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Pseudoscalar-Vector Meson Scattering Amplitude Reduced from Six-Point Veneziano Model

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An extension of the Veneziano model to *Ps-V* elastic (inelastic) scattering amplitude is presented, using the generalized Veneziano amplitude. The amplitude has the following natural properties: consistency with the original Veneziano model for $\pi\pi\to\pi\omega$ scattering amplitude and reasonable treatment for spin complications. Then one obtains relations for the coupling constants for three-vector mesons. These predicted relations are shown to be consistent with the $SU(3)$ symmetry.

§ 1. Introduction

The problem of extension of the Veneziano formula[1] to the spinning particles is a non-trivial one. Several authors[2] have noticed that a simple-mined extension of the formula to pseudoscalar-vector elastic scattering gives rise to the so-called parity doubling phenomenon, and have observed that in order to eliminate parity doubling, several higher terms were needed.

On the other hand the Veneziano formula has been extended to processes with n spinless particles,[3] so that one could think *a priori* that the best way of treating the spin complications is to start from the n-body amplitude. Along these lines of thought Bardakci and Ruegg[4] have constructed a beautiful model for the $(K\overline{K}\pi\pi\pi)$ and $(K\overline{K}K\overline{K}\pi)$ systems. The four-point amplitudes reduced from them have as good properties as the original Veneziano model.

In this note we further study the spin complications in Veneziano-type formulae for pseudoscalar-vector elastic (inelastic) scattering, starting from the six-point Veneziano model for the $(K\overline{K}\pi\pi\pi\pi)$ process.

In § 2, we construct an amplitude for the process $(K\overline{K}\pi\pi\pi\pi)$, paying attention to the consistency with the original Veneziano model and the five-point Veneziano amplitude of Bardakci and Ruegg. There the consistency is shown by an example.

In § 3 reduced amplitudes are presented for the scattering $K\rho\to K\rho$, $\pi\rho\to\pi\rho$ and $\pi K\to\rho K^*$. The general properties of these amplitudes are studied.

Relations for the coupling constants for V-V-V are given in § 4, and in the last section some general remarks are given.

§ 2. Construction of an amplitude for the process ($K\overline{K}\pi\pi\pi\pi$)

The six-point function for spinless particles has been constructed by Chan Hong-Mo et al.[3] It satisfies the following conditions:

(a) analyticity,
(b) crossing symmetry,
(c) Dolen-Horn-Schmid duality,[5]
(d) resonance poles on linearly rising trajectories,
(e) Regge asymptotic behaviours in all Mandelstam channels.

We require the absence of the $I=\frac{3}{2}$, $I=2$ and $S=2$ resonances and the lowest spin of parent to be 1^-. Accordingly, we consider the $\omega(A_2)$ trajectory for three-pion resonances only, but we can add a $\pi(A_1)$-trajectory for them following the prescriptions described in the last section.

Hence one can write the amplitude as a sum of terms each corresponding to a particular permutation of the external lines.

$$A(K\overline{K}\pi\pi\pi\pi)=g\sum_{P(3456)} I_{123456}K_2^+\tau_{i_3}\tau_{i_4}\tau_{i_5}\tau_{i_6}K_1B_6(1-\alpha_{12}^{\rho},1-\alpha_{23}^{K^*},$$
$$1-\alpha_{34}^{\rho},1-\alpha_{45}^{\rho},1-\alpha_{56}^{\rho},1-\alpha_{61}^{K^*};1-\alpha_{234}^{K^*},1-\alpha_{345}^{\omega},1-\alpha_{456}^{\omega}), \qquad (1)$$

where

$$I_{123456}=\left\{\begin{array}{l} \delta_{\mu_1\mu_2}[\delta_{\mu_3\mu_4}\delta_{\mu_5\mu_6}-\delta_{\mu_3\mu_5}\delta_{\mu_4\mu_6}+\delta_{\mu_3\mu_6}\delta_{\mu_4\mu_5}] \\ -\delta_{\mu_1\mu_3}[\delta_{\mu_2\mu_4}\delta_{\mu_5\mu_6}-\delta_{\mu_2\mu_5}\delta_{\mu_4\mu_6}+\delta_{\mu_2\mu_6}\delta_{\mu_4\mu_5}] \\ +\delta_{\mu_1\mu_4}[\delta_{\mu_2\mu_3}\delta_{\mu_5\mu_6}-\delta_{\mu_2\mu_5}\delta_{\mu_3\mu_6}+\delta_{\mu_2\mu_6}\delta_{\mu_3\mu_5}] \\ -\delta_{\mu_1\mu_5}[\delta_{\mu_2\mu_3}\delta_{\mu_4\mu_6}-\delta_{\mu_2\mu_4}\delta_{\mu_3\mu_6}+\delta_{\mu_2\mu_6}\delta_{\mu_3\mu_4}] \\ +\delta_{\mu_1\mu_6}[\delta_{\mu_2\mu_3}\delta_{\mu_4\mu_5}-\delta_{\mu_2\mu_4}\delta_{\mu_3\mu_6}+\delta_{\mu_2\mu_5}\delta_{\mu_3\mu_4}] \end{array}\right\}$$
$$\times p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_4^{\mu_4}p_5^{\mu_5}p_6^{\mu_6}$$

with $p_1+p_2+p_3+p_4+p_5+p_6=0$. The indices 1, $\cdots$, 6 label the particles in Fig. 1. The sum is over all permutations of the four pions. The kinematical factor is imposed by parity and gives a correct Regge behaviour, as well as satisfying Adler's self-consistency relations for soft mesons. The isospin factor in front satisfies the requirements about the absence of exotic resonances, K_i being an isospinor and τ the Pauli matrices. The function B_6 is defined in reference 3). It can be written in many particular forms, one of which is

$$B_6(1-\alpha_{12},1-\alpha_{23},1-\alpha_{34},1-\alpha_{45},1-\alpha_{56},1-\alpha_{61};1-\alpha_{234},1-\alpha_{345},1-\alpha_{456})$$
$$=\iiint_0^1 du_1du_5dv_3\left(\frac{v_3}{u_6^{\,2}}\right)u_1^{-\alpha_{12}}u_2^{-\alpha_{23}}u_3^{-\alpha_{34}}u_4^{-\alpha_{45}}u_5^{-\alpha_{56}}u_6^{-\alpha_{61}}$$
$$\times v_1^{-\alpha_{234}-1}v_2^{-\alpha_{345}-1}v_3^{-\alpha_{456}-1}, \qquad (2)$$

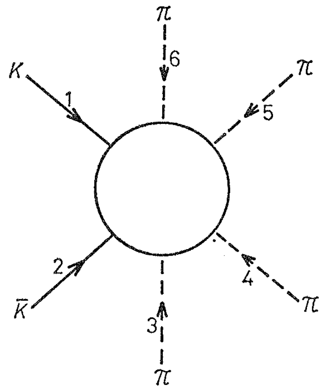

Fig. 1.

