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The Proximity Potential for Heavy Ion Reactions on Deformed Nuclei

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DISCLAIMER

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I. INTRODUCTION

The usual treatment of the deformed optical model for analysis of heavy ion induced inelastic scattering data involves a deformed (target) radius, a spherical (projectile) radius and a potential strength dependent on the surface separation along the line between the two centers. Expressed in terms of the target deformation parameters β_L the center line potential has the form

$$V(r,\theta) = V_o\, f(r-R(\theta)) \tag{1}$$

where

$$R(\theta) = R_P + R_T\Big(1 + \sum_L \beta_L\, Y_L^0(\cos\theta)\Big) \tag{2}$$

with R_P the spherical projectile radius, R_T for the target, and f the radial form factor, often taken to be a Woods-Saxon shape

$$f(r-R(\theta)) = \frac{1}{1+\exp\dfrac{r-R(\theta)}{a}}\,. \tag{3}$$

Several authors using various approaches have shown that this center line potential is geometrically inadequate especially for description of higher L deformation parameters probed in heavy ion induced inelastic scattering experiments[1-3]. Yet despite its inadequacy the center-line potential continues to be commonly used. Perhaps it is because the center-line prescription works fairly well for the most commonly measured β_2 deformation parameter, and none of the more sophisticated treatments is perceived as being either definitive (they still contain serious geometrical approximations) or easily related to the Woods-Saxon potential (they are expressed in density folding or Thomas Fermi forms).

In this paper we work out a quantitatively adequate form of the deformed proximity potential suitable for use with a coupled channels reaction code in the analysis of inelastic scattering data above the Coulomb barrier. A major objective is to be able to extract reliably higher deformed multipole moments from such data. The deformed potential calculated in the folding model will serve as a geometrically exact benchmark to evaluate the accuracy of the proximity potential prescriptions.

II. THE SPHERICAL PROXIMITY POTENTIAL

The proximity potential as discussed by Blocki et al.[4] and by Brink and Stancu[5] is based on a Thomas-Fermi treatment of the energy density of the ion-ion system. However we are interested in the tail of the potential for inelastic scattering analysis, and it is in just that region that the Thomas-Fermi method breaks down. Thus we will frame our development in terms of a model which should have its greatest validity in the tail of the potential, namely the folding model. We are not particularly interested in the normalization of the basic ingredients of the folding model since the object of our consideration is the geometry of the deformed heavy ion potential. The normalization can be obtained from a fit to data. We take a single folding

approach in which the nucleon-projectile optical potential (obtained empirically) is folded over the density of the target. We have for the folded potential V_F

$$V_F(R) = \int d^3r \; V_p(\vec{r}-\vec{R})\rho_T(\vec{r}) \tag{4}$$

where $V_p(\vec{r}-\vec{R})$ is the nucleon-projectile real optical potential and $\rho_T(\vec{r})$ is the target density. This integral may be evaluated exactly on a computer. To obtain its proximity form we follow the approach taken by Brink and Stancu in the related Thomas-Fermi problem. Due to spherical symmetry Eq. (4) can be reduced to a two-dimensional integral over r_1 and r_2

$$V_F(R) = \frac{2\pi}{R} \int r_T dr_T r_p dr_p \; V_p(r_T-R_T)\rho_T(r_p-R_p) \tag{5}$$

with the limits of integration

$$r_T + r_p > R \qquad |r_T-r_p| < R \quad .$$

From this form Brink and Stancu obtain a two term proximity potential

$$V_{P2}(R) = 2\pi \frac{R_T R_P}{(R_T+R_P+S)} \left[\varepsilon_0(S) + \frac{R_T+R_P}{2R_T R_P} \varepsilon_1(S)\right] \tag{6}$$

where S is the distance betwen the two surfaces,

$$\varepsilon_n(S) = \int_S^\infty S'^n e(S')dS' \; , \tag{7}$$

and in our folding potential case

$$e(S') = \int_{-\infty}^{\infty} V_P(u_1)\rho_T(S'-u_1)du_1 \ . \tag{8}$$

This form contains the next correction for finite curvature in comparison to more approximate form of Blocki et al.

