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Public Debt, Economic Growth and Nonlinear Effects: Myth or Reality?

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Public debt, economic growth and nonlinear effects: Myth or reality?

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Abstract

This paper puts the Reinhart-Rogoff dataset to a formal econometric testing to see whether public debt has a negative nonlinear effect on growth if public debt exceeds 90% of GDP. Using nonlinear threshold models, we show that the negative nonlinear relationship between debt and growth is very sensitive to modelling choices. We also show that when nonlinearity is detected, the negative nonlinear effect kicks in at much lower levels of public debt (between 20% and 60% of GDP). These results, based on bivariate regressions on secular time series, are confirmed on a shorter dataset (1960-2010) using a multivariate growth framework.

JEL classification codes: E6 ; F3 ; F4 ; N4

Keywords: public debt; economic growth; nonlinearity; threshold effects

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1. Introduction

The financial and economic crisis prompted by the unwinding US subprime mortgage market resulted in deep economic recession in many countries of the world. Governments and central banks reacted to the Great Recession by firing heavy artillery: fiscal and monetary policy expansion, unprecedented in size and in the way they were co-ordinated across countries, were swiftly enacted in advanced and emerging markets, and banking sector bailouts prevented the collapse of the financial system. While these actions certainly helped smooth the cycle, discretionary fiscal loosening and banking sector bail-outs contributed to a large extent to a sharp increase in many countries' public debt-to-GDP ratio. It is against this background that Reinhart and Rogoff (2010) pointed out the existence of strong negative effects of high public debt on economic growth. Using simple descriptive statistics, they demonstrated forcefully that economic growth slows down considerably if the public debt-to-GDP ratio exceeds 90%.

There are a number of channels through which public debt is likely to hamper long-term growth. First, tax hikes needed to service a higher public debt crowd out private investment by reducing disposable income and saving, raise the distortionary costs of taxation, and are likely to result in non-neutral tax treatment within and across asset classes, thus amplifying distortions. Second, soaring public debt will push up long-term sovereign yields in a nonlinear fashion, as the likelihood of default increases. High long-term rates crowd out productive public investment, and, more importantly, reduce private investment by increasing the cost of capital. Reduced investment in R&D will have long-lasting negative impacts on growth (Elmeskov and Sutherland, 2012). Third, public authorities, especially in countries with weak institutions, may decide to inflate away debt, and high inflation has a notoriously detrimental effect on growth (Kumar and Woo, 2010).

Many recent empirical papers sought to pin down and explain the possibly nonlinear negative relationship between public debt and growth. Most of these papers broadly confirm that the turning point beyond which economic growth slows down sharply is around 90% of GDP. Cecchetti *et al*. (2011) find a threshold of 86% of GDP for a panel of 18 OECD countries and for the period from 1980 to 2010. Padoan *et al*. (2012) report similar effects for a similar group of countries but a longer period (1960 to 2010). Covering a mix of advanced and emerging market economies, Kumar and Woo (2010) finds a turning point at 90% of GDP. Checherita and Rother (2010) and Baum *et al*. (2012) report similar results for a set of euro area countries. But

Caner *et al*. (2010) and Elmeskov and Sutherland (2012) show that the tipping point is probably lower: 77% for a set of 77 countries, and 66% for a dozen of OECD countries, respectively. Finally, in a recent contribution, Panizza and Presbitero (2012) argue that a negative correlation between debt and growth does not imply causality, as lower growth can result in a higher public debt to GDP ratio.

This paper seeks to contribute to this literature by putting a variant of the Reinhart-Rogoff dataset to a formal econometric testing by first using the thresholds proposed by Reinhart and Rogoff (2010) and then identifying the thresholds endogenously on the basis of the testing procedure proposed by Hansen (1999) for the period 1790 to 2009 and 1946 to 2009. We then embed the growth-debt relationship in a general multivariate growth framework and combine it with Bayesian model averaging to gauge the impact of model uncertainty on the presence of threshold effects for 1960 to 2010.

We find some evidence in favour of a negative nonlinear relationship between debt and growth. But these results are very sensitive to the time dimension and country coverage considered, data frequency (annual data *vs*. multi-year averages) and assumptions on the minimum number of observations required in each nonlinear regime. We also show that nonlinear effects are likely to kick in at much lower levels of public debt (between 20% and 60% of GDP). These results, based on bivariate regressions on secular time series, are largely confirmed on a shorter dataset (1960-2010) when using a multivariate growth framework that accounts for traditional drivers of long-term economic growth and model uncertainty.

The paper is organised as follows. Section 2 provides some stylised facts about the public debt-growth nexus by focusing on the Reinhart-Rogoff dataset. Section 3 presents empirical results for the variant of the Reinhart-Rogoff dataset. Section 4 embeds the debt-growth relationship in a multivariate growth framework and provides results using Bayesian model averaging. Finally, section 5 summarises and provides some policy implications.

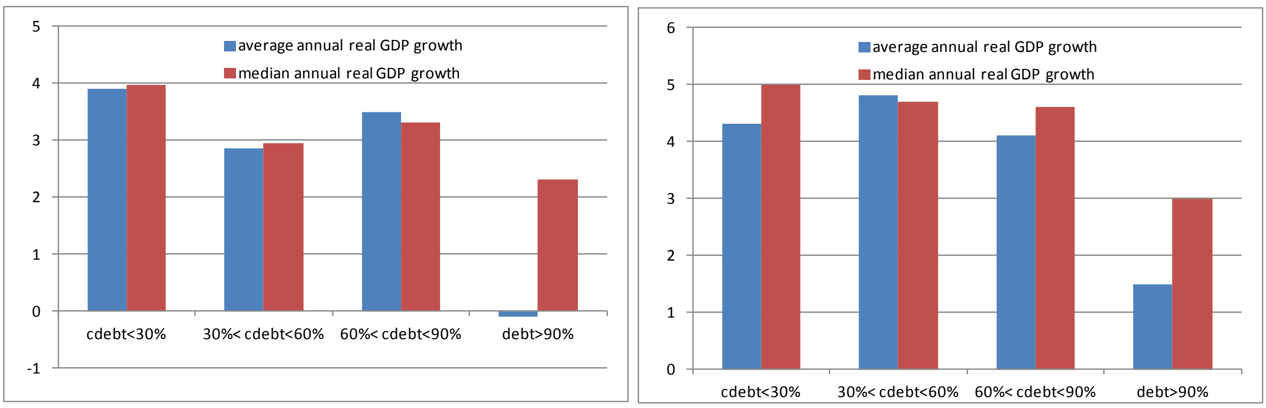
2. Stylised facts

In their influential paper, Reinhart and Rogoff (2010) rely on descriptive statistics to show that public debt as a share of GDP may have a detrimental effect on the rate of growth of real GDP. More specifically, they argue that the crucial public debt-to-GDP ratio is 90%, beyond which growth slows down considerably. Their charts and tables are meant to prove this claim: average and median annual GDP growth rates are shown relative to the level of the central government debt-to-GDP ratio for the period from 1946 to 2009. For a group of selected advanced countries, average GDP growth drops from around 3% to below 2% as public debt passes the threshold of 90% of GDP. The fall is more dramatic if growth is measured in terms of the median, rather than the average: a public debt-to-GDP ratio higher than 90% is associated with zero GDP growth. A similar pattern can be observed if only data for the US are considered: public debt exceeding the threshold of 90% goes in tandem with a decline in annual growth from about 3.5% to well below zero. The drop is again more pronounced if the median and not the average growth rate is looked at. The conclusion is strikingly similar for a group of selected emerging market economies, for which growth slows down by an annual 2 percentage points when public debt moves from below to above 90% of GDP. Figure 1 below reproduces these results for the group of advanced and emerging market economies.

Figure 1. **Annual real GDP growth and central government debt as a % of GDP, 1946-2009**

Results of Reinhart and Rogoff (2010)

Developed countries | Emerging market economies



Source: Reinhart and Rogoff (2010).

While Reinhart and Rogoff (2010) do not provide the data used in their paper, the data on central government debt can be obtained from the data appendix of another paper of the same authors (Reinhart and Rogoff, 2011). Real GDP growth rates are available for a number of countries for the same time period from the Barro-Ursúa macroeconomic dataset (Barro and Ursúa, 2011). Putting these two datasets together enables us to broadly replicate the the Reinhart and Rogoff data coverage on selected advanced economies. But there are some differences. First, our data excludes Ireland and includes Switzerland. Second, the emerging market country coverage of our data differs substantially from theirs. We have data on GDP growth for 16 out of the 24 countries included in their empirical investigation. But we also have data for five additional developing countries. Finally, and importantly, our series often start considerably earlier. This gives us more observations for the full sample period, whereas our coverage is almost identical for the period 1946-2009. Table A1 in the appendix lists the differences.

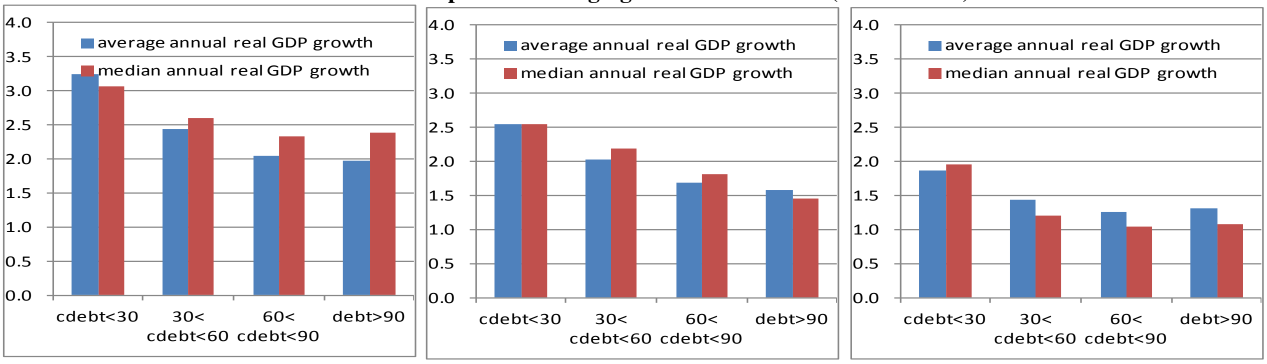
We use our dataset to replicate and extend the results of Reinhart and Rogoff (2010). Figures hereafter show average and median real GDP growth as the central government debt-to-GDP ratio varies for our entire sample, the group of advanced economies and two groups of emerging markets, the first including the 16 countries covered in Reinhart and Rogoff (2010) and the second containing all emerging countries for which data are available. In addition to the whole sample (1790-2009) and the post-war period (1946-2009), we also show numbers for the period from 1790 to 1939.

