# Prudence Demands Conservatism 

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## Prudence Demands Conservatism


#### Abstract

We define accounting systems as conditionally conservative (or liberal) if they produce finer information at lower (or higher) expected earning levels. We study the preference for differing levels of conservatism among decision-maker (DM)'s with varying risk aversion and prudence. Similar to risk aversion, prudence is a metric based on DM's indirect utility; prudent DM's save more as income becomes riskier. In a model of precautionary savings with information, we show that prudent DM's (those with positive prudence measures) prefer more conservative accounting systems and that imprudent DM's (those with negative prudence measures) prefer liberal accounting systems. We also show that conservative accounting may be preferred to a perfect (i.e., perfectly fine) information system if the signals are costly. We provide cases demonstrating the generality of our results, including examples where conservatism is preferred with increasing, constant and decreasing risk aversion.


## 1 Introduction

Conditional conservatism has been described as responding to news or information in a manner that results in reporting lower income, but deferring the recognition of higher income; in the extreme case, this has been interpreted to mean 'anticipate no profits but anticipate all losses. ${ }^{3}$ While conditionally conservative accounting methods are prevalent in current accounting standards, standard setters seem opposed to designing standards to be explicitly conservative. ${ }^{4}$ For example, in rerewriting chapter 3 of Statement of Financial Accounting Concepts No. 8, 'Qualitative Characteristics of Useful Financial Information', the Financial Accounting Standards Board explicitly exclude conservatism as a useful characteristic of financial information, judging it to be 'inconsistent with neutrality 'and stating that 'an admonition to be prudent is likely to lead to a bias. ${ }^{5}$ With due respect to this viewpoint, we believe an alternative perspective where conservatism does not lead to bias is possible.

Conditional conservatism has generated considerable interest, which in turn has resulted in a considerable amount of research on the topic. Despite this effort, there remains a dearth of simple formal explanations for the existence and preference for conditional conservative accounting methods. We know conservative methods exist and, despite the perspective of some researchers and standard setters, we feel practical accountants like or prefer these methods, but we do not have a simple, formal model to explain why. Our objective is to develop such a model.

We define information systems as being conditionally conservative if they produce finer information at lower earnings levels and, analogously, we define information systems as being conditionally liberal if they produce finer information at higher earnings levels. In doing so, we provide an alternative perspective where conservatism does not necessarily lead to bias; rather conservatism leads to differential timing in recognizing good and bad news, but in an unbiased manner. We measure prudence as the sensitivity of expected utility maximizing decision makers (DM's) to the variability of a decision variable to risk. Similar to the Arrow - Pratt measure of risk aversion, the measure of prudence depends on a DM's basic preference ordering as quantified by a DM's indirect utility funtion. More specifically, prudence is the

[^1]negative of the ratio of the second and third derivatives of the utility function and measures the sensitivity of a DM's savings decision to risk; prudent DM's save more as income becomes riskier while imprudent DM's save less as income becomes riskier. Using this definition of conservatism and this measure of prudence, we derive the following results.

First, in a model of conservatism with precautionary savings, we show that prudent DM's (those with positive prudence measures) prefer conservative accounting systems to liberal accounting systems. Second, we show that DM's with prudence neutral preferences (prudence measure equal to zero) are indifferent to the conservative or liberal reporting systems. Third, we show that imprudent DM's (those with negative prudence measures) prefer liberal accounting systems. Fourth, we show that conservatism is preferred by prudent DM's over an accounting system that fully recognizes income based on perfect signals, if the per signal cost is sufficiently high. Last, we provide cases demonstrating the generality of our results, including situations where conservatism is preferred with increasing, constant and decreasing risk aversion. These results demonstrate that decreasing risk aversion is not required for conservatism to be preferred nor does risk aversion alone suffice to explain the demand for conservative accounting methods; only prudence demands conservatism.

## 2 Background and Literature Review

Recent studies offer differing definitions of accounting conservatism and argue that conservatism is valuable for different reasons. ${ }^{6}$ In this section, we discuss the alternative formal models that argue why conservatism in accounting is demanded and explain how our attempt to explicitly model 'conditional conservatism' differs from these alternative arguments. We start, in subsection 2.1, by describing the general definitions of conservatism, emphasizing the definition of conditional conservatism. In subsections 2.2 and 2.3 , we summarize the most common models of conservatism, the agency and debt models, respectively. We propose a model in which a fairly general population of DM's demand conditionally conservative accounting for reasons not related to agency or debt considerations; in subsection 2.4 we compare our model to other non-agencey, non-debt models of conservatism in the literature. Finally, prior to presenting our model in section 3 , we use section 2.5 to describe how empirical work may be designed to differentiate between our model and other models of conservatism.

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### 2.1 General discussion and definitions

Most modern definitions of conservative accounting originate from Devine [1963] who defines conservatism as prompt revelation of unfavorable circumstances and reluctant revelation of favorable circumstances. If DM's have an asymmetric loss function, with bad news affecting the DM's' utility more strongly than good news, conservatism will be preferred. While various models impose such a loss function, we generate an intrinsic reason for such functions using the need for precautionary savings.

More recently, Watts (2003a and 2003b) reviews notions of conservatism in accounting. He gives reasons why conservatism is prevalent in accounting and gives alternate definitions. The main definition offered is that 'higher degree of verification is required for recognition of profits versus losses'. The explanations he suggests for conservatism are contracting, shareholder litigation, taxation, and accounting regulation and expects the first two to be most important. He also discusses the role of earnings management though it is not a prime explanation. In the contracting explanation, conservatism will ameliorate self-interested managerial behavior and will offset managerial misreporting. The second explanation is that by understating results, conservatism reduces the litigation costs expected by the firm. We do not use any of these reasons given for conservatism in this paper, though the notion of 'timeliness' offered by Watts (2003a and 2003b) is relevant for this paper. In this paper we use an alternate reason - DM's demand conservatism as it gives better information for their consumption-savings decisions. Hence, in addition to the demand by contracting parties for conditionally conservative accounting methods, we show that DM's will also prefer conditional conservatism in reporting when making simple, general decisions, such as investing or consuming.

Building a precise economic model of verifiability and timing of recognition is somewhat tricky. For example, we need to answer questions such as the following. Is verifiability a choice variable, is it chosen ex - ante or ex - post, is it chosen after observing some outcomes, and if so, what outcomes are relevant? Further, in any market with rational market participants, DM's will still properly infer information concerning signals that they do not directly observe. For example if a firm does not recognize a loss when the accounting method requires losses be recognized, the market will assume no loss existed. Instead the market will assume there were profits, but that the firm did not recognize these profits for some reason. In this study, we construct a model of accounting systems that are chosen ex ante and that are well defined and relatively simplistic in their structure in order to focus on the conservatism versus liberalism trade-off.

Debt contracting has been used most often as the reasons for demanding conservatism. Debt holders will trigger default if performance is bad and call the loan (Beneish and Press 1993). Further, debt holders will attempt to restrict managerial actions so as to increase the value of assets; in this attempt, conservative accounting rules will be enforced. This shareholders - bondholders conflict is shown by Ahmed et al. (2001) to lead to conservatism as an efficient contracting mechanism

If information acquisition and disclosure costs, including disposal costs, are zero, DM's will prefer to have all the information. In reviewing the development of theory relating to conservatism, Watts (2003a and 2003b) points out the importance of costs. 'If information is free and there are no agency costs, then there is no role for accountants or financial reports. Accounting and reporting exist because of such costs.' This paper's model shows that if full information setup is costly then DM's will, in some cases, prefer a conservative accounting system over both a liberal system as well as over a system of full recognition.

### 2.2 Agency theory based models of conservatism

Kwon et al. (2001) use limited liability arguments to motivate conservatism. With limited penalties in a principal agent setting, they show that conservative reporting is more efficient in motivating agents. In a related paper Kwon (2005) shows that the principal can implement better effort choices by the manager with conservatism in reporting.

Gigler and Hemmer (2001) show that more conservatism lowers levels of the risk - sharing benefits derived from timely disclosure. In this paper we only show that conservatism in accounting is more valuable than liberalism; conservatism is definitely less valuable than full recognition with perfect signals, if they are of same cost. However, since conservatism in our model results in fewer signals, we show that conservatism may be more valuable than a system that provides full recognition of perfect signals if they have the same cost per signal.

Bagnoli and Watts (2005) use managerial discretion in choosing conservatism as a signal of firm value. The market can use management's reporting policy choice to infer management's private information. However, their model differs from ours in how they construct the accounting systems. Our model does not allow for bias in reporting, as our accounting system is based on a coarsening of the underlying state space, while their system introduces noise into the signals depending on whether the
good or bad signal is reported. ${ }^{7}$ While we conjecture that our results may extend to their definition of conservatism, this remains an open question.

Raith (2009) models a two period principal - agent model and shows that the manager will be paid in the first period at a 'conservative' (i.e., lower) rate for the first period outcome. Though the model setup and definition of conservatism are quite different from this paper, Raith (2009) is similar to our paper as it has the element of saving and consumption over two periods, albeit with exponential utility functions. However Raith conflates the formal notions of prudence and conservatism in stating 'The theory supports the intuition that conservatism as prudence is a response to (symmetric) uncertainty about future cash flows'. In this paper we keep them separate; prudence is characteristic of DM behavior, while conservatism is a property of accounting systems. Conservatism is a solution for a prudent DM's choice of an accounting system.

### 2.3 Debt contracting based models of conservatism

In a debt contracts setting, Gigler et. al. (2007) show that conservatism in accounting is less efficient than the alternatives. They define conservatism through probability of reporting a high report when the true economic cash flow is low. Wrong liquidation decisions that cause inefficiency are more likely with conservatism in their model. In this study of the accounting information, we consider a model that has no conservative bias, but one that does have different levels of accuracy for good versus bad outcomes. Li, Ningzhong (2008) also uses debt - contracting to analyze accounting conservatism as asymmetric timeliness in recognition of losses and gains for unverifiable information. Li, Jing (2008) looks at the impact of accounting conservatism on the efficiency of debt contracts and shows that, with high costs of renegotiation, conservatism decreases efficiency.

Chen and Deng (2010) add the choice level of conservatism as a signaling device. Better firms with lower value of high outcomes can use conservatism as a signal to mitigate cross-subsidization costs as a substitute for debt covenants. They show that conservative accounting sometimes may emerge as a signaling device even if contracting efficiency is not increased.

[^3]Guay and Verrecchia (2007) model a potentially informed manager in a setting where the market is not sure whether the manager is informed. As is often the case, the manager in this setting discloses only high firm values of private information. They define conservatism as a reporting system where low firm values are reported at their actual realization, whereas high firm values are reported 'conservatively', close to the definition of conservatism used in this paper. They show conservatism forces all informed firms to release the information and substitutes for commitment.

Gox and Wagenhofer (2009) study the optimal accounting policy choice of a financial constrained firm that pledges collateral to raise debt capital. In their model, the accounting system generates information about the value of the assets pledged as collateral. They find that, absent regulation, the optimal accounting policy is conditionally conservative. Conditionally conservative means that the system reports asset values if the asset is impaired and the value is below a specified threshold, but does not report unrealized gains in asset value.

### 2.4 Non-agency, non-debt models of conservatism

The group of models of conditional conservatism that do not use agency theory and do not rely on debt contracting to generate demand for conservative accounting is sparsely populated; other than our model, we found only Suijs (2008) in this group. Suijs (2008) considers an overlapping generations model of risk averse investors trading in a risky stock market where risk is partly determined by the volatility of stock price set by current shareholders selling to the new generation of shareholders. It is shown that asymmetric reporting of good and bad financial information has different value implications as it affects how risk is allocated among the future generation of shareholders. The main result of the model is that, when the discount rate is not too high, an accounting system that reports bad news more precisely than good news will more efficiently share risk among generations of shareholders and hence, firm value will be higher under this accounting system.

By not relying on debt or contracting based arguments, we believe that our model of conservatism offers valuable contributions to the literature for a few reasons. First, they offer additional arguments that complement the debt and contracting based models that show demand for conditionally conservative accounting policies. Second, our model suggests potentially testable empirical hypotheses that differ from the hypotheses suggested by the debt and contracting based models. Third, by showing that the demand for conditionally conservative financial reporting policies may
arise due to investor preferences, we show that the demand for conservtism does not depend on the DM's having a specific contractual relation. Regulators seem to believe that unbiased accounting is preferred for most decision making unless contracting aspects complicate the issues. Our results show that conservative accounting may be preferred due simply to the preferences of a broadly defined segment of the investing population; contracting aspects do not play a role in this preference. This suggests that regulators may wish to reconsider their emphasis on promoting unbiased or neutral accounting and replace this emphasis with a more open-minded perspective that considers the value of other, especially conservative, accounting policies.