$$V_{P1}(R) = 2\pi \frac{R_T R_P}{R_T+R_P} \ \varepsilon_0(S) \tag{9}$$

If we assume that both the nucleon-projectile potential and the target density have the same Fermi function form (with the same diffuseness a)

$$V_P(r_T) = \frac{V_P^0}{1+\exp\left(\frac{r_T-R_T}{a}\right)} \tag{10}$$

$$\rho_T(r_p) = \frac{\rho_T^0}{1+\exp\left(\frac{r_R-R_p}{a}\right)} \tag{11}$$

Then Eq. (5) has the analytical solution

$$e(S) = V_p^0\rho_T^0 \frac{S}{\exp\frac{S}{a}-1} \tag{12}$$

Brink[6] suggested this form for the one term proximity form of the folding potential Eq. (9), which becomes

$$V^F_{P1}(R) = 2\pi V^0_P \rho^0_T \frac{R_T R_P}{R_T+R_P} \int_{R-R_T-R_P}^{\infty} \frac{S\, dS}{\exp\frac{S}{A} - 1} \quad . \tag{13}$$

We have adapted Eq. (12) also for the two term proximity potential Eq. (6) and obtain

$$V^F_{P2}(R) = 2\pi V^0_P \rho^0_T \frac{R_T R_P}{R} \left[\int_{R-R_T-R_P}^{\infty} \frac{S\, dS}{\exp\frac{S}{A} - 1} + \frac{R_T+R_P}{2R_T R_P} \int_{R-R_T-R_P}^{\infty} \frac{S^2 dS}{\exp\frac{S}{A} - 1} \right] \quad . \tag{14}$$

To test the validity of the proximity potential approximation we have made comparisons of the one and two term proximity potentials generated by Eq. (13) and (14) with exact results obtained by numerical folding on a computer. Five representative cases are taken, spanning the region of heavy ion reactions: $^{16}O + {}^{16}O$, $^{16}O + {}^{58}Ni$, $^{16}O + {}^{208}Pb$, $^{58}Ni + {}^{208}Pb$, and $^{208}Pb + {}^{208}Pb$. In all cases r_o is set to 1.2 and a to .65 for both target and projectile Woods-Saxon densities.

The results seen in Figure 1 are conclusive: the proximity potential in its two term form is in excellent agreement with the folding model for all heavy ion reactions, not only in shape, but in absolute magnitude. Furthermore, with a slightly increased normalization, the one term proximity potential is also

essentially identical to the folding model. Brink and Stancu[5] make the statement that the one term proximity potential should be a less accurate approximation in the folding model than in their extended Thomas-Fermi case, but in fact our accuracy is comparable to their case. The two term expression is even more suited to the folding model than to the more complicated angle dependent Thomas-Fermi case of Brink and Stancu.

III. THE DEFORMED PROXIMITY POTENTIAL

A natural picture to use in representing the deformed optical potential is that of the proximity potential because of its dependence on the radius of curvature of the two interacting nuclei. We generalize from the spherical to the deformed proximity potential by assuming that the operative curvature of the deformed nucleus is taken at the point on the surface closest to the spherical nucleus and that the separation distance is also taken at this point. This is the approach taken by Randrup and Vaagen[3], but approximations that they took for the curvature make their numerical results quantitatively inadequate for extraction of higher multipole moments.

In discussing an optical potential involving a deformed target nucleus one must carefully define the orientation of the deformed nucleus as well as its separation from the spherical projectile. We will express the potential for a given orientation in terms of the radius of curvature of the spherical nucleus and in terms of two local principal radii of curvature at the point on the surface of the deformed nucleus closest to the spherical nucleus (see Fig. 2). Our approach will be semi-numerical: while we will derive analytical expressions for the radii of curvature, we use a computer search to find the point on the deformed nucleus closest to the spherical nucleus. It is from this point that we calculate the radii of curvature, R_1 and R_2, as well as the separation, S, between the two nuclei.

