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Public Debt as Private Wealth

PUBLIC DEBT AS PRIVATE WEALTH
SOME EQUILIBRIUM CONSIDERATIONS

EKKEHART SCHLICHT

ABSTRACT. Government bonds are interest-bearing assets. Increasing public debt increases income, wealth, and consumption demand. The smaller government expenditure is, the larger consumption demand must be in equilibrium, and the larger must be public debt. Conversely, lower public debt implies higher government spending and taxation.

Public debt plays, thus, an important role in establishing equilibrium. It distributes output between consumers and government. In case of insufficient demand, a larger public debt entails higher private consumption and less public spending. If upper bounds on public debt are introduced (as in the Maastricht treaty), such constraints place lower bounds on taxation and public spending or may even rule out the existence of macroeconomic equilibrium altogether.

Domar (1944) and Gehrels (1957) have discussed similar issues in an unemployment setting. In contrast, this note considers the full employment case and looks at adjustments in debt, taxes and government spending that sustain full employment. The explicit modelling of some adjustment processes that have not been considered in the earlier contributions leads to somewhat different and, in a sense, more "debt-friendly" results.

INTRODUCTION

Government bonds are interest-bearing assets. As such, they constitute part of private wealth. This note considers the case of a closed economy that grows at a certain natural rate. With a given rate of interest, weak consumption demand necessitates a certain amount of public debt to establish long-term equilibrium at full employment. It turns out that there exists a trade-off between public debt on the one hand and government spending and taxation on the other: The higher the level of public debt, the lower will be taxation and government spending in equilibrium. As a consequence, *upper* bounds on public debt (as in the Maastricht treaty) imply *lower* bounds on government spending and taxation.

The issue of public debt is often discussed in a setting where full employment prevails without any need for without public debt, as in Barro (1974). In such cases, there is obviously and by assumption no purpose public debt might serve. (The idea that public debt has no impact at all on macroeconomic performance is reminiscent, though, of the metallists' conclusion that, due to backward induction,

pure paper money cannot exist (Moll, 1922).) The argument presented here considers the case of full employment but introduces the restriction of a fixed rate of interest that precludes full employment without government intervention. Government must boost demand either by increased government spending or by reduced taxation. It is shown that such policies lead to stable outcomes with stable shares of government spending and government debt and a stable rate of taxation. The case of a fixed rate of interest may apply if the rate of interest is so low that it cannot be reduced any further, or if the central bank stabilizes the rate of interest by a kind of Taylor rule, or if the rate of interest is to be kept at a minimum level in order to prevent dynamic inefficiency.

Other contributions, most notably Domar (1944) and Gehrels (1957), have discussed similar issues in a framework of unemployment where Keynesian multipliers are of relevance. The following considerations pursue similar questions in a full employment setting: Government succeeds in maintaining full employment by adjusting taxes, government expenditure, and public debt appropriately while observing its budget constraint, and a globally stable steady state obtains.

The argument is presented in terms as simple as possible. A closed economy is assumed. The rate of investment is just right (given the prevailing rate of interest, wages, population growth, and technical progress), and private income and wealth has a positive effect on consumption demand. The direct effect of wealth on consumption is formalized in the spirit of Pigou (1947) and Gehrels (1957) by adding wealth as an argument in the consumption function. This is an entirely conventional approach, which has been used in many expositions such as Blinder and Solow (1974, 4, 51) or Branson (1989, 525-30). The case of a constant propensity to save is covered as a special case. Here the interest income generated by public debt is of relevance only, and the wealth effect is absent, yet the overall result is maintained.

In Section 1, the algebra is outlined. In Section 2, the case of fixed government spending with a variable tax rate is analyzed, and in Section 3 the case of a constant tax rate with variable government spending is discussed. It is proved that the dynamics lead to stable outcomes in both cases. Section 4 discusses the economic function of public debt and considers cases of zero government spending and zero taxation. Section 5 discusses the effects of upper bounds on public debt of the type enforced by the Maastricht treaty in Europe. Section 6 discusses the effect of the rate of interest briefly. Section 7 gives some numerical illustration for the case of a constant savings rate and comments on the difference between the present analysis and that of Domar (1944). The note concludes with some cautionary remarks.

1. The model

Consider a simple macroeconomic model of a closed economy with a government that levies an income tax with a proportionate rate τ. Net product at full employment (potential value added, the sum of wages and profits minus depreciation) is

X. The purpose of the tax is to finance government expenditure G and interest payments for government debt D. The purpose of government expenditure, debt, and taxation is to provide public goods and maintain full employment. We write:

$$G = X - I - C \tag{1}$$

with I denoting net investment and C denoting consumption.

We assume an economy growing at the natural rate n. The rate of interest i is at first assumed constant. As the rate of interest determines capital intensity, it determines capital productivity.[1] With a constant rate of interest, we have, thus, a constant rate of net capital productivity (as the inverse of the capital-output ratio):

$$x \quad := \quad \frac{X}{K}. \tag{2}$$

The growth of the capital stock is given by

$$\dot{K} = I \tag{3}$$

with I as net investment. Full employment requires the capital stock to grow at the rate n, implying investment demand

$$I = nK. \tag{4}$$

Regarding consumption, we follow Gehrels (1957), Clower and Johnson (1968), and others and take consumption as a function of current disposable income Y_D and wealth V:

$$C = C(Y_D, V), \quad C, Y_D, V \geq 0 \tag{5}$$

with consumption increasing in income and wealth:

$$0 < \frac{\partial C}{\partial Y_D} < 1, \quad \frac{\partial C}{\partial V} \geq 0. \tag{6}$$

This consumption function may be rationalized in various ways. One alternative would be to assume with Clower and Johnson that wealth enters the utility function directly; another would be that wealth affects the inter-temporal budget constraint of households, and this impinges on consumption, as in Modigliani and Brumberg (1954), or, more recently, in Woodford (1998). In this sense, the specification (5) is fairly flexible.

