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PSO-BASED FAST OPTIMIZATION ALGORITHM FOR BROADBAND ARRAY ANTENNA BY USING THE CUBIC SPLINE INTERPOLATION

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Abstract—This paper describes a fast pattern synthesis method for a broadband array antenna using the particle swarm optimization (PSO) and cubic spline interpolation (CSI). Being an indispensable part of a high speed space-division communication system, the array antenna operates in a wide frequency band (200–400 MHz) and has stable patterns with 60-degree half power beam width (HPBW) in the whole frequency band. Firstly, by establishing a versatile objective function, the complex excitations of the circular array at the selected seven frequency points are determined via the PSO algorithm. Then, the complex excitations of the circular array at arbitrary frequency points in the whole working frequency band are calculated effectively using the CSI method. A uniform circular array with six broadband dipole elements is examined. The broadband patterns with 60-degree HPBW and the accuracy of the interpolation method are demonstrated.

1. INTRODUCTION

With the development and wide application of mobile and multimedia commutations, the user number and requirements of data transmission rate are increasing rapidly. It poses a new challenge for the capacity of communication system. Many schemes have been utilized to exploit the frequency resources to enlarge the capacity [1, 2]. The traditional schemes are Frequency Division Multiple Access (FDMA), Time

Division Multiple Access (TDMA), and Code Division Multiple Access (CDMA). Recently, Space Division Multiple Access (SDMA) [1, 2] has received more and more attentions due to its following advantages: the base station antenna of SDMA is array antenna, whose coverage area is much greater than that of other single antenna; interferences coming from other systems can be reduced significantly. SDMA employs fixed beam servicing to different users, and can be flexibly used in different coverage area. The space is divided into six coverage sectors in the presented SDMA system. As a result, the base station antenna should have a 60° half power beam width (HPBW) and adaptive main lobe direction. Many array antenna pattern synthesis methods have been studied [3, 4, 7, 8, 10, 11], however, most literatures mainly focus on single frequency or narrowband optimization. In modern communication system, frequency hopping spread spectrum is usually adopted to further increase the data transmission rate. That requires a broadband array antenna which can cover one or two times frequency band and even wider. However, the question of optimizing the broadband antenna array has not yet to be determined. Finding a fast and effective synthesis method now comes to be a new difficulty challenge.

In this paper, we present a novel method for optimization of broadband antenna array, which seeks the complex excitations (amplitudes and phases) of circular array elements in the working frequency band (200–400 MHz) for finding an adaptive shaped beam with a 60° HPBW. First, the particle swarm optimization algorithm is used to maximize the antenna gain in the main direction of desired signal and form 60° space coverage for HPBW. Then, the cubic spline interpolation (CSI) method is utilized to obtain the optimized complex excitations at arbitrary frequency points within the working frequency band.

2. PSO ALGORITHM

In 1995, Kennedy and Eberhart firstly introduced particle swarm optimization (PSO), which is a robust stochastic evolutionary computation technique based on the movement and intelligence of swarms [3–12]. The PSO algorithm is based on a social-psychological principle. Compared with other stochastic optimization techniques like genetic algorithms (GAs) and simulated annealing (SA), PSO has fewer complicated operations and fewer defining parameters, and can be implemented easily. Because of these advantages, the PSO method has attracted a wide spread attention and has been widely used in electromagnetic community, especially for array antenna analysis.

The PSO algorithm randomly initializes the locations and velocities of particles in the D-dimensional search space. Each particle flies to the target area and adjusts its position according to its own experience and the best experience of its topological neighbour. The position and velocity of the i-th particle are represented as $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$ and $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$, respectively. The personal previous position is recorded and expressed as $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$. The index of the best particle among all the particles is represented by the symbol g. The positions and velocities of particles are updated according to the following equations:

$$v_{id} = w * v_{id} + c_1 * rand() * (p_{id} - x_{id}) + c_2 * rand() * (p_{gd} - x_{id}) \quad (1)$$

$$x_{id} = x_{id} + v_{id}, 1 \leq d \leq D \quad (2)$$

where w is the inertia weight. c_1 and c_2 are acceleration constants, both of which are positive. $rand()$ denotes a random number in the range $[0, 1]$. The first part of formula (1) is the initial velocities of particles; the second part is "cognition", which expresses the cogitation of particles; the third part is "social", which expresses the registration of message and cooperation among particles.

3. ARRAY ANTENNA OPTIMIZATION

3.1. Objective Function

For a practical problem, an objective function must be defined to link the design requirements with the optimization algorithm. In the proposed SDMA communication system, the maximum gain of the desired signal direction and the 60° HPBW coverage are the key factors. By the way, a low side lobe level (SLL) is also needed to reduce the interferences from undesired directions. Considering all the above factors, the objective function is defined as follows:

$$\begin{aligned} f \; = \; & c_1 \min\{1/G_0\} + c_2 \min\{|G_0 - G_{\max}|\} + c_3 \min\{\max\{SLL\}\} \\ & + c_4 \min\{|0.5 * G_0 - G(phamax \pm 30)|\} \end{aligned} \quad (3)$$

where G_0 is the gain of the desired signal direction, $G_{\max}$ is the maximum gain for each solution. SLL and $phamax$ denote the side lobe level and the direction of the desired signal, respectively. The constants c_1, c_2, c_3 and c_4 are the adjustable weights for each part of the objective function and determined by numerical experiment.