One final point related to our model concerns the claim that conservative accounting is always biased. Bias is defined as being the opposite of neutral, i.e., reporting that is 'slanted, weighted, emphasized, deemphasized, or otherwise manipulated to increase the probability that financial information will be received favorably or unfavorably by users. ${ }^{8}$ The conservative and liberal accounting systems in this paper do not slant, weight, emphasize, deemphasize or manipulate the reported outcomes, rather they recognize gains and losses differentially depending on the level of assurance that the outcome will be realized. As we describe in more detail in the following sections, especially in section 3.3 and 3.4 , conservative accounting recognizes losses because these outcomes are sufficiently assured to occur while deferring the recognition of gains because the gain outcomes lack sufficient assurance. While some instances of conservative accounting may be biased, we do not believe our model of conservative (or liberal) accounting meet the preceding definition of being biased.

Next, in the final subsection of this section, we discuss the empirical literature related to both conservatism and prudence, focusing on how our empirical research may distinguish demand for conservatism based on the competing models.

### 2.5 Related empirical research

We will show that, under the model presented in the next section, prudent DM's prefer conditionally conservative accounting methods. By showing that prudence drives the demand for conservatism, our model produces potentially testable empirical hypotheses that cannot and will not arise from any other existing theoretical model. Hence, our model suggests a natural test of the validity and usefulness of our results in advancing our understanding of the practical phenomonon affecting the demand for conservative accounting policies in the everyday business environment. We make

[^4]this argument via a three step approach.
First, we review the current empirical literature relating to the debt and agency models. Second, we identify potentally testable empirical hypotheses within the exisitng empirical designs that will distinguish our model from the competing models. In this second step, we also identify some of the empirical difficulties that we expect would need to be addressed to design empirical tests to assess whether the prudence model or the debt or agency models better explain the demand for conservative accounting policies. Third, we briefly review the extensive literature in economics relating to prudence, explaining how results in this literature may influence the task of designing empirical tests to address the issues described in step 2.

Building on the survey articles Ball (2003a and 2003b) mentioned earlier, there has been numerous empirical studies on conservatism. These studies investigate how conservatism relates to accrual accounting, earnings management, or accounting choice, but these studies have not provided conclusive evidence that either debt or agency based models can be ruled out as justification for the prevalence of conditonally conservative accounting policies. For example, Ball and Shivakumar (2006) investigate the role of accrual accounting in the timely recognition of losses and find that recognizing gains and losses in a timely fashion is major role of accrual accounting. ${ }^{9}$ While Ball and Shivakumar (2006) focus primarily on how conservatism affects the relation of accrual accounting to stock prices, their results also suggest connections between conservatism and earnings management. In a more recent study, Jackson and Liu (2010) study the role of conservatism on earnings management directly via the allowance for doubtful accounts and bad debt expense. They find that the allowance is conservative, has become more conservative over time, but that conservatism may facilitate and even accentuate the extent of earnings management. Moving to accounting choice, Gormley, Kim and Martin (2012) investigate the impact of changes in the banking industry to adoption of conservative accounting policy. They find that entry by foreign firms into the banking market in India in the 1990's is associated with more timely loss recognition. Also, they find that this positive association is positively related to a firm's subsequent debt levels, indicating support for the models of conservatism based debt related arguments. In another international study, Ball, Robin and Sadka (2008) find that size of debt amrkets, but not equity markets, is associated with higher levels of timely recognition and conservatism, indicating additional support for debt based models.

[^5]While providing valuable insight into the relation of conservatism to other aspects of accounting choice and managerial behavior, this work cannot be said to definitively distinguish between debt and agency based models. While some of the empirical studies provide evidence of a link between debt and conservatism, these studies do not explicitly rule out a role for the agency based models. Some of the studies, for example, those on earnings quality, may actually be interpreted as support for the agency-based theory. So one difficulty that needs addressing is to design addition empirical to measure the relative importance of debt versus agency based arguments in driving the demand for conservatism. Introducing our model that shows demand for conservatism follows from prudence seems to complicate the design task even further, but this may not be the case. For example, if we can identify a positive association between prudence and conservatism that does not require debt or agency conflict, then this would provide support for the prudence model. Further, such evidence might be consistent with other arguments, so that it may be that demand for conservatism may increase for debt and agency reasons, in addition to the demand generated by the prudence of DM's. The critical design task here then is to measure both conservatism and prudence; while the accounting literature has been busy addressing the first measurement task, the economics literature has been busy on the second task.

As we discuss in more detail in the next section, we can trace the origins of prudence back to the precautionary savings problem of Leland (1968); the empirical studies related to this concept date even earlier. ${ }^{10}$ More recently, macroeconomic studies including Lee and Sawada (2008), Gollier (2008), Carroll (2009) and Menegatti (2010) have studied the prevalence and importance of the prudence of investors on savings and consumption. Lee and Sawada (2008) update the standard analysis of prudence testing from the 1990's to show that prudence may be more importantly empirical than earlier assessed. ${ }^{11}$ Gollier (2008) relaxes the assumption of serially independent discount rates and shows that, with prudent investors, shocks on aggregate consumption may result in decreasing term structure for discount rates. Carroll (2009) inves-

[^6]tigates the marginal propensity to consume permanent (MPCP) consumption shocks. He first observes that in the simplest case, MPCP should equal one; however, it has been empirically estimated to be less than one. Carroll argues that with both permanent and temporary shocks, the presence of prudent investors will drive the MPCP below one as these investors seek to maintain a wealth-to-permanent-income ratio instead an optimal consumption level. Menegatti (2010) analyses evidence about prudence in OECD countries in the period 1955-200 using an alternative measure of uncertainty and finds the evidence supports the presence of prudence, i.e., that a greater degree of uncertainty increases savings. These studies show first, that a basic approach to measuring prudence currently exists in the literature and second, that various branches to this research are being developed, so that measuring prudence has a strong empirical foundation.

From the perspective of the current research topic, the studies outlined in the preceding paragraph demonstrate the feasiblity of measuring prudence. The empirical work on conservatism demonstrates that we can measure conservatism, even though we may not be able to always distinguish the differentially impact of debt and agency considerations on the level of conservatism demanded. What remains an open question is how to couple the empirical testing of prudence with that of conservatism. While distinguishing among the models will be difficult, it seems designing an empirical approach to assess the relation between prudence and conservatism is, at first glance, a feasible task.

## 3 Prior Results, Definitions and Model Development

We make our main argument using two facts and one result. First, we argue that income, gains and losses are recognized only when the information is sufficiently precise, so that we are assured the outcome will be realized. Second, we argue that conservative accounting recognizes lower income outcomes sooner while deferring the recognition of higher income levels and that liberal accounting does the opposite. Third, we will show that prudent investors prefer information systems that provide more precise information for lower levels of income over information systems that provide more precise information for higher levels of income. Combining these two facts with our result, we argue that this shows that prudent investors prefer conservative accounting.

While we want to analyze how accounting information systems differ by their relative conservatism and show that prudent DM's prefer conservative accounting
systems to liberal ones, first we need to introduce some preliminaries. In the first subsection, we provide background notation and basic definitions. In the second subsection we summarize the precautionary savings problem and review some prior results related to this problem. In the third subsection, we introduce the precautionary savings problem with information, which includes developing the model of different accounting information systems.

### 3.1 Background notation, definitions and example

We start by introducing some basic notation and background definitions and restate some important results upon which we intend to build. This model is based on the model of prudence of Kimball (1990), extended to include information systems, so we start first with the model excluding the information systems.

Let $v_{j}(x)$ be a utility function, defined over the consumption variable $x \in X$, for an expected utility maximizing decision - maker $j=1,2 .{ }^{12}$ We assume that both DM's are risk averse in the sense of $\operatorname{Pratt}$ (1964), where $\mathrm{DM} j=1$ is more risk averse than a 2 nd $\mathrm{DM}, j=2$, if for every risk, his (the 1 st DM's) cash equivalent (the amount for which he would exchange the risk) is smaller than her (the 2nd DM's) cash equivalent. The cash equivalent amount that will leave the DM as well off after imposing the risk as he or she was before is called the risk premium. As Pratt (1964) shows, this sense of risk aversion can be expressed formally as follows.

D1: Definition of risk aversion: $\mathbf{D M} j=1$ is more risk averse than $\mathbf{D M} j=2$ if there exists a monotonically increasing and concave function, $g(\cdot)$, where $v_{2}(x)=$ $g\left(v_{1}(x)\right)$ holds for all $x \in X$.

The Arrow - Pratt measure of risk aversion, defined as negative of the ratio of the second and first derivatives of the utility function, was then shown to measure levels of risk aversion. More specifically, the following proposition was shown to hold.

R1: Prior result 1, Pratt (1964) ${ }^{13}$ : Define the absolute risk aversion measure (also known as the Arrow - Pratt measure of absolute risk aversion) as follows:

[^7]\[

$$
\begin{equation*}
h_{j}(x) \equiv-\frac{\partial^{2} v_{j}(x) / \partial x^{2}}{\partial v_{j}(x) / \partial x}=-\frac{v^{\prime \prime}{ }_{j}(x)}{v_{j}^{\prime}(x)} \text { for } j=1,2 . \tag{1}
\end{equation*}
$$

\]

Then $\mathrm{DM} j=1$ is more risk averse than $\mathrm{DM} j=2$ if and only if $h_{1}(x)>h_{2}(x)$ holds for all $x \in X$.

These definitions continue to form the basis for our understanding of risk aversion with $h_{j}(x)$ and $x h_{j}(x)$ being referred to as DM $j^{\prime} s$ measure of absolute risk aversion and relative risk aversion, respectively. ${ }^{14}$

In addition to studying risk aversion, precautionary saving in response to risk has been studied. Precautionary savings represent the additional savings required by a utility maximizing agent if their future income is random instead of being known. ${ }^{15}$ More generally, the focus is to study a DM's reaction to a choice or control variable that affects the utility; we can characterize the interaction of the choice variable and the utility function and its effect on the DM's behavior in a manner analogous to how we measure risk aversion. The study of risk aversion began by using the notion of an amount, called the risk premium, that left the DM as well off after imposing the risk as the DM was without the additional risk. In the same way, we use the precautionary saving amount to measure the cost to the DM of additional risk. Continuing the analogy, we see that the precautionary savings chosen in response to additional risk is related to the convexity of the marginal utility function, or a positive third derivative of the expected utility function.

The general model starts by using the general framework for choice under uncertainty due to Rothschild - Stiglitz (1971). Assume a DM's utility can be represented by a function of two variables, a choice variable $\delta$ and an exogenous random variable $\theta$, so that the DM chooses to maximize expected utility $E V(\theta, \delta)$. More explicitly, the DM's problem is based on the following optimization situation.

$$
\begin{equation*}
\max _{\delta} E[V(\theta, \delta)] \text { using the first - order condition (FOC), } E[\partial V / \partial \delta]=0 \tag{2}
\end{equation*}
$$

We will refer to the function, $V(\theta, \delta)$, as the indirect utility function of a DM. Assuming that $E[\partial V / \partial \delta]=0$ is convex in $\theta$, then increases in the variability of $\theta$ will result in increases in the optimal choice of $\delta$. Briefly, just as the concavity of the utility function can be used to measure risk aversion, convexity of the FOC can be used to measure

[^8]the optimal response of choice variables to risk.
The next step in the general model of the theory of the optimal response of choice variables to risk is to get a measure, analogous to the Arrow - Pratt measure of risk aversion, which measures the sensitivity of the optimal choice variable to risk. Such a measure, called a measure of 'prudence', was offered in Kimball (1990). Prudence is described as the propensity to prepare and forearm oneself in the face of uncertainty. In that article it was shown that the cross - partial derivative, $\partial^{2} V(\theta, \delta) / \partial \theta \partial \delta$, is the key building block in the construction of the prudence measure. More specifically, if the cross - partial derivative function, $\partial^{2} V(\theta, \delta) / \partial \theta \partial \delta$, is uniformly positive or uniformly negative, then define the measure $\eta(\theta, \delta)$ as follows.

D2: Definition of the measure of absolute prudence : Define the absolute prudence measures as follows:

$$
\eta(\theta, \delta) \equiv-\frac{\partial^{3} V(\theta, \delta) / \partial \theta^{2} \partial \delta}{\partial^{2} V(\theta, \delta) / \partial \theta \partial \delta}
$$

We call $\eta(\theta, \delta)$ the measure of absolute prudence of the DM. We will refer to an investor for whom $\eta(\theta, \delta)>0, \eta(\theta, \delta)<0$, and $\eta(\theta, \delta)=0$ as a prudent investor, as an imprudent investor and as a prudent-neutral investor, respectively. Absolute prudence is a good measure of the sensitivity of the optimal choice variable to risk for similar reasons that the Arrow - Pratt measure is a good measure of absolute risk aversion. ${ }^{16}$ More specifically, if one DM's FOC is a concave or convex transformation of another DM's FOC, then we can characterize how the two DM's differ in their degree of sensitivity of the optimal choice variable to risk.

