

## PSEUDOMONOTONE VARIATIONAL INEQUALITIES AND FIXED POINTS

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**Abstract.** We introduce two new iterative algorithms with line-search process for solving a variational inequality problem with pseudomonotone and Lipschitz continuous mapping and a common fixed-point problem of an asymptotically nonexpansive mapping and a strictly pseudocontractive mapping. The proposed algorithms are based on inertial subgradient extragradient method with line-search process, hybrid steepest-descent method, and viscosity approximation method. Under mild conditions, we prove strong convergence of the proposed algorithms in a real Hilbert space.

**Key Words and Phrases:** Inertial subgradient extragradient method, pseudomonotone variational inequality, nonexpansive mapping, strictly pseudocontractive mapping.

**2020 Mathematics Subject Classification:** 47H05, 90C30, 47H10.

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*Received: July 10, 2019; Accepted: May 16, 2020.*