which is vertical on one side, or it has an angle which is acute on the other side.

(C) Every card has a square which is black on one side, or it has a line which is crooked on the other side.

(D) Every card has a number which is even on one side, or it has a letter which is a vowel on the other side.

Each of these four types of material was, in addition, negated in its first clause so that it corresponded to ($\bar{p}$ v q), e.g. "Every card has a number which isn't Roman on one side, or it has a letter which is capital on the other side."

These eight sentences were typed on separate cards (5 × 3 in.). It will be noted that the sentences are not strictly binary, but the stimuli used in conjunction with them made them binary. For each of the sentences eight stimulus cards were constructed consisting of exemplars of p or $\bar{p}$ on one side and q or $\bar{q}$ on the other side. The stimuli used for the four sets of lexical material were as follows:

(A) Roman numbers, Arabic numbers; capital letters, lower case letters.

(B) Vertical rectangles, horizontal rectangles; acute angles, obtuse angles.

(C) Black squares, white squares; crooked lines, straight lines.

(D) Even numbers, odd numbers; vowels, consonants.

Table I shows the logical values on the stimulus cards associated with each of the four experimental conditions. The values given first of all are those on the front of the cards, and those given second in brackets are those on the back of the cards.

TABLE I

The logical values on the stimulus cards and the number of cards associated with each experimental condition

	p(p)	q(p)	p($\bar{q}$)	$\bar{q}$(p)	$\bar{p}$(q)	q($\bar{p}$)	$\bar{p}$($\bar{q}$)	$\bar{q}$($\bar{p}$)	
(p v q)*T*	1	1	1	2	2	1	—	—	8
(p v q)*F*	1	1	1	—	—	1	2	2	8
($\bar{p}$ v q)*T*	2	1	—	—	1	1	1	2	8
($\bar{p}$ v q)*F*	—	1	2	2	1	1	1	—	8

The explanation for the choice of the particular set of values for each condition is as follows. A necessary constraint is that each of the values p, $\bar{p}$, q, $\bar{q}$, should appear twice on the front of the cards, making a total of eight cards. Thus in the (p v q)T condition it is necessary for $\bar{q}$(p) and $\bar{p}$(q) to occur twice because inclusion of $\bar{p}$($\bar{q}$) or $\bar{q}$($\bar{p}$) would render the rule false. A second constraint is that under the false conditions all four correct cards should render the rule false. Thus for the (p v q)F condition it is necessary that $\bar{q}$(p) and $\bar{p}$(q) be omitted. It will readily be seen that similar principles apply to the stimuli in the ($\bar{p}$ v q)T and ($\bar{p}$ v q)F conditions. Inspection of the table also shows that the correct stimulus cards are those which are doubly represented in each condition.

Procedure

The first 12 subjects were presented with the lexical material in the order A, B, C, D, and the second 12 subjects in the order D, C, B, A. Under each of the four experimental conditions the appropriate sentence was presented and the eight corresponding stimulus cards placed on the desk in front of the subject in a random array. It was explained to the subject that each sentence was a rule. Each condition had three phases:

(1) The subject was told that the rule was true (false), and that he had to pick out the four cards which he needed to turn over in order to prove this. He was told that there were only four cards which it was necessary to inspect. The subject was given as much time as he needed to think, and then selected four cards.

(2) The experimenter slowly turned over the cards selected by the subject, one at a time, and the subject was asked to tell him as soon as he was sure that the rule had been proved true (false).

(3) The subject was asked why he had not chosen the four remaining cards.

Results

Each subject was given a score from 0 to 4 for each condition according to the number of correct cards which he had selected. The mean scores were as follows: $(\text{p v q})T = 2{\cdot}9$, $(\text{p v q})F = 3{\cdot}3$, $(\bar{\text{p}} \text{ v q})T = 2{\cdot}6$, $(\bar{\text{p}} \text{ v q})F = 2{\cdot}7$. The prediction that the negative conditions would be more difficult than the affirmative ones was confirmed ($P = 0{\cdot}04$, Wilcoxon test, one-tailed). There was no significant difference between the true and false conditions and the predicted interaction was not confirmed.

Table II shows the frequency of the selection of the logical values for each condition.

TABLE II

Frequency of the selection of logical values for each condition

	$\bar{\text{p}}\bar{\text{q}}$	pq	$\bar{\text{p}}$q	p$\bar{\text{q}}$	Others	Totals
$(\text{p v q})T$	17	6	0	0	1	24
$(\text{p v q})F$	19	3	2	0	0	24
$(\bar{\text{p}} \text{ v q})T$	3	2	6	13	0	24
$(\bar{\text{p}} \text{ v q})F$	4	2	4	12	2	24

The frequencies in Table II, except those classified under "others," refer to the four selected cards, two for each value specified, e.g. the frequency of 17, under $(\text{p v q})T$ for $\bar{\text{p}}\bar{\text{q}}$, means that in 17 cases two values of $\bar{\text{p}}$ and two of $\bar{\text{q}}$ were selected. The selections classified under "others" consisted of two cases of subjects selecting one value of each of the four values, p, $\bar{\text{p}}$, q, $\bar{\text{q}}$, and in one case of selecting two values of q and two of $\bar{\text{q}}$. The value of χ^2 was computed independently for each condition and was significant in each case. It will be noted that, under all conditions, a correct selection was made more frequently than any other selection. Ignoring truth and falsity, $\bar{\text{p}}\bar{\text{q}}$ was selected 36 times out of 48 for (p v q), and p$\bar{\text{q}}$ was selected 25 times out of 48 for ($\bar{\text{p}}$ v q). Only four out of the 24 subjects were correct for all four conditions.