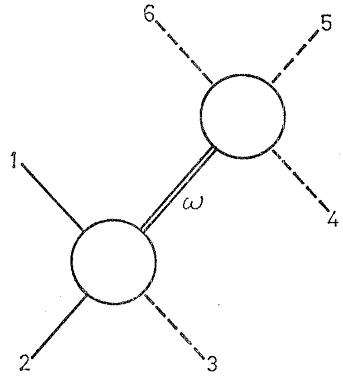

Fig. 2.

where

$$u_1 = 1 - u_6 u_2 v_1 , \quad u_4 = 1 - u_3 u_5 v_1 , \quad v_1 = 1 - v_2 v_3 u_1 u_4 ,$$
$$u_2 = 1 - u_1 u_3 v_2 , \quad u_5 = 1 - u_4 u_6 v_2 , \quad v_2 = 1 - v_3 v_1 u_2 u_5 ,$$
$$u_3 = 1 - u_2 u_4 v_3 , \quad u_6 = 1 - u_5 u_1 v_3 , \quad v_3 = 1 - v_1 v_2 u_3 u_6 .$$

One can convince himself that the amplitude (1) is consistent with the original Veneziano model and has correct kinematical factors. It is easily seen by considering the process described in Fig. 2. The reduction is done using the formula

$$\begin{aligned} \mathrm{Res}_{\alpha_{123}=1}[B_6(1-\alpha_{12}, \cdots, 1-\alpha_{61}; 1-\alpha_{234}, 1-\alpha_{345}, 1-\alpha_{456})] \\ = B(1-\alpha_{12}, 1-\alpha_{23}) B(1-\alpha_{45}, 1-\alpha_{56}) \end{aligned} \tag{3}$$

and the identities

$$\varepsilon_{i_1 i_2 i_3} \tau_{i_4} = (\varepsilon_{i_1 i_2 k} \delta_{i_3 i_4} + \varepsilon_{i_2 i_3 k} \delta_{i_1 i_4} - \varepsilon_{i_1 i_3 k} \delta_{i_2 i_4}) \tau_k , \tag{4}$$

$$\begin{aligned} \tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4} = \delta_{i_1 i_2} \delta_{i_3 i_4} + \delta_{i_2 i_3} \delta_{i_1 i_4} - \delta_{i_1 i_3} \delta_{i_2 i_4} + i (\delta_{i_1 i_2} \varepsilon_{i_3 i_4 k} \\ + \delta_{i_3 i_4} \varepsilon_{i_1 i_2 k} + \delta_{i_2 i_3} \varepsilon_{i_1 i_4 k} + \delta_{i_1 i_4} \varepsilon_{i_2 i_3 k} - \delta_{i_1 i_3} \varepsilon_{i_2 i_4 k} - \delta_{i_2 i_4} \varepsilon_{i_1 i_3 k}) \tau_k . \end{aligned} \tag{5}$$

In terms of these identities one gets

$$\begin{aligned} A(K\overline{K}\pi\pi\pi\pi)|_{\alpha_{123}=1} \Rightarrow & B(1-\alpha^{\rho}_{12}, 1-\alpha^{K^*}_{23}) \\ & \times [B(1-\alpha^{\rho}_{45}, 1-\alpha^{\rho}_{56}) \{I_{123456} \tau_{i_3} \tau_{i_4} \tau_{i_5} \tau_{i_6} + I_{123654} \tau_{i_3} \tau_{i_6} \tau_{i_5} \tau_{i_4}\} \\ & \quad + B(1-\alpha^{\rho}_{46}, 1-\alpha^{\rho}_{56}) \{I_{123465} \tau_{i_3} \tau_{i_4} \tau_{i_6} \tau_{i_5} + I_{123564} \tau_{i_3} \tau_{i_5} \tau_{i_6} \tau_{i_4}\} \\ & \quad + B(1-\alpha^{\rho}_{45}, 1-\alpha^{\rho}_{46}) \{I_{123546} \tau_{i_3} \tau_{i_5} \tau_{i_4} \tau_{i_6} + I_{123645} \tau_{i_3} \tau_{i_6} \tau_{i_4} \tau_{i_5}\}] \\ & \qquad + B(1-\alpha^{\rho}_{12}, 1-\alpha^{K^*}_{13}) \cdots \\ = & -2i (\delta_{i_3 i_4} \varepsilon_{i_5 i_6 k} + \delta_{i_3 i_6} \varepsilon_{i_4 i_5 k} - \delta_{i_3 i_5} \varepsilon_{i_4 i_6 k}) \tau_k \varepsilon_{\mu_1 \mu_2 \mu_3 \nu} \varepsilon_{\mu_4 \mu_5 \mu_6 \nu} \prod p_i^{\mu_j} \\ & \times \{B(1-\alpha^{\rho}_{12}, 1-\alpha^{K^*}_{23}) + B(1-\alpha^{\rho}_{12}, 1-\alpha^{K^*}_{13})\} \end{aligned}$$

$$\times \{B(1-\alpha_{45}^{\rho}, 1-\alpha_{56}^{\rho}) + B(1-\alpha_{56}^{\rho}, 1-\alpha_{46}^{\rho}) + B(1-\alpha_{46}^{\rho}, 1-\alpha_{45}^{\rho})\}$$
$$+\text{anomalous terms.} \tag{6}$$