Several conclusions can be drawn from Figure 2, which shows GDP growth along rises in public debt as a share of GDP. First, for all countries and for the group of developed countries, growth remains broadly stable as public debt increase from the range of 60% to 90% of GDP to above 90%. This suggests the absence of any sudden change (fall) in growth rates beyond 90% of the public debt-to-GDP ratio. As a matter of fact, for these two samples, growth rates appear to decline gradually with the rise in public debt from the range 0% to 30% to above 90%. Growth even seems to increase slightly once the debt ratio is above 90% for some periods. Second, for the period 1946-2009, growth slows down for the two groups of emerging market economies as public debt moves from the range of 60% to 90% to beyond 90% of GDP. But GDP growth rates associated with debt levels above 90% are higher than when debt varies between 30% and 60% of GDP. Finally, contrary to Reinhart and Rogoff (2010), some of these observations are not very sensitive to the alternative use of the median of annual growth rates and the average growth rate. The difference only matters for emerging markets and for the whole and the pre-war periods (1790-2009 and 1790-1939): for the group of emerging markets, higher debt goes in tandem with a increase in the median growth rate (Panel C and D of Figure 2).

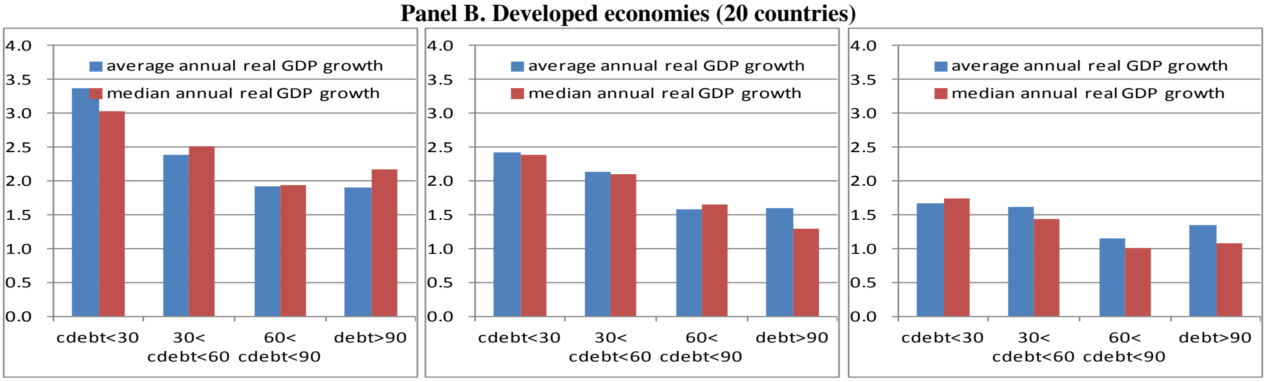
Figure 2. **Annual real GDP growth and central government debt as a % of GDP**

1790-2009 1790-1939 1946-2009

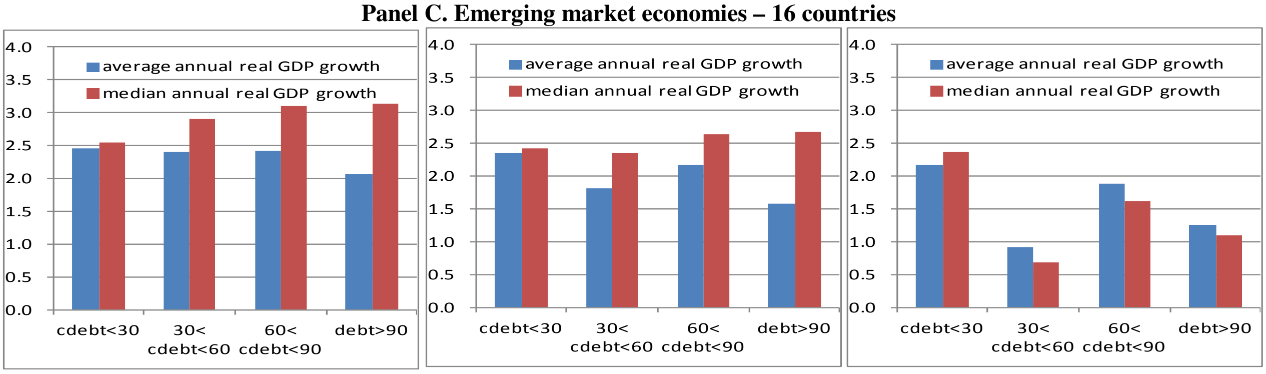
Panel A. Developed and emerging market economies (41 countries)



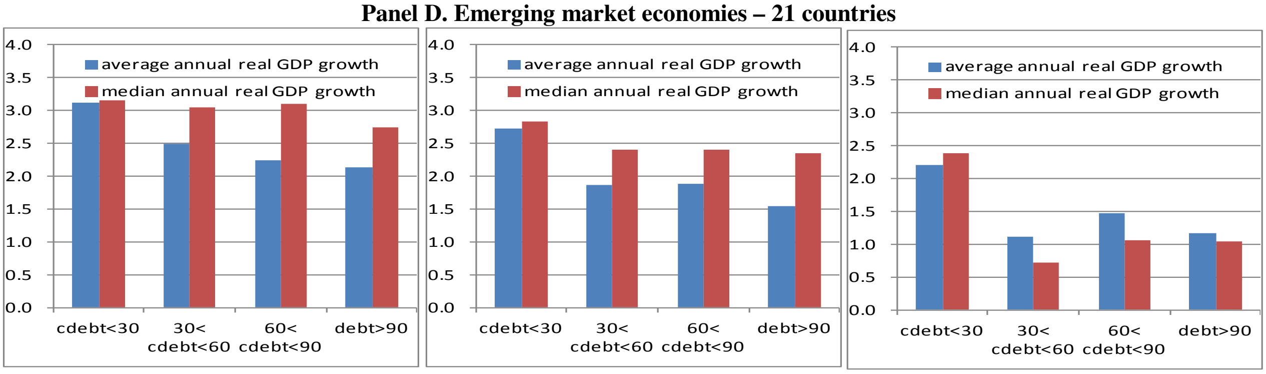
Panel B. Developed economies (20 countries)



Panel C. Emerging market economies – 16 countries



Panel D. Emerging market economies – 21 countries

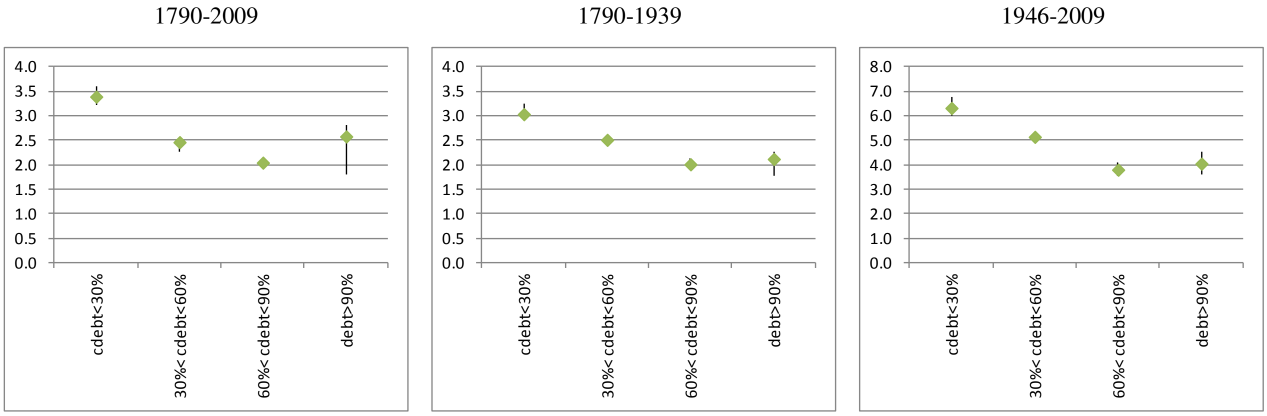


Source: Author's calculations.

We carry out a sensitivity analysis to see the extent to which the average growth rates are subject to the influence of individual countries. Jackknifing the sample of the selected advanced economies (that is recalculating the average growth rates by omitting one country at a time) indicates that the average for the whole advanced economy sample is not influenced by outlier countries if the debt ratio is below 90%. But the uncertainty is relatively large when debt is higher than 90% of GDP: the gap between the minimum and maximum of the average growth rate is one percentage point for the entire sample period (1790-2009) and half a percentage point for the two sub-periods (Figure 3).

Figure 3. **Annual real GDP growth and central government debt as a % of GDP sensitivity analysis**

Developed economies, minimum, maximum and average



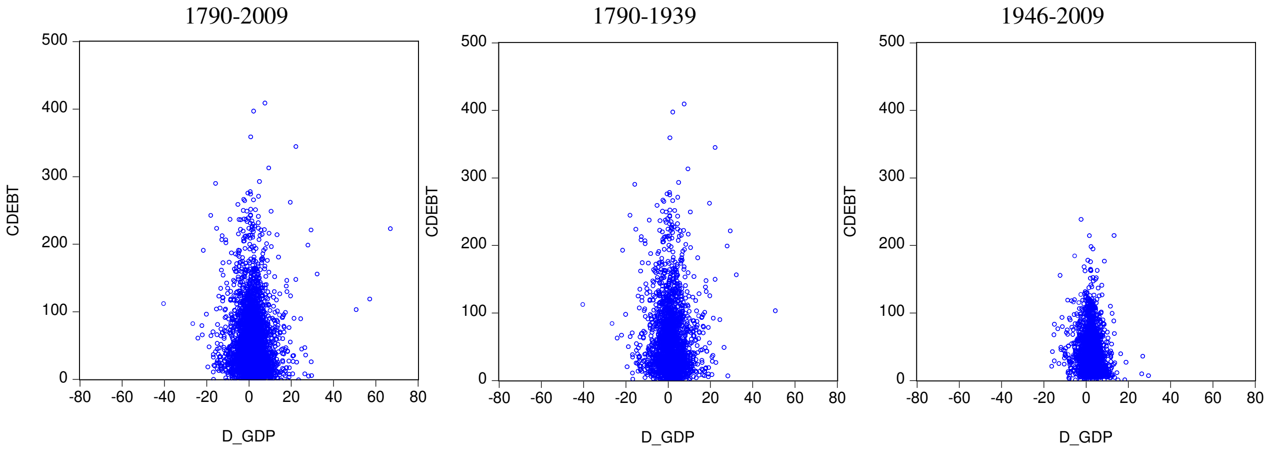
Source: Author's calculations.