### 3.2 The precautionary savings problem and basic results

The final step in the presentation of the background material involves introducing the concrete problem which forms the heart of the initial analysis of prudence, which is the precautionary savings problem. As mentioned earlier, our model builds on the model of precautionary savings of Kimball (1990), which we then extend to include information systems, so the following is simply a summary of some of the analysis in that paper.

Consider a two period model of an expected utility maximizing DM facing a de-

[^9]cision about how much of his wealth to consume in period one. The DM has total beginning wealth of $w_{0}$ that is known at date 0 (beginning of period 1) and a random labor income of $\tilde{y}$ received at date 2(i.e., at the end of period 2). The DM must choose how much to consume at dates 1 and 2 (end of periods 1 and 2 ). The amount the DM consumes at date 1 is denoted as $c$ while he consumes the remainder of his wealth, $w_{0}-c+\tilde{y}$, at date 2 . More specifically, the DM faces the following optimization problem. ${ }^{17}$
\[

$$
\begin{equation*}
\max _{c}\left\{v(c)+E\left[v\left(w_{0}-c+\tilde{y}\right)\right]\right\} \tag{3}
\end{equation*}
$$

\]

Next, we impose additional assumptions to simplify the presentation, analogous to Kimball (1990).

Assume that the random 2nd period income is the sum of a noise variable, $\tilde{\varepsilon}$, plus a known constant, $\bar{y}$, so that $\tilde{y}=\bar{y}+\tilde{\varepsilon}$. Also, we introduce a constant, $w=w_{0}+\bar{y}$, to denote the known portion of the DM's wealth. The introduction of this constant allows us to rewrite the DM's optimization problem and FOC as follows.

$$
\begin{gather*}
\max _{c}\{v(c)+E[v(w-c+\tilde{\varepsilon})]\}  \tag{4}\\
\partial v(c) / \partial c=\frac{E[\partial v(w-c+\tilde{\varepsilon}]}{\partial c} \tag{5}
\end{gather*}
$$

As has been noted in previous work, it is clear that the uncertainty in the 2nd period income affects the choice of the 1st period consumption only insofar as it affects 2 nd period expected marginal utility. Also, the focus shifts to the savings variable, which is defined as $s=w-c$.

Next, to put the precautionary savings problem in the general Rothschild - Stiglitz framework used to introduce the prudence measure earlier, rewrite the DM's objective function as follows.

$$
\begin{equation*}
V(w+\tilde{\varepsilon}, c)=v(c)+E[v(w-c+\tilde{\varepsilon})] \tag{6}
\end{equation*}
$$

Here the consumption variable $c$ is the decision variable while the sum $w+\tilde{\varepsilon}$ is the uncertain or random variable. Using this objective, we can rewrite the analysis leading to the prudence measure shown earlier. We replace $E[\partial V / \partial \delta]=0$ with the DM's FOC, which we write as $V^{\prime}=v^{\prime}-E\left[v^{\prime}\right]=0$, replace $\partial^{2} V / \partial \delta \partial \theta$ with $-v^{\prime}$, which is always positive for risk averse DM's, and replace $\partial^{3} V / \partial \delta \partial \theta^{2}$ with $v^{\prime \prime}$. Since $v^{\prime}$ is constant for each fixed value of the decision variable, and since $\partial v(w-c+\tilde{\varepsilon}) / \partial c$ is a function of decision variable $c$ only through $s=w-c$, we can rewrite the prudence measure in terms of savings as follows.

[^10]\[

$$
\begin{equation*}
\eta(w, c)=\eta(s) \equiv \frac{\partial^{3} v(s) / \partial s^{3}}{\partial^{2} v(s) / \partial s^{2}} \tag{7}
\end{equation*}
$$

\]

To summarize, let $h(s) \equiv-\left(v^{\prime \prime}(s) / v^{\prime}(s)\right.$ and $\eta(s) \equiv-\left(v^{\prime \prime \prime}(s) / v^{\prime \prime}(s)\right.$ denote the risk aversion and prudence measures, respectively. This facilitates a comparison of prudence and risk aversion as we describe next.

Kimball (1990), summarizing Dreze and Modiglian (1972), showed that the relative values of the prudence and risk aversion measures can be characterized in terms of the first derivative of the risk aversion measure. More specifically, we have the following general result.

R2: Prior result 2, Kimball (1990) ${ }^{18}$ : Assuming the DM is strictly risk - averse with a positive absolute risk aversion measure, then the prudence measure is greater than, equal to, or less than the risk aversion measure as the absolute risk aversion is decreasing, constant or increasing (abbreviated as DARA, CARA and IARA, respectively), i.e., the following are true:

1. If $h^{\prime}(s)<0$ holds for all $s$, (DARA), then $\eta(s)>h(s)$ [Eq. 6.a]
2. If $h^{\prime}(s)=0$ holds for all $s$, (CARA), then $\eta(s)=h(s)$ [Eq. 6.b]
3. If $h^{\prime}(s)>0$ holds for all $s$, (IARA), then $\eta(s)<h(s)$ [Eq. 6.c]

The case of DARA is often seen as the most important case, and in this case, the measure of prudence exceeds the measure of risk aversion, which is positive. Alternatively, DARA implies the negative of the derivative of the utility function is more risk averse than the utility function, or $-v^{\prime}(s)$ is more risk averse than $v(s)$.

The preceding provides the background and foundation for our analysis. Next we introduce some additional notation and definitions that pertain to the concept of conservatism as understood in accounting.

### 3.3 Notation and definitions for conservatism

In our model, an accounting information system involves the receipt of a signal by the DM which will be used by the DM in his or her decision making. The signal

[^11]provides information about the arguments in the indirect utility function and the DM uses this information when making his or her decision. We wish to focus on the relative demand for accounting information systems, where accounting systems may be defined as either conservative or liberal.

While there are many ways to describe conditional conservatism, recognizing losses while deferring the recognition of gains is included in virtually every description of the concept. In our definition of a conservative accounting system, we formalize the notion of 'recognition' by assuming the signal is perfect. We distinguish between conservative and liberal accounting systems by assuming recognition occurs for the conservative system at lower levels of income and for the liberal system at higher levels. More specifically, we assume the conservative system generates perfect signals for lower levels of second period income in the precautionary savings problem, but imperfect signals at higher levels. The liberal system does the reverse. The probability distribution and the earnings levels are chosen to ensure symmetry in the two systems except for the distinction that we make to distinguish between conservative and liberal systems. ${ }^{19}$ We explain our assumptions about the payoffs and the signals received from the information systems in more detail next.

We assume that the DM has zero initial income, but can borrow costlessly. The DM consumes at both dates one and two, as in the precautionary savings model described above. The uncertain second period earnings, denoted as $w \in W$, takes one of four equally likely values, so that $w_{n} \in W=\left\{w_{1}, w_{2}, w_{3}, w_{4}\right\}$, and $\left\{w_{1}, w_{2}, w_{3}, w_{4}\right\}=$ $\{2 U+\delta, 2 U-\delta, 2 L-\delta, 2 L-\delta\}$, where $U-L>\delta>0$. The evolution of earnings is shown in Figure 1.

Figure 1: Evolution of Earnings

$$
t=1
$$

$$
\begin{aligned}
& w_{1}=2 U+\delta \\
& w_{2}=2 U-\delta \\
& w_{3}=2 L+\delta \\
& w_{4}=2 L-\delta \\
& t=2
\end{aligned}
$$

[^12]As Figure 1 suggests, earnings results can be grouped into good and bad outcomes, where good earnings are represented by the set $G=\left\{w_{1}, w_{2}\right\}=\{2 U+\delta, 2 U-\delta\}$ and bad earnings are represented by the set $B=\left\{w_{3}, w_{4},\right\}=\{2 L-\delta, 2 L-\delta\}$.

The DM must choose how much to consume at dates 1 and 2 , (end of periods 1 and 2). The DM has zero total beginning wealth and must decide how much of the random earnings of $w \in W$ received at date 2 (i.e., at the end of period 2) to borrow and consume at date 1 . (Without loss of generalization, we set the borrowing rate at zero.) The amount the DM consumes at date 1 is denoted as $c$ while he consumes the remainder of his wealth, $w_{n}-c$, at date 2 . Before choosing his consumption at date 1, the DM observes a signal from one of four possible accounting information systems denoted as $i=$ Full, Part, Csv, Lib for full, partial, conservative and liberal accounting, respectively. The Full system generates perfect signals, allowing full recognition, while the Partial system generates imperfect signals, resulting in recognition based on expected values. The conservative and liberal accounting systems generate a mixed set of perfect and imperfect signals, as discussed more fully below. The signal for each system is denoted as $z_{n}^{i} \in Z^{i}$, where the superscript is used to indicate the accounting information system, the subscript indexes the signal and the lower case (or upper case) indicates the realized signal (set of potential signals). The set of signals in each system is discussed next, beginning with the full recognition system.

For the full recognition system, the signal is perfect so the signal set has four possible signals and is denoted as $Z^{\text {Full }}=\left\{z_{1}^{\text {Full }}, z_{2}^{\text {Full }}, z_{3}^{\text {Full }}, z_{4}^{\text {Full }}\right\}=\left\{w_{1}, w_{2}, w_{3}, w_{4}\right\}$. The recognition of earnings under the full recognition accounting system is shown in Figure 2.

## Figure 2: Full Recognition of Earnings



Figure 2 shows that full recognition accounting describes a situation where a perfect signal is generated. Since the second period income is known with certainty at date 1 , the DM will choose to consume exactly half the income. This means that the DM optimally consumes $U+0.5 \delta, U-0.5 \delta, L+0.5 \delta$, or $L-0.5 \delta$, in each period, as the DM sees the signal $z_{1}^{\text {Full }}, z_{2}^{\text {Full }}, z_{3}^{\text {Full }}$ or $z_{4}^{\text {Full }}$, respectively. Since there is no uncertainty, the DM chooses a precautionary savings level equal to zero. Next we turn to the situation with imperfect signals of earnings.

Throughout the paper we assume that the DM always has a partial information system, where one of two possible signals is observed. The signal set is denoted as $Z^{\text {Part }}=\left\{z_{1}^{\text {Part }}, z_{2}^{\text {Part }}\right\}=\{G, B\}$ and the earnings information under the partial accounting system is shown in Figure 3.

## Figure 3: Partial Information about Earnings



Figure 3 describes a situation where imperfect signals are generated for good and for bad news. Even if good earnings are observed, there is still some uncertainty about earnings and so the accounting system may not reccognize any earnings in the financial statements. Even if this imprecise information is disclosed, the DM's utility will be lower then under the perfect signal of Figure 2, as the optimal consumptionsavings decisions will not be chosen. If $G$ is disclosed, each period's consumption on average should be $U$, but some savings (i.e., the amount that consumption is below $U$ in the first period), may occur. This is the precautionary savings when the signal $G$ is reported as $s_{U}$. The DM consumes $c_{G}=U-s_{U}$ in period one and, in the second period, he or she consumes whatever is generated in the second period, less the first period consumption. But the expected consumption in the second period is always greater than the consumption in the first period as savings is positive.

We now allow for the possibility that the DM can force the accounting system to identify the exact value of the second period earnings in the first period conditional on whether 'Good' or 'Bad' earnings signal was observed in the first period. Finding out the exact value and recognizing one signal, either good or bad news, in the first period is always optimal. We first focus on the choice between two less than full recognition systems, which recognize only either good or bad news.

In the conservative accounting system, the signal set has three possible signals, and is denoted as $Z^{C s v}=\left\{z_{1}^{C s v}, z_{2}^{C s v}, z_{3}^{C s v}\right\}=\left\{G, w_{3}, w_{4},\right\}$. The disclosure of earnings under the conservative accounting system is shown in Figure 4.

## Figure 4: Conservative Accounting of Earnings



Figure 4 shows that, under conservative accounting, the system generates the imperfect signal $G=\left\{w_{1}, w_{2}\right\}=\{2 U+\delta, 2 U-\delta\}$ if the earnings are good but generates the perfect signals of either $z_{2}^{C s v}=w_{3}$ or $z_{3}^{C s v}=w_{4}$ if the earnings are bad. So bad earnings are recognized in the first period, which means they are recognized earlier than good earnings. If the earnings are good, only in the second period will the exact amount be identified and recognized. ${ }^{20}$ We have designed the model so that no savings occur for perfect signals, but they may occur for imperfect ones, as we next discuss.