The consumption function is assumed linear homogeneous. Using this property, we can re-write it as

$$C = \varphi(\omega) V \tag{7}$$

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[1] A special case would be the standard neoclassical case: if $f(k)$ denotes output per efficiency unit of labor as a function of capital per efficiency unit of labour, marginal productivity theory would require $f'(k) = i$ with $\frac{dk}{di} = \frac{1}{f''} < 0$ under the standard assumption $f'' < 0$. As capital productivity is $x = \frac{f(k)}{k}$ with $\frac{dx}{dk} = -\frac{1}{k}\left(\frac{f}{k} - f'\right) < 0$ under standard assumptions, capital productivity would be an increasing function of the rate of interest. The case of a constant rate of interest implies a constant capital productivity, therefore. The case of a varying rate of interest - and an entailed varying capital productivity - is briefly dealt with in Section 6.

with

$$\omega \quad := \quad \frac{Y_D}{V} \tag{8}$$

as the income-wealth ratio and

$$\varphi(\omega) \quad := \quad C(\omega, 1) \geq 0, \quad \varphi' > 0 \text{ for } \omega \geq 0 \tag{9}$$

as the consumption-wealth ratio. This implies together with (6)

$$\varphi' = \frac{\partial C}{\partial Y_D}, \qquad 0 < \varphi' < 1, \tag{10}$$

$$\varphi - \varphi'\omega = \frac{\partial C}{\partial V}, \qquad \varphi - \varphi'\omega \geq 0. \tag{11}$$

Note that the case of a constant average propensity to consume, as discussed by Domar (1944, 801-2), is covered with

$$\varphi(\omega) = \mu \cdot \omega, \quad 0 < \mu < 1 \tag{12}$$

and μ as the propensity to consume.

Private wealth V is the sum of the capital stock K and financial wealth in the form of government debt D:

$$V = K + D. \tag{13}$$

The returns to real and financial wealth are the same and are given by the rate of interest i. Hence profit income is

$$P = iK, \tag{14}$$

wage income is

$$W = X - iK, \tag{15}$$

and the interest income from government debt is

$$iD. \tag{16}$$

In order to have positive wage income, the rate of interest must satisfy the condition

$$i < x. \tag{17}$$

Total net income is the sum of wage income, profit income, and income from government debt:

$$Y = X + iD. \tag{18}$$

Disposable income is income minus taxes, and therefore

$$Y_D = (1 - \tau)(X + iD). \tag{19}$$

Tax receipts are

$$T = \tau(X + iD). \tag{20}$$

The budget constraint of government is given by the condition that government expenditure plus interest payments must be equal to tax revenue and the increase in government debt:

$$G + iD = T + \dot{D}. \tag{21}$$

4

For the subsequent argument it is convenient to express income, disposable income, government expenditure, and government debt relative to the capital stock, using the corresponding lower-case symbols:

$$y := \frac{Y}{K}, \quad y_D := \frac{Y_D}{K}, \quad g := \frac{G}{K}, \quad d := \frac{D}{K}. \tag{22}$$

As the capital stock grows at the natural rate n, we have $\dot{D} = \left(\dot{d} + nd\right)K$ and (20) and (21) imply the budget constraint of government

$$\tau y = g + id - nd - \dot{d}. \tag{23}$$

With (22) and (23), equations (19) and (8) can be written as

$$y_D = (1 - \tau)(x + id) \tag{24}$$
$$= x + nd - g + \dot{d}, \tag{25}$$
$$\omega = \frac{(1 - \tau)(x + id)}{1 + d} \tag{26}$$
$$= \frac{x + nd - g + \dot{d}}{1 + d}. \tag{27}$$

For the consumption-capital ratio

$$c := \frac{C}{K} \tag{28}$$

we obtain together with (7)

$$c = \varphi\left(\frac{(1 - \tau)(x + id)}{1 + d}\right)(1 + d). \tag{29}$$

or alternatively

$$c = \varphi\left(\frac{x + nd - g + \dot{d}}{1 + d}\right)(1 + d), \tag{30}$$

depending on whether we wish to parametrize by government expenditure g or the tax rate τ.

Collecting terms, we can re-write (1) in two alternative but equivalent ways:

$$\left(\varphi\left(\frac{(1 - \tau)(x + id)}{1 + d}\right) + n\right)(1 + d) - (1 - \tau)(x + id) + \dot{d} = 0, \tag{31}$$

$$\varphi\left(\frac{x + nd - g + \dot{d}}{1 + d}\right)(1 + d) + n + g = x. \tag{32}$$

Equation (31) allows for an analysis of the evolution of government debt under the assumption that the tax rate is constant and the government runs surpluses or deficits such as to maintain full employment; equation (32) permits an analysis of the dynamics of public debt under the assumption that government expenditure is kept constant and the tax rate is adjusted to maintain full employment.

Equations (31) and (32) both imply the equilibrium condition

$$\varphi\left(\bar{\omega}\right) + n = \bar{\omega} \tag{33}$$

with $\bar{\omega}$ as the *equilibrium income-wealth ratio.*

Proposition 1. *The equilibrium income-wealth ratio is an increasing function of the growth rate and independent of other variables such as the rate of government spending, taxation, the rate of interest, or capital productivity.*

Note that (33) reduces to $\bar{\omega} = \frac{n}{1-\mu}$ for the case of a constant propensity to consume μ.