In order to derive principal radii of curvature of the deformed nucleus we choose a coordinate system with the z axis along the symmetry axis of the deformed nucleus (Fig. 2). The center of the sphere is in the y-z plane. We assume the shape of the deformed nucleus is a spheroid with the radius given by $R(\theta) = R_0(1+ \Sigma\ \beta_L Y_L^0(\theta))$ where θ = arctan (z,y). The radius of curvature in the y-z plane (R_1) is given by the well known expression

$$R_1 = \left| \frac{[R^2+(dR/d\theta)^2]^{3/2}}{R^2+2(dR/d\theta)^2-R(d^2R/d\theta^2)} \right| \quad . \tag{15}$$

The second principal radius of curvature is then determined by the intersection of the spheroid and a plane perpendicular to the y-z plane and normal to $R(\theta)$. The form of this radius of curvature may be determined algebraically or geometrically and the final expression obtained is

$$R_2 = \left| \frac{R\sin\theta[R^2+(dR/d\theta)^2]^{1/2}}{(dr/d\theta)\cos\theta - R\sin\theta} \right| \quad . \tag{16}$$

Generation of the deformed one term proximity potential is then straightforward. One takes a geometric mean of the two perpendicular curvature factors to obtain

$$V_{P1}(R,\theta) = 2\pi\sqrt{\frac{R_1(\theta)R_P}{R_1(\theta)+R_P}}\sqrt{\frac{R_2(\theta)R_P}{R_2(\theta)+R_P}}\ \varepsilon_0(S) \quad . \tag{17}$$

In an analogous fashion a two term proximity potential can be defined as the geometric mean of the two spherical proximity potentials corresponding to the two perpendicular radii of curvature of the deformed nucleus.

$$V_{P2}(R,\theta) = 2\pi \sqrt{\frac{R_1(\theta)R_P}{R_1(\theta)+R_P+S}} \sqrt{\frac{R_2(\theta)R_P}{R_2(\theta)+R_P+S}} \left[\left(\varepsilon_0(S) + \frac{R_1(\theta)+R_P}{2R_1(\theta)R_P} \varepsilon_1(S) \right) \right.$$
$$\left. \times \left(\varepsilon_0(S) + \frac{R_2(\theta)+R_P}{2R_2(\theta)R_P} \varepsilon_1(S) \right) \right]^{1/2} \qquad (18)$$

In both these expressions $\varepsilon_0(S)$ and $\varepsilon_1(S)$ are the one dimensional slab on slab functions of the separation (Eq. 7,8,12) of the two nuclei.

Up to now we have only really treated the real part of the optical potential. One does not expect the folding picture to have much validity for the imaginary potential. We will assume, however, that the geometrical picture of the deformed proximity potential can be taken over to the imaginary potential in conjunction with a Woods-Saxon or other empirical potential for the slab on slab radial form. In fact one can show in the proximity picture that the Woods-Saxon potential form can be seen to arise out of a surface-surface folding. This picture does not seem geometrically unreasonable for the generation of a heavy ion imaginary potential. Adopting a one term proximity potential treatment of the deformed imaginary potential we obtain

$$W_{P1}(R,\theta) = W(S) \sqrt{\frac{R_1(\theta)R_P}{R_1(\theta)+R_P}} \sqrt{\frac{R_2(\theta)R_P}{R_2(\theta)+R_P}} \left(\frac{R_T+R_P}{R_P R_T} \right) \qquad (19)$$

where $W(S)$ is any empirical potential form and R_T is the spherical radius of the deformed radius $R_1(\theta)$.

This potential form might also be utilized for the real potential if one did not want to abandon empirical Woods-Saxon potentials, but nevertheless wanted to provide a geometrically more correct treatment of deformation.

For comparison we will calculate the angular momentum components of the angle dependent optical potentials for both folded and proximity potentials

$$V_L(R) = \int d\Omega Y_L^0(\Omega) V(R,\theta) \quad . \tag{20}$$

These components are evaluated on a computer using eight gauss points per quadrant.

To investigate various aspects of the deformed proximity potential we have chosen a case for which data exist and for which a coupled channels analysis[7] has been performed: 72 MeV ^{16}O + ^{152}Sm. Geometrically, this is a typical case, a light heavy ion projectile on a more massive deformed target.

In numerical computations analogous to the results of Figure 1, optical potential components generated in the two term proximity potential are practically indistinguishable from the folding model both in radial shape of all the components and in absolute magnitude. The one term proximity potential is slightly lower in magnitude for all components.