The total expected utility at date 1 given $G$ is given as follows.

$$
\begin{equation*}
E[v \mid G]=v\left(U-s_{U}\right)+0.5 v\left(U+\delta+s_{U}\right)+0.5 v\left(U-\delta+s_{U}\right) \tag{8}
\end{equation*}
$$

[^13]The FOC after differentiating with respect to savings is given as follows.

$$
\begin{equation*}
\frac{\partial E[v \mid G]}{\partial s_{U}}=-v^{\prime}\left(U-s_{U}\right)+0.5 v^{\prime}\left(U+\delta+s_{U}\right)+0.5 v^{\prime}\left(U-\delta+s_{U}\right)=0 \tag{9}
\end{equation*}
$$

The optimal savings when signal $G$ is observed will be chosen to solve equation (??). If either of the bad earnings, $2 L+\delta$ or $2 L-\delta$, are disclosed, consumption will be half the total earnings to be realized in period 2, so $c=L+0.5 \delta$ or $c=L-0.5 x \delta$, depending on which signal is observed. Hence, under conservative accounting disclosure, the total ex - ante expected utility at date 0 is given as follows.

$$
\begin{align*}
E\left[v \mid Z^{C s v}\right] & =0.5\left[v\left(U-s_{U}\right)+0.5 v\left(U+\delta+s_{U}\right)+0.5 v\left(U-\delta+s_{U}\right)\right] \\
& +0.5[v(L-0.5 \delta)+v(L+0.5 \delta)] \tag{10}
\end{align*}
$$

Next we turn to the liberal accounting information system. The liberal accounting information system is defined in a manner exactly analogous to the conservative system, except now the recgnition with perfect information occurs only when good earnings are reported. Hence, under liberal accounting, the signal set again has three possible signals, and is denoted as $Z^{L i b}=\left\{z_{1}^{L i b}, z_{2}^{L i b}, z_{3}^{L i b}\right\}=\left\{w_{1}, w_{2}, B\right\}$. The disclosure of earnings under the conservative accounting system is shown in Figure 5.

## Figure 5: Liberal Accounting of Earnings



In liberal accounting, just as under conservative accounting, the signal set includes both perfect and imperfect signals, but now perfect signals are generated for good news and imperfect signals are generated for bad news. As with the perfect signals under the conservative system, it is again the case that no savings occur for perfect
signals under the liberal system, but they may occur for imperfect ones, as we next discuss.

In the first period, if $B=\left\{w_{3}, w_{4}\right\}$ is disclosed, the actual earnings in the second period are either $2 L+\delta$ or $2 L-\delta$ with equal probability. Hence, the expected earnings and total consumption amounts are $2 L$, and the average consumption in each period is $L$. The total expected utility at date 1 given $B$ is given as follows.

$$
\begin{equation*}
E[v \mid B]=v\left(L-s_{L}\right)+0.5 v\left(L+\delta+s_{L}\right)+0.5 v\left(L-\delta+s_{L}\right) \tag{11}
\end{equation*}
$$

The FOC after differentiating with respect to savings is given as follows.

$$
\begin{equation*}
\frac{\partial E[v \mid B]}{\partial s_{L}}=-v^{\prime}\left(L-s_{L}\right)+0.5 v^{\prime}\left(L+\delta+s_{L}\right)+0,5 v^{\prime}\left(L-\delta+s_{L}\right)=0 \tag{12}
\end{equation*}
$$

The optimal savings ${ }^{21}$ when signal $B$ is observed will be chosen to solve equation (??). If either of the good earnings, $2 U+\delta$ or $2 U-\delta$, is disclosed, consumption will be half the total earnings to be realized in period 2 , so $c=U+0.5 x \delta$ or $c=U-0.5 x \delta$, depending on which signal is observed. Hence, under liberal accounting, the total ex - ante expected utility at date 0 is given as follows.

$$
\begin{align*}
E\left[v \mid Z^{L i b}\right] & =0.5[0.5 v(U-0.5 \delta)+0.5 v(U+0.5 \delta)] \\
& +0.5\left[v\left(L-s_{L}\right)+0.5 v\left(L+\delta+s_{L}\right)+0.5 v\left(L-\delta+s_{L}\right)\right] \tag{13}
\end{align*}
$$

Finally, we have the formal definition of accounting information systems.
D3 - Definition of accounting systems: Define the accounting systems $Z^{i}$ for $i=$ Full, Part, Csv, Lib, in the following manner:

| Full | $Z^{\text {Full }}=$ | $\left.\left[z_{1}^{\text {Full }}, z_{2}^{\text {Full }}, z_{3}^{\text {Full }}\right], z_{4}^{\text {Full }}\right]$ | $=\left[w_{1}, w_{2}, w_{3}, w_{4}\right]$ |
| :--- | :--- | :--- | :--- |
| Partial | $Z^{\text {Part }}=$ | $\left[z_{1}^{\text {Part }}, z_{2}^{\text {Part }}\right]$ | $=[G, B]$. |
| Conservative | $Z^{\text {Csv }}=$ | $\left[z_{1}^{\text {Csv }}, z_{2}^{\text {Csv }}, z_{3}^{\text {Csv }}\right]$ | $=\left[G, w_{3}, w_{4}\right]$. |
| Liberal | $Z^{\text {Lib }}=$ | $\left[z_{1}^{\text {Lib }}, z_{2}^{\text {Lib }}, z_{3}^{\text {Lib }}\right]$ | $=\left[w_{1}, w_{2}, B\right]$. |

This clarifies the symmetry between the conservative and liberal accounting systems; next, we provide an example to demonstrate how each accounting system would affect financial reporting. We use this example to show how our model could be interpreted to approximate conditionally conservative accounting methods found in practice.

[^14]
### 3.4 Example of conservative financial reporting

In our model, the DM is both manager and investor; we do not build an explicit financial reporting framework. By having a single agent player both observing the information and making the decision, we ignore any tension that may exist between the player deciding what to report in the financial reports and how the player reading these reports interprets them. While this modeling choice simplifies the analysis, it does little to illuminate how the accounting systems relate to accounting policy in the real world. To clarify this connection, we provide a simple, yet concrete, example of financial reporting to demonstrate that our conservative accounting information resembles what is usually interpreted as conservatism. We use inventory profitability to show that accounting information system $Z^{C s v}$ produces financial reports that resemble accounting under a lower of cost or market regime.

First, we assume that the outcomes represent the net profit from selling inventory. Let $U=10, L=-10$ and $\delta=5$, so that the outcomes are calculated as follows: $\left[w_{1}, w_{2}, w_{3}, w_{4}\right]=[25,15,-15,-25]$. We assume that recognition occurs, in general, only if an outcome is more likely than not to occur. We operationalize the term "more likely than not" as an outcome whose probability, after observing the signal, is strictly greater than fifty percent. Hence, by construction in our model, recognition occurs only when a perfect signal is observed. Also by construction, when an imperfect signal is observed, there is no recognition at the interim date.

For example, under the information system $Z^{\text {Full }}$, we have income or loss being recognized under each of the signals that might be reported. If signal $z_{1}^{\text {Full }}$ is observed, then we know that outcome $w_{1}=25$ will be realized, hence, we recognize an additional 25 dollars in the income statement at the interim date. Similarly, if signal signal $z_{2}^{\text {Full }}$ is observed, then we know that outcome $w_{2}=15$ will be realized so reported income will increase by 15 dollars, while if either signal $z_{3}^{\text {Full }}$ or $z_{4}^{\text {Full }}$ is observed, then we know that a loss of either 15 or 25 , respectively, will occur and these losses are recognized. While some income or loss is recognized for each signal that may be observed under the $Z^{\text {Full }}$ information system, no recognition occurs under the $Z^{\text {Part }}$ accounting information system. A disclosure may or may not be made, perhaps depending on whether signal $z_{1}^{\text {Part }}$ or signal $z_{2}^{\text {Part }}$ is observed, but no income or loss is recognized and reported on the interim income statement. Perhaps more interesting are the cases when either accounting system $Z^{C s v}$ or accounting system $Z^{L i b}$ is used.

Under the conservative accounting system, if signal $z_{1}^{C s v}$ is observed, then we know either income of 25 dollars or 15 dollars will be realized in the future. However,
we do not know which of these outcomes will occur (both are equally likely), and since the outcome is not known with sufficient confidence, no income is recognized. Instead, the recognition is deferred to a later date. On the other hand, if either signal $z_{2}^{C s v}$ or signal $z_{3}^{C s v}$ is observed, then the manager knows that a loss of 15 dollars or 25 dollars, respectively, will be realized. Since the manager is sufficiently confident that these losses will be realized in the future, he recognizes each of these losses, depending on which signal is observed. Even under the conservative accounting system, if no losses are recognized, the DM will infer that $z_{1}^{C s v}$ has occurred and good earnings can be expected the second period. Conversely, under the liberal accounting system, if signal $z_{3}^{L i b}$ is observed, then the outcome is not known with sufficient confidence, and no income or loss is recognized, even though we know a loss of either 25 dollars or 15 dollars will be realized in the future. Also in an analogous fashion to what happens under the conservative system with low realizations, if either of the high signals, signal $z_{1}^{L i b}$ or signal $z_{2}^{L i b}$, is observed, then the manager knows that a gain of 25 dollars or 15 dollars, respectively, will be realized and these gains are recognized.

In this example, the manager only recognizes gains or losses if he is sufficiently confident that the outcome will be realized. We operationalize the phrase "sufficiently confident" to mean that the outcome is "more likely than not to occur;" for simplicity, we model this as certainty. An accounting system is conservative if it produces signals that are more informative about lower value outcomes than about higher value outcomes and interpret a liberal accounting system as one that produces information that has the reverse relative informativeness. Since conservative accounting produces signals that are more informative about losses, we recognize losses more readily under conservative accounting and defer the recognition of gains until they are realized. The opposite occurs under a liberal accounting system. While our model does not explicitly model financial reporting, the "accounting systems" developed in our model can easily be related to accounting observed in practice, as this example demonstrates.

Our subsequent results do not depend on either this example itself nor the manner in which we operationalize the recognition criteria. The example shows that our definitions of conservative and liberal accounting systems correspond to the intuitive and natural interpretations given to conditional conservatism in the real world. We used write-down of inventory (see Accounting Standards Codification, or ASC, Topic 330) to illustrate conditional conservatism, however we could have used another example of conditionally conservative accounting to illustrate our point. These other examples include other-than-temporary impairments on financial instruments (ASC Topic 320), goodwill impairments (ASC Topic 350), impairments of property (ASC

Topic 360), losses on purchase commitments (ASC Topic 440), loss contingencies (ASC Topic 450), loss contingencies on guarantees (ASC Topic 460) or provisions for losses on contracts (ASC Topic 605, Subtopice 35, Section 25, paragraphs 45-50) to name a few.

The accounting in these areas, and the conditionally conservative accounting in general, have the following similar features. An initial determination is made to ascertain whether or not a loss may have occurred. If there is an initial indication that a loss has occurred, additional work is done to ascertain the amount of the loss and the loss is recognized, while no additional work is done to ascertain whether a gain has occurred, nor is any gain recognized. As we noted earlier (see note 17 above), this is reflected in how we model the information system. We next turn to our results.

## 4 Results

We will analyze how accounting information systems differ by their relative conservatism and show that prudent DM's always prefer conservative accounting systems to liberal ones. In the first subsection, we show the basic result that the conservative system is preferred by prudent DM's over the liberal one. In the second subsection, we extend the results to consider the impact of introducing cost to the accounting systems. In the third subsection, we use simple cases to demonstrate the generality of our results.

### 4.1 Simple model of precautionary savings with conservatism

As mentioned earlier, our model starts with the model of precautionary savings of Kimball (1990) extended to include information systems. While we have introduced the background notation in section 3, we now need to make explicit the actual problem that we solve and how we measure preferences. For our first step, we have the DM choose the optimal savings that solves the precautionary savings problem with information. We state this problem formally as P1 below.

P1: Precautionary savings problem with information: Problem where a DM with utility function $v(s)$ wishes to maximize the following objective function

$$
\begin{equation*}
\max _{s}\left\{v(s)+E\left[v(s) \mid Z^{i}\right]\right\} \text { for } i=C s v, \text { Lib } \tag{14}
\end{equation*}
$$

Here we define savings as $s=w-c$. The expectation is taken over the set of four equally likely uncertain 2nd period earnings amounts, denoted as follows $w \in W=$
$\left\{w_{1}, w_{2}, w_{3}, w_{4}\right\}=\{2 U+\delta, 2 U-\delta, 2 L+\delta, 2 L-\delta\}$, and the information system, systems $Z^{i}$ are as defined in D3.

While problem P1 is the critical one that DM solves, our real focus is on which accounting system the DM prefers. We use expected utility to measure preferences and define the preference ordering, " $\succ$ ", based on the relative expected utility achieved under each system. More explicitly, we define the preference ordering as follows.

D5-Definition of preferences: A DM prefers one accounting system to another, denoted as " $\succ$ ", if, under the optimal choice of savings, the DM has a higher expected utility under the first accounting system than under the second, i.e., denoting the optimal savings given signal $z_{n}^{i}$ as $s\left(z_{n}^{i}\right)$, then the following are true.