2. FIXED GOVERNMENT SPENDING

Consider first the case in which government controls demand by adjusting the tax rate while keeping government spending g fixed over time. The following proposition states that in case demand is insufficient with fully tax financed government spending in absence of public debt, government can maintain full employment by adjusting taxes. Under this policy, public debt will build up and will eventually grow at the natural rate at which income grows.

Proposition 2. *In case of insufficient demand at zero debt*

$$\varphi\left(x - g\right) + n + g < x \tag{34}$$

there exists a unique equilibrium debt level $\bar{d} > 0$ which is approached if the government pursues a full employment policy through tax adjustment.

Proof. Equilibria are given by (32) with $\dot{d}$ put to zero:

$$\varphi\left(\frac{x + nd - g}{1 + d}\right)\left(1 + d\right) + n + g = x. \tag{35}$$

By assumption the left-hand side is smaller than the right-hand side for $d = 0$. With $d \uparrow \infty$, we have $\varphi \longrightarrow \varphi\left(n\right)$ and the left-hand side becomes arbitrarily large. For continuity reasons there must exist an equilibrium debt level $d = \bar{d} > 0$ such that (35) is satisfied.

Equation (32) can be solved for $\dot{d}$, and the derivative with respect to d can be evaluated at $\dot{d} = 0$:

$$\left.\frac{\partial \dot{d}}{\partial d}\right|_{\dot{d}=0} = -\frac{\varphi' n + \left(\varphi - \varphi'\omega\right)}{\varphi'} < 0. \tag{36}$$

This establishes stability and uniqueness since $\dot{d}$, considered as a function of d, must pass the abscissa with negative slope which can occur only once. $\qquad \square$

With (27), (33), and (35), the equilibrium debt rate $\bar{d}$ can be expressed in terms of the equilibrium income-wealth ratio as

$$\bar{d} = \frac{x - g - \bar{\omega}}{\bar{\omega} - n} > 0 \tag{37}$$

with

$$\frac{d\bar{d}}{dg} = -\frac{1}{\bar{\omega} - n} < 0 \tag{38}$$

where $\bar{\omega} > n$ is implied by (33). (Note that the numerator in (37) is positive because of (34) and (10)).

The corresponding equilibrium tax rate is

$$\bar{\tau} = \frac{g(\bar{\omega} - n) + (x - g - \bar{\omega})(i - n)}{x(\bar{\omega} - n) + i(x - g - \bar{\omega})} \tag{39}$$

with

$$\frac{d\bar{\tau}}{dg} = \frac{(x - i)(\bar{\omega} - n)\bar{\omega}}{(i(x - g - \bar{\omega})x(\bar{\omega} - n))^2} > 0 \tag{40}$$

Putting things together we obtain

Proposition 3. *Equilibrium debt $\bar{d}$ is decreasing in government spending g and the equilibrium tax rate $\bar{\tau}$ is increasing in government spending g.*

Note that

$$1 - \bar{\tau} = \frac{(x - g - n)\bar{\omega}}{(x - i)(\bar{\omega} - n) + (x - g - n)i} > 0. \tag{41}$$

The equilibrium tax rate will thus always be below 100% as long as there is insufficient demand without government debt, *viz.* as long as government spending satisfies (34).

Note further that the equilibrium tax rate will always be positive if the rate of interest exceeds the growth rate ($i > n$) but may be negative if the rate of interest is smaller ($i < n$) and government spending is sufficiently small. In this case, equilibrium would require a debt-financed tax subsidy. More specifically, in the case of zero government expenditure ($g = 0$), equations (37) and (39) specialize to

$$\bar{d}\big|_{g=0} = \frac{x - \bar{\omega}}{\bar{\omega} - n} > 0, \tag{42}$$

$$\bar{\tau}\big|_{g=0} = \frac{(x - \bar{\omega})(i - n)}{x(\bar{\omega} - n) + i(x - \bar{\omega})} \tag{43}$$

and establish positive debt together with a tax rate that is positive for $i > n$ and negative for $i < n$ in equilibrium.

3. A CONSTANT TAX RATE

Consider next the case of a constant tax rate. Equation (31) defines a differential equation for relative government debt:

$$\dot{d} = \left(\frac{(1 - \tau)(x + id)}{1 + d} - \varphi\left(\frac{(1 - \tau)(x + id)}{1 + d}\right) - n\right)(1 + d). \tag{44}$$

The following proposition states that in case demand is insufficient for some sufficiently high tax rate τ and government expenditure $g = \tau(x + id)$ equal to tax receipts, there exists a unique stable equilibrium level of relative government debt

which will eventually be reached if government maintains full employment by adjusting its spending.

Proposition 4. *In case of insufficient demand*

$$\varphi\left(\left(1-\tau\right)x\right)+n+\tau x < x \tag{45}$$

and a sufficiently high rate of taxation τ there exists a unique and globally stable equilibrium debt level (given as a solution to (44)) that will be approached if government pursues a full employment policy by adjusting government spending.