On the other hand from the original Veneziano model one can construct the amplitude as

$$(\varepsilon_{\mu_1\mu_2\mu_3\nu} p_1^{\mu_1} p_2^{\mu_2} p_3^{\mu_3} e_\omega^{\ \nu} i\tau_{i_3}) g_{\omega K\bar{K}\pi} A(\pi K \to \omega K) \frac{1}{s_{123} - m_\omega^{\ 2} + i\varepsilon}$$
$$\times (\varepsilon_{\mu_4\mu_5\mu_6\lambda} p_4^{\mu_4} p_5^{\mu_5} p_6^{\mu_6} e_\omega^{\ \lambda} \varepsilon_{i_4 i_5 i_6}) g_{\pi\pi\pi\omega} A(\pi\pi \to \pi\omega),$$

where

$$A(\pi K \to \omega K) = B(1-\alpha_{12}^{\rho}, 1-\alpha_{23}^{K^*}) + B(1-\alpha_{12}^{\rho}, 1-\alpha_{13}^{K^*}),$$
$$A(\pi\pi \to \pi\omega) = B(1-\alpha_{45}^{\rho}, 1-\alpha_{56}^{\rho}) + B(1-\alpha_{56}^{\rho}, 1-\alpha_{46}^{\rho}) + B(1-\alpha_{46}^{\rho}, 1-\alpha_{45}^{\rho}). \tag{7}$$

With the identity (4), Eq. (6) is equivalent to Eq. (7) with the anomalous terms discarded. The reason for discarding the anomalous terms is: when one obtains the amplitudes for $PsPs \to PsT$ from five-point Veneziano model (Appendix A), one encounters the same difficulties. We consider these circumstances always occur in applying the multi-point Veneziano model. However, one can expect that these difficulties are evaded by taking into account the higher order Veneziano terms. This is supported by the fact that a simple extension of the Veneziano model for PsV-scattering amplitudes fails to give reasonable amplitudes without taking into account higher order Veneziano terms.[2)]

§ 3. Reduced amplitudes

By reduction of the five-point Veneziano amplitude one gets the amplitudes for $PsPs \to PsV(T)$ (see Appendix A). In the same way the amplitude (1) reduces to ones for $PsV \to PsV$.

3-1 $K(p_1)\rho^i(p_3) \to K(p_2)\rho^j(p_4)$

This process corresponds to Fig. 3, and can be reduced using[3)]

$$\mathrm{Res}_{\alpha_{34}=1, \alpha_{56}=1}[B_6(1-\alpha_{12}, \cdots, 1-\alpha_{61}; 1-\alpha_{234}, 1-\alpha_{345}, 1-\alpha_{456})]$$
$$= B(1-\alpha_{12}, 1-\alpha_{234}), \qquad \text{etc.} \tag{8}$$

Then the amplitude is given as follows:

$$T = \delta_{ji} A^{(+)} + \tfrac{1}{2}[\tau_j, \tau_i] A^{(-)},$$
$$A^{(\pm)} = (X+Y+Z) B(1-\alpha^{\rho}(t), 1-\alpha^{K^*}(s))$$
$$\pm (X-Y-Z) B(1-\alpha^{\rho}(t), 1-\alpha^{K^*}(u)), \tag{9}$$

where

$$4X = (p_1 p_2)[(e_{\rho_3} p_4)(e_{\rho_4} p_3) - (e_{\rho_3} e_{\rho_4})(p_3 p_4)],$$

$$8(Y+Z)=-(p_1-p_2,p_3)(e_{\rho_3}p_4)(e_{\rho_4},p_1+p_2)+(p_1+p_2,e_{\rho_3})(p_3e_{\rho_4})(p_4,p_1-p_2)$$
$$-(p_1+p_2,p_3)(e_{\rho_3}e_{\rho_4})(p_4,p_1-p_2)+(p_1-p_2,e_{\rho_3})(p_3p_4)(e_{\rho_4},p_1+p_2)$$
$$-(p_1+p_2,e_{\rho_3})(p_3p_4)(e_{\rho_4},p_1-p_2)+(p_1-p_2,p_3)(e_{\rho_3}e_{\rho_4})(p_4,p_1+p_2)$$
$$-(p_1-p_2,e_{\rho_3})(p_3e_{\rho_4})(p_4,p_1+p_2)+(p_1+p_2,p_3)(e_{\rho_3}p_4)(e_{\rho_4},p_1-p_2).\tag{10}$$

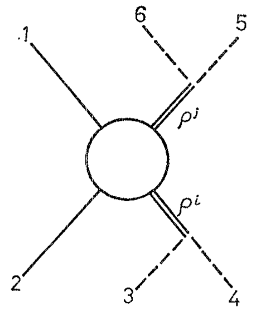

Fig. 3.

e_{ρ_3} and e_{ρ_4} are the polarization vectors of particle 3 and 4, and $p_1+p_2+p_3+p_4=0$, $s=(p_1+p_3)^2$, $t=(p_1+p_2)^2$ and $u=(p_1+p_4)^2$. (Derivations of these are given in Appendix B.)

The coupling for V-V-V is easily seen by taking the pole at $(p_1+p_2)^2=m_\rho^2$ in the amplitude $A^{(-)}$ and using coupling for Ps-Ps-V as $(Ps\overleftrightarrow{\partial}_\mu Ps)V^\mu$. Then the residue of $A^{(-)}$ at the pole $(p_1+p_2)^2=m_\rho^2$ suggests that one can take the $V_AV_BV_C$ vertex as

$$Y+Z\Rightarrow\frac{1}{8}\Big\{\frac{1}{4}(e_Ap_{BC})(e_Bp_{CA})(e_Cp_{AB})$$
$$-\frac{m_\rho^2}{2}[(e_Ae_B)(e_Cp_{AB})+(e_Be_C)(e_Ap_{BC})+(e_Ce_A)(e_Bp_{CA})]\Big\},$$

where $p_{AB}=p_A-p_B$, and e_A is the polarization vector of the particle A,[6] etc. This is the momentum representation of the coupling,

$$\tfrac{1}{8}f_A^{\mu\nu}f_B^{\nu\lambda}f_C^{\lambda\mu},\tag{11}$$

where $f_A^{\mu\nu}=\partial^\mu V_A^\nu-\partial^\nu V_A^\mu$, etc.