Another way of having a preliminary idea about the correlation between growth and debt is to plot the annual GDP growth rates against the debt ratio. The scatter plots presented in Figure 4 are striking: GDP growth and the public debt ratio do not appear to have any relationship with one another for any of the country groups and sub-periods considered. The general picture does not change if public debt is plotted with a lag of one year (Figure A1 in the appendix).

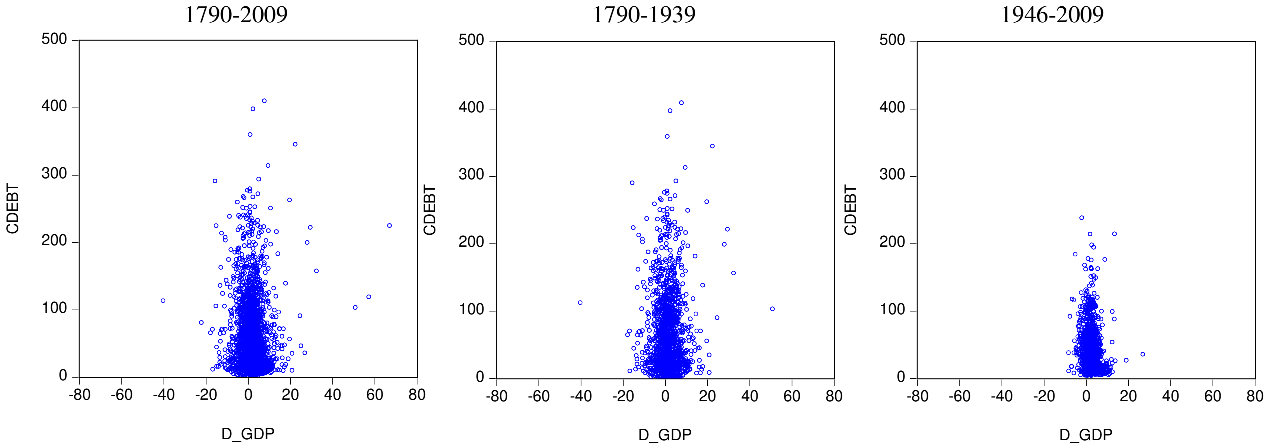
Multi-year averages eliminate cyclical and other short-term effects, which may contaminate the scatter plots of annual figures. Therefore, Figure 4b plots non-overlapping 5-, 8- and 10-year averages for growth and public debt for the period 1946-2009. Yet eyeball econometrics does not suggest an apparent negative correlation between debt and growth, especially for the full sample and the group of emerging market economies. Using debt with a one period lag confirms these observations (Figure A2 in the appendix).

Figure 4a. **Annual real GDP growth and central government debt as a % of GDP**

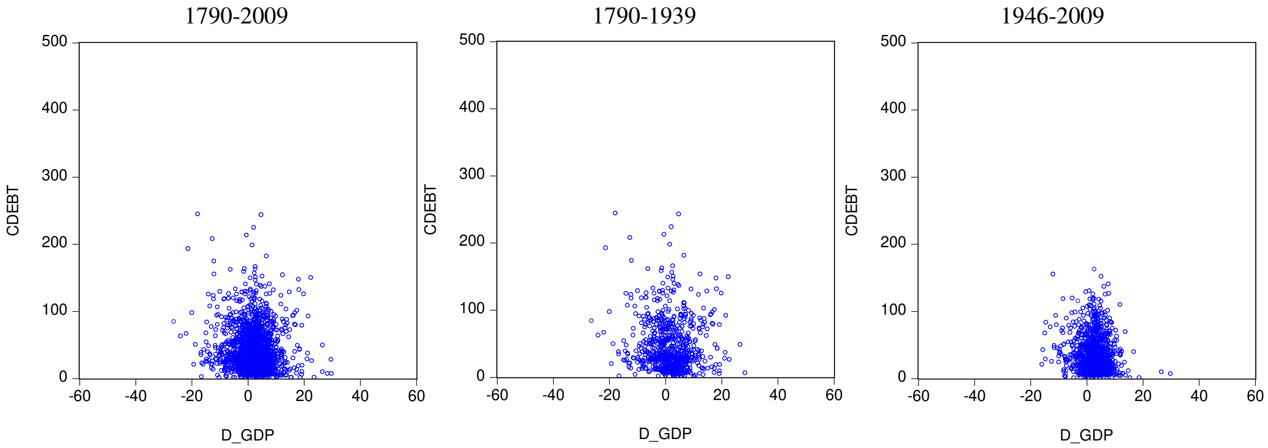
Panel A. Developed and emerging market economies



Panel B. Developed economies



Panel C. Emerging market economies – 21 economies

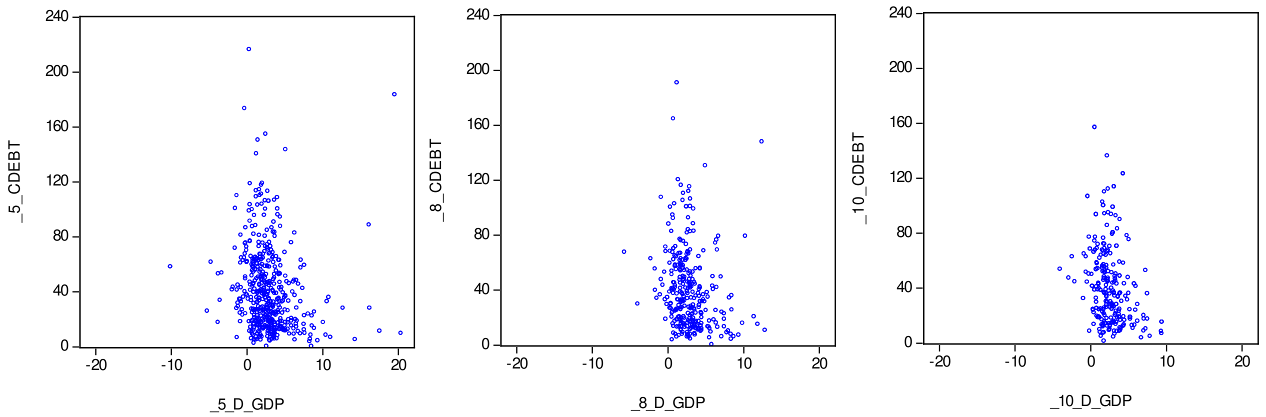


Source: Author's calculations.

Figure 4b. **Non-overlapping multi-year real GDP growth and central government debt as a % of GDP, 1946-2009**

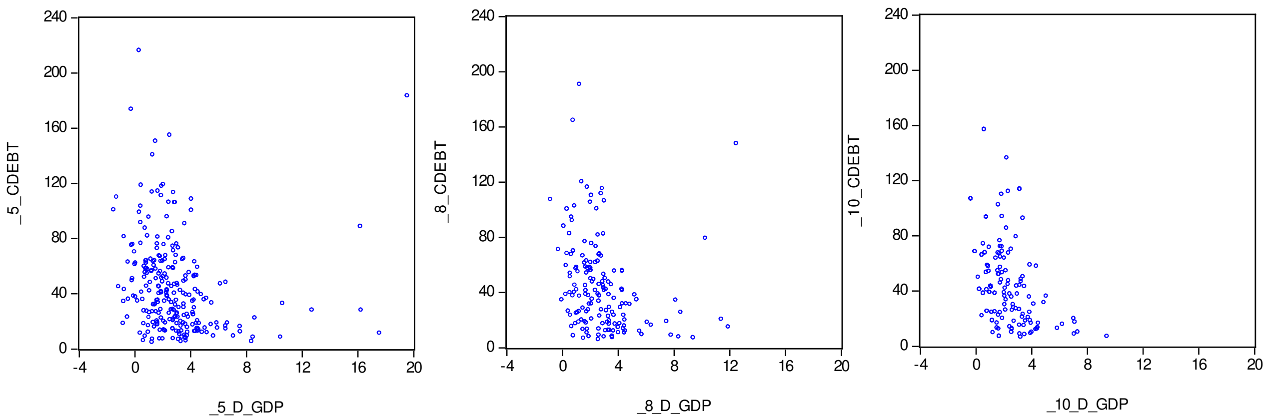
Panel A. Developed and emerging market economies

5-year averages | 8-year averages | 10-year averages



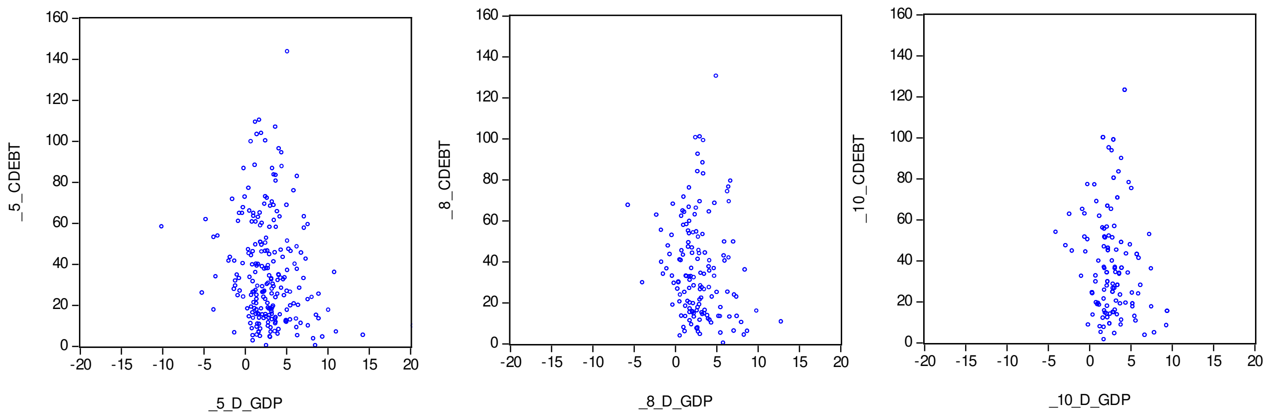
Panel B. Developed economies

5-year averages | 8-year averages | 10-year averages



Panel C. Emerging market economies – 21 economies

5-year averages | 8-year averages | 10-year averages



Source: Author's calculations.

3. A formal test of nonlinearity

Econometric issues

We apply a multi-step approach to our secular dataset covering the period from 1790 to 2009 to formally analyse the link between public debt and growth. We first start by looking at the bivariate linear relationship by estimating the following regression for growth and debt:

$$\Delta y_t = \alpha + \beta debt_t + \varepsilon_t \qquad (1)$$

where Δy is annual real GDP growth and *debt* stands for the central government debt-to-GDP ratio. Equation (1) is estimated for a pooled panel and with country fixed effects.