Since the V_0 component is ultimately fit to elastic data, the ratio of computed components to the L = 0 component must be reliably calculated for reliable extraction of multipole moments. And since numerical investigations indicate that the one term proximity potential is a reasonably good representation of the folding model for all L components when renormalized, we will only use the one term proximity potential in the following calculations.

The crucial point of this whole development is how the proximity model treatment differs from the centerline prescription when higher multipole components are extracted. We have plotted in Figure 3 the ratios of angular momentum components to the real monopole potential using the centerline prescription and the Woods-Saxon proximity potential of Eq. 19. Assuming the Woods-Saxon proximity potential has the more correct relative geometry for the given deformation lengths ($\beta_2 R_N^2 = 1.65$, $\beta_4 R_N^4 = .29$), we find that V_4/V_0 and V_6/V_0 are significantly overpredicted in the centerline prescription for the $\beta_4 R_N^4$ value of .29, implying a true value somewhat larger. In fact there is a discrepancy between this value obtained in Kim's heavy ion analysis and the values of $\beta_4 R^4 = .52$ and .53 from electron scattering and Coulomb excitation respectively. If we keep $\beta_2 R_N^2$ at 1.65 and set $\beta_4 R^4$ to .52 then we obtain ratios for the Woods-Saxon proximity model which are very close to the ratios of the center line prescription in Figure 3 probed by Kim's analysis.

To further test these observations coupled channels calculations have been performed for this case using the coupled channels code QUICC[8]. In Figure 4 the solid line is a repetition of Kim's original calculation for the 4^+ angular distribution, using the centerline prescription and $\beta_2 R_N^2 = 1.65$, $\beta_4 R_N^4 = .29$, which fits the data. The dotted line is a Woods-Saxon proximity potential calculation with the same parameters. The dashed line is a Woods-Saxon proximity potential calculation with $\beta_4 R_N^4 = .52$ and all other parameters the same. Clearly the proximity potential calculation with $\beta_4 R_N^4 = .52$ corresponds better to the centerline calculation of $\beta_4 R_N^4 = .29$ and thus to the data. Using the more correct proximity prescription has caused the discrepancy with the electron scattering and Coulomb excitation results to disappear.

IV DISCUSSION

We have mentioned three previous improved treatments of the interaction of a spherical projectile on a deformed target. The approach of Moffa et al.[1] was a full folding of a spherical density with a deformed density, but the deformation of the density was only taken to first order as was appropriate for the vibrational nuclei which they considered. Hendrie's work[2] considered the nuclei as touching, for his correction to the multipole moments generated by an angular shift relative to the centerline prescription. However he did not include effects of the angular change in the force due to the changing radius of curvature. Randrup and Vaagen[3] showed the importance of the local radius of curvature as well as the effective angular shift, but considered the effect for only the β_2 deformation and for only one separation of the two nuclei. Furthermore, their approximations for the radii of curvature, while qualitatively instructive, are not quantitatively useful for extracting multipole moments. For example at $\theta = 0$, these authors' first order approximation for the local radius of curvature $R_1(\theta = 0)$ yields .6216 R for their parameters while our exact value is .8050 R for the same parameters.

In conclusion we feel that we have presented a quantitatively adequate treatment of the deformed optical potential for extraction of multipole moments from heavy ion induced inelastic scattering on deformed nuclei.