1. $Z_{C s v} \succ Z_{L i b}$, if and only if $E\left[v\left(s\left(z_{n}^{C s v}\right)\right) \mid Z^{C s v}\right]>E\left[v\left(s\left(z_{n}^{L i b}\right)\right) \mid Z^{L i b}\right]$
2. $Z_{C s v} \prec Z_{L i b}$, if and only if $E\left[v\left(s\left(z_{n}^{C s v}\right)\right) \mid Z^{C s v}\right]<E\left[v\left(s\left(z_{n}^{L i b}\right)\right) \mid Z^{L i b}\right]$
3. $Z_{C s v} \approx Z_{L i b}$, if. and only if $E\left[v\left(s\left(z_{n}^{C s v}\right)\right) \mid Z^{C s v}\right]=E\left[v\left(s\left(z_{n}^{L i b}\right)\right) \mid Z^{L i b}\right]$

Having defined the precautionary savings problem with information and how we measure the DM's relative preference for the two accounting systems, we now turn to our results. Our main result, Theorem 1, characterizes the DM's preference between the two information systems.

Theorem 1: For all expected utility maximizing DM's facing the precautionary savings problem described in P1 above, the follow are true.

- (a) DM's prefer the conservative accounting system if they are prudent, i.e., $Z^{C s v} \succ Z^{L i b}$, if $\eta(s)>0, \forall s$.
- (b) DM's prefer the liberal accounting system if they are imprudent i.e., $Z^{C s v} \prec$ $Z^{L i b}$, if $\eta(s)<0, \forall s$.
- (c) DM's are indifferent between the liberal and the conservative systems if they are prudent neutral, i.e., $Z^{C s v} \approx Z^{L i b}$, if $\eta(s)=0, \forall s$.

When the DM observes a perfect signal, as he does when the earnings are high under liberal accounting or when earnings are low under conservative accounting, the DM chooses perfectly smoothed consumption, i.e., consumption that it is the same in each period. When the signal is imperfect, as it is for low earnings under liberal
accounting and high earnings under conservative accounting, the DM is forced to deviate from smoothed consumption. The prudent DM saves a positive amount when faced with uncertainty; in our problem, he will save and consume less than the average expected earnings in the first period. The intuition for the results in Theorem 1 can best be understood by examining Figures 6-8, which show the choices being faced by the DM.

The manager prefers a finer signal; we see the intuition for this using Figure 6. Figure 6 shows the utility when earnings are low with both perfect and imperfect signals. Under conservative accounting, bad earnings are signaled perfectly, i.e., the total earnings at the end of the second period will be $2 L+\delta$ (or $2 L-\delta$ ). The DM is able to smooth consumption perfectly. He consumes at point $\mathrm{E}=L+0.5 \delta$ (or point $\mathbf{D}=L-0.5 \delta$ ) on the graph in both periods, if signal $z_{1}^{L i b}=w_{1}$ (or signal $z_{2}^{L i b}=w_{2}$ ) is reported. The expected utility when earnings are low under conservative accounting is the average of D and E , which is shown as point J in Figure 6.

Under liberal accounting, the signal is imperfect when earnings are low, so the DM knows that earnings are either $2 L+\delta$ or $2 L-\delta$, but not which one. The expected earnings are $2 L$, so the DM will consume an average of $L$ and he will save $s_{L}$ in the first period for additional consumption of $s_{L}$ in the second period. Let the consumption be at point $\mathbf{A}=L-s_{L}$ in the first period and either $\mathbf{B}=L+\delta+s_{L}$ or $\mathbf{C}=L-\delta+s_{L}$ in the second period, where the savings $s_{L}$ are chosen to equate the marginal utilities in equation (??) introduced earlier. The utility, conditional on realization $w_{4}=2 L-\delta$, is the utility expected from consuming at points $\mathrm{A}\left(L-s_{L}\right)$ and $\mathrm{C}\left(L-\delta+s_{L}\right)$ and is shown as point F in Figure 6. The expected utility, conditional on realizing $w_{3}=2 L+\delta$, is the utility expected from consuming at points $\mathbf{A}\left(L-s_{L}\right)$ and $\mathbf{B}\left(L+\delta+s_{L}\right)$ and is shown as point $G$ in Figure 6. The average expected utility from good news under conservatism is the average of F and $G$, shown as H in the graph. Since $J$ always exceeds $H$ in utility terms, the DM has higher utility from conservative accounting (the perfect signal) than from liberal accounting (the imperfect signal) when the earnings are low.

The preceding discussion provides the intuition for why the manager prefers the finer signal. Since conservative (or liberal) accounting produces a finer signal at lower (or higher) earnings, it follows that the manager prefers conservative (or liberal) accounting if the earnings are lower (or higher); next we provide the intuition for why the manager prefers conservative accounting overall. The manager prefers conservatism to liberalism because the relative value, in utility terms, of having the finer signal at lower earnings levels exceeds the value of having the finer signal at higher earnings levels. This ranking of relative utility works because we assume that the
marginal utility function is convex, which means the concavity of the utility function is increasing. We often speak of the second derivative in terms of its effect on the first derivative; e.g., we say the concave utility function means utility increases at a decreasing rate. Prudence means that the rate of the decrease is itself increasing. Showing how this works is a little more complicated; we use all three figures, 6-8, but especially figures 7 and 8, to accomplish this task.

Figure 6 showed that, with low earnings, the expected utility from a perfect signal, as represented by point J , exceeds the expected utility from an imperfect signal, as represented by point $H$. The difference, $J-H$, relies on all the consumption points, A to E, where the DM realizes J under conservative accounting and H under liberal accounting. Though not shown here, we will have an analogous group of points for high earnings, e.g., $\mathrm{A}^{\prime}-\mathrm{E}^{\prime}$, that is centered around $U>L$, and an analogous difference in utility for the perfect and imperfect signals, e.g., J' - H'. These points will be similar to the basic setup in Figure 6, except that the points are shifted to the right. However, for high earnings, the DM realizes H' under conservatism and J' under liberalism. To prove that conservatism is preferred, we must show that the condition that the J-H difference decreases as the group of relevant consumption points, A-E, shifts to the right; this is shown in Figure 7. Alternatively, the excess in expected utility from better information under conservative accounting, $\mathrm{J}-\mathrm{H}$, realized when earnings are low, exceeds the excess in expected utility under liberalism, J' - H', realized when earnings are high. To see why prudence implies that this condition holds, consider figure 7.

Figure 7 shows the marginal utility of each of the consumption points introduced in the discussion of figure 6, but using lower case, so that the marginal utility of A is $a$, the marginal utility of $B$ is $b$, etc. First we see that the marginal utility of $A, a$, equals $1 / 2$ the marginal utility of $C$ plus $1 / 2$ the marginal utility of $B, a=1 / 2 c+1 / 2$ b ; as mentioned, this represents the solution to equation (??) above. Perhaps more importantly, Figure 8 shows how the difference, $J-H$, changes as the earnings levels change. The rate of change of $\mathrm{J}-\mathrm{H}$ equals $0.5[\mathrm{~d}+\mathrm{e}-(\mathrm{a}+0.5 \mathrm{c}+0.5 \mathrm{~b})]$

The change in the difference $\mathrm{J}-\mathrm{H}$ can be decomposed into the change in two other differences relating to the two income realizations. The first change difference relates to income $w_{4}=L-\delta+s_{L}$ and represents the change in the difference between consuming at point D each period versus consuming at points A and C , as represented by point F. The rate of change in this difference is represented in Figure 7 as the difference $d-f$. The second change difference represents the change in the difference between consuming at E each period or consuming at A and B, as represented by point G. Analogously, the rate of change in this difference is represented in Figure 7 as the
difference e-g. Figure 7 shows that these rate of change differences are negative.
Figure 8 represents these observations showing that the difference J - H will decrease as expected earnings increase. DM's will always prefer full recognition to partial recognition. Conservative accounting is full recognition for lower expected earnings levels and partial recognition for higher earnings levels ( $J$ ' and $H^{\prime}$ ). The figure is drawn for prudent DM's and shows that prudence implies the DM prefers conservative accounting to liberal accounting.

While we have the basic result that prudence drives the demand for conservatism, this result raises other issues, such as what role, if any, does risk aversion and changing risk aversion have on the demand for conservatism. We turn to some of these issues in the next subsection

### 4.2 Extension to the basic result on conservatism

From prior research (see prior result R2 above), we know that risk averse DM's with decreasing absolute risk aversion (abbreviated as DARA) will be prudent. We use this result to extend the basic result on prudence and conservatism in the following corollary.

Corollary 1.1: Under conditions of Theorem 1, if the DM is strictly risk averse and has DARA, then he or she prefers the conservative system, i.e., if for all savings levels, $h(s)>0$ and $h^{\prime}(s)<0$ both hold, then $Z_{C s v} \succ Z_{L i b}$ holds.

Kimball (1990) noted that risk averse DM's who exhibit DARA are also prudent, as shown in result R2 introduced earlier. It follows immediately that these DM's prefer conservative accounting to liberal accounting. This result is important, as risk averse investors with DARA form a group of DM's that are arguably one of the most important groups analyzed in economic theory.

We next consider the situation where accounting systems are costly to employ. It is reasonable to assume that generating signals requires incurring costs; in the following analysis, we assume that the cost of a signal generating system is linear in the number of signals it generates. Under these conditions, we find that conservative accounting is preferred to full recognition for costs that are sufficiently high. We present this result in the following theorem.

Theorem 2: Let the conditions of Theorem 1 hold and consider the set of prudent investors, so that we suppose prudence $\eta(s)>0, \forall s$, and also now assume each ac-
counting system costs the DM a cost of $C>0$ per signal generated. Then there exists cut - off costs, $0<C_{1}<C_{2}$, such that the following hold.
(a) For $0<C<C_{1}$,
$Z^{\text {Full }} \succ Z^{\text {Csv }}$
(b) For $C=C_{1}$,
$Z^{\text {Full }} \approx Z^{\text {Csv }}$
(c) For $C_{1}<C<C_{2}$,
$Z^{\text {Csv }} \succ Z^{\text {Full }}$
(d) For $C=C_{2}$,
$Z^{\text {Csv }} \approx Z^{\text {Part }}$
(e) For $C_{2}<C$,
$Z^{\text {Part }} \succ Z^{\text {Csv }}$

Figure 9 illlustrates this result. Even though the cost of full recognition increases faster than the cost of conservative accounting with an increase in per signal cost, comparing the relative expected utility under the two systems is complicated because the optimal precautionary savings level also changes. As the wealth of the DM decreases with the increase in cost, the DM saves more. Theorem 2 shows that the change in the savings level does not prevent the relative expected utility of conservative accounting versus full recognition from rising and eventually causing it to turn positive.

We know that full recognition is preferred if $C$ is low and Theorem 2 tells us that for sufficiently high cost, first full recognition and then conservative accounting is preferred by prudent DM's. Theorem 1 and 2 together indicate that the preference for accounting systems is not balanced; prudent DM's prefer conservatism while imprudent DM's prefer liberalism at some cost levels. One is tempted to conjecture that a balanced or neutral system would never be preferred by prudent DM's for all costs.

For example, consider the following accounting system, which we refer to as partial recognition accounting system with costly auditing. At date 1 , the DM pays a cost of $C>0$, and receives a signal reported under the partial recognition system, so that the DM knows whether good news or bad news will occur. Further, with probability $1>\gamma_{i}>0$ for $i=B, G$, an audit, or additional investigation, will reveal the perfect signal. A neutral accounting system with auditing would have $\gamma_{B}=\gamma_{G}$, so that the recognition is equally likely under a neutral system. Our conjecture is that, regardless of the cost, a DM would prefer a neutral system only if that DM had a prudence measure of zero. We formalize this conjecture in the following corollary.

Corollary 2.1: Under conditions of Theorem 1, suppose the DM is offered a costly partial recognition system, with perfect information revealed under good and bad news with probability $\gamma_{G}$ and $\gamma_{B}$, respectively. Then prudent DM's would not prefer a neutral system for intermediate cost levels. DM's prefer systems with higher prob-
ability of recognizing bad news (or good news) if they are prudent (or imprudent). Prudence $\eta(s) \neq 0$ implies $0<\gamma_{B}=\gamma_{G}<1$ is never optimal. $\eta(s)>0$ and $\eta(s)<0$ implies $\gamma_{B}>\gamma_{G}$ and $\gamma_{B}>\gamma_{G}$ is preferred, respectively.

Corollary 2.1, follows immediately using proof techniques similar to those used to prove Theorem 2. It formalizes the idea that balanced partial recognition will not, in general, be preferred. This result is contrary to much of the current discussion in academic and regulatory circles. Most arguments are framed in terms of risk neutral DM's, for whom unbiased accounting information is preferred. Yet, as Corollary 2.1 shows, prudent DM's will not prefer unbiased accounting, at least where unbiased accounting means that income is recognized with the same probability at both high and low earnings levels. Corollary 2.1 instead suggests that biased or unbalanced recognition, where either high or low income is recognized with greater probability, will be preferred. This also suggests that unbiased accounting may be more common in practice.