We then estimate threshold models, in which the effect of debt on growth depends on the level of debt. In these models, the thresholds put forth by Reinhart and Rogoff (2010) are imposed. A two-regime model is estimated where the coefficient on public debt can be different below and above the threshold of 90% of the debt ratio. We also estimate a three-regime model, in which the three regimes are separated by the 60% and 90% debt thresholds. Finally, a four-regime model account for all 3 thresholds proposed by Reinhart and Rogoff: 30%, 60% and 90% of central government debt. This four-regime model can be written as follows:

$$\Delta y_t = \begin{cases} \alpha_1 + \beta_1 DEBT_t + \varepsilon_t & if \quad DEBT < 30\% \\ \alpha_2 + \beta_2 DEBT_t + \varepsilon_t & if \quad 30\% \le DEBT < 60\% \\ \alpha_3 + \beta_3 DEBT_t + \varepsilon_t & if \quad 60\% \le DEBT < 90\% \\ \alpha_4 + \beta_4 DEBT_t + \varepsilon_t & if \quad DEBT \ge 90\% \end{cases} \qquad (2)$$

A shortcoming of this approach is that the choice of the number of the regimes and the value of the thresholds are necessarily arbitrary and we do not know whether any of the nonlinear models provides a better fit for the underlying data than alternative linear and nonlinear specifications. The testing procedure developed by Hansen (1999) helps solve these problems because it first determines the threshold values endogenously through a grid search, and second, it tests the different models sequentially against one another using bootstrapping methods. The linear specification is tested against a two-regime model. If the null hypothesis of the linear model can be rejected against the alternative of a two-regime model, the null of a two-regime model is tested against the alternative of a three-regime model. The two-regime and three-regime models can be written as follows.

$$\Delta y_t = \begin{cases} \alpha_1 + \beta_1 \cdot DEBT_t + \varepsilon_t & if \quad DEBT < T \\ \alpha_2 + \beta_2 \cdot DEBT_t + \varepsilon_t & if \quad DEBT \ge T \end{cases} \qquad (3a)$$

$$\Delta y_t = \begin{cases} \alpha_1 + \beta_1 DEBT_t + \varepsilon_t & if \quad DEBT < T_1 \\ \alpha_2 + \beta_2 DEBT_t + \varepsilon_t & if \quad T_2 > DEBT \geq T_1 \\ \alpha_3 + \beta_3 DEBT_t + \varepsilon_t & if \quad DEBT \geq T_2 \end{cases} \tag{3b}$$

T is the value of the threshold of debt in the two-regime model and T_1 and T_2 are the lower and upper threshold values of debt in the three-regime model. A grid search with steps of 1% of the distribution is carried out to find the value of the threshold variable (public debt) that minimises the sum of squared residuals of the estimated two-regime model. The grid search starts at 20% of the distribution and stops at 80% to ensure that a sufficient number of observations falls into each regime.

The three-regime model is estimated based on two threshold values of the threshold variable that minimise the sum of squared residuals across the estimated models. The threshold from the two-regime model is held fixed and a grid search is used to identify the second threshold. We impose the restriction that the two thresholds should be separated at least by 20% of our sample observations. Once the second threshold is identified, a backward grid search is performed to identify the first threshold as suggested by Hansen (1999).

We can proceed with the sequential testing of the models, once the thresholds are identified. Hansen (1999) shows that the null hypothesis of $\beta_1 = \beta_2$ from equations (3a) can be tested using a likelihood ratio test. Given that the likelihood ratio test statistic does not follow a standard asymptotic distribution as the threshold value is not identified under the null hypothesis, the distribution of the test statistic is obtained through bootstrapping with random draws with replacement. The bootstrap test was carried out using *N*=500 replications. If the likelihood ratio test statistic rejects the null hypothesis of the linear model against the two-regime model (on the basis of the bootstrapped critical values), whether there are three different regimes rather than only two regimes is also analysed. The bootstrap procedure described above is applied to the two-regime and three-regime models.

Estimation results

Simple bivariate panel regressions yield a negative link between growth and public debt. The coefficient is always negative but its size is not particularly large in economic terms: a 10 percentage increase in the public debt ratio is associated with 0.1 to 0.2 percentage point lower economic growth. In addition, the statistical significance of this result varies very much across different country samples and time periods. Results tend to be statistically significant for the whole period and for the post-war period but not for 1790 to 1939. The result is also sensitive to country coverage: for the two sub-periods (1790-1939 and 1946-2010), the estimate is not significant for the smaller sample of emerging markets but it is when five countries are added to the sample (Table 1).

As for the nonlinear specifications estimated using threshold values taken from Reinhart and Rogoff (2010), the results again show some instability. The estimated coefficients are not significant at the 10% level for 1790-1939 for the samples including all countries and the advanced countries only and for 1946-2009 for the smaller group of emerging countries.

While the estimated coefficients of the public debt ratio variable is almost always negative for 1790-2009 and 1946-2009, indicating that higher debt relates to slower growth, the size of the coefficients decreases by a factor of 2 to 5 as the debt ratio rises. This could imply that the harmful effect of public debt on growth diminishes with rising debt, but it could also well be the case that lower coefficients indicate that a one percentage point increase in the public debt-to-GDP ratio means a lower rate of growth of debt for higher levels of debt. We re-run the equations using the rate of growth rather than the level of the debt-to-GDP ratio as independent (nonlinear) variable[2]. For the two- and three-regime models, it indeed seems to be the case that the negative coefficients of the growth rate of debt increases with a rise of the level of debt. But for the 4-regime models, the negative coefficient for debt ranging from 60% to 90% of GDP is lower if debt is lower than 60% or higher than 90% (Table 1).

2. But we still use the debt-to-GDP ratio as the threshold variable.

Table 1. **The nonlinear relation between public debt and growth, 1790-2009, annual data**

EXOGENOUSLY IMPOSED DEBT THRESHOLDS (30%, 60%, 90% of government debt)

	ALL COUNTRIES			**DEVELOPED COUNTRIES**			**16 EMERGING COUNTRIES**			**21 EMERGING COUNTRIES**		
	1790-2009	**1790-1939**	**1946-2009**	**1790-2009**	**1790-1939**	**1946-2009**	**1790-2009**	**1790-1939**	**1946-2009**	**1790-2009**	**1790-1939**	**1946-2009**
Nonlinear variable = central government debt as a % of GDP												
Linear model	-0.009 **	-0.001	-0.022 **	-0.006 *	0.003	-0.020 **	-0.019 **	-0.021	-0.023 **	-0.017 **	-0.015	-0.026 **
2-regime model												
debt<90%	-0.016 **	-0.001	-0.029 **	-0.013 **	0.006	-0.030 **	-0.022 **	-0.019	-0.025 **	-0.023 **	-0.018	-0.029 **
debt>=90%	-0.009 **	-0.001	-0.021 **	-0.007 **	0.003	-0.019 **	-0.018 **	-0.021	-0.022 **	-0.016 **	-0.015	-0.024 **
3-regime model												
debt<=60%	-0.016 **	0.003	-0.035 **	-0.007	0.017	-0.038 **	-0.041 **	-0.065 **	-0.030 **	-0.035 **	-0.041	-0.032 **
60%<=debt <90%	-0.016 **	0.000	-0.029 **	-0.014 **	0.005	-0.029 **	-0.022 **	-0.024	-0.024 **	-0.023 **	-0.022	-0.029 **
debt>=90%	-0.009 **	0.000	-0.022 **	-0.006 *	0.004	-0.021 **	-0.021 **	-0.029 *	-0.023 **	-0.018 **	-0.020	-0.025 **
4-regime model												
debt<30%	-0.021	0.011	-0.057 **	0.012	0.033	-0.019	-0.084 **	-0.105 *	-0.082 **	-0.076 **	-0.069	-0.096 **
30%<=debt <60%	-0.017 **	0.004	-0.040 **	-0.003	0.019	-0.033 **	-0.051 **	-0.075 **	-0.042 **	-0.044 **	-0.048	-0.046 **
60%<=debt <90%	-0.017 **	0.001	-0.033 **	-0.011 *	0.007	-0.026 **	-0.030 **	-0.033	-0.034 **	-0.030 **	-0.027	-0.040 **
debt>=90%	-0.009 **	0.000	-0.024 **	-0.004	0.005	-0.019 **	-0.027 **	-0.035 *	-0.030 **	-0.023 **	-0.023	-0.033 **
Nonlinear variable = rate of growth of central government debt as a % of GDP												
Linear model	-0.009 **	-0.001	-0.022 **	-0.006 *	0.003	-0.020 **	-0.019 **	-0.021	-0.023 **	-0.017 **	-0.015	-0.026 **
2-regime model												
debt<90%	-0.011 **	-0.018 **	-0.009 **	-0.016 **	-0.014 **	-0.020	-0.010 **	-0.042 **	-0.008 **	-0.010 **	-0.029 **	-0.008 **
debt>=90%	-0.072 **	-0.069 *	-0.083 **	-0.041	-0.023	-0.216 **	-0.099 **	-0.118 **	-0.080 **	-0.093 **	-0.117 **	-0.074 **
3-regime model												
debt<=60%	-0.011 **	-0.018 **	-0.008 **	-0.019 **	-0.013 **	-0.054 **	-0.009 **	-0.056 **	-0.007 **	-0.009 **	-0.033 **	-0.007 **
60%<=debt <90%	-0.016	-0.018	-0.016	-0.007	-0.023	-0.002	-0.040	-0.002	-0.077 **	-0.041	-0.008	-0.071 **
debt>=90%	-0.072 **	-0.069 *	-0.083 **	-0.041	-0.023	-0.219 **	-0.099 **	-0.119 **	-0.080 **	-0.093 **	-0.117 **	-0.075 **
4-regime model												
debt<30%	-0.008 **	-0.014 **	-0.006 **	-0.013 **	-0.010 **	-0.046 **	-0.007 **	-0.069 **	-0.005 **	-0.007 **	-0.056 **	-0.006 **
30%<=debt <60%	-0.045 **	-0.033 **	-0.061 **	-0.074 **	-0.098 **	-0.072 **	-0.054 **	-0.041 **	-0.062 **	-0.035 **	-0.021 *	-0.057 **
60%<=debt <90%	-0.016	-0.018	-0.016	-0.008	-0.023	-0.002	-0.040	-0.001	-0.077 **	-0.041	-0.008	-0.071 **
debt>=90%	-0.072 **	-0.069 *	-0.084 **	-0.041	-0.023	-0.219 **	-0.100 **	-0.118 **	-0.082 **	-0.093 **	-0.117 **	-0.076 **

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively. The estimations are carried out with country fixed effects.