Proof. Write

$$\omega = \frac{\left(1-\tau\right)\left(x+id\right)}{1+d} \tag{46}$$

as in (26) and note

$$\frac{\partial\omega}{\partial d} = -\frac{1-\tau}{1+d}\left(x-i\right) < 0. \tag{47}$$

From (44) and (46) we conclude

$$\dot{d}\left\{\begin{matrix}\leqq\\\geqq\end{matrix}\right\}0 \Leftrightarrow \varphi\left(\omega\right)-\omega\left\{\begin{matrix}\geqq\\<\end{matrix}\right\}n. \tag{48}$$

Regarding existence of an equilibrium we must look at a solution ω to $\varphi\left(\omega\right)-\omega = n$. At $d=0$ we have $\omega=\left(1-\tau\right)$ and (45) and (48) imply $\dot{d}>0$ at $d=0$. For $d\to\infty$, we have $\omega\to\left(1-\tau\right)i$. If the tax rate is sufficiently high (say $\tau>\frac{i-n}{i}$) we obtain

$$\left(1-\tau\right)i - \varphi\left(\left(1-\tau\right)i\right) \quad < \quad n \tag{49}$$

which implies $\dot{d}<0$ for sufficiently large government debt d. For continuity reasons there exists an equilibrium debt $\bar{d}$ such that $\dot{d}=0$. As ω is decreasing in debt and $\varphi\left(\omega\right)-\omega$ is decreasing in ω, we have $\dot{d}>0$ for $d<\bar{d}$ and $\dot{d}<0$ for $d>\bar{d}$. This establishes stability and uniqueness. □

As in the case of fixed government spending discussed in the previous section, equilibrium debt is characterized by the unique income-wealth ratio $\bar{\omega}$ defined in (33), and the equilibrium debt rate $\bar{d}$ can be expressed as

$$\bar{d} = \frac{\left(1-\tau\right)x-\bar{\omega}}{\bar{\omega}-\left(1-\tau\right)i} \tag{50}$$

with

$$\frac{d\bar{d}}{d\tau} = -\frac{\left(x-i\right)\bar{\omega}}{\left(\bar{\omega}-\left(1-\tau\right)i\right)^{2}} < 0. \tag{51}$$

The corresponding equilibrium rate of government spending $\bar{g}$ is

$$\bar{g} = \frac{\left(1-\tau\right)\left(x-i\right)\bar{\omega}}{\left(x-n\right)\omega+\left(1-\tau\right)\left(n-i\right)x} \tag{52}$$

with

$$\frac{d\bar{g}}{d\tau} = \frac{\left(x-i\right)\left(x-n\right)\bar{\omega}^{2}}{\left(\left(x-n\right)\omega+\left(1-\tau\right)\left(n-i\right)x\right)^{2}} > 0. \tag{53}$$

Proposition 5. *Equilibrium debt $\bar{d}$ is decreasing in the tax rate τ and equilibrium government spending $\bar{g}$ is increasing in the tax rate τ.*

For a zero tax rate, equations (50) and (52) specialize to

$$\bar{d}\big|_{\tau=0} = \frac{x - \bar{\omega}}{\bar{\omega} - i} \tag{54}$$

and

$$\bar{g}\big|_{\tau=0} = \frac{(x - i)\,\bar{\omega}}{(x - n)\,\omega + (n - i)\,x}. \tag{55}$$

Proposition 6. *A zero tax rate requires positive debt and positive government spending in equilibrium.*

4. THE FUNCTION OF PUBLIC DEBT

The two policies discussed in the previous sections (controlling the economy either by fixing the tax rate and adjusting government spending, or by fixing government spending and adjusting taxation) are basically equivalent: Ultimately, we will have $C + I + G = X$ and

$$c + n + g = x. \tag{56}$$

If, as in the preceding argument, a fixed level of investment is assumed, the government decides on the ratio c/g of private consumption to public spending, either by selecting g directly or by selecting the corresponding tax rate τ. If the level of government spending is decided on, the tax level serves to adjust consumption to $x - n - g$. This usually implies a changing debt level until equilibrium is reached. By fixing a tax rate and by adjusting government spending, the same steady state will be obtained.

The relation between the tax rate τ, government spending g, and equilibrium debt $\bar{d}$ follows from propositions 3 and 5: Selecting an increased rate of government spending implies an increased tax rate in equilibrium, and vice versa, and selecting an equilibrium with a higher rate of government spending and a matching tax rate *decreases* equilibrium government debt $\bar{d}$.

Public debt serves an important function in economic co-ordination: It controls the ratio of private consumption to public spending. A higher share of private consumption implies a higher rate of public debt in equilibrium, associated with a lower level of public spending.

In the case of insufficient demand presupposed throughout, there is still the possibility to go either for zero taxation ($\tau = 0$) or for zero government spending ($g = 0$). These two policies are compatible if and only if the rate of interest coincides with the growth rate. Otherwise these two policies are mutually exclusive. Regarding their implications for public debt, equations (42) and (54) imply

$$\bar{d}\big|_{\tau=0} - \bar{d}\big|_{g=0} = \frac{x - \bar{\omega}}{(\bar{\omega} - i)\,(\bar{\omega} - n)}\,(i - n). \tag{57}$$

Proposition 7. *Equilibrium debt will be higher with zero taxation than with zero government spending if and only if the rate of interest exceeds the growth rate.*

5. MAASTRICHT CONSTRAINTS

The Maastricht treaty places two constraints on public debt: One is that the share of government debt in GDP is not to exceed a certain threshold α, and the other requires the growth of public debt not to exceed a certain fraction β of income. Assuming a rate of depreciation δ, gross product is $X + \delta K$, and the constraints can be written as

$$d \leq \alpha(x + \delta), \tag{58}$$

$$\dot{D} \leq \beta(x + \delta)K. \tag{59}$$

Equation (58) gives the *level restriction*, and (59) gives the *increment restriction*. In the Maastricht treaty the constants are fixed at $\alpha = 0.6$ and $\beta = 0.03$, respectively. Consider what happens if one or the other restriction is binding.