Unfortunately the amplitude $A^{(+)}$ has small residue at the pole $\alpha^\rho(t)=1$; but as pointed out in the previous section, if we take into account a contribution of higher order terms in the direct channel, this difficulty can be avoided without changing the coupling for V-V-V.

3-2 $\pi^{i_1}(p_1)\rho^{i_4}(p_4)\to\pi^{i_2}(p_2)\rho^{i_3}(p_3)$

This process can be reduced by taking the pole at $\alpha_{K\bar{K}}=1$ and $\alpha_{\pi\pi}=1$, then the very amplitude is obtained as (for details of the derivation, see Appendix C)

$$T=\delta_{i_1i_2}\delta_{i_3i_4}A+\delta_{i_1i_3}\delta_{i_2i_4}B+\delta_{i_2i_3}\delta_{i_4i_1}C,$$
$$A=(X+Y+Z)B(1-\alpha^\omega(s),1-\alpha^\rho(t))+(X-Y-Z)B(1-\alpha^\omega(u),1-\alpha^\rho(t))$$
$$-(X+Y-Z)B(1-\alpha^\omega(s),1-\alpha^\omega(u)),$$
$$B=-(X+Y+Z)B(1-\alpha^\omega(s),1-\alpha^\rho(t))+(X-Y-Z)B(1-\alpha^\omega(u),1-\alpha^\rho(t))$$
$$+(X+Y-Z)B(1-\alpha^\omega(s),1-\alpha^\omega(u)),$$
$$C=(X+Y+Z)B(1-\alpha^\omega(s),1-\alpha^\rho(t))-(X-Y-Z)B(1-\alpha^\omega(u),1-\alpha^\rho(t))$$
$$+(X+Y-Z)B(1-\alpha^\omega(s),1-\alpha^\omega(u)),\tag{12}$$

where

$$4X=(p_1p_2)[(e_{\rho_3}p_4)(e_{\rho_4}p_3)-(e_{\rho_3}e_{\rho_4})(p_3p_4)],$$
$$4Y=-(p_1p_3)[(e_{\rho_3}p_4)(e_{\rho_4}p_2)-(e_{\rho_3}e_{\rho_4})(p_4p_2)]$$
$$+(p_1e_{\rho_3})[(p_3p_4)(e_{\rho_4}p_2)-(p_3e_{\rho_4})(p_4p_2)],$$
$$4Z=(p_1p_4)[(e_{\rho_4}p_3)(e_{\rho_3}p_2)-(e_{\rho_4}e_{\rho_3})(p_3p_2)]$$
$$-(p_1e_{\rho_4})[(p_4p_3)(e_{\rho_3}p_2)-(p_4e_{\rho_3})(p_3p_2)]. \tag{13}$$

X, Y and Z are equal to those in Eq. (10) for $K\rho\to K\rho$. The vertex for V-V-V is given in the same manner as in the $K\rho$-elastic scattering case.

3-3 $\pi^i(p_2)K(p_1)\to\rho^j(p_4)K^*(p_3)$

Reduction for this process is made as the previous subsections, and the result is given,

$$T=\delta_{ji}A^{(+)}+\tfrac{1}{2}[\tau_j,\tau_i]A^{(-)},$$
$$A^{(\pm)}=(X+Y+Z)B(1-\alpha^{K^*}(s),1-\alpha^{\omega}(t))$$
$$\pm(X+Y-Z)B(1-\alpha^{K^*}(u),1-\alpha^{\omega}(t)), \tag{14}$$

where X, Y and Z are the same for the $\pi\rho\to\pi\rho$ scattering case with e_{ρ_4} replaced by e_{K^*} in Eq. (13).

This amplitude gives also the same coupling for V-V-V as ones for $K\rho\to K\rho$ and $\pi\rho\to\pi\rho$.

§ 4. Coupling constants of three-vector mesons

In terms of the four- and five-point Veneziano models, relations for the coupling constants for Ps-Ps-$V(T)$ and Ps-V-$V(T)$ have been obtained and those give $\omega\varphi$ and ff' mixing angles by $\mathrm{tg}^2\,\theta=\frac{1}{2}$.[4)]

Using these results, our model amplitude provides partial $SU(3)$ symmetric three-vector-meson couplings. For example,

$$g^2_{\omega\rho\pi}=g_{\rho\pi\pi}g_{\rho\rho\rho},$$
$$2g^2_{K^*K\rho}=g_{\rho KK}g_{\rho\rho\rho}, \qquad \text{etc.} \tag{15}$$

and

$$g_{\rho\rho\rho}=2g_{K^*K^*\rho}, \tag{16}$$

where the normalization of the g's can be read off from the couplings

$$g_{\rho\pi\pi}\tfrac{1}{2}(\pi_i\overleftrightarrow{\partial}_\mu\pi_j)\rho_k{}^\mu\varepsilon^{ijk},$$
$$g_{\omega\rho\pi}\tfrac{1}{4}\varepsilon_{\lambda\mu\nu\sigma}(\partial^\lambda\omega^\mu)(\partial^\nu\rho_i{}^\sigma)\pi^i,$$
$$g_{\rho_A\rho_B\rho_C}\tfrac{1}{8}f^{\mu\nu}(\rho_A)_if^{\nu\lambda}(\rho_B)_jf^{\lambda\mu}(\rho_C)_k\varepsilon^{ijk},$$

where

$$f^{\mu\nu}(\rho_A)_i = \partial^\mu (\rho_A)_i^\nu - \partial^\nu (\rho_A)_i^\mu, \qquad \text{etc.} \tag{17}$$

(Note that these symmetrical couplings for three-vector mesons will be changed if Ps-meson exchanges are included.)

§ 5. Discussion

Requiring the six-point Veneziano amplitude to reduce the original Veneziano model for $\pi\pi\to\pi\omega$, we have constructed the amplitude for the $(K\bar{K}\pi\pi\pi\pi)$ system and derived amplitudes for $K\rho$-, $\pi\rho$-elastic scattering and $\pi K\to\rho K^*$ scattering.