A serious problem with the correlation between public debt and growth is that any change in the growth rate of real GDP will have a mechanical effect on the debt-to-GDP ratio. Therefore, we re-estimate the nonlinear specifications using the lagged public debt-to-GDP ratio. Indeed, the previous results can be confirmed only for two sub-samples: for all countries from 1946 to 2009 and for the advanced country group from 1946 to 2009. The statistically significant negative nonlinear relationship between debt and growth disappears for the other sub-samples (Table A2 in the appendix). Again, we check whether a decrease in the negative coefficients on the level of the debt-to-GDP ratio implies a declining or increasing negative impact as the level of debt rises. Astonishingly, the results show the absence of any negative correlation between the growth rate of debt and economic growth for debt levels exceeding 90% of GDP. The only exception is the group of advanced countries for 1946 to 2009.

These disappointing results may be due to the fact that the choice of the debt thresholds is arbitrary. So it is natural to try to figure out the values of the thresholds in a data-driven approach. There is evidence for nonlinear effects both for the entire period (1790-2009) and for the post-war period (1946-2009) for the relation between growth and contemporaneous debt. The estimated negative coefficients tend to decrease for higher debt levels. But an important difference is that the debt thresholds are very low: slightly below 20% of GDP for the sample encompassing all countries and slightly above 30% for the group of emerging markets. The thresholds picked in the three-regime model for advanced economies are also considerably lower than the 90% put forward by Reinhart and Rogoff (2010): the first threshold is about 30% and the second threshold is around 60% GDP (Table A3 in the appendix).

But again, the evidence obtained for the debt ratio lagged one year is much less convincing. Table 2 shows that while the estimated coefficients are negative for the high-debt regime for the entire sample and the developed country group (1790-2009), they are not statistically significant and their size is very small. For the same period, no nonlinearity could be detected for emerging market economies. When the time coverage is restricted to 1946 to 2009, the tests of nonlinearity show that the relation between debt and growth differs depending on the level of debt. For the advanced countries, a low level of debt is associated with a nontrivial positive effect of debt on growth and with a small negative impact above the debt threshold of 20% of GDP: a 10 percentage point rise in the debt-to-GDP ratio goes in tandem with 0.08 percentage point decline in economic growth. For the other country groups, the coefficient estimates are statistically insignificant in the high-debt regime (Table 2). Results obtained using the growth rate of the public debt-to-GDP ratio are not much more reassuring. It is only for the period 1790 to 2009 and for developed countries that a high level of debt is associated with a lower growth performance (Table 3): a 1% increase in the debt ratio goes hand in hand with an almost 0.4% drop in economic growth. Yet this result cannot be confirmed for the period 1946-2009 for which the null hypothesis of a linear specification cannot be rejected against the alternative of threshold nonlinearities (Table 3).

Using non-overlapping multi-year averages can potentially eliminate short-term noises from our data series. Results based on such data for advanced countries (1946-2009) are reported in Table 4. There is no empirical evidence for a negative nonlinear relation between debt and growth if we use 5-year averages. For 8-year averages, regressions based on the growth rate of debt indicate strong nonlinear effects (Table 4): there is no correlation between growth and debt if debt is roughly below 40% of GDP and a negative relationship above this threshold, which imply that a 1% increase in debt is associated with a 0.1% decline in growth. For 10-year averages, the negative effect is even stronger above the threshold of 67% of GDP: a 0.3% decline in growth. These findings are somewhat sensitive to how the threshold models are parameterised (the minimum number of observations required in each regime) but the overall conclusions remain unaltered (Table A5 in the appendix).

Finally, we also investigate whether alternative forms of nonlinearity do a better job of describing the debt-growth relation. A polynomial trend of the debt variable (equation 4) would allow for instance a smooth transition around the turning point:

$$\Delta y_t = \alpha + \beta debt_t + \gamma debt_t^2 + \varepsilon_t \qquad (4)$$

Table 2. **The nonlinear relationship between public debt and growth, annual data - Endogenous debt thresholds**

Nonlinear variable = lagged public debt; Threshold variable = lagged public debt

	All countries		Advanced countries		16 Emerging countries	21 Emerging countries	
1790-2009							
Test of nonlinearity	Bootstrapped p-value						
H0: linear *vs*. H1: 2-regimes	**0.064**		**0.018**		0.134	0.218	
H0: 2 regimes *vs*. H1: 3-regimes	0.218		**0.010**		0.146	0.226	
Coefficients							
Low debt	0.011		0.061	**			
Middle debt	-0.001		0.015	*			
High debt			-0.001				
Debt thresholds (%)							
Threshold 1	40.51		20.38				
Threshold 2			55.35				
Country fixed effects	YES		YES		YES	YES	
No. obs	4700		2881		1634	1880	
1946-2009							
Test of nonlinearity	Bootstrapped p-value						
H0: linear *vs*. H1: 2 regimes	**0.012**		**0.000**		**0.060**	**0.050**	
H0: 2 regimes *vs*. H1: 3 regimes	0.116		0.104		0.304	0.376	
Coefficients							
Low debt	0.047	**	0.063	**	-0.021	-0.032	*
Middle debt							
High debt	-0.004		-0.008	*	0.004	-0.002	
Debt thresholds (%)							
Threshold 1	16.42		20.38		38.02	36.10	
Country fixed effects	YES		YES		YES	YES	
No. obs	2220		1236		896	1037	

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively.

Table 3. **The nonlinear relationship between public debt and growth, annual data**
Endogenous debt thresholds

Nonlinear variable = lagged rate of growth of public debt
Threshold variable = lagged public debt

	All countries	Advance countries	16 Emerging countries	21 Emerging countries
1790-2009				
Test of nonlinearity	bootstrapped p-value			
H0: linear vs. H1: 2 regimes	**0.024**	**0.076**	**0.034**	**0.030**
H0: 2 regimes vs. H1: 3 regimes	**0.030**	0.338	**0.054**	**0.036**
Coefficients				
Low debt	0.000	-0.002	0.001	0.000
Middle debt	-0.024 **		-0.026 **	-0.023 **
High debt	-0.004	-0.036 *	0.000	0.002
Debt thresholds (%)				
Threshold 1	28.90	73.73	17.45	17.39
Threshold 2	51.72		54.18	50.99
1946-2009				
Test of nonlinearity	bootstrapped p-value			
H0: linear vs. H1: 2 regimes	**0.004**	0.104	**0.000**	**0.004**
H0: 2 regimes vs. H1: 3 regimes	**0.000**	0.024	**0.028**	**0.028**
Coefficients				
Low debt	-0.001		-0.001	-0.001
Middle debt	-0.033 **		-0.045 **	-0.036 **
High debt	-0.002		-0.010	-0.007
Debt thresholds (%)				
Threshold 1	18.39		18.46	17.71
Threshold 2	60.83		50.57	53.84

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively.

Table 4. **Advanced OECD countries, 1946-2009, non-overlapping multi-year averages**

	Nonlinear variable = public debt/GDP Threshold variable = public debt/GDP			Nonlinear variable = growth rate of public debt/GDP Threshold variable = public debt/GDP		
	5-year	8-year	10-year	5-year	8-year	10-year
Test of nonlinearity	bootstrapped p-value					
H0: linear vs. H1: 2 regimes	**0.004**	**0.024**	**0.004**	**0.044**	**0.018**	**0.006**
H0: 2 regimes vs. H1: 3 regimes	**0.002**	0.106	**0.012**	**0.086**	0.106	0.134
Coefficients						
Low debt	0.098 **	-0.034 **	0.11 **	0.018	-0.005	-0.068 **
Middle debt	0.034 *		0.031	-0.064 **		
High debt	-0.005	-0.017 **	-0.01	-0.008	-0.095 **	-0.289 **
Debt thresholds (%)						
Threshold 1	18.61	58.17	18.41	17.47	42.56	67.04
Threshold 2	33.27		36.30	39.13		
Country fixed effects	YES	YES	YES	YES	YES	YES
No. obs	238	140	140	237	139	139
No. countries	20	20	20	20	20	20

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively.

Table 5. **The nonlinear effect of public debt on growth**

Polynomial functional form of nonlinearity

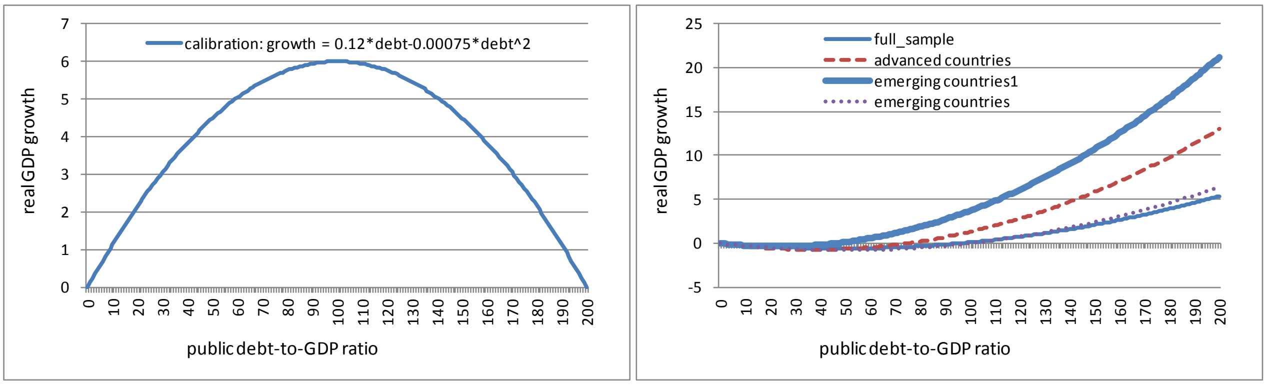
	All countries		Advanced countries		16 Emerging countries		21 Emerging countries	
	$\Delta y_t = \alpha + \beta debt_t + \gamma debt_t^2 + \varepsilon_t$							
	α	β	α	β	α	β	α	β
1790-2009	-0.026**	0.00008**	-0.027**	0.00009**	-0.001	-0.00014	-0.013	-0.00003
1790-1936	-0.015	0.00005	-0.014	0.00006	0.051	-0.00046**	0.014	-0.00016
1946-2009	-0.070**	0.00038*	-0.076**	0.00042*	-0.044**	0.00017	-0.048**	0.00019
	$\Delta y_t = \alpha + \beta debt_{t-1} + \gamma debt_{t-1}^2 + \varepsilon_t$							
	α	β	α	β	α	β	α	β
1790-2009	-0.011*	0.00004	-0.019**	0.00006*	-0.010	0.00014	-0.003	0.00006
1790-1936	0.013	-0.00002	0.005	0.00000	0.026	0.00000	0.026	-0.00003
1946-2009	-0.025**	0.00011**	-0.039**	0.00015**	-0.032*	0.00037**	-0.032*	0.00032**

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively. The estimations are carried out with country fixed effects.