We next present some cases to provide intuition and insight into the relative preferences of different types of DM's.

### 4.3 Common utility functions and their preferences for conservatism

We use this section to discussion examples that provide insight into prudence and conservatism. We start with a discussion of some common utility functions and describe whether these functions represent DM's that are prudent. We follow this discussion by providing additional formal results that clarify how risk aversion and changing risk aversion affect preferences for conservatism. The point of these discussions is to emphasize that it is prudence, and not risk aversion or changing risk aversion, that drives the preference for conservatism.

Our discussion first identifies whether or not some common utility functions have prudence. DM's having many common utility functions are prudent. ${ }^{22}$ For example, DM's who have the natural logarithm utility function, $v(x)=\ln (x)$ for $x>0$, or the power utility function, $v(x)=x^{\gamma}$ for $1>\gamma>0$, are both risk averse DM's who exhibit DARA. Hence, by Corollary 1.1, we know they are prudent and prefer conservative accounting. However, prudence and the preference for conservative accounting hold despite the fact that the relations between prudence and risk aversion differ for these

[^15]utility functions. For example, the natural logarithm utility has prudence that is always twice the level of absolute risk aversion, or $\eta(x)=2 h(x)$. As another example, the power utility has prudence that is always a constant multiple of the level of risk aversion. More specifically, for power utility with parameter $1>\gamma>0$, we have $\eta(x)=\left(\frac{2-\gamma}{1-\gamma}\right) h(x)$. These cases indicate that our main result, that prudence implies a preference for conservatism, holds under many relations between prudence and risk aversion.

Next we directly address questions regarding the role of risk aversion and changing risk aversion. First, one might think that risk aversion alone will drive the demand for conservative accounting. Second, one might think that changing risk aversion, in particular, decreasing absolute risk aversion (DARA), is what drives the demand for conservatism. In Theorem 3, we demonstrate that, while prudence insures preferences for conservatism, risk aversion is not sufficient and DARA is not necessary.

Theorem 3: For all expected utility maximizing DM's facing the precautionary savings problem described in P 1 above, the following are true.

- (a) DARA is not necessary to ensure that the DM prefers the conservative system, i.e., there exists a utility function where a non-DARA DM prefers conservative accounting, or where $Z^{C s v} \succ Z^{L i b}$ and $h^{\prime}(s)>0, \forall s$ both hold.
- (b) Risk aversion is not sufficient to insure that the DM prefers a conservative accounting system, i.e., there exists a utility function where a risk averse DM prefers liberal accounting, or where $Z^{C s v} \prec Z^{L i b}$ and $h(s)>0, \forall s$ both hold.

Probably the most commonly utilized utility function is the negative exponential. DM's with negative exponential utility functions, $v(x)=-\gamma e^{-\gamma x}$ for $\gamma>0$, are also prudent and prefer conservatism. As is well known, the negative exponential utility function exhibits CARA (constant absolute risk aversion), so this is an example that demonstrates part (a) of Theorem 3. With CARA utility, we have absolute risk aversion equal to a constant; for the negative exponential, we also have prudence equal to this same constant, which is the parameter $\gamma>0$. Since risk aversion does not change with changes in the DM's wealth, CARA utility such as the negative exponential function are seen as attractive in many types of analyses where we wish to isolate the wealth effects. We see from this case that even with no wealth effects (i.e., even when prudence remains unchanged with changing levels of the DM's wealth), the DM still prefers conservatism.

For part b, we show that a DM can be risk averse and imprudent, so that this DM
prefers liberalism, i.e., part b holds. This occurs because the first derivative of the utility function is concave; as usual, the first derivative is decreasing (i.e., the utility function is concave), but now it decreases at a decreasing rate. Hence risk aversion does not by itself imply that the DM is prudent. In summary, DARA is not necessary and risk aversion is not sufficient; as our paper title indicates, it is prudence that drives the demand for conservatism.

## 5 Summary and conclusions

Our research objective was to show how conservative accounting may be demanded by decision makers (DM's) based simply on their characteristics. We show that prudent DM's will prefer conservative accounting systems over liberal accounting systems where these accounting systems are equivalent in the informational sense that they provide the same level of disaggregated signals, but that they differ only in whether that disaggregation applies to higher or lower earnings levels. If the better (i.e., disaggregated or finer) information is provided for lower earnings levels, we call the system conservative. Alternatively, if the better information is provided for higher earnings levels, we call the system liberal. We find that prudent DM's, i.e., those whose marginal utility decreases at an increasing rate, put higher value (in terms of expected utility) on better information at lower earnings levels than on better information at higher earnings levels. The reverse holds for imprudent DM's, i.e., those whose marginal utility decreases at a decreasing rate. Prudent DM's save more if risk increases.

We see at least two important implications from our findings. First, it is prudence, not risk aversion or changes in risk aversion, that drive the demand for conservatism. While we find this to be of theoretical interest, it also offers opportunities for potentially testable empirical hypotheses. There has been much empirical research testing for the presence of conservative accounting; our research suggests that conservative accounting should be found where prudent DM's represent a strong presence. Firms whose stockholders are prudent are likely to demand conservative accounting principles. Further, there is significant empirical research being conducted in economics to find prudent DM's. Our results suggest that conservative accounting may offer another approach to identifying where prudent DM's may be present.

The second implication from our findings may have an even more important practical impact. We show that the demand for conservative accounting can be explained by fundamental characteristics of the DM's themselves, without resorting to argu-
ments based on contracting or even capital structure. Most alternative explanations for the demand for conservatism rely upon contracting related arguments that often involve asymmetric information and debt related interactions. Our model provides an argument that conservatism is valued by DM's due to their intrinsic characteristics; that is, simply because they are prudent. Regulators sometimes argue that the business community prefers accounting that produces unbiased reports, where "unbiased reports" is interpreted to mean that gains and losses are equally likely to be recognized early; our results show that such arguments fail to hold if the business persons involved are prudent. The key aspect of our model is that prudent DM's use the information differentially, relying more heavily on the information about lower earnings levels than they do on higher earnings levels. We feel that this fundamental manner of explaining the demand for conservatism offers a strong, hitherto unidentified, reason that conservative accounting methods are so prevalent in practice. Further, it gives pause to regulators' and standard setters' preference for unbiased accounting standards. Our findings suggest that, ceterus paribus, the average (i.e., prudent) investor will value conservative accounting more than unbiased accounting, so that conservative accounting should be the goal of standard setters.

We have a number of issues that we hope to address in subsequent research. First, we characterize the accounting systems in a very structured manner. On the one hand, this allows us to isolate the impact of the conservatism versus liberalism trade - off. Often one argues that conservatism involves greater bias in reporting; we believe that our characterization of the conservative and liberal accounting methods does not endow either method with greater bias, so that bias does not drive the relative preferences. However, most accounting systems are more complicated than those developed in our model. One area of further research involves relaxing the structure of the model to allow for greater flexibility in the characterization of bias in the accounting systems.

A related topic for future research is to compare and contrast our results with other analytic results in a more formal manner. In particular, other research characterizes conservative accounting methods differently than we do. One obvious question that we would like to answer is whether or not our results extend to these alternative definitions of conservative accounting methods.

Figure 6: Utility of consumption with perfect and imperfect signals.


Figure 7: Marginal utility of consumption with perfect and imperfect signals.


Figure 8: Expected utility of consumption with perfect and imperfect signals.


Figure 9: Consumption with perfect and imperfect signals.


## 6 References

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## 7 Appendix A: Proofs of Results

In this appendix we present the proofs of our formal results.

Theorem 1: For the information systems, $Z^{i}$ for $i=C s v, L i b$, defined in definition D3 and for all expected utility maximizing DM's facing the precautionary savings problem, the following are true.

$$
\begin{align*}
& a . Z^{C s v} \succ Z^{L i b}, \text { if } \eta(s)>0, \forall s .  \tag{15}\\
& b . Z^{C s v} \prec Z^{L i b}, \text { if } \eta(s)<0, \forall s .  \tag{16}\\
& c . Z^{C s v} \approx Z^{L i b}, \text { if } \eta(s)=0, \forall s . \tag{17}
\end{align*}
$$

Proof of Theorem 1: We start with the proof of part a.; the proofs of parts (b) and (c) will follow in an analogous fashion. To prove $Z_{C s v} \succ Z_{L i b}$ holds, if $\eta(s)>0, \forall s$, it suffices to show if $\eta(s)>0, \forall s$ implies that the DM has greater expected utility under the conservative rather than liberal accounting, or that the following inequality holds.

$$
\begin{equation*}
E\left[v \mid Z^{C s v}\right]>E\left[v \mid Z^{L i b}\right] \tag{18}
\end{equation*}
$$

From equations (??) and (??) in the text, the fully written inequality is as follows.

$$
\begin{align*}
& 0.5\left[v\left(U-s_{U}\right)+0.5 v\left(U+\delta+s_{U}\right)+0.5 v\left(U-\delta+s_{U}\right)\right]  \tag{19}\\
& +0.5[v(L-0.5 \delta)+v(L+0.5 \delta)]  \tag{20}\\
> & 0.5\left[v\left(L-s_{L}\right)+0.5 v\left(L+\delta+s_{L}\right)+0.5 v\left(L-\delta+s_{L}\right)\right]  \tag{21}\\
& +0.5[0.5 v(U-0.5 \delta)+0.5 v(U+0.5 \delta)] \tag{22}
\end{align*}
$$

From equations (??) and (??) in the text, the FOC for choosing the optimal savings, $s_{U}$ for conservative and $s_{L}$ for liberal accounting, are shown in the following equations.

$$
\begin{gather*}
\frac{\partial E[v \mid G]}{\partial s_{U}}=-v^{\prime}\left(U-s_{U}\right)+0.5 v^{\prime}\left(U+\delta+s_{U}\right)+0.5 v^{\prime}\left(U-\delta+s_{U}\right)=0  \tag{23}\\
\frac{\partial E[v \mid B]}{\partial s_{L}}=-v^{\prime}\left(L-s_{L}\right)+0.5 v^{\prime}\left(L+\delta+s_{L}\right)+0,5 v^{\prime}\left(L-\delta+s_{L}\right)=0 \tag{24}
\end{gather*}
$$

Equations (??) and (??) are used repeatedly in the following proof. Begin by considering the problem when $\delta=0$. As the FOC of equations (??) and (??) show, the optimal savings are $s_{U}=0=s_{L}$ and equation (??) holds with equality. Next, letting $\delta$ increase, we have the following equations.

$$
\begin{align*}
\frac{\partial E\left[v \mid Z^{C v s}\right]}{\partial \delta} & =0.25\left\{v^{\prime}\left(U+\delta+s_{U}\right)-v^{\prime}\left(U-\delta+s_{U}\right)-v^{\prime}(L-0.5 \delta)+v^{\prime}(L+0.5 \delta)\right\} \\
& +\frac{d E\left[v \mid z^{C s v}\right]}{d s_{U}} \frac{d s_{U}}{d \delta}  \tag{25}\\
\frac{\partial E\left[v \mid Z^{L i b}\right]}{\partial \delta} & =0.25\left\{v^{\prime}\left(L+\delta+s_{L}\right)-v^{\prime}\left(L-\delta+s_{L}\right)-v^{\prime}(U-0.5 \delta)+v^{\prime}(U+0.5 \delta)\right\} \\
& +\frac{d E\left[v \mid z^{L i b}\right]}{d s_{L}} \frac{d s_{L}}{d \delta} \tag{26}
\end{align*}
$$

From the FOC of equations (??) and (??), $\frac{d E\left[v \mid z^{C s v}\right]}{d s_{U}}=0=\frac{d E\left[v \mid z^{L i b}\right]}{d s_{L}}$. To show that (??) holds, it suffices to show that (??) exceeds (??), or, rearranging and simplifying, it suffices to show that the following inequality holds.

$$
\begin{align*}
& v^{\prime}\left(U+\delta+s_{U}\right)-v^{\prime}\left(U-\delta+s_{U}\right)-v^{\prime}(L-0.5 \delta)+v^{\prime}(L+0.5 \delta) \\
> & v^{\prime}\left(L+\delta+s_{L}\right)-v^{\prime}\left(L-\delta+s_{L}\right)-v^{\prime}(U-0.5 \delta)+v^{\prime}(U+0.5 \delta) \tag{27}
\end{align*}
$$

Next, consider the problem if we let $U=L$. In this case, the FOC of equations (??) and (??) imply $s_{U}=s_{L}$ holds. This means that, with $U=L$, (??) will hold with equality. Our next step is to investigate (??) as we increase $U$. Differentiating each side of (??) with respect to $U$ gives the following equations.

$$
\begin{align*}
& \frac{\partial^{2} E\left[v \mid Z^{C v s}\right]}{\partial \delta \partial U}=0.25\left[v^{\prime \prime}\left(U+\delta+s_{U}\right)-v^{\prime \prime}\left(U-\delta+s_{U}\right)\right]+\frac{d^{2} E\left[v \mid z^{C s v}\right]}{d \delta d s_{U}} \frac{d s_{U}}{d U}  \tag{28}\\
& \frac{\partial^{2} E\left[v \mid Z^{L i b}\right]}{\partial \delta \partial U}=0.25\left[-v^{\prime \prime}(U-0.5 \delta)+v^{\prime \prime}(U+0.5 \delta)\right]+\frac{d^{2} E\left[v \mid z^{L i b}\right]}{d \delta d s_{L}} \frac{d s_{L}}{d U} \tag{29}
\end{align*}
$$

From the first order condition that showed $\frac{d E\left[v \mid Z^{C s v}\right]}{d s_{U}}=0=\frac{d E\left[\left.v\right|^{L i b}\right]}{d s_{L}}$, it further follows that we also have $\frac{d^{2} E\left[v \mid z^{C s v}\right]}{d \delta d s_{U}}=0=\frac{d^{2} E\left[v \mid z^{L i b}\right]}{d \delta d s_{L}}$. To show that (??) holds, it suffices to show that (??) exceeds (??), or, rearranging and simplifying, it suffices to show that the follwing holds

$$
\begin{align*}
\frac{\partial^{2} E\left[v \mid Z^{C v s}\right]}{\partial \delta \partial U} & >\frac{\partial^{2} E\left[v \mid Z^{L i b}\right]}{\partial \delta \partial U} \Leftrightarrow \\
0 & \left.>\left[v^{\prime \prime}\left(U-\delta+s_{U}\right)-v^{\prime \prime}(U-0.5 \delta)\right]+\left[v^{\prime \prime}(U+0.5 \delta)\right]-v^{\prime \prime}\left(U+\delta+s_{U}\right)\right] \tag{30}
\end{align*}
$$

The final steps in the proof show that, for prudent DM's, equation (??) holds by showing that the two differences on the right hand side of the final inequality are both negative.