Then one can study the spin complications reasonably and can predict the V-V-V coupling constants to be consistent with F-type coupling in $SU(3)$ (even if the amplitudes have higher order terms).

Now we would like to mention some other consequences concerning gauge conditions and effects of higher order terms.

1) Gauge conditions: Writing the invariant amplitude for $\pi\rho$-elastic scattering,

$$T = (e_{\rho_3}P)(e_{\rho_4}P)A + \{(e_{\rho_3}P)(e_{\rho_4}Q) + (e_{\rho_4}P)(e_{\rho_3}Q)\}B + (e_{\rho_3}Q)(e_{\rho_4}Q)C + (e_{\rho_3}e_{\rho_4})D,$$

where

$$P = \tfrac{1}{2}(p_1 - p_2) \qquad \text{and} \qquad Q = \tfrac{1}{2}(p_4 - p_3),$$

Itabashi[7] has obtained the "non-zero mass gauge" conditions

$$\left\{(s+m_\rho^2-\mu^2)^2 - 4m_\rho^2 s + \frac{t}{2}(s+m_\rho^2-\mu^2)\right\}A^N - \frac{t}{2}(s+m_\rho^2-\mu^2)B^N + 4m_\rho^2 D^N = 0\,, \tag{18}$$

$$\left\{(s+m_\rho^2-\mu^2)^2 - 4m_\rho^2 s + \frac{t}{2}(s+m_\rho^2-\mu^2)\right\}B^N - \frac{t}{2}(s+m_\rho^2-\mu^2)C^N - 4(s-\mu^2)D^N = 0\,, \tag{19}$$

where $A^N \sim D^N$ stand for the s-channel normal-parity parts of $A\sim D$. In our amplitude the s-channel normal-parity part, for example ω-pole term, has a residue $X+Y$ (this is equal to that of standard perturbation calculation). Then one can fix the invariant amplitudes $A^N \sim D^N$,

$$A^N(s=m_\omega^2) = -\tfrac{1}{2}(t-2m_\rho^2),$$
$$B^N(s=m_\omega^2) = u-\mu^2-m_\rho^2+\tfrac{1}{2}(t-2m_\rho^2),$$
$$C^N(s=m_\omega^2) = -\left[\tfrac{1}{2}(t-2m_\rho^2) - 2(s-m_\rho^2+\mu^2)\right],$$

$$D^N(s=m_\omega^2) = -\tfrac{1}{4}[(u-\mu^2-m_\rho^2)-(t-2\mu^2)(t-2m_\rho^2)].$$

These actually satisfy the conditions (17) and (18). Moreover this is true if one adds higher order terms. So our amplitude has reasonable features for s- (and u-) channel normal-parity parts concerning the non-zero mass gauge conditions.

2) Effects of higher order terms: The amplitudes derived in the previous sections have favorable features without free parameters, but have defects for spin doubling. Here we will make explicit the simplest possibility for resolving them.

Requiring Regge behaviours at high energies, one would for example replace Eq. (12) as follows:

$$\begin{aligned}
A\to A' =& (X+Y+Z)B(1-\alpha(s),1-\alpha(t))+(X-Y-Z)B(1-\alpha(t),1-\alpha(u))\\
&-(X+Y-Z)B(1-\alpha(s),1-\alpha(u))\\
&+(X+Y-Z)B(1-\alpha(s),2-\alpha(t))+(-X-Y-Z)B(1-\alpha(t),2-\alpha(u))\\
&-(X+Y+Z)B(1-\alpha(s),2-\alpha(u))\\
&+(-X+Y+Z)B(2-\alpha(s),1-\alpha(t))+(X+Y-Z)B(2-\alpha(t),1-\alpha(u))\\
&-(X-Y-Z)B(2-\alpha(s),1-\alpha(u)),\\
B\to B' =& -(X+Y+Z)B(1-\alpha(s),1-\alpha(t))+(X-Y-Z)B(1-\alpha(t),1-\alpha(u))\\
&+(X+Y-Z)B(1-\alpha(s),1-\alpha(u))\\
&-(X+Y-Z)B(1-\alpha(s),2-\alpha(t))+(-X-Y-Z)B(1-\alpha(t),2-\alpha(u))\\
&+(X+Y+Z)B(1-\alpha(s),2-\alpha(u))\\
&-(-X+Y+Z)B(2-\alpha(s),1-\alpha(t))+(X+Y-Z)B(2-\alpha(t),1-\alpha(u))\\
&+(X-Y-Z)B(2-\alpha(s),1-\alpha(u)),\\
C\to C' =& (X+Y+Z)B(1-\alpha(s),1-\alpha(t))-(X-Y-Z)B(1-\alpha(t),1-\alpha(u))\\
&+(X+Y-Z)B(1-\alpha(s),1-\alpha(u))\\
&+(X+Y-Z)B(1-\alpha(s),2-\alpha(t))-(-X-Y-Z)B(1-\alpha(t),2-\alpha(u))\\
&+(X+Y+Z)B(1-\alpha(s),2-\alpha(u))\\
&+(-X+Y+Z)B(2-\alpha(s),1-\alpha(t))-(X+Y-Z)B(2-\alpha(t),1-\alpha(u))\\
&+(X-Y-Z)B(2-\alpha(s),1-\alpha(u)).
\end{aligned}$$

In these ways one can get rid of the difficulty involved in the spin-doubling phenomenon. Notice that in terms of the inclusion of higher order terms as above, the results for coupling constants in the previous section are not altered. Of course, our method is a term by term one, so this is nothing more than a speculation and we do not claim that it actually happens in nature.

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Appendix A

Amplitudes reduced from the Bardakci-Ruegg amplitude[4)]

Reducing the Bardakci-Ruegg model for $(K\bar{K}\pi\pi\pi)$ process, one can obtain the amplitudes for $\pi\pi\to\pi A_2$ and $\pi K\to\pi K^{**}$, assuming the degeneracy $\alpha_\omega=\alpha_{A_2}$ and $\alpha_{K^*}=\alpha_{K^{**}}$. Here we list the amplitudes for $\pi\pi\to\pi A_2$ and $\pi K^*\to\pi K^{**}$ scattering.