Ideally, a hump-shaped pattern of growth around the turning point such as plotted in Panel A of Figure 5 would be in line with a disruptive debt-growth relationship. Nevertheless, the estimation results, both for the contemporaneous and lagged level of debt, indicate that higher debt is accompanied by higher economic growth (Panel B of Figure 5). Not quite the results we were looking for.

Figure 5. **Polynomial functional form of nonlinearity**

Panel A. Calibration **Panel B. Estimated parameters**



Note: Panel B uses the estimated parameters of the model $\Delta y_t = \alpha + \beta debt_t + \gamma debt_t^2 + \varepsilon_t$ reported in Table 55. The results are almost identical if the estimated parameters of the model based on lagged public debt is used.

4. Accounting for other drivers of growth and model uncertainty

Nonlinearity embedded in a model averaging framework

We now embed the public debt-growth nexus into a general growth framework. In accordance with a general production function approach, the level of per capita GDP (rather than real GDP growth) is a function of human and physical capital and labour input. Formulating this long-run relationship in terms of an error correction model yields a model where the rate of growth of per capita GDP depends on lagged per capita level (cap_{-1}) and lagged values of the other explanatory variables. Following the growth literature, physical capital can be proxied by the investment-to-GDP ratio (*inv*), human capital by average years of schooling (*edu*) and labour input by population growth (Δpop). Additional controls used in the literature are inflation (*infl*) and openness (*open*).

Hansen's threshold modelling framework can be applied to this set of covariants to analyse the possible nonlinear relationship between economic growth and debt:

$$\Delta cap_t = \begin{cases} \alpha_1 + \sum_{j=1}^{n-1} \beta_j \cdot X_{j,t-1} + \varphi_1 \cdot debt_{t-1} + \varepsilon_t & if \quad debt < T \\ \alpha_2 + \sum_{j=1}^{n-1} \beta_j \cdot X_{j,t-1} + \varphi_2 \cdot debt_{t-1} + \varepsilon_t & if \quad debt \geq T \end{cases} \tag{5a}$$

$$\Delta cap_t = \begin{cases} \alpha_1 + \sum_{j=1}^{n-1} \beta_j \cdot X_{j,t-1} + \varphi_1 \cdot debt_{t-1} + \varepsilon_t & if \quad debt < T_1 \\ \alpha_2 + \sum_{j=1}^{n-1} \beta_j \cdot X_{j,t-1} + \varphi_2 \cdot debt_{t-1} + \varepsilon_t & if \quad T_1 \leq debt < T_2 \\ \alpha_3 + \sum_{j=1}^{n-1} \beta_j \cdot X_{j,t-1} + \varphi_3 \cdot debt_{t-1} + \varepsilon_t & if \quad debt \geq T_2 \end{cases} \tag{5b}$$

where *debt* is general government debt and $\bar{X}$ is a vector of independent variables. But a high number of independent variables poses the problem of model uncertainty. To address this issue, Bayesian averaging of classical estimates (BACE), which provides estimates for all possible combinations of the (K) candidate explanatory variables, is given by 2^K [3]. The BACE technique shows whether the inclusion of a candidate variable improves the fit of the model (Sala-i-Martin *et al.*, 2004). BACE determines the posterior probability attributed to each single model M_j that includes the variable of interest and is conditioned on the underlying dataset ($P(M_j|y)$).

$$P(M_j|y) = \frac{P(M_j)T^{-k_j/2}SSE_j^{-T/2}}{\sum_{i=1}^{2^K} P(M_i)T^{-k_i/2}SSE_i^{-T/2}} \tag{6}$$

where SSE is the sum of squared residuals, T is the number of observations, k denotes the number of explanatory variables included in the specific model and K is the number of all explanatory variables considered. Expression (6) gives the contribution of a given model to explaining the dependent variable as

3. Or some subset of models. If the number of models to be estimated is too large, techniques such as Markov-Chain Monte-Carlo, stochastic search variable selection, or random sampling are alternative approaches to estimating all possible models. Given the relatively low number of potential explanatory variables used here, we estimate all possible combinations.

compared to the other models. Expression (6) is then summed over the models that contain the variable of interest to obtain the posterior inclusion probability of this variable. If the posterior inclusion probability is higher than the prior inclusion probability, one can conclude that the candidate variable should be included in the estimated models.[4]

The posterior mean and the square root of the variance (standard error) conditional on inclusion can be used to obtain t-statistics and to determine the significance of the individual variables upon inclusion. The posterior mean conditional on inclusion ($E(\beta|y)$) is the average of the individual OLS estimates weighted by $P(M_j|y)$. As the unconditional posterior mean considers all regressions (even those without the variable of interest), the unconditional posterior mean of any given variable can be derived as the product of the conditional posterior mean and the posterior inclusion probability. The posterior variance of β ($Var(\beta|y)$) can be calculated as follows:

$$Var(\beta|y) = \sum_{j=1}^{2^K} P(M_j|y)Var(\beta|y, M_j) + \sum_{j=1}^{2^K} P(M_j|y)(\hat{\beta}_j - E(\beta|y))^2 \quad (7)$$

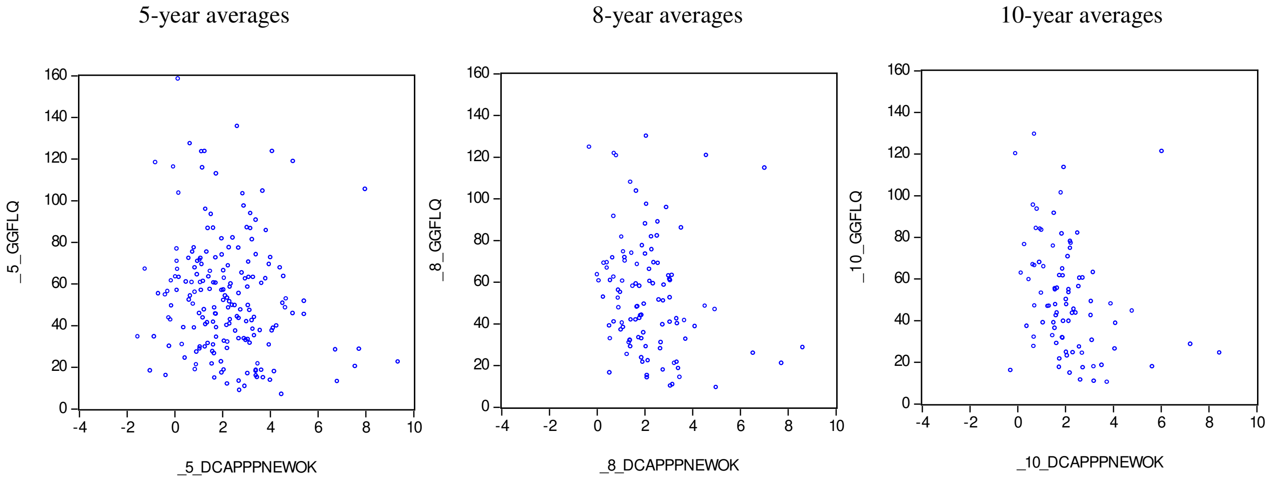
The linear BACE approach can be extended to possible nonlinearities between growth and public debt by including equations (5a) and (5b) into the model space (Crespo-Cuaresma and Doppelhofer, 2007). In the spirit of model averaging, we estimate all possible combinations of the candidate explanatory variables. For each combination, the linear, two-regime and three-regime models are estimated. The selection between linear and nonlinear models is done by using Hansen's (1999) bootstrapping method described earlier. An advantage of this methodology is that only a single linear or nonlinear model is selected for a given set of explanatory variables.

Estimation results

We assemble a dataset, drawn from the OECD's *Economic Outlook* database, including non-overlapping multi-year (5-year, 8-year and 10-year) averages for 29 OECD countries covering the period 1960-2010. As the series start for a number of countries only in the 1970s and 1980s, we also define a narrow sample including only 13 OECD countries for which time series start in the 1960s. Figure 6 below gives a flavour on the possible correlation between lagged general government debt and growth. The scatter plots do not show a clear cut negative relationship.

4. Sala-i-Martin *et al.* (2004) compare the posterior inclusion probability to a prior inclusion probability for their 67 explanatory variables in 7 variable models. The prior inclusion probability is then 7/67=0.1044.

Figure 6. **Non-overlapping multi-year real GDP per capita growth (DCAPPNEWOK) and lagged general government debt (GGFLQ) as a % of GDP, 1960-2010, 29 OECD countries**



Source: Author’s calculations.

Bayesian averaging of classical estimates augmented for nonlinearities helps us evaluate the uncertainty stemming from model specification. The estimation results suggest the presence of a strong negative nonlinear relation between lagged general government debt and growth. For the sample of 29 OECD countries, the posterior inclusion probability is higher than 50% for the three-regime model for the 5-year, 8-year and 10-year averages (Table 4a). This indicates that the three-regime model prevails over the linear and two-regime models. But there is one exception: the debt variable, either in a linear specification or in a nonlinear fashion, is not included in the final model space for 8-year averages if each regime has to contain at least 10% of the observations. Results for the posterior mean conditional on inclusion exhibit a large amount of instability. The nature of nonlinearity differs depending on whether 5-, 8- or 10-year averages are used. For 5-year averages, there is a negative relationship between growth and debt in all three regimes, but the largest negative effect occurs when public debt is the lowest. For 8-year averages, public debt and growth have a positive correlation if the debt-to-GDP ratio is roughly below 35% but the relation turns negative above this threshold. Finally, the correlation between debt and growth is very volatile for 10-year averages: the correlation can be negative or positive depending on how many observations are allowed in particular regimes (Table 4a).

Turning now to the group of 13 OECD countries, Table 4b shows that whether the two- or the three-regime model is selected depends on the type of the multi-annual averages (5-, 8- and 10-year averages). The posterior mean is negative for 8-year averages and increases with higher public debt. For 10-year averages, the negative relationship is the largest for the low-debt regime, which contradicts the Reinhart-Rogoff prediction. Finally, for 5-year averages, there is a positive relation between debt and growth if the debt–to-GDP ratio increases. Another strong contradiction with the Reinhart-Rogoff result is that if there is a negative nonlinear link between debt and growth, they kick in at much lower debt levels: between 20% and 60% of GDP. The results are much shakier if we replace the level of the debt-to-GDP ratio by its growth rate. There is much less evidence for nonlinearity in this case, and a strong negative correlation emerges at higher levels of debt only for the group of 13 OECD countries and if 8-year averages are used (Tables 4c and 4d).