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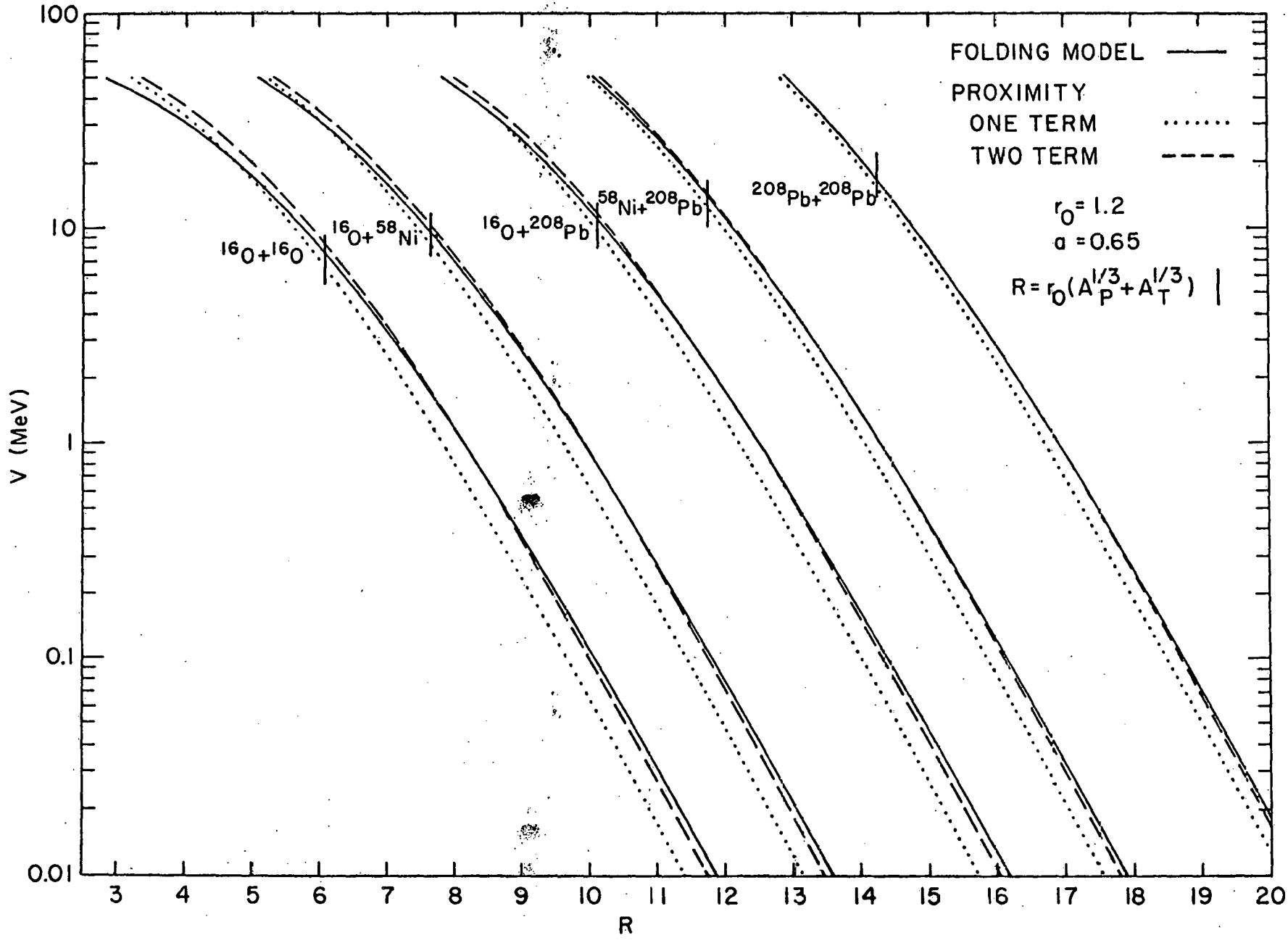
Figure Captions