(a) $\pi^{i_1}(p_1)\pi^{i_2}(p_2)\to\pi^{i_3}(p_3)A_2{}^{i_4}(p_4)$

$$T=g_{A_2\pi\pi\pi}\varepsilon_{\mu\nu\lambda\sigma}e^{\mu\beta}_{A_2}p_1{}^\nu p_2{}^\lambda p_3{}^\sigma$$

$$\left\{\begin{aligned} &p_1{}^\beta[\delta_{i_4i_1}\delta_{i_2i_3}A^{(-)}+(\delta_{i_4i_2}\delta_{i_3i_1}-\delta_{i_4i_3}\delta_{i_1i_2})A^{(+)}]\\ &+p_2{}^\beta[\delta_{i_4i_2}\delta_{i_3i_1}B^{(-)}+(\delta_{i_4i_3}\delta_{i_1i_2}-\delta_{i_4i_1}\delta_{i_2i_3})B^{(+)}]\\ &+p_3{}^\beta[\delta_{i_4i_3}\delta_{i_1i_2}C^{(-)}+(\delta_{i_4i_1}\delta_{i_2i_3}-\delta_{i_4i_2}\delta_{i_3i_1})C^{(+)}]\end{aligned}\right\},$$

$$A^{(\pm)}=B(2-\alpha(u),1-\alpha(t))\pm B(2-\alpha(s),1-\alpha(t)),$$

$$B^{(\pm)}=B(2-\alpha(s),1-\alpha(u))\pm B(2-\alpha(t),1-\alpha(u)),$$

$$C^{(\pm)}=B(2-\alpha(t),1-\alpha(s))\pm B(2-\alpha(u),1-\alpha(s)),$$

where $p_1+p_2+p_3+p_4=0$, $s=(p_1+p_2)^2$, $t=(p_1+p_3)^2$ and $u=(p_1+p_4)^2$. $e^{\mu\beta}_{A_2}$ is the polarization tensor of A_2-meson,[8)] and $\alpha=\alpha^\rho$.

(b) $\pi^i(p_1)K(k)\to\pi^j(p_2)K^{**}(q)$

$$T=g_{K^{**}K\pi\pi}\varepsilon_{\mu\nu\lambda\sigma}e^{\mu\beta}_{K^{**}}k^\nu p_1{}^\lambda p_2{}^\sigma\{\delta_{ji}A_\beta{}^{(+)}+\tfrac{1}{2}[\tau_j,\tau_i]A_\beta{}^{(-)}\},$$

$$\begin{aligned}A_\beta{}^{(\pm)}=&k_\beta[B(2-\alpha^{K^*}(s),1-\alpha^\rho(t))+B(2-\alpha^{K^*}(u),1-\alpha^\rho(t))]\\ &\pm(p_1)_\beta B(2-\alpha^\rho(t),1-\alpha^{K^*}(u))-(p_2)_\beta B(2-\alpha^\rho(t),1-\alpha^{K^*}(s)),\end{aligned}$$

where $p_1+p_2+k+q=0$ and $e^{\mu\beta}_{K^{**}}$ is the polarization tensor of K^{**}-meson.

Appendix B

Derivation of the amplitude for $K\rho\to K\rho$

Taking poles at $\alpha_{34}=1$ and $\alpha_{56}=1$ and using the reduction formula

$$\mathrm{Res}_{\alpha_{34}=1,\alpha_{56}=1}[B_6(1-\alpha_{12},\cdots,1-\alpha_{61};1-\alpha_{234},1-\alpha_{345},1-\alpha_{456})]=B(1-\alpha_{12},1-\alpha_{234}),$$

one can get the amplitude.

The factor corresponding to a term $B(1-\alpha_{12},1-\alpha_{234})$ is

$$I_{123456}\tau_{i_3}\tau_{i_4}\tau_{i_5}\tau_{i_6}+I_{123465}\tau_{i_3}\tau_{i_4}\tau_{i_6}\tau_{i_5}+I_{124365}\tau_{i_4}\tau_{i_3}\tau_{i_6}\tau_{i_5}$$
$$+I_{124356}\tau_{i_4}\tau_{i_3}\tau_{i_5}\tau_{i_6}=(\alpha+\beta)(X+Y+Z)+\cdots, \qquad (\text{B}\cdot 1)$$

where*)

$$\alpha=\delta_{i_4i_5}\delta_{i_3i_6}-\delta_{i_3i_5}\delta_{i_4i_6},$$
$$\beta=i(\delta_{i_4i_5}\varepsilon_{i_3i_6k}+\delta_{i_3i_6}\varepsilon_{i_4i_5k}-\delta_{i_3i_5}\varepsilon_{i_4i_6k}-\delta_{i_4i_6}\varepsilon_{i_3i_5k})\tau_k,$$
$$X=\delta_{\mu_1\mu_2}(-\delta_{\mu_3\mu_5}\delta_{\mu_4\mu_6}+\delta_{\mu_3\mu_6}\delta_{\mu_4\mu_5})\prod p_i,$$
$$Y+Z=[\delta_{\mu_1\mu_3}(\delta_{\mu_2\mu_5}\delta_{\mu_4\mu_6}-\delta_{\mu_2\mu_6}\delta_{\mu_4\mu_5})-\delta_{\mu_1\mu_4}(\delta_{\mu_2\mu_5}\delta_{\mu_3\mu_6}-\delta_{\mu_2\mu_6}\delta_{\mu_3\mu_5})$$
$$-\delta_{\mu_1\mu_5}(\delta_{\mu_2\mu_3}\delta_{\mu_4\mu_6}-\delta_{\mu_2\mu_4}\delta_{\mu_3\mu_6})+\delta_{\mu_1\mu_6}(\delta_{\mu_2\mu_3}\delta_{\mu_4\mu_5}-\delta_{\mu_2\mu_4}\delta_{\mu_3\mu_5})]\prod p_i,$$

where

$$\prod p_i=p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_4^{\mu_4}p_5^{\mu_5}p_6^{\mu_6}. \qquad (\text{B}\cdot 2)$$