Table 4a. **Nonlinear model averaging – General government debt and growth, 29 OECD countries, 1960-2010**

Nonlinear variable = lagged public debt; Threshold variable = lagged public debt

	5-year averages			8-year averages			10-year averages		
	Minimum % of observations in one regime								
	10%	20%	30%	10%	20%	30%	10%	20%	30%
Posterior inclusion probability									
Linear regime	0.009	0.016	0.052	0.071	0.002	0.001	0.000	0.000	0.000
2-regimes	0.003	0.284	0.344	0.158	0.457	0.416	0.076	0.003	0.002
3-regimes	**0.790**	**0.700**	**0.605**	0.077	**0.541**	**0.583**	**0.911**	**0.997**	**0.998**
Posterior mean conditional on inclusion									
Low debt	-0.041	-0.017	-0.019	--	0.010	0.008	-0.034	0.000	0.000
Middle debt	-0.011	-0.002	-0.004	--	0.031	0.029	-0.011	0.013	0.012
High debt	-0.021	-0.008	-0.010	--	-0.014	-0.015	-0.021	0.066	0.065
Debt thresholds (%)									
Threshold 1	27.70	24.54	21.20	--	23.21	25.00	36.95	42.14	41.93
Threshold 2	53.02	46.98	40.58	--	34.02	36.63	64.53	70.64	70.72

Note: Bold figures indicate that the estimated posterior inclusion probability is higher than 0.50.

Table 4b. **Nonlinear model averaging–- General government debt and growth, 13 OECD countries, 1960-2010**

Nonlinear variable = lagged public debt; Threshold variable = lagged public debt

	5-year averages			8-year averages			10-year averages		
	Minimum % of observations in one regime								
	10%	20%	30%	10%	20%	30%	10%	20%	30%
Posterior inclusion probability									
Linear	0.000	0.227	0.227	0.000	0.007	0.007	0.000	0.000	0.001
2-regimes	**0.686**	**0.773**	**0.773**	0.000	0.186	0.186	0.000	0.001	0.001
3-regimes	0.000	0.000	0.000	**0.839**	**0.807**	**0.807**	**0.999**	**0.998**	**0.997**
Posterior mean conditional on inclusion									
Low debt	-0.015	-0.022	-0.022	-0.004	-0.005	-0.005	-0.045	-0.045	-0.045
Middle debt				-0.005	0.034	0.033	-0.018	-0.018	-0.018
High debt	-0.008	0.007	0.007	-0.013	-0.061	-0.061	-0.028	-0.029	-0.028
Debt thresholds (%)									
Threshold 1	42.69	49.73	49.73	37.48	35.42	35.44			
Threshold 2				51.59	48.99	49.02	40.93	40.92	40.87

Note: Bold figures indicate that the estimated posterior inclusion probability is higher than 0.50.

Table 4c. **Nonlinear model averaging – he rate of growth of general government debt and growth, 29 OECD countries, 1960-2010**

Nonlinear variable = lagged growth rate of public debt
Threshold variable = lagged public debt

	5-year averages			8-year averages			10-year averages		
	Minimum % of observations in one regime								
	10%	20%	30%	10%	20%	30%	10%	20%	30%
Posterior inclusion probability									
Linear	**0.888**	0.315	0.315	**0.950**	0.114	0.114	0.000	0.000	0.000
2 regimes	0.000	0.232	0.232	0.000	0.478	0.478	**0.988**	**0.943**	**0.987**
3 regimes	0.000	0.453	0.453	0.000	0.408	0.408	0.012	0.057	0.013
Posterior mean conditional on inclusion									
Low debt	0.034			0.038			0.031	0.030	0.031
Middle debt									
High debt							0.150	0.143	0.150
Debt thresholds (%)									
Threshold 1							68.82	66.441	69.51
Threshold 2									

Note: Bold figures indicate that the estimated posterior inclusion probability is higher than 0.50.

Table 4d. **Nonlinear model averaging – The rate of growth of general government debt and growth, 13 OECD countries, 1960-2010**

Nonlinear variable = lagged growth rate of public debt
Threshold variable = lagged public debt

	5-year averages			8-year averages			10-year averages		
	Minimum % of observations required in one regime								
	10%	20%	30%	10%	20%	30%	10%	20%	30%
Posterior inclusion probability									
Linear	0.000	0.035	0.035	0.000	0.004	0.004	0.015	0.134	0.189
2-regimes	**0.961**	**0.965**	**0.965**	0.016	0.157	0.154	0.190	**0.772**	**0.737**
3-regimes	0.000	0.000	0.000	**0.941**	**0.839**	**0.843**	**0.669**	0.094	0.074
Posterior mean conditional on inclusion									
Low debt	0.010	0.008	0.008	0.008	-0.004	-0.004	0.026	0.025	0.049
Middle debt				0.071	0.042	0.043	0.057		
High debt	0.080	0.079	0.079	-0.082	-0.070	-0.070	0.009	0.068	0.060
Debt thresholds (%)									
Threshold 1	42.05	43.14	43.14	40.49	36.62	36.76	27.42	53.51	45.86
Threshold 2				57.13	50.93	51.14	47.02		

Note: Bold figures indicate that the estimated posterior inclusion probability is higher than 0.50.

5. Conclusions

The aim of this paper was to contribute to the empirical literature on the debt threshold beyond which negative effects for economic growth appear. We put a variant of the Reinhart-Rogoff dataset to a formal econometric test. Using nonlinear threshold models, we found some evidence in favour of a negative nonlinear relationship between debt and growth. But these results are very sensitive to the time dimension and country coverage considered, data frequency (annual data *vs*. multi-year averages) and assumptions on the minimum number of observations required in each nonlinear regime. We also showed that nonlinear effects can kick in at much lower levels of public debt (between 20% and 60% of GDP). These results, based on bivariate regressions on secular time series, are largely confirmed on a shorter dataset (1960-2010) when using a multivariate growth framework that accounts for traditional drivers of long-term economic growth and model uncertainty.

Previous empirical papers, validating the Reinhart-Rogoff result of a 90% public debt ceiling beyond which economic growth slows significantly, called for debt reduction to improve long-term growth. The implications of our results are that 90% is not a magic number. The threshold can be lower and the nonlinearity can change across different samples and specifications. Nonlinear effects might be more complex and difficult to model than previously thought. Instability might be a result of nonlinear effects changing over time, across countries and economic conditions. Further research is certainly needed to fully understand the link between public debt and growth.

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Appendix

Table A1. **Data coverage: Reinhart and Rogoff (2010) *vs*. the dataset used in the paper**

	Reinhart and Rogoff (2010)	Our dataset, which draws on Reinhart and Rogoff (2011) for the level of central government debt Barro and Ursúa (2012) for real GDP growth
Developed countries		
Australia	1902-2009	1861-2009
Austria	1880-2009	1880-2009
Belgium	1835-2009	1847-2009
Canada	1925-2009	1871-2009
Denmark	1880-2009	1880-2009
Finland	1913-2009	1914-2009
France	1880-2009	1880-2009
Germany	1880-2009	1880-2009
Greece	1884-2009	1848-2009
Ireland	1949-2009	--
Italy	1880-2009	1862-2009
Japan	1885-2009	1872-2009
Netherlands	1880-2009	1814-2009
New Zealand	1932-2009	1831-2009
Norway	1880-2009	1880-2009
Portugal	1851-2009	1851-2009
Spain	1850-2009	1850-2009
Sweden	1880-2009	1801-2009
Switzerland	--	1880-2009
United Kingdom	1830-2009	1831-2009
USA	1790-2009	1791-2009
Emerging market economies		
Argentina	1900-2009	1876-2009
Bolivia	1950-2009	--
Brazil	1980-2009	1861-2009
Chile	1900-2009	1861-2009
China	--	1982-2009
Colombia	1923-2009	1906-2009
Costa Rica	1950-2009	--
Egypt	--	1895-2009
Ecuador	1939-2009	--
El Salvador	1939-2009	--
Ghana	1952-2009	--
India	1950-2009	1868-2009
Indonesia	1972-2009	1911-2009
Kenya	1963-2009	--
Korea	--	1913-2009
Malaysia	1955-2009	1949-2009
Mexico	1917-2009	1896-2009
Nigeria	1990-2009	--
Peru	1917-2009	1897-2009
Philippines	1950-2009	1948-2009
Russia	--	1885-2009
Singapore	1969-2009	1969-2009
South Africa	1950-2009	1912-2009
Sri Lanka	1950-2009	1871-2009
Thailand	1950-2009	1987-2009
Turkey	1933-2009	1875-2009
Uruguay	1935-2009	1971-2009
Venezuela	1921-2009	1914-2009

Table A2. **The nonlinear relation between lagged public debt and growth, 1790-2009, annual data**

EXOGENOUSLY IMPOSED DEBT THRESHOLDS (30%, 60%, 90% of government debt)