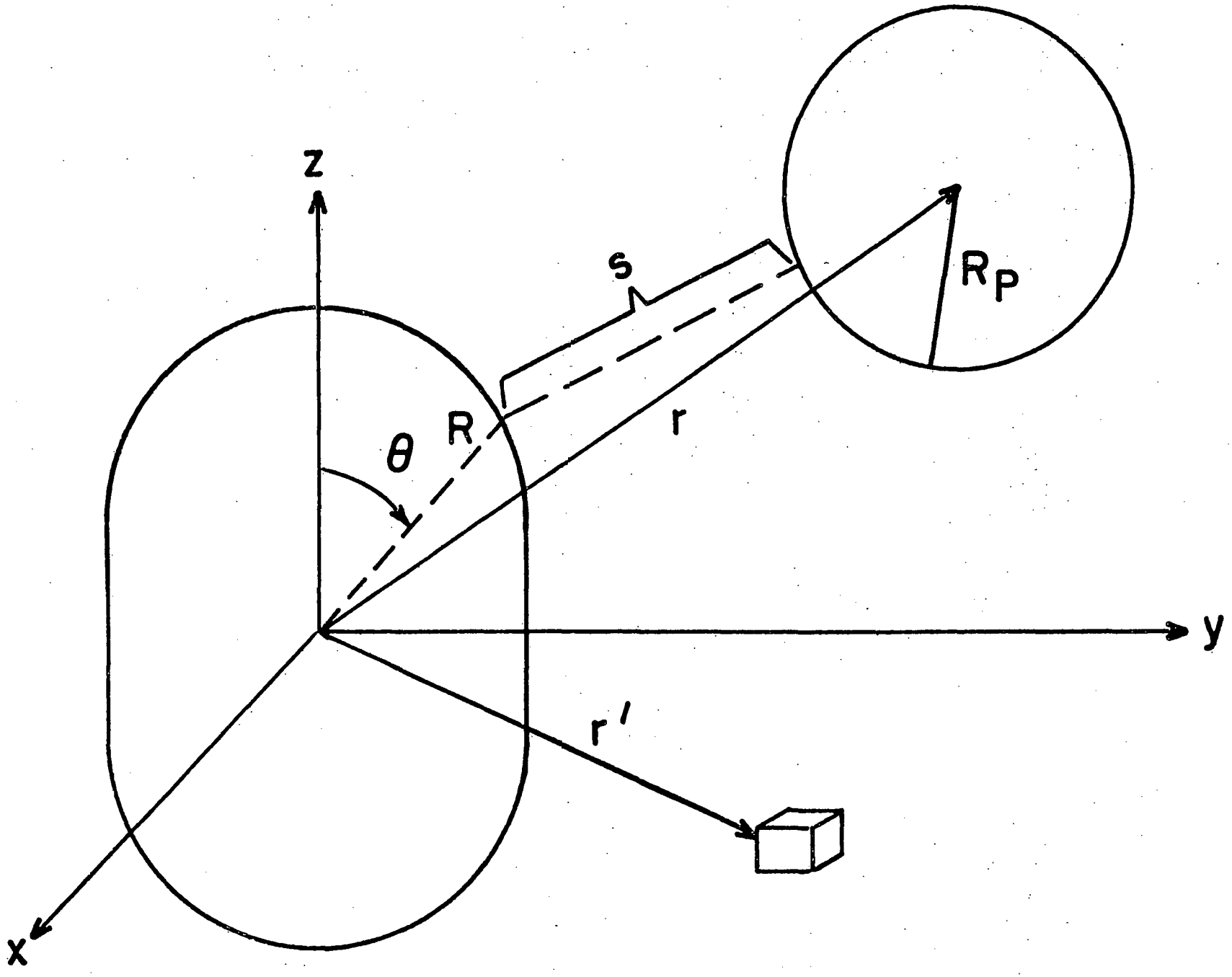
Fig. 1 Comparison of proximity and folding model calculations of the ion-ion potential. Both V_p^0 and ρ_T^0 are normalized to unity.