1. The level restriction. Consider the level restriction (58) first. If equilibrium debt satisfies

$$\bar{d} \leq \alpha(x + \delta), \tag{60}$$

equilibrium can be attained; otherwise it is ruled out. As equilibrium debt is inversely related both to government spending and the tax level (proposition 3) government may raise government spending and taxes in tandem in order to meet the equilibrium constraint. The level restriction places *lower* bounds on government spending and taxation:

$$g \geq x - \omega - (x + \delta)(\bar{\omega} - n)\alpha, \tag{61}$$

$$\tau \geq \frac{x - \bar{\omega} - (x + \delta)(\bar{\omega} - i)\alpha}{x(1 + i\alpha) + i\alpha\delta}. \tag{62}$$

These lower bounds *increase* if the level constraint is tightened (reduced). In this sense, a tightening of the Maastricht level constraint *enforces* higher government spending and taxation in equilibrium. This occurs because increased government spending entails lower consumption spending and lower public debt.

With regard to dynamic adjustment we have to consider several cases:

Case 1. Equilibrium debt $\bar{d}$ and initial debt d_0 both satisfy the Maastricht level constraint. In this case, the level constraint will not bind and adjustment will remain unaffected and will remain stable.

Case 2. Equilibrium debt $\bar{d}$ exceeds the Maastricht level constraint and initial debt d_0 satisfies the Maastricht level constraint. In this case, the government must *raise* government spending and taxation in order to obtain the state of affairs of case 1, and adjustment proceeds in the same way. If there exists, for some level of government spending, an equilibrium, it can be reached in this manner.

Case 3. Equilibrium debt $\bar{d}$ satisfies the Maastricht level constraint and initial debt d_0 exceeds the Maastricht level constraint. In this case, debt must be reduced. The change

of debt is given by the government's budget constraint (23) and is to be negative, say, $-r$:

$$\dot{d} = g + ((1 - \tau) i - n) d - \tau x = -r. \tag{63}$$

This must be consistent with the differential equations (31) and (32), and this gives the implied rates of taxation and government expenditure. Replacing $\dot{d}$ by $-r$ in (31) implies

$$\frac{\partial \tau}{\partial r} = \frac{1}{(x + id) \varphi'} > 0. \tag{64}$$

In a similar way, (32) implies

$$\frac{\partial g}{\partial r} = \frac{\varphi' - n}{1 - \varphi'} \tag{65}$$

which will be positive for the relevant case $\varphi' > n$ as well. *Hence government will have to increase both spending and taxation* in order to reduce the debt level to the Maastricht requirement while maintaining full employment. At the same time, the associated equilibrium debt level will be reduced. In the end, the Maastricht level constraint will be met and adjustment can proceed as in Case 1.

Case 4. Neither equilibrium debt $\bar{d}$ nor initial debt d_0 satisfy the Maastricht level constraint. In this case, debt reduction would be required as in Case 3. This would involve increasing government spending and taxation, and this would decrease equilibrium debt. In the end, an equilibrium will be attained provided the constraint is not so tight as to rule out equilibrium.

2. The increment restriction. Consider the increment constraint next. From the definition of d as D/K we have $\dot{D} = \left(\dot{d} + nd \right) K$. In equilibrium ($\dot{d} = 0$) the increment constraint (59) reads

$$\bar{d} \leq \frac{\beta}{n} (x + \delta). \tag{66}$$

The increment constraint is, with regard to equilibrium, equivalent to some level constraint. If $\alpha n = \beta$ holds true, the constraints (58) and (59) amount to the same. For the Maastricht case with $\alpha = 0.6$ and $\beta = 0.03$, a growth rate of $n = .05$ would define this break-even point. The level constraint will be binding in equilibrium for growth rates below five *per cent.*, and the increment constraint will be binding for higher growth rates.

Regarding adjustment, the increment constraint reads

$$\dot{d} \leq \beta (x + \delta) - nd. \tag{67}$$

If it is binding, it eventually enforces a debt level $\frac{\beta}{n} (x + \delta)$. If the equilibrium debt level is below this threshold, the increment constraint ceases to bind and adjustment to the equilibrium debt level obtains. If the equilibrium debt level is larger, government spending and taxation must be increased such that the equilibrium debt level meets the equilibrium increment constraint (66).

In conclusion, the two Maastricht constraints both constrain the level of public debt. At best, they do not bind. Otherwise they enforce high government spending and taxation or rule out equilibrium.

A note to the purists: In case of insufficient demand, any Maastricht constant of zero rules out the existence of equilibrium for sure, as both $\alpha = 0$ and $\beta = 0$ enforce $\bar{d} = 0$.

Note that all these considerations have been made under the assumption that full employment is maintained throughout. Additional formidable problems occur with regard to dynamic adjustment if unemployment is taken into account. The Maastricht treaty is not geared to the potential output x but rather to realized output x^r which may be lower. In case of a negative demand shock, the Maastricht level constraint would thus force a reduction in government spending or an increase in taxation, driving aggregate demand further down, and the Maastricht increment constraint would enforce a reduction in government borrowing, a reduction of government spending and an increase in taxation. While the full-employment policies pursued in absence of the Maastricht constraints would induce a stable adjustment, the Maastricht constraints may induce instability in this sense, and it will hurt future generations by reducing capital formation and on-the-job training. All in all, the Maastricht treaty seems not to be a strike of genius.