	ALL COUNTRIES			**DEVELOPED COUNTRIES**			**16 EMERGING COUNTRIES**			**21 EMERGING COUNTRIES**		
	1790-2009	**1790-1939**	**1946-2009**	**1790-2009**	**1790-1939**	**1946-2009**	**1790-2009**	**1790-1939**	**1946-2009**	**1790-2009**	**1790-1939**	**1946-2009**
Nonlinear variable = lagged central government debt; Threshold variable = lagged central government debt												
Linear model	-0.003	0.008 *	-0.007 **	-0.006 **	0.005	-0.013 **	0.009	0.026 *	0.010	0.007	0.020 *	0.004
2-regime model												
debt<90%	0.005	0.031 **	-0.004	0.000	0.026 **	-0.016 **	0.017 **	0.046 **	0.014 *	0.014 **	0.042 **	0.008
debt>=90%	-0.003	0.011 **	-0.008 **	-0.005 *	0.009	-0.013 **	0.006	0.023	0.003	0.005	0.020 *	0.000
3-regime model												
debt<=60%	-0.003	0.018	-0.017 **	-0.001	0.018	-0.029 **	-0.004	0.025	0.002	-0.003	0.029	0.000
60%<=debt <90%	-0.005	0.014	-0.013 **	-0.007	0.011	-0.022 **	0.002	0.027	0.005	0.000	0.025	-0.001
debt>=90%	-0.003	0.009 *	-0.007 **	-0.005 *	0.006	-0.014 **	0.010	0.026	0.013 *	0.007	0.021 *	0.008
4-regime model												
debt<30%	-0.001	0.009	-0.016	0.024	0.039	-0.015	-0.027	-0.028	0.003	-0.023	-0.021	-0.010
30%<=debt <60%	-0.003	0.017	-0.017 **	0.004	0.021	-0.026 **	-0.009	0.013	0.002	-0.008	0.018	-0.002
60%<=debt <90%	-0.005	0.013	-0.013 **	-0.003	0.013	-0.020 **	-0.002	0.016	0.005	-0.003	0.016	-0.003
debt>=90%	-0.003	0.009 *	-0.007 *	-0.003	0.007	-0.012 **	0.007	0.019	0.013	0.005	0.016	0.007
Nonlinear variable = lagged rate of growth of central government debt; Threshold variable = lagged central government debt												
Linear model	-0.002	-0.002	-0.003	-0.004	-0.001	-0.004	-0.001	0.006	-0.003	-0.001	-0.004	-0.002
2-regime model												
debt<90%	-0.001	0.000	-0.003	-0.002	-0.001	-0.004	-0.001	0.002	-0.003	-0.001	0.003	-0.002
debt>=90%	-0.005	0.006	-0.001	-0.030	-0.013	-0.026	0.014	0.022	0.005	0.015	0.022	0.002
3-regime model												
debt<=60%	-0.002	-0.002	-0.003	-0.002	0.000	-0.010	-0.001	0.006	-0.002	-0.002	-0.007	-0.002
60%<=debt <90%	-0.007	-0.011 *	-0.002	-0.003	-0.006	-0.001	-0.022 **	-0.023 **	-0.018	-0.016 **	-0.021 **	-0.009
debt>=90%	0.001	0.014	0.000	-0.031	-0.010	-0.082 **	0.024	0.038	0.008	0.023	0.039	0.005
4-regime model												
debt<30%	0.000	0.000	-0.001	-0.001	0.000	0.000	0.000	-0.001	-0.001	0.000	-0.002	-0.002
30%<=debt <60%	-0.020 **	-0.011	-0.033 **	-0.018 *	-0.014	-0.027 **	-0.018	0.015	-0.041 **	-0.020 **	-0.010	-0.035 **
60%<=debt <90%	-0.007	-0.011 *	-0.002	-0.003	-0.006	-0.001	-0.022 **	-0.023 **	-0.018	-0.016 **	-0.021 **	-0.009
debt>=90%	0.001	0.014	-0.001	-0.031	-0.010	-0.082 **	0.023	0.039	0.008	0.023	0.039	0.004

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively. The estimations are carried out with country fixed effects.

Table A3. **The nonlinear effect of public debt on growth, annual data**

DEBT THRESHOLDS DETERMINED ENDOGENOUSLY

	1790-2009			**1946-2009**		
	Nonlinear variable =public debt/GDP Threshold variable = public debt/GDP			Nonlinear variable = public debt/GDP Threshold variable = public debt/GDP		
	All countries	**Developed countries**	**21 emerging markets**	**All countries**	**Developed countries**	**21 emerging markets**
Test of nonlinearity			Bootstrapped p-value			
H0: linear *vs*. H1: 2-regimes	**0.000**	**0.002**	**0.006**	**0.000**	**0.000**	**0.002**
H0: 2-regimes *vs*. H1: 3-regimes	0.194	**0.022**	**0.020**	0.104	**0.020**	0.104
Coefficients						
Low debt	0.039 **	0.069 **	-0.093 **	0.037 *	0.005	-0.082 **
Middle debt		0.017	-0.053 **		-0.029 **	-0.035 **
High debt	-0.006 **	-0.002	-0.027 **	-0.018 **	-0.018 **	
Debt thresholds (%)						
Threshold 1	19.26	20.40	29.01	17.48	28.21	32.97
Threshold 2		40.68	55.57		62.32	

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively.

Table A4. **Sensitivity check, advanced OECD countries, 1946-2009, annual data, lagged public debt**

	Nonlinear variable = lagged public debt/GDP Threshold variable = lagged public debt/GDP						Nonlinear variable = lagged growth rate of public debt/GDP Threshold variable = lagged public debt/GDP					
	Minimum % of observations required in one regime											
	30%		10%		5%		30%		10%		5%	
Test of nonlinearity	Bootstrapped p-value											
H0: linear *vs*. H1: 2-regimes	**0.000**		**0.000**		**0.000**		0.258		**0.034**		**0.022**	
H0: 2-regimes *vs*. H1: 3-regimes	0.190		0.118		**0.000**		0.018		**0.028**		**0.012**	
Coefficients												
Low debt	0.048	**	0.063	**	0.032				0.017		0.017	
Middle debt					-0.019	**			-0.023	**	-0.023	**
High debt	-0.008	**	-0.008	*	-0.006				-0.002		-0.002	
Debt thresholds (%)												
Threshold 1	22.37		20.38		20.38				13.64		13.64	
Threshold 2					106.33				60.82		60.82	

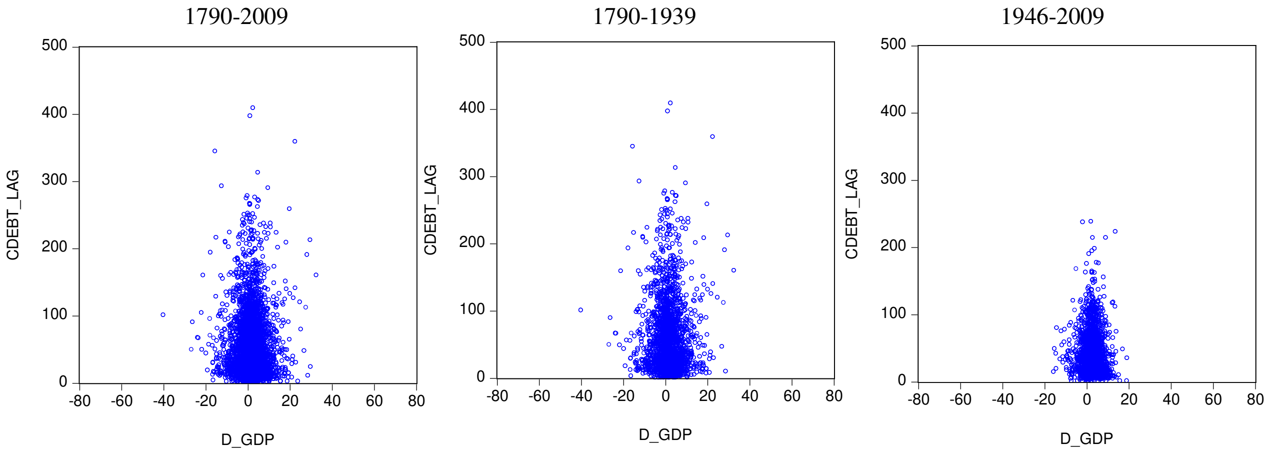
Note: * and ** denote statistical significance at the 10% and 5% levels, respectively.

Table A5. **Advanced OECD countries, 1946-2009, non-overlapping multi-year averages, sensitivity check**

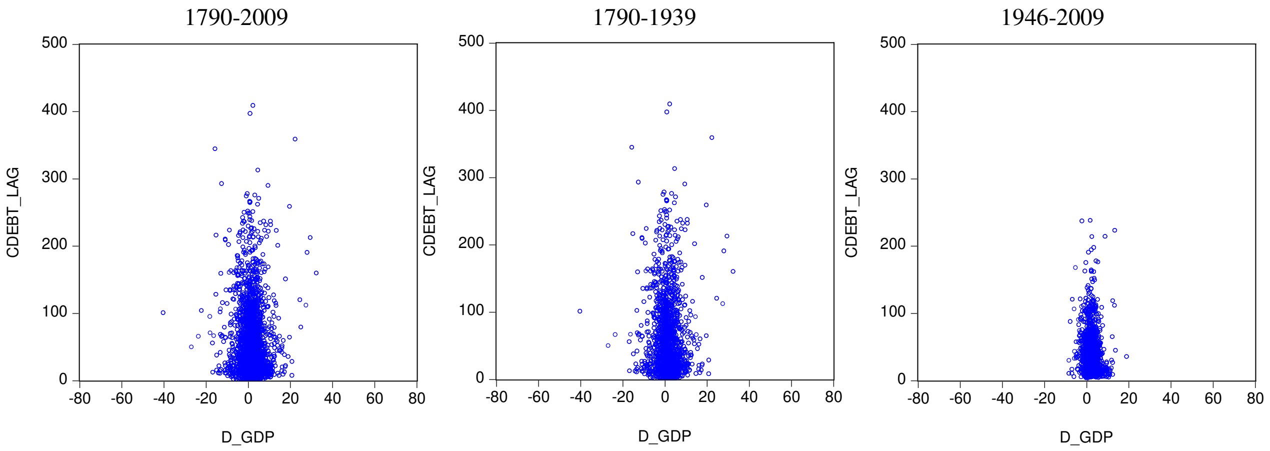
	Minimum % of observations required in one regime								
	30%			10%			5%		
	Nonlinear variable = level of the public debt to GDP ratio **Threshold variable= public debt to GDP ratio**								
	5-year	8-year	10-year	5-year	8-year	10-year	5-year	8-year	10-year
Test of nonlinearity	Bootstrapped p-value								
H0: linear *vs*. H1: 2-regimes	**0.002**	**0.048**	**0.010**	**0.002**	**0.024**	**0.004**	**0.002**	**0.020**	**0.008**
H0: 2-regimes *vs*. H1: 3-regimes	0.246	0.252	**0.006**	**0.002**	**0.004**	**0.002**	**0.004**	**0.008**	**0.006**
Coefficients									
Low debt	0.032 *	0.028	-0.003	0.186 **	0.226 **	0.189 **	0.186 **	0.226 **	0.189 **
Middle debt		-0.009	-0.04 **	0.046 **	0.047 **	0.041 *	0.046 **	0.047 **	0.041 *
High debt	-0.01 *		-0.022 **	-0.004	-0.003	-0.009	-0.004	-0.003	-0.009
Debt thresholds (%)									
Threshold 1	32.47	31.56	36.30	12.51	11.98	12.30	12.51	11.98	12.30
Threshold 2			54.53	33.27	31.56	36.30	33.27	31.56	36.30
	Nonlinear variable = rate of growth of the public debt to GDP ratio **Threshold variable= public debt to GDP ratio**								
	5-year	8-year	10-year	5-year	8-year	10-year	5-year	8-year	10-year
Test of nonlinearity	Bootstrapped p-value								
H0: linear *vs*. H1: 2-regimes	**0.022**	**0.034**	**0.018**	**0.022**	**0.016**	**0.020**	**0.044**	**0.024**	**0.010**
H0: 2-regimes *vs*. H1: 3-regimes	0.282	0.374	0.210	**0.088**	0.100	**0.074**	**0.090**	**0.088**	**0.096**
Coefficients									
Low debt	-0