In the same way the coefficient for a term $B(1-\alpha_{12}, 1-\alpha_{134})$ is

$$I_{125634}\tau_{i_5}\tau_{i_6}\tau_{i_3}\tau_{i_4}+I_{126534}\tau_{i_6}\tau_{i_5}\tau_{i_3}\tau_{i_4}+I_{126543}\tau_{i_6}\tau_{i_5}\tau_{i_4}\tau_{i_3}$$
$$+I_{125643}\tau_{i_5}\tau_{i_6}\tau_{i_4}\tau_{i_3}=(\alpha-\beta)(X-Y-Z)+\cdots. \qquad (\text{B}\cdot 3)$$

Combining Eqs. (B·1) and (B·3), one gets the amplitude:

$$A=\alpha[(X+Y+Z)B(1-\alpha_{12}, 1-\alpha_{234})+(X-Y-Z)B(1-\alpha_{12}, 1-\alpha_{134})]$$
$$+\beta[(X+Y+Z)B(1-\alpha_{12}, 1-\alpha_{234})-(X-Y-Z)B(1-\alpha_{12}, 1-\alpha_{134})]. \qquad (\text{B}\cdot 4)$$

On the other hand, isospin amplitudes are given for this process,

$$\varepsilon_{i_3i_4i}\varepsilon_{i_5i_6j}(\delta_{ji}A^{(+)}+i\varepsilon_{jik}\tau_kA^{(-)})=(\delta_{i_3i_5}\delta_{i_4i_6}-\delta_{i_3i_6}\delta_{i_4i_5})A^{(+)}$$
$$+i(\delta_{i_4i_6}\varepsilon_{i_3i_5k}+\delta_{i_3i_5}\varepsilon_{i_4i_6k}-\delta_{i_3i_6}\varepsilon_{i_4i_5k}-\delta_{i_4i_5}\varepsilon_{i_3i_6k})\tau_kA^{(-)}. \qquad (\text{B}\cdot 5)$$

Comparing Eqs. (B·4) and (B·5) one has

$$A^{(\pm)}=(X+Y+Z)B(1-\alpha_{12}, 1-\alpha_{234})\pm(X-Y-Z)B(1-\alpha_{12}, 1-\alpha_{134}). \qquad (\text{B}\cdot 6)$$

This is the form of Eq. (9) in § 3. X, Y and Z are easily expressed as in the text. For example,

$$8(Y+Z)=-(p_1-p_2, p_3+p_4)(p_3-p_4, p_5+p_6)(p_5-p_6, p_1+p_2)$$
$$+(p_1+p_2, p_3-p_4)(p_3+p_4, p_5-p_6)(p_5+p_6, p_1-p_2)$$
$$-(p_1+p_2, p_3+p_4)(p_3-p_4, p_5-p_6)(p_5+p_6, p_1-p_2)$$
$$+(p_1-p_2, p_3-p_4)(p_3+p_4, p_5+p_6)(p_5-p_6, p_1+p_2)$$
$$+(p_1-p_2, p_3+p_4)(p_3-p_4, p_5-p_6)(p_5+p_6, p_1+p_2)$$
$$-(p_1+p_2, p_3-p_4)(p_3+p_4, p_5+p_6)(p_5-p_6, p_1-p_2)$$

*) We have used the coupling for $\rho\pi\pi$ as $(\pi\overset{\leftrightarrow}{\partial}_\mu\pi)\rho^\mu$.

$$+(p_1+p_2, p_3+p_4)(p_3-p_4, p_5+p_6)(p_5-p_6, p_1-p_2)$$
$$-(p_1-p_2, p_3-p_4)(p_3+p_4, p_5-p_6)(p_5+p_6, p_1+p_2).$$

Then by use of the coupling $(\pi\overleftrightarrow{\partial}_\mu\pi)\rho^\mu$ for the two $\rho\pi\pi$ vertices, this reduces to Eq. (10), rewriting the initial and final rho-meson momenta $p_3(\leftarrow p_3+p_4)$ and $p_4(\leftarrow p_5+p_6)$ respectively.

Appendix C

Derivation of the amplitude for $\pi\rho\to\pi\rho$

Taking poles at $\alpha_{12}=1$ and $\alpha_{34}=1$ in the amplitude $(K\overline{K}\pi\pi\pi\pi)$, one has an amplitude for $\pi\rho$ elastic scattering.

$$\begin{aligned}
&B(1-\alpha_{56}, 1-\alpha_{345})[I_{123456}\tau_{i_3}\tau_{i_4}\tau_{i_5}\tau_{i_6}+I_{124356}\tau_{i_4}\tau_{i_3}\tau_{i_5}\tau_{i_6}\\
&\qquad\qquad +I_{126534}\tau_{i_6}\tau_{i_5}\tau_{i_3}\tau_{i_4}+I_{126543}\tau_{i_6}\tau_{i_5}\tau_{i_4}\tau_{i_3}]\\
&+B(1-\alpha_{56}, 1-\alpha_{346})[I_{123465}\tau_{i_3}\tau_{i_4}\tau_{i_6}\tau_{i_5}+I_{124365}\tau_{i_4}\tau_{i_3}\tau_{i_6}\tau_{i_5}\\
&\qquad\qquad +I_{125634}\tau_{i_5}\tau_{i_6}\tau_{i_3}\tau_{i_4}+I_{125643}\tau_{i_5}\tau_{i_6}\tau_{i_4}\tau_{i_3}]\\
&+B(1-\alpha_{345}, 1-\alpha_{346})[I_{125346}\tau_{i_5}\tau_{i_3}\tau_{i_4}\tau_{i_6}+I_{125436}\tau_{i_5}\tau_{i_4}\tau_{i_3}\tau_{i_6}\\
&\qquad\qquad +I_{126345}\tau_{i_6}\tau_{i_3}\tau_{i_4}\tau_{i_5}+I_{126435}\tau_{i_6}\tau_{i_4}\tau_{i_3}\tau_{i_5}]\\
&=\alpha[(X+Y+Z)B(1-\alpha_{56}, 1-\alpha_{345})+(X-Y-Z)B(1-\alpha_{56}, 1-\alpha_{346})\\
&\qquad +(X+Y-Z)B(1-\alpha_{345}, 1-\alpha_{346})]\\
&\quad+\beta[(X+Y+Z)B(1-\alpha_{56}, 1-\alpha_{345})-(X-Y-Z)B(1-\alpha_{56}, 1-\alpha_{346})\\
&\qquad +(X+Y-Z)B(1-\alpha_{345}, 1-\alpha_{346})]\\
&\quad+\gamma[(X+Y+Z)B(1-\alpha_{56}, 1-\alpha_{345})-(X-Y-Z)B(1-\alpha_{56}, 1-\alpha_{346})\\
&\qquad -(X+Y-Z)B(1-\alpha_{345}, 1-\alpha_{346})]\\
&\quad+\cdots,
\end{aligned} \tag{C·1}$$