The number of cards which the subjects needed to turn over in phase 2 before making a decision that the rule was true, or false, was computed independently for the *T* and *F* conditions. Only correctly selected cards were included because under the *F* conditions incorrectly selected cards do not falsify the rule. Each subject had two trials in the T condition and two in the *F* condition, and since a subject may make an incorrect selection on one or both trials, mean values over the two were computed. Table III shows the number of subjects associated with each of these mean values. The correct number of cards to turn over in order to verify the rule is four, and the correct number to falsify it is one. Hence it will be noted that most subjects made a good approximation to the rational strategy.

TABLE III

Mean number of cards turned over to prove that the rule is true or false

Mean number of cards turned over	Number of subjects True	False
4·0	11	3
3·0	1	2
2·5	1	2
2·0	2	0
1·5	0	1
1·0	3	13
Subjects making incorrect selections on both trials	6	3

At the end of the experiment an informal test was made of whether the subjects had been reasoning by inclusive or exclusive disjunction. The remaining cards were turned over by the experimenter to see whether a card which had *both* the values mentioned in the rule was counted by the subject as consistent or inconsistent with the rule. In fact only three subjects gave evidence of reasoning exclusively.

Discussion

It is evident that the element of negation, which is inevitably present in the logical structure of implication, does, to some extent, contribute towards its difficulty. 52·1 per cent of the subjects reason correctly when implication is expressed in disjunctive form, compared with 75 per cent when disjunction is not negated.

It is of interest to observe the types of error made in this experiment. Under the (p v q) conditions matching responses (mirror images of the correct selections) accounted easily for the majority of errors—9 out of 12. On the other hand, under the ($\bar{p}$ v q) conditions such responses accounted for only 10 out of 21 errors. It seems likely that the negative component induces a confusion which manifests itself in a greater variety of erroneous responses. There were 12 matching responses under the *T* conditions as opposed to 7 under the *F* conditions—a trend which is consistent with the predicted interaction between the truth of a rule and its logical form. Contrary to the prediction, however, the difference between the matching responses for the *T* and *F* conditions was more pronounced for (p v q) than for ($\bar{p}$ v q).

When correct selections of cards had been made the reasons given for rejecting the remaining cards tended to reveal an appreciation of disjunction. "Those can be seen to prove at least half the rule." "Those four satisfy the conditions; so I do not need to turn them." When incorrect selections had been made the reasons reflect the erroneous matching principle and, in at least one case, a conflation between disjunction and conjunction. "Those are straight lines and white squares. The rule requires crooked lines and black squares." "Those are already incorrect on one side. I don't know what is on the other side. You just can't do it with four cards." "I took the four which had an even number and a vowel so that if it had the other on the back it would prove the rule true."

A very interesting effect due to the truth of the rule was observed. Four subjects matched on both the true conditions, but three of them made correct selections on both the false conditions. When the rule was true the following were the reasons for rejecting the remaining cards. "You are trying to prove the rule is true and these might disprove it." "Those others are not what is stated in the rule and the rule is true." "I chose the true ones. No matter what is on the other side, these would proved the rule." When the rule was false, these subjects made the correct selections for the wrong reasons. "If I am trying to prove the rule false, then it is obvious you wouldn't choose the ones which prove it correct." "Those others are what is stated in the rule and the rule is supposed to be false." "Those others are half of the rule which I'm trying to disprove."

The results of this experiment, when compared with those of Wason (1968), show that expressing implication as the disjunction, "either not-p or q", makes it easier to grasp than expressing it as the conditional, "if p then q." Just over half the subjects reasoned efficiently about the disjunctive expression, but in the previous experiments only 16·7 per cent of the subjects selected $\bar{q}$. Thus the verbal guise, or description, of a rule has a marked effect on the way it is understood.

The most striking differences, however, are not just quantitative; they are qualitative. When the subjects are given the rule in the form "if p then q," they tend to go wrong in a decisive way. And little that one can do will evidently prevent them falling into error. No such feeling of finality attached to the responses in the present experiment. Sixteen out of the 24 subjects spontaneously doubted that some of their selections were correct. These doubts were expressed on 17 incorrect selections and on 4 correct ones. "I should have chosen the capitals instead of the lower case letters. They would prove the rule false" (selection right, comment wrong). "I was working on the wrong assumption. I could have done it more easily if I had chosen the other four" (selection right, comment wrong). "My mind went blank. Please let me think again" (selection wrong, comment right).

The labile quality of this kind of reasoning seemed to be associated with both affirmatively and negatively expressed disjunctions. One consequence was that the subjects were ready both to admit error and to gain spontaneous insight into their mistakes after they had made them. Very frequently the act of committal to a choice immediately evoked misgivings. It is as if the "either . . . or" expression itself creates uncertainty. It breaks up the "direction" which seems to be strongly imposed by the conditional, "if . . . then," sentence. With a conditional the individual is likely to be confident but wrong; with a disjunction he is more likely to be unconfident but right. The meaning of a conditional gives no hint of the negation or falsity which underlies its logic. The disjunctive expression makes this element explicit, but this seems to weaken the grounds upon which any inference can be made. In conclusion, a tentative answer may be given to the question posed in the Introduction. There would seem to be inherent difficulties in the concept of implication, but the words in which it is expressed also effect both its ease of understanding and the manner in which it is construed.

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Received 6 *August* 1968