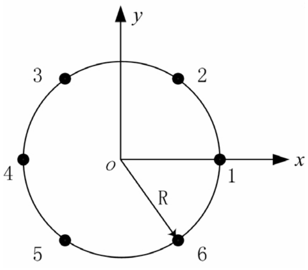


Figure 1. Geometry of a 6-element circular array.

3.2. 6-element Circular Array Antenna

The basic geometry of a 6-element circular array antenna is shown in Fig. 1. The array is composed of six broadband dipoles [13], which are identical and perpendicular to the array plane. The array factor of the circular array is given in the following:

$$AF(\theta,\varphi)=\sum_{n=1}^{N} I_n e^{j[ka\sin\theta\cos(\varphi-\varphi_n)+\alpha_n]} \tag{4}$$

where N is the number of the circular array elements, $\theta\in[-\pi/2,\pi/2]$, $\varphi\in[0,2\pi]$, $\phi_n=2(n-1)\pi/N$. I_n, α_n are amplitude and phase of each element, respectively. $X=(I_1,I_2,\ldots,I_N,\alpha_1,\alpha_2,\ldots,\alpha_N)$ is the vector variable of PSO. For an N-element circular array antenna, X is $2N$ dimensional vector.

Due to the symmetrical configuration of the array, if the required radiation pattern is also symmetrical, only half of the complex excitation coefficients have to be optimized. A smaller excitation vector consequently leads to a faster code.

4. CUBIC SPLINE INTERPOLATION

In the frequency hopping spread spectrum communication system, the hopping frequency interval is usually chosen as 0.5 MHz or even less. In our study, the working frequency covers from 200 MHz to 400 MHz, which has at least 400 frequency points to be optimized. Performing such huge number of optimization is an extremely complicated work. This requires too much memory and time to render it in real time. In order to adapt to the trend of high speed and large capacity communication, a fast calculation method should be utilized. In this paper, we present a novel method. Firstly, seven frequency points are selected, which distribute in the working frequency band. Secondly, the amplitudes and phases are optimized using the proposed method in Section 2 at each selected frequency point, respectively. Finally, the amplitudes and phases at arbitrary frequency points are calculated out using the cubic spline interpolation method described as follows.

Cubic spline interpolation (CSI) is an effective method that offers true continuity between the segments [14, 15]. We take a series of points $[x_i, y_i]$ $(i = 0, 1, 2, \ldots, n)$ for the function $y = f(x)$. That makes $n + 1$ points and n intervals between them. The assumed form of the cubic polynomial curve fit for each segment is:

$$s_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i i = 0, 1, 2, \ldots, n \quad (5)$$

The routine fits $n$equations subjecting to the boundary conditions of $n+1$ data points. The liner boundary conditions are given as follows:

1) The cubic polynomial matches the values of the table at both ends of the interval:

$$s_i(x_i) = y_i, \quad s_{i+1}(x) = y_{i+1} \quad (6)$$

2) In order to make the interpolation as smooth as possible, the first and second derivatives should be continuous:

$$s'_{i-1}(x_i) = s'_i(x_i) \quad s''_{i-1}(x_i) = s''_i(x_i), \quad i = 1, 2, \ldots, n - 1. \quad (7)$$

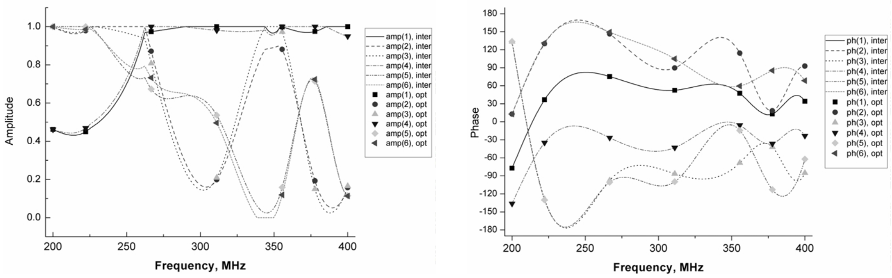
3) There are some standard choices in the following:

$$s''_0(x_0) = 0, \quad s''_{n-1}(x_n) = 0 \quad \text{(natural)} \quad (8)$$

$$s'_0(x_0) = f'(x_0), \quad s'_{n-1}(x_n) = f'(x_n) \quad \text{(clamped)} \quad (9)$$

The choice of (8) or (9) depends on the practical applications, and the natural condition is used here.