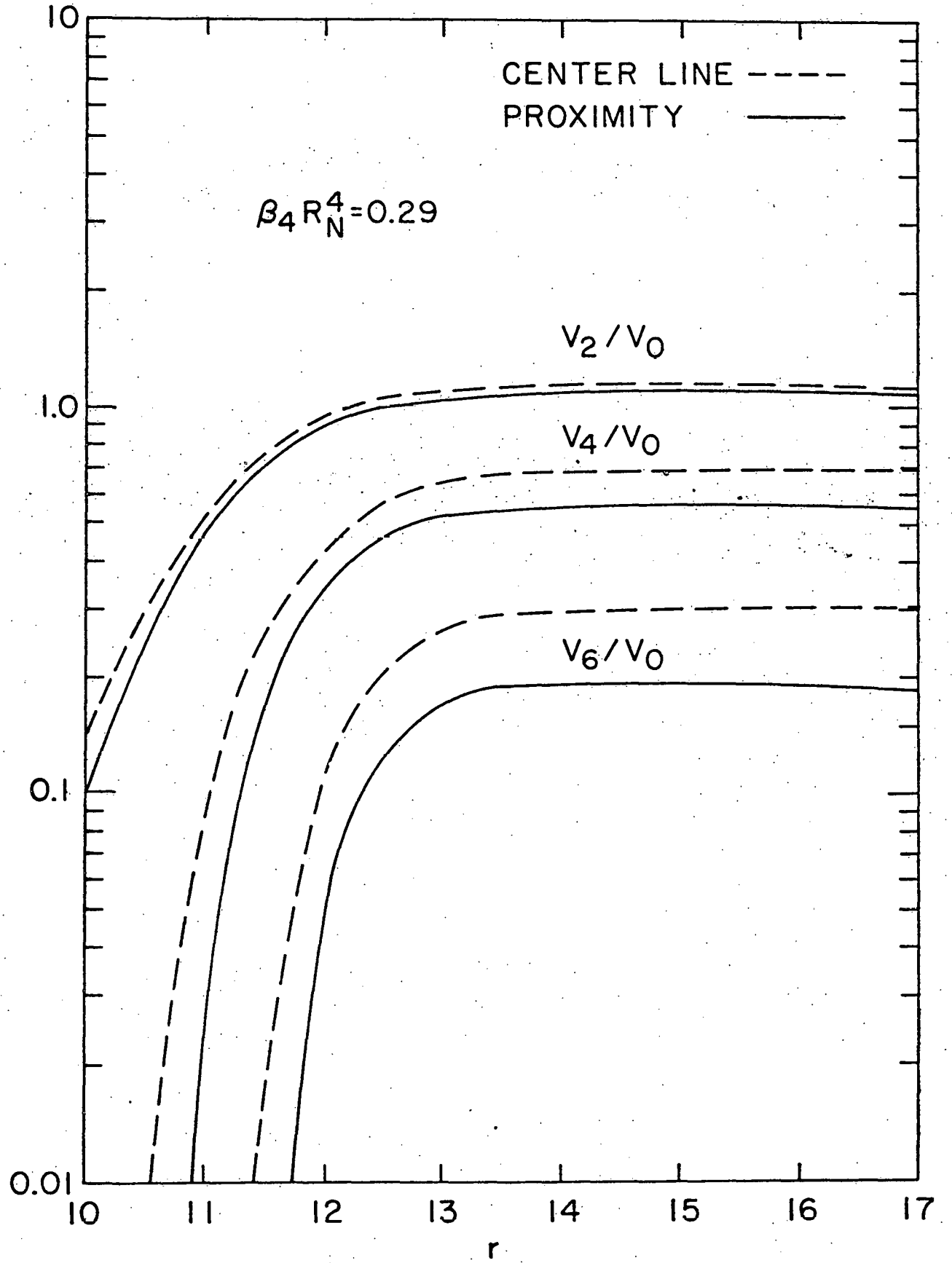
Fig. 2 Coordinate system for a deformed optical potential including a volume element for folding.