First, we show that $0.5 \delta>s_{U}>0$. Since the DM is prudent, by assumption, we know that $s_{U}>0$. To show $0.5 \delta>s_{U}$ hold, suppose that $0.5 \delta=s_{U}$. Substituting into the FOC equation (??), we get the following.

$$
\begin{equation*}
-v^{\prime}(U-0.5 \delta)+0.5\left\{v^{\prime}(U+1.5 \delta)+v^{\prime}(U-0.5 \delta)\right\}<0 \tag{31}
\end{equation*}
$$

Hence, it must be that $0.5 \delta>s_{U}$ to increase the left hand side of equation ??. But since the DM is prudent, we know that $v^{\prime}()$ convex, and $v "()$ is increasing, and by risk aversion, is negative. Hence the following are true.

$$
\begin{align*}
& v^{\prime \prime}\left(U-\delta+s_{U}\right)-v^{\prime \prime}(U-0.5 \delta)<0 \\
& v^{\prime \prime}(U+0.5 \delta)-v^{\prime \prime}\left(U+\delta+s_{U}\right)<0 \tag{32}
\end{align*}
$$

This shows that equation (??) holds, completing the proof of part (a) of Theorem 1.

Corollary 1.1: Under conditions of Theorem 1, if for all savings levels, $h(s)>0$ and $h^{\prime}(s)<0$ both hold, then $Z_{C s v} \succ Z_{L i b}$ holds.

Proof of Corollary 1.1: Corollary 1.1 follows immediately from Theorem 1 when we use the prior result R1. Prior results R1 says that $h^{\prime}(s)<0$ implies $\eta(s)>h(s)$. Since by assumption $h(s)>0$, it follows that $\eta(s)>h(s)$, which implies that $Z_{C s v} \succ Z_{L i b}$, completing the proof of corollary 1.1.

Theorem 2: Let the conditions of Theorem 1 hold, suppose prudence $\eta(s)>0, \forall s$, and also now assume each accounting system costs the DM a cost of $C>0$ per signal generated. Then there exists cut - off costs, $0<C_{1}<C_{2}$, such that,
(a) For $0<C<C_{1}$,

$$
Z^{\text {Full }} \succ Z^{C s v}
$$

(b) For $C=C_{1}, \quad \quad Z^{\text {Full }} \approx Z^{\text {Csv }}$
(c) For $C_{1}<C<C_{2}, \quad \quad Z^{C s v} \succ Z^{\text {Full }}$
(d) For $C=C_{2}, \quad Z^{C s v} \approx Z^{\text {Part }}$
(e) For $C_{2}<C, \quad Z^{\text {Part }} \succ Z^{C s v}$

Proof of theorem 2: We begin the proof by first providing some additional equations for expected utility with under different signals. Let $R(U)$ and $P R(U)$ be the expected utility when earnings are good under full and partial disclosure, respectively, so that $R(U)$ and $P R(U)$ are given as follows.

$$
\begin{align*}
R(U) & \equiv 0.5 E\left[v \mid z_{1}^{\text {Full }}\right]+0.5 E\left[v \mid z_{2}^{\text {Full }}\right]=2[0.5 v(U-0.5 \delta)+0.5 v(U+0.5 \delta)] \\
P R(U) & \equiv \max _{s_{U}} E[v \mid G]=\max _{s_{U}}\left[v\left(U-s_{U}\right)+0.5 v\left(U+\delta+s_{U}\right)+0.5 v\left(U-\delta+s_{U}\right)\right] \tag{33}
\end{align*}
$$

Let $R(L)$ and $P R(L)$ be the expected utility when earnings are high under full and partial disclosure, respectively, so that $R(L)$ and $P R(L)$ are given as follows.

$$
\begin{align*}
R(L) & \equiv 0.5 E\left[v \mid z_{1}^{\text {Full }}\right]+0.5 E\left[v \mid z_{2}^{\text {Full }}\right]=v(L-0.5 \delta)+v(L+0.5 \delta) \\
P R(L) & \equiv \max _{s_{L}} E[v \mid B]=\max _{s_{L}}\left[v\left(L-s_{L}\right)+0.5 v\left(L+\delta+s_{L}\right)+0.5 v\left(L-\delta+s_{L}\right)\right] \tag{34}
\end{align*}
$$

We see that $R^{\prime}(U)>0, P R^{\prime}(U)>0$, and $R^{\prime}(U)-P R^{\prime}(U)<0$, where the final inequality follows if $v^{\prime}(\cdot)$ is convex, which holds if the DM is prudent. Similar inequalities hold for low earnings .

Without loss of generality, we assume partial recognition has zero cost, liberal and conservative accounting have a cost of $C>0$ and full recognition has a cost of $2 C$. Since $R(L)-P R(L)>0$, the expected utility function are monotonic increasing and concave, there exists a cost, $C_{2}>0$, at which the investor is indifferent between conservative accounting and partial recognition. This means the following equation holds.

$$
\begin{equation*}
P R(L)+P R(U)=R\left(L-C_{2}\right)+P R\left(U-C_{2}\right) \tag{35}
\end{equation*}
$$

Concavity ensures that the expected utility under conservatism must cross the constant level of expected utility from the partial recognition system for some cost sufficiently low. This shows that part (d) holds, while part (e) follows immediately. Also, we know that for sufficiently low cost, we have $Z^{\text {Full }} \succ Z^{C s v}$, so part (a) hold. Hence, to complete the proof, we need only show that parts (b) and (c) hold, i.e., that there exists $C=C_{1}$ such that $Z^{\text {Full }} \approx Z^{C s v}$ holds and that for $C_{1}<C<C_{2}$, that $Z^{\text {Csv }} \succ Z^{\text {Full }}$ holds.

We have shown that the expected utility under both conservative accounting and full recognition is decreasing concave functions of cost. Hence, since part (a) holds, we need only show that there exists a $C>0$ where $Z^{C s v} \succ Z^{\text {Full }}$ holds. Suppose, at $C=C_{2}$, the DM is currently observing signals reported under conservatism, but is given the opportunity to change how the signals are reported for good news. He is given the opportunity to adopt the following uncertain system: receive a perfect signal with probability $\tau>0$ and to continue to receive the imperfect signal with probability $1-\tau$. He can make this change at a cost of $\tau C_{2}>0$. If we can show that the expected utility of this opportunity is negative, then $Z^{C s v} \succ Z^{\text {Full }}$ holds, completing the proof.

Consider the expected utility for the DM under the proposed new uncertain accounting system as a function of the probability $\tau$. This expected utility, denoted as $W(\tau)$, is given as follows.

$$
\begin{equation*}
W(\tau)=R\left(L-C_{2}-\tau C_{2}\right)+(1-\tau) P R\left(U-C_{2}-\tau C_{2}\right)+\tau R\left(U-C_{2}-\tau C_{2}\right) \tag{36}
\end{equation*}
$$

We see that for $\tau=0$, the expected utility equals the expected utility under conservative accounting, while for $\tau=1$, the expected utility equals the expected utility under
full recognition. Showing $W^{\prime}(\tau)<0$, is sufficient to show that $Z^{\text {Csv }} \succ Z^{\text {Full }}$ holds at $C=C_{2}$. Taking the derivative of equation (??) with respect to $\tau$, we have the following equation.

$$
\begin{align*}
W^{\prime}(\tau) & =\left[R\left(U-C_{2}-\tau C_{2}\right)-P R\left(U-C_{2}-\tau C_{2}\right)\right] \\
& -C_{2}\left[R^{\prime}\left(L-C_{2}-\tau C_{2}\right)+(1-\tau) P R^{\prime}\left(U-C_{2}-\tau C_{2}\right)+\tau R^{\prime}\left(U-C_{2}-\tau C_{2}\right)\right]  \tag{37}\\
& <\left[R\left(L-\tau C_{2}\right)-P R\left(L-\tau C_{2}\right)\right] \\
& -C_{2}\left[R^{\prime}\left(L-C_{2}-\tau C_{2}\right)+(1-\tau) P R^{\prime}\left(U-C_{2}-\tau C_{2}\right)+\tau R^{\prime}\left(U-C_{2}-\tau C_{2}\right)\right]  \tag{38}\\
& =\left[R\left(L-\tau C_{2}\right)+P R(L)+P R(U)-R\left(L-C_{2}\right)-P R\left(U-C_{2}\right)-P R\left(L-\tau C_{2}\right)\right] \\
& -C_{2}\left[R^{\prime}\left(L-C_{2}-\tau C_{2}\right)+(1-\tau) P R^{\prime}\left(U-C_{2}-\tau C_{2}\right)+\tau R^{\prime}\left(U-C_{2}-\tau C_{2}\right)\right] \tag{39}
\end{align*}
$$

Inequality (??) follows from the observations that $R^{\prime}(U)-P R^{\prime}(U)>0$ and $L<U-C_{2}$. We used equation (??) for the final equality.

Next, we use the concavity of the expected utility functions to produce two useful inequalities. In general, for any increasing, concave function $g(x)$, for $y \leq x-k$ we have the following inequality.

$$
-k g^{\prime}(y)+[g(x)-g(x-k)]<0
$$

Hence, the concavity of $R(U)$ and $P R(U)$ ensure that the following inequalities hold.

$$
\begin{align*}
& R\left(L-\tau C_{2}\right)-R\left(L-C_{2}\right)<C_{2}(1-\tau)\left[R^{\prime}\left(L-C_{2}-\tau C_{2}\right)\right]  \tag{40}\\
& \quad P R(U)-P R\left(U-C_{2}\right)<C_{2}(1-\tau)\left[P R^{\prime}\left(U-C_{2}-\tau C_{2}\right)\right] \tag{41}
\end{align*}
$$

We make the following substitutions in the equation to simplify subsequent derivations.

$$
\begin{align*}
& A_{1}=\left[R^{\prime}\left(L-C_{2}-\tau C_{2}\right)+P R^{\prime}\left(U-C_{2}-\tau C_{2}\right)\right]  \tag{42}\\
& A_{2}=\left[R^{\prime}\left(U-C_{2}-\tau C_{2}\right)+P R^{\prime}\left(U-C_{2}-\tau C_{2}\right)\right] \tag{43}
\end{align*}
$$

Using these variables equation (??) can be rewritten as follows.

$$
\begin{align*}
W^{\prime}(\tau) & <-C_{2}\left[A_{1}+\tau A_{2}\right] \\
& +\left[R\left(L-\tau C_{2}\right)+P R(L)+P R(U)-R\left(L-C_{2}\right)-P R\left(U-C_{2}\right)-P R\left(L-\tau C_{2}\right)\right]  \tag{44}\\
& =-\tau C_{2}\left[A_{1}+A_{2}\right]-(1-\tau) C_{2} A_{1} \\
& +\left[R\left(L-\tau C_{2}\right)+P R(L)+P R(U)-R\left(L-C_{2}\right)-P R\left(U-C_{2}\right)-P R\left(L-\tau C_{2}\right)\right] \tag{45}
\end{align*}
$$

Using equations (??) and (??), we have the following inequality.

$$
\begin{equation*}
-(1-\tau) C_{2} A_{1}+\left[R\left(L-\tau C_{2}\right)+P R(U)-R\left(L-C_{2}\right)-P R\left(U-C_{2}\right)\right]<0 \tag{46}
\end{equation*}
$$

Substituting into (??) using equation (??), we get the following inequality.

$$
\begin{align*}
W^{\prime}(\tau) & <-\tau C_{2}\left[A_{1}+A_{2}\right]+\left[P R(L)-P R\left(L-\tau C_{2}\right)\right]  \tag{47}\\
& =-\tau C_{2}\left[R^{\prime}\left(L-C_{2}-\tau C_{2}\right)+R^{\prime}\left(U-C_{2}-\tau C_{2}\right)\right]  \tag{48}\\
& +\left[P R(L)-P R\left(L-\tau C_{2}\right)\right]<0 \tag{49}
\end{align*}
$$

The final inequality holds at $\tau=0$, completing the proofs of parts b and c and completing the proof of Theorem 2.