6. THE RATE OF INTEREST AND PUBLIC DEBT

Up to now, a given rate of interest has been presupposed. This may be adequate in case that the interest rate has reached such a low level that it cannot be reduced anymore, or that it is not advisable from an allocational point of view to reduce it further. (If the rate of interest i is below the natural rate n, this can be interpreted as implying dynamic inefficiency, for instance. Note also that modern monetary policy tends to stabilize the real rate of interest at a certain level.)

We may ask, however, what may happen if the rate of interest can be changed. The easiest way is to think about a neoclassical growth model where the rate of interest is positively related to capital productivity[2]:

$$x = x(i), \quad x' > 0. \tag{68}$$

An increase in i reduces capital intensity and increases capital productivity.

Recall that the equilibrium income-wealth ratio $\bar{\omega}$, as defined by (33), is to be constant in equilibrium, independently of i. As $x = x(i)$, equation (50) defines the equilibrium debt ratio $\bar{d} = d(i)$ as a function of i:

$$d(i) = \frac{(1 - \tau) x - \bar{\omega}}{\bar{\omega} - (1 - \tau) i}. \tag{69}$$

Its derivative is

---

[2]See footnote 1

$$
\begin{aligned}
d'(i) &= \frac{(1-\tau)\left(x'+\bar{d}\right)}{\bar{\omega}-(1-\tau)\,i} \\
&= \frac{\left(1+\bar{d}\right)\left(x'+\bar{d}\right)}{x-i} > 0.
\end{aligned}
\tag{70}
$$

Hence equilibrium debt $\bar{d} = d(i)$ is an increasing function of the rate of interest, given any rate of taxation τ.

With a varying rate of interest it is not very appropriate, though, to study the effect of such changes on the debt-capital ratio d, because an increasing rate of interest goes along with a decrease in effective capital intensity and an increase in capital productivity. We may be more interested in the share of public debt in the net product x, which will be denoted by q:

$$
q(i) \quad := \quad \frac{d(i)}{x(i)}.
\tag{71}
$$

This implies

$$
\begin{aligned}
\frac{q'}{q} &= \frac{d'}{d} - \frac{x'}{x} \\
&= \frac{(1-\tau)}{\bar{\omega}-(1-\tau)\,i} + \frac{\bar{\omega}x'}{\left((1-\tau)\,x-\bar{\omega}\right)x} > 0.
\end{aligned}
\tag{72}
$$

With a fixed tax rate, the equilibrium share of government debt in income will increase with an increasing rate of interest.

Next turn to the case of fixed government spending and a variable tax rate. The equilibrium income-wealth ratio $\bar{\omega}$ implies

$$
\bar{d} = \frac{x-g-\bar{\omega}}{\bar{\omega}-n}.
\tag{73}
$$

As x is increasing in i, equilibrium debt $\bar{d}$ will increase with an increase in the rate of interest if government spending is kept constant. With fixed government spending, the equilibrium share of debt in income is

$$
q = \frac{x-g-\bar{\omega}}{(x-g-n)\,\bar{\omega}}
\tag{74}
$$

with

$$
\frac{\partial q}{\partial x} = \frac{\bar{\omega}-n}{(x-n-g)^2\,\bar{\omega}} > 0.
\tag{75}
$$

As x is increasing in i, an increase in the rate of interest increases the share of debt in income if the share of government spending in production is kept constant.

If public debt is required to meet some constraints, as in the Maastricht treaty, a varying rate of interest may enforce changes in government spending and taxation which appear quite arbitrary from an efficiency point of view. Consider the case of an equilibrium where the Maastricht level constraint is just met, and consider an increase in the rate of interest. This would enforce an increase in taxation and

government spending as in Case 4 above (on page 11) even if the initial ratio of government spending to private spending was just right, or even excessive.

7. SOME CALCULATIONS IN THE SPIRIT OF DOMAR

Domar (1944, 823-5, Cases 1 and 3) assumes a constant propensity to consume, denoted here by μ, which implies $\varphi = \mu \cdot \omega$. Equation (32) reduces to

$$\dot{d} = \frac{1-\mu}{\mu}\left(x-g\right) - \frac{1}{\mu}n - nd. \tag{76}$$

This differential equation has the globally stable solution

$$\bar{d} = \frac{1-\mu}{n\mu}\left(x-g\right) - \frac{1}{\mu} \tag{77}$$

with the implied tax rate

$$\bar{\tau} = 1 - \frac{n\left(x-g-n\right)}{\left(x-g-n\right)\left(1-\mu\right)i + n\mu\left(x-i\right)}. \tag{78}$$

For zero growth, this tax rate is 100 *per cent.* - even if government spending is zero. (This result corresponds to Domar's Case 1.) The derivative of (78) with respect to g is

$$\frac{\partial \bar{\tau}}{\partial g} = - \frac{\left(x-i\right)\mu n^2}{\left(\left(x-g-n\right)\left(1-\mu\right)i + n\mu\left(x-i\right)\right)^2} > 0 \tag{79}$$

which reconfirms the positive association between government spending and taxation. However, Domar looks at the ratio of government borrowing to production

$$\gamma \quad := \quad \frac{\dot{D}}{X} = \frac{\dot{d}+nd}{x} \tag{80}$$

and keeps this fixed. (Our γ stands for Domar's α.) Equation (80) gives rise to the differential equation

$$\dot{d} = \gamma x - nd \tag{81}$$

with the unique stable equilibrium solution

$$\bar{d} \quad = \quad \frac{\gamma x}{n} \tag{82}$$

which is equivalent to Domar's equation (10). Equating this with (77) yields equilibrium government spending parametrized by the Domar constant γ

$$g \quad = \quad \frac{x\left(\left(1-\mu\right)n - \gamma\mu\right) - n}{\left(1-\mu\right)n} \tag{83}$$

which shows the inverse relationship between government borrowing - as expressed by the Domar constant γ - and government spending. Because of the positive association between government spending and taxation given in (79), this result carries over to taxation: A higher rate of government borrowing implies a lower rate of taxation.