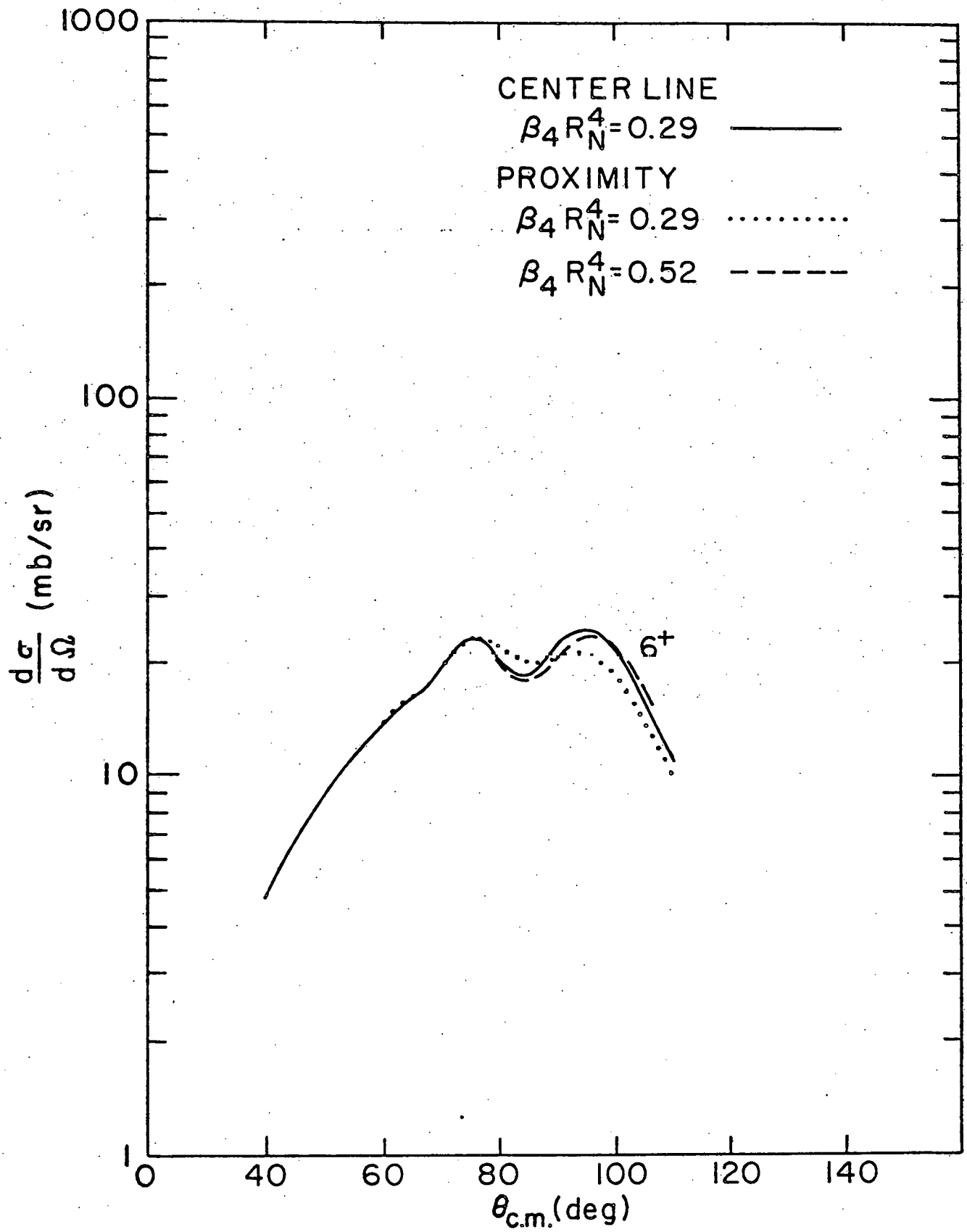
Fig. 3 Comparison of ratios of L = 2, 4, 6 components of the Woods-Saxon potential of Ref. 7 to the L = 0 component using the centerline prescription and the one term proximity prescription.

Fig. 4 Comparison of $^{152}Sm(^{16}O,^{16}O')^{152}Sm(4^+)$ angular distributions. The value of the Woods-Saxon proximity real and imaginary potential depths have been reduced by 6% to correspond to the L = 0 potential strength of the conventional Woods-Saxon potential.


FOLDING MODEL
PROXIMITY
ONE TERM
TWO TERM
$r_0 = 1.2$
$a = 0.65$
$R = r_0(A_P^{1/3} + A_T^{1/3})$
$^{16}O+^{16}O$
$^{16}O+^{58}Ni$
$^{16}O+^{208}Pb$
$^{58}Ni+^{208}Pb$
$^{208}Pb+^{208}Pb$
V (MeV)
R
100
10
1
0.1
0.01
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20


z
s
R_P
θ
R
r
y
r'
x


CENTER LINE
PROXIMITY
$\beta_4 R_N^4 = 0.29$
V_2 / V_0
V_4 / V_0
V_6 / V_0
10
1.0
0.1
0.01
10
11
12
13
14
15
16
17
r


1000
100
10
1
0
40
60
80
100
120
140
CENTER LINE
$\beta_4 R_N^4 = 0.29$
PROXIMITY
$\beta_4 R_N^4 = 0.29$
$\beta_4 R_N^4 = 0.52$
6+
$\frac{d\sigma}{d\Omega}$ (mb/sr)
$\theta_{c.m.}$ (deg)