where

$$\alpha=i\delta_{i_5i_6}\varepsilon_{i_3i_4k}\tau_k, \qquad \beta=i(\delta_{i_3i_6}\varepsilon_{i_4i_5k}-\delta_{i_4i_6}\varepsilon_{i_3i_5k})\tau_k, \qquad \gamma=i(\delta_{i_4i_5}\varepsilon_{i_3i_6k}-\delta_{i_3i_5}\varepsilon_{i_4i_6k})\tau_k,$$

and

$$X=-\delta_{\mu_5\mu_6}(\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}-\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3})\prod p_j^{\mu_i},$$
$$Y=[\delta_{\mu_2\mu_5}(\delta_{\mu_1\mu_3}\delta_{\mu_4\mu_6}-\delta_{\mu_1\mu_4}\delta_{\mu_3\mu_6})-\delta_{\mu_1\mu_5}(\delta_{\mu_2\mu_3}\delta_{\mu_4\mu_6}-\delta_{\mu_2\mu_4}\delta_{\mu_3\mu_6})]\prod p_j^{\mu_i},$$
$$Z=[-\delta_{\mu_3\mu_5}(\delta_{\mu_2\mu_4}\delta_{\mu_1\mu_6}-\delta_{\mu_2\mu_6}\delta_{\mu_1\mu_4})+\delta_{\mu_4\mu_5}(\delta_{\mu_2\mu_3}\delta_{\mu_1\mu_6}-\delta_{\mu_2\mu_6}\delta_{\mu_1\mu_3})]\prod p_j^{\mu_i},$$

while isospin amplitudes are taken for this process

$$\begin{aligned}
&i\tau_{i_{12}}[\delta_{i_{34}i_{12}}\delta_{i_5i_6}A+\delta_{i_{34}i_6}\delta_{i_{12}i_5}B+\delta_{i_{34}i_5}\delta_{i_{12}i_6}C]\varepsilon_{i_3i_4i_{34}}\\
&\qquad=i\delta_{i_5i_6}\varepsilon_{i_3i_4k}\tau_k(A+B+C)+i(\delta_{i_3i_5}\varepsilon_{i_4i_6k}-\delta_{i_4i_5}\varepsilon_{i_3i_6k})\tau_kB\\
&\qquad\qquad+i(\delta_{i_3i_6}\varepsilon_{i_4i_5k}-\delta_{i_4i_6}\varepsilon_{i_3i_5k})\tau_kC\,.
\end{aligned} \tag{C·2}$$

Comparing Eqs. (C·1) and (C·2), one has

$$A=(X+Y+Z)B(1-\alpha_{56},1-\alpha_{345})+(X-Y-Z)B(1-\alpha_{56},1-\alpha_{346})$$
$$-(X+Y-Z)B(1-\alpha_{345},1-\alpha_{346}),$$
$$B=-(X+Y+Z)B(1-\alpha_{56},1-\alpha_{345})+(X-Y-Z)B(1-\alpha_{56},1-\alpha_{346})$$
$$+(X+Y-Z)B(1-\alpha_{345},1-\alpha_{346}),$$
$$C=(X+Y+Z)B(1-\alpha_{56},1-\alpha_{345})-(X-Y-Z)B(1-\alpha_{56},1-\alpha_{346})$$
$$+(X+Y-Z)B(1-\alpha_{345},1-\alpha_{346}).$$

These are Eq. (12) in § 3. X, Y and Z can be written as in the text, rewriting

$$4X=(p_5p_6)[(p_1-p_2,p_3+p_4)(p_3-p_4,p_1+p_2)-(p_1-p_2,p_3-p_4)(p_3+p_4,p_1+p_2)],$$
$$4Y=-(p_1+p_2,p_5)[(p_1-p_2,p_3+p_4)(p_3-p_4,p_6)-(p_1-p_2,p_3-p_4)(p_3+p_4,p_6)]$$
$$+(p_1-p_2,p_5)[(p_1+p_2,p_3+p_4)(p_3-p_4,p_6)-(p_1+p_2,p_3-p_4)(p_3+p_4,p_6)],$$
$$4Z=(p_3+p_4,p_5)[(p_1+p_2,p_3-p_4)(p_1-p_2,p_6)-(p_1-p_2,p_3-p_4)(p_1+p_2,p_6)]$$
$$-(p_3-p_4,p_5)[(p_1+p_2,p_3+p_4)(p_1-p_2,p_6)-(p_1-p_2,p_3+p_4)(p_1+p_2,p_6)].$$

(In § 3, we have replaced $p_5(p_6)\to p_1(p_2)$, $p_1-p_2(p_3-p_4)\to e_{p_3}(e_{p_4})$ and $p_1+p_2(p_3+p_4)\to p_3(p_4)$.)

Of course, one can easily convince oneself that these amplitudes involve natural spin complications. For example, consider at the pole $\alpha_{345}^{\omega}=1$, then this case involves a factor $X+Y$ according to the isospin symmetry in Eq. (C·2). On the other hand the pole diagram

$$K(p_1)\overline{K}(p_2)\pi(p_6)\to\rho(p_1+p_2)\pi(p_6)\to\omega(p_1+p_2+p_6=-p_3-p_4-p_5)$$
$$\to\rho(p_3+p_4)\pi(p_5)\to\pi(p_3)\pi(p_4)\pi(p_5)$$

has the same factor $X+Y$.

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