Domar defines the "tax rate" differently, however. It is, in our terminology,

$$\tau_{Domar} \quad := \quad \frac{di}{x + di} \tag{84}$$

and in equilibrium

$$\bar{\tau}_{Domar} \quad = \quad \frac{i\gamma}{n + i\gamma}, \tag{85}$$

whereas (83) and (78) imply a lower tax burden:

$$\bar{\tau} \quad = \quad \frac{i\gamma - \gamma\frac{\mu}{(1-\mu)} - \frac{n}{x(1-\mu)} - n(1-\gamma)}{(n + i\gamma)} < \bar{\tau}_{Domar}. \tag{86}$$

The difference comes about mainly because Domar has identified the additional tax burden with interest payments for public debt. Yet in a growing economy, government debt will grow at the natural rate as well. A larger public debt implies larger government borrowing, therefore, which offsets part of the interest payments on public debt. This renders it possible that equilibrium taxation is *less* than interest payments on public debt. Further, Domar's partial analysis has neglected the constraints on government spending and taxation coming from the full employment requirement which will imply a reduction of government borrowing over time. A constant rate of government borrowing will occur only in equilibrium.

As a numerical illustration consider the case of a rate of savings of 10 *per cent.*, a growth rate of 2 *per cent.*, a capital-output ratio of 3 and a share of government spending in the net product of 20 *per cent.* This implies $\mu = 0.9$, $n = 0.02$, $x = 0.33$, and $g = 0.2 \cdot x = 0.066$. Equation (76) now reads

$$\dot{d} = 1.33 - 0.02d \tag{87}$$

with the equilibrium debt ratio $\bar{d} = 0.356$ and the solution

$$d(t) = 0.356 + (d_0 - 0.356) e^{-0.02 \cdot t} \tag{88}$$

where d_0 denotes initial debt. The associated development of the tax rate is given by the government budget constraint (23) as

$$\tau(t) \quad = \quad \frac{g + (i - n) d(t) - \dot{d}(t)}{x + i d(t)}. \tag{89}$$

These are plotted in Figure 1.

The associated share of government expenditure in net production (as in (71)) is $q(t) = \frac{d(t)}{x}$, roughly the threefold of d. Its equilibrium value is $\bar{q} = 1.07$.

Varying the rate of government spending changes the equilibrium level of public debt as indicated in Figure 2. (To calculate gross capital productivity $x + \delta$, a rate of depreciation of 3 *per cent.* is assumed.)
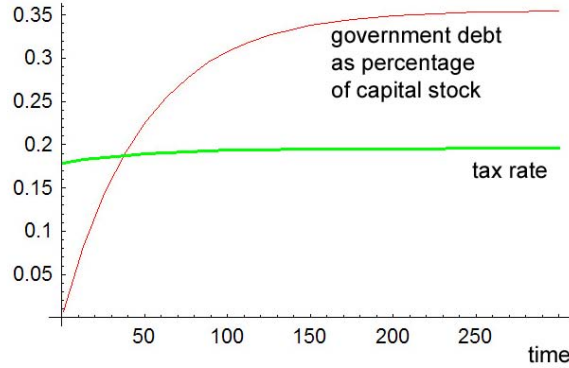
15

FIGURE 1. Public debt and the tax rate with a share of government spending of 20 *per cent.* in net production. Assumed parameter values: 10% savings rate, 2% growth rate, 0.33 capital productivity, 20% share of government spending in net production, 2% interest rate, initial debt of zero.
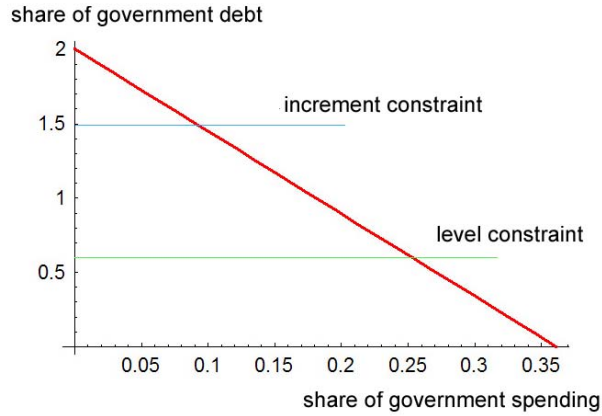

FIGURE 2. The share of government spending in gross production and the share of public debt in gross production. A rate of depreciation of 3% is assumed, the other parameter values are as in Figure 1. The Maastricht increment constraint requires a share of government spending above 10% (increment constraint) and 25% (level constraint).

The Maastricht constraints imply in equilibrium $d \leq 0.22$ (level constraint) and $d \leq 0.45$ (increment constraint). As shares of gross product, we have $\frac{d}{x+\delta} \leq 0.6$ and $\frac{d}{x+\delta} \leq 1.5$, respectively.