Corollary 2.1: Suppose the DM is offered an opportunity to adopt a partial recognition system with costly auditing under conditions of Theorem 1 , with perfect information revealed under good and bad news with probability $\gamma_{G}$ and $\gamma_{B}$, respectively. Then prudent DM's would never prefer a neutral system where $0<\gamma_{B}=\gamma_{G}<1$.

For intermediate cost levels, DM's prefer systems with higher probability on recognizing bad news or good news, depencing on whether they are prudent or imprudent, respectively. i.e., $\eta(s) \neq 0$ implies $0<\gamma_{B}=\gamma_{G}<1$ is never optimal, while $\eta(s)>0$ and $\eta(s)<0$ implies $\gamma_{B}>\gamma_{G}$ and $\gamma_{B}>\gamma_{G}$ is preferred, respectively.

Proof of Corollary 2.1: First, let $Z^{A u d}\left(\gamma_{B}, \gamma_{G}\right)$ denote a partial recognition system with costly auditing under probabilities $0<\gamma_{B}=\gamma_{G}<1$. Using the notation introduced in the proof of Theorem 2, the expected utility at date 0 under system $Z^{A u d}\left(\gamma_{B}, \gamma_{G}\right)$ is given as follows.

$$
\begin{align*}
E\left[v \mid Z^{A u d}\left(\gamma_{B}, \gamma_{G}\right)\right] & =.05\left[\gamma_{B} R(L-C)+\left(1-\gamma_{B}\right) P R(L-C)\right] \\
& +.05\left[\gamma_{G} R(U-C)+\left(1-\gamma_{G}\right) P R(U-C)\right] \tag{50}
\end{align*}
$$

Next, let $0<\gamma_{B}=\gamma+\tau<1$ and $0<\gamma_{G}=\gamma-\tau<1$, so that we start with a neutral system (with $0=\tau$ ) and consider the impact of shifting probability on recognition from good news to bad news or vice versa. Taking the derivative of the expected utility with respect to $0=\tau$, we get the following equation.

$$
\begin{equation*}
0.5[R(L-C)-P R(L-C)]-0.5[R(U-C)-P R(U-C)] \tag{51}
\end{equation*}
$$

Equation (??) will be positive (negative) as $R(U)-P R(U)$ is decreasing (or increasing), indicating that prudent (or imprudent) DM's will prefer to increase (or decrease) $\gamma_{B}=$ $\gamma+\tau$ relative to $\gamma_{G}=\gamma-\tau$, while neither will wish to keep $0=\tau$, completing the proof of corollary 2.1. Q.E.D. on Corollary 2.1.

Theorem 3: For all expected utility maximizing DM's facing the precautionary savings problem described in P1 above, the following are true.

- (a) Decreasing risk aversion is not necessary to ensure that the DM prefers the conservative system, i.e., there exists a utility function with where $Z^{C s v} \succ Z^{L i b}$ and $h^{\prime}(s)>0, \forall s$ both hold.
- (b) Risk aversion is not sufficient to ensure that the DM prefers a conservative accounting system, i.e., there exists a utility function where $Z^{C s v} \prec Z^{L i b}$ and $h(s)>0, \forall s$ both hold.

Proof of Theorem 3: We use examples based on the utility function, $v(x)=a-b(k-x)^{\gamma}$ to prove both parts. For part a. we use $\gamma=3$ while for part (b) we use $\gamma=1.5$. See appendix B below for the detail for these cases.

## 8 Appendix B: Checking preferences for conservatism using prudence for common utility functions

In this appendix we present some examples of common utility functions and the relative preferences for conservative versus liberal accounting information systems. The preferences are obtained by checking prudence of the utility functions.

Case 1: Natural logarithmic utility function. We start with the natural logarithm utility function, which is a function that exhibits both absolute risk aversion everywhere as well as decreasing absolute risk aversion (DARA). For all $x>0$, define,

$$
\begin{equation*}
v(x)=\ln x ; v^{\prime}(x)=x^{-1} ; v^{\prime \prime}(x)=-x^{-2} ; v^{\prime \prime \prime}(x)=2 x^{-3} . \tag{52}
\end{equation*}
$$

Since $v^{\prime \prime}(x)>0$ always, $v^{\prime}(x)$ is convex.
Risk Aversion $=h(x)=x^{-1}$; Risk aversion is always decreasing ; DARA
Prudence $=\eta(x)=2 x^{-1}>h(x)=$ Risk Aversion $=x^{-1}>0$
Conservative accounting is always preferred as logarithmic utility function is always prudent.

Case 2: Exponential utility function. For all $x$, with $\gamma>0$ define,

$$
v(x)=-e^{-\gamma x} ; v^{\prime}(x)=\gamma e^{-\gamma x} ; v^{\prime \prime}(x)=-\gamma^{2} e^{-\gamma x} ; v^{\prime \prime \prime}(x)=\gamma^{3} e^{-\gamma x}
$$

Since $v^{\prime \prime}(x)>0$ always, $v^{\prime}(x)$ is convex.

$$
\begin{aligned}
\text { Risk Aversion } & =h(x)=\gamma ; \text { Risk aversion is constant; CARA } \\
\text { Prudence } & =\eta(x)=\gamma=h(x)=\text { Risk Aversion }=\gamma>0
\end{aligned}
$$

Conservative accounting is always preferred as exponential utility function is always prudent.

Case 3: Power utility function. For all $x$, with $1>\gamma>0$ define,

$$
v(x)=x^{\gamma} ; v^{\prime}(x)=\gamma x^{\gamma-1} ; v^{\prime \prime}(x)=\gamma(\gamma-1) x^{\gamma-2} ; v^{\prime \prime \prime}(x)=\gamma(\gamma-1)(\gamma-2) x^{\gamma-3}
$$

Since $v^{\prime \prime}(x)>0$ always, $v^{\prime}(x)$ is convex.
Risk Aversion $=h(x)=-(\gamma-1) / x$; Risk aversion is decreasing; DARA
Prudence $=\eta(x)=-(\gamma-2) / x>h(x)=$ Risk Aversion $=-(\gamma-1) / x>0$
Conservative accounting is always preferred as power utility function is always prudent.

Case 4: Polynomial utility function. For all $x<K$, with $\gamma>1$ define,

$$
\begin{array}{lr}
v(x)=K^{\gamma}-(K-x)^{\gamma} ; & v^{\prime}(x)=\gamma(K-x)^{\gamma-1} \\
v^{\prime \prime}(x)=-\gamma(\gamma-1)(K-x)^{\gamma-2} ; & v^{\prime \prime}(x)=\gamma(\gamma-1)(\gamma-2)(K-x)^{\gamma-3}
\end{array}
$$

$v^{\prime \prime \prime}(x)>0$ only if $\gamma>2$. So $v^{\prime}(x)$ is convex only if $\gamma>2$. It is linear if $\gamma=2$ and concave otherwise.

Risk Aversion $=h(x)=(\gamma-1) /(K-x)$; Risk aversion is always increasing; IARA

$$
\text { Prudence }=\eta(x)=(\gamma-2) /(K-x)<h(x)=\text { Risk Aversion }=(\gamma-1) /(K-x)>0
$$

But prudence $>0$ only if $\gamma>2$. Conservative accounting will then be preferred. Even if we have increasing absolute risk aversion, conservative accounting is preferred as long as prudence is positive.


[^0]:    ${ }^{1}$ College of Business Administration, University of Illinois at Chicago, 601 South Morgan Street, (MC 006), Chicago, Illinois, rooms 2309 and 2302, respectively.
    ${ }^{2}$ We would like to thank Judson Caskey, Mingcherng Deng, Frank Gigler, Bjorn N. Jorgensen, Chandra Kanodia, Carolyn Levine, Ivan Marinovic, Jeroen Suijs, seminar participants at Universities of Illinois at Chicago, University of Minnesota and Colorado at Boulder, the 2009 Financial Economics and Accounting Conference at Rutger University for helpful comments on earlier drafts.

[^1]:    ${ }^{3}$ See Watts (2003a).
    ${ }^{4}$ See section 3.4 below for more detailed discussion of the conditionally conservative accounting methods prevalent in current US GAAP.
    ${ }^{5}$ See FASB (2010), paragraphs BC3.27 and BC3.28, respectively. Also, in these paragraphs, FASB seems to conflate prudence and conservatism, interpreting them as interchangeable concepts. In contrast, we distinguish between conservatism, which is a characteristic of the accounting system, and prudence, which describes a DM's preferences.

[^2]:    ${ }^{6}$ Watts 2003a and 2003b provide descriptions of the various definitions and a survey of the relevant literature up to 2003 .

[^3]:    ${ }^{7}$ More specifically, Bagnoli and Watts (2005) define conservative accounting as accounting where the bad outcome is reported perfectly but the good outcome is reported with noise. This means that a good report means a good outcome has occurred, but a bad report indicates either a good or bad outcome has occurred. Hence, conservative accounting produces reports with noise in the lower signals but no noise in the high signals.

[^4]:    ${ }^{8}$ See paragraph QC. 14 in FASB (2010).

[^5]:    ${ }^{9}$ Consistent with usage that is common in the literature (e.g., see Gormley Kim and Martin (2012), we use timely loss recognition to mean conditional conservatism.

[^6]:    ${ }^{10}$ See e.g., Leland (1968), note 5 on page 471. As with more current work, the empirical study investigated other economic issues; in this case the empirical work focused on testing the permanent income hypotheses and Leland interpreted results as support for the existence of precautionary savings. Also, while not formerly equivalent to the precautionary savings problem, which has been also called the buffer stock problem, we ignore the differences as they do not affect our subquent analysis.
    ${ }^{11}$ Lee and Sawada (2008) state that while theoretical economcis research increasingly stress the importance of prudence in savings decisions, the empirical research in the past decade or so has failed to find it to be empirically significant. In fact, estimates of prudence seem to conflict with accepted beliefs about risk aversion (see Lee and Sawada (2007), note 1 on page 196). Lee and Sawada suggest, as an answer to this riddle, that prior research may have overlooked an omitted-variable bias in the consumption equation.

[^7]:    ${ }^{12}$ In our notation, we follow previous studies as much as possible, but variations in the notation force us to change some of the notation. We aim for consistency and choose notation closest to Pratt (1964) and Kimball (1990), when possible. We restrict the mention of multiple DM's to our discussion of risk aversion.
    ${ }^{13}$ See Theorem 1, page 128; the prior result itnroduced above represents only part of Theorem 1 in Pratt (1964).

[^8]:    ${ }^{14}$ While we focus only on the measure of absolute risk aversion and absolute prudence, we believe many of our results extend to the relative measures as well.
    ${ }^{15}$ See Leland (1968) for this definition.

[^9]:    ${ }^{16}$ While Kimball goes on to analyze a second measure called relative prudence and defined analogously to relative risk aversion as $\theta \eta(\theta, \delta)$, we focus solely on the measure of absolute prudence.

[^10]:    ${ }^{17}$ Kimball (1990) used the more general equation, $\max _{c}\left\{u(c)+E\left[v\left(w_{0}-c+\tilde{y}\right)\right]\right\}$, but we assume the same utility function for each period.

[^11]:    ${ }^{18}$ See equation 20, page 65.

[^12]:    ${ }^{19} \mathrm{We}$ recognize that we employ a number of strong restrictions in our modeling of the information systems; we expand upon these restrictions, and their impact on our results, in our conclusion.

[^13]:    ${ }^{20} \mathrm{We}$ can think of the manager getting initial information that is either good or bad, but only expending additional effort to verify the actual outcome when earnings are bad. This is in fact how managers implement conditionally conservative accounting methods such as those discussed in Section 3.4 below.

[^14]:    ${ }^{21}$ We asuume throughout $L>\delta$ so that the second period consumption is nonnegative.

[^15]:    ${ }^{22}$ For more detailed analysis of prudence and preference for conservative accounting for common utility functions, see Appendix B.

