## Pseudo-differential Operators and the Nash–Moser Theorem

## Serge Alinhac Patrick Gérard

Translated by Stephen S. Wilson

Graduate Studies in Mathematics

Volume 82



American Mathematical Society Providence, Rhode Island



## Contents

Preface	e to the English edition	vii
Genera	l introduction	1
Chapter 0. Notation and review of distribution theory		5
$\S1.$	Spaces of differentiable functions and differential operators	5
§2.	Distributions on an open set of $\mathbb{R}^n$	6
§3.	Convolution	8
§4.	Kernels	9
§5.	Fourier analysis on $\mathbb{R}^n$	10
Chapter I. Pseudo-differential operators		15
§1.	Introduction	15
§2.	Symbols	20
§ <b>3</b> .	Pseudo-differential operators in $\mathcal{S}$ and $\mathcal{S}'$	24
§4.	Composition of operators	28
§5.	Action of pseudo-differential operators and Sobolev spaces	29
§6.	Operators in an open subset of $\mathbb{R}^n$	34
§7.	Operators on a manifold	37
§8.	Appendix	41
Commentary on Chapter I		49
Exercises for Chapter I		50
Chapte	er II. Nonlinear dyadic analysis, microlocal analysis, energy estimates	77

. .

§A.	Nonlinear dyadic analysis	77
§Β.	Microlocal analysis: wave front set and pseudo-differential	
	operators	89
§C.	Energy estimates	98
Commentary on Chapter II		106
Exercises for Chapter II		107
Chapter III. Implicit function theorems		121
§Α.	Implicit function theorem and elliptic problems	121
§Β.	Two examples of the use of the fixed-point method	128
§C.	Nash–Moser theorem	135
Commentary on Chapter III		153
Exercises for Chapter III		154
Bibliography		161
Main notation introduced		165
Index		167