The relationship between government's share and the rate of taxation is not very sensitive to changes in the rate of growth or in the rate of interest. This is illustrated
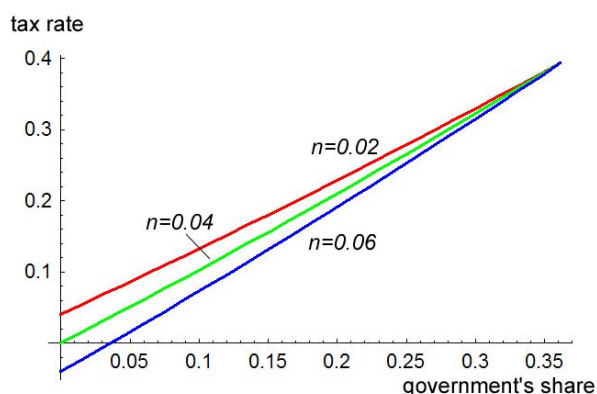
FIGURE 3. The relation between government's share in gross production and the tax rate for alternative growth rates and 4% rate of interest. The other parameters are as in Figure 1.
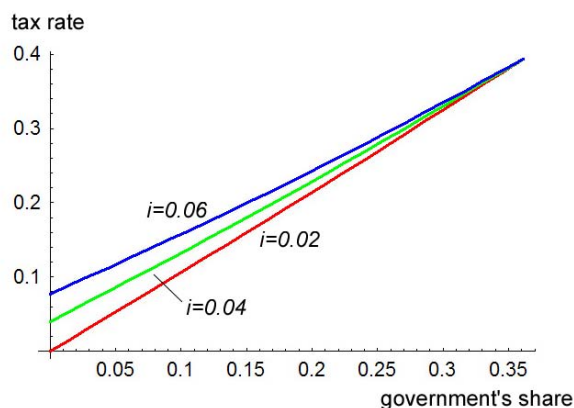

FIGURE 4. The relation between government's share in gross production and the tax rate for alternative rates of interest with a 2% growth rate. The other parameters are as in Figure 1.

for the case of a fixed capital productivity in Figures 3 and 4. This runs somewhat against the grain of Domar's argument.

8. CONCLUDING COMMENTS

The argument makes the point that government debt bears an important macroeconomic function: It allocates output between private consumption and public spending. Lower taxation and lower public spending imply higher public debt in equilibrium. Upper bounds on public debt may enforce high government spending and taxation and may undermine macroeconomic equilibrium.

The argument has been presented for the case of "insufficient demand" character-ized by conditions (34) and (45). A symmetric case of "excessive demand" would require negative public debt - that is, the accumulation of financial assets - by gov-ernment. For the case of a constant rate of savings this can be seen directly from (77).

Further, the argument has been presented for a closed economy. It is apparent, though, that foreign assets that are entailed by export surpluses perform a similar role as public debt.

The argument should not be seen, however, as an unabashed defense of "Key-nesian" policies of uncontrolled public spending, since an important feature of reality is suppressed here, namely, the possibility of inflation - even accelerating inflation - at low levels of employment. This phenomenon will curtail attempts to maintain full employment through Keynesian measures, as well as supply-side measures. The phenomenon is well illustrated by the West German experience where unemployment increased from below 1 *per cent.* in the seventies to the cur-rent 9 *per cent.* without any noticeable effect on wage formation, and in spite of weakening labor unions, increased flexibility, and increasing international com-petitiveness. This phenomenon has been neglected by Keynes (1936, 298-303) who thought that inflation would build up near full employment only, and recent "nat-ural rate" theorists share this problematic Keynesian assumption. The new classi-cal economists, on the other hand, cannot be reproached in this manner as a lack (or excess) of aggregate demand would be disregarded by them, being inconsis-tent with their theoretical approach.

The argument may, however, be read as a defense of Keynesian full employment policies if proper safeguards against inflation are implemented at all employment levels.

Equilibrium exercises as the one presented here may arguably be quite irrelevant for economic policy, but then arguments that object to government debt in terms of long-term outcomes are irrelevant as well. It has been shown here that such ob-jections are severely misleading in any case, as they entirely neglect an important equilibrating function of public debt.

REFERENCES

Barro, Robert J. 1974. Are Government Bonds Real Net Wealth? *Journal of Political Economy*, **82**(6), 1095–1117.

Blinder, Alan S., and Solow, Robert M. 1974. Analytical Foundations of Fiscal Policy. *Pages 3–115 of:* Gordon, Kermit (ed), *The Economics of Public Finance*. Washington D.C.: The Brookings Institution.

Branson, William H. 1989. *Macroeconomic Theory and Policy*. 3 edn. New York: Harper and Row.

Clower, Robert W., and Johnson, M. Bruce. 1968. Income, Wealth, and the Theory of Consumption. *Pages 45–96 of:* Wolfe, J. N. (ed), *Value, Capital, and Growth. Papers in honour of Sir John Hicks*. Edinburgh University Press.

Domar, Evsey D. 1944. The 'Burden of the Debt' and the National Income. *American Economic Review*, **34**(4), 798–827.

Gehrels, Franz. 1957. Government Debt as a Generator of Economic Growth. *The Review of Economics and Statistics*, **39**(2), 183–92.

Keynes, John Maynard. 1936. *The General Theory of Employment, Interest, and Money*. Harcourt, Brace and World.

Modigliani, Franco, and Brumberg, R. 1954. Utility Analysis and the Consumption Function. *Pages 388–436 of:* Kurihara, Kenneth (ed), *Post-Keynesian Economics*. Rutgers University Press.

Moll, Bruno. 1922. *Logik des Geldes*. 2 edn. Leipzig and Munich: Duncker und Humblot. first edition 1917.

Pigou, A.C. 1947. Economic Progress in a Stable Environment. *Economica*, **14**(55), 180–88.

Woodford, Michael. 1998. *Public Debt and the Price Level*. Bank of England Conference on Governemnt Debt and Monetary Policy, Online: `www.princeton.edu/~woodford/BOE.pdf`, 5-15-2004.

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