In this paper, we take seven frequency points into account. The amplitudes and phases have been optimized by PSO at each point. Cubic spline interpolation (CSI) is implemented using FORTRAN code program, the calculation results are shown in Fig. 2. Fig. 2(a) depicts the amplitudes of the array elements versus frequency, and all the values of them are normalized to 1 at each frequency point; Fig. 2(b) depicts the phases of the complex excitations versus frequency, which are set from −180 to 180 degree. From these figures, we can get complex excitations (amplitudes: amp(1,. . . ,6) and phases: ph(1,. . . ,6)) for arbitrary frequency points in the working frequency band. And, it is clearly seen that the seven selected optimized results fall on the curves of interpolation, which proves that the interpolation method is correct and effective.



(a) (b)

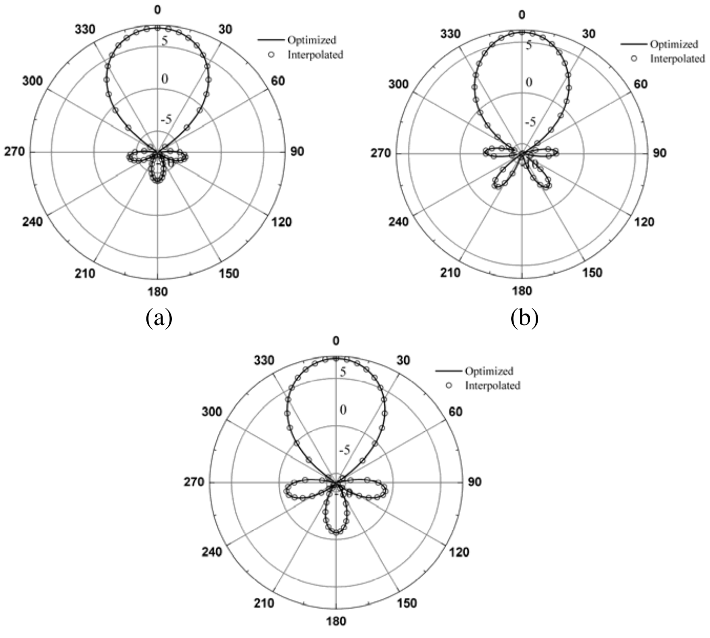
Figure 2. Complex excitations of circular array antenna. (a) Amplitudes of the circular array elements. (b) Phases of the circular array elements.

5. ANTENNA ARRAY DESIGN AND NUMERICAL RESULTS

A 6-element circular array antenna working in a broad-band frequency (200–400 MHz) is studied here. The radius of the circular array R is selected as 0.32 meter. The desired user direction is considered at $\theta = 90°$ and $\phi = 0°$. The HPBW is designed as 60° to suit the requirements of SDMA.

The performance of the presented arrays is optimized by the PSO algorithm using FORTRAN code program. The array elements are identical and horizontal omnidirectional. The mainlobe may points to the arbitrary direction, and the radiation pattern is only symmetrical in the specifically condition. Thus, all complex excitations coefficients are selected to be optimized. The vector variable for the PSO algorithm is $X = (I_1, I_2, I_3, I_4, I_5, I_6, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)$. The inertia weight w is set to 0.7. The acceleration constants c_1 and c_2 are both 2.0. The variation range of I is from 0 to 1.0 and α is from -180 to 180 degree. Seven frequency points including starting and ending frequency and another five points among them are selected for optimization. The radiation pattern of the starting and ending frequency are depicted in Figs. 3(a) and 3(b). From the two pictures, it can be found easily that the optimized array antenna has a radiation pattern with 60° HPBW and the main lobe direction points to the desired user. Furthermore, the SLL of the presented array antenna is quite low. The radiation patterns of the other five points are similar to Fig. 3 and are not given here for the sake of simplicity.

By using the optimized results at the seven selected frequency points, the CSI method is implemented to calculate out the complex



(a) (b)

(c)

Figure 3. Radiation patterns for 6-element circular array antenna, (a) 200 MHz, (b) 400 MHz, (c) 310 MHz.

excitations at arbitrary frequency points in the working frequency band. Figs. 3(a) and 3(b) give comparisons between the optimized results and the interpolated results at 200 MHz and 400 MHz. From the figures, we can see that both of them have preferable good agreement. In order to further prove the correctness and efficiency of the CSI method, arbitrary sampling frequency points are chosen, and taking 310 MHz as an example. Good agreement is also achieved between optimization and interpolation, as shown in Fig. 3(c).

6. CONCLUSION

A novel method for optimization of broadband antenna array in space-division communication system is proposed in this paper. The PSO algorithm is firstly used to optimize the amplitudes and phases of a 6-element circular array antenna to form adaptive shaped beam at several frequency points. By utilizing the cubic spline interpolation method, fast broadband array antenna pattern synthesis becomes possible. A 6-element circular array antenna is constructed and satisfactory results are obtained. This method can be effectively adopted in smart antenna system, especially in space-division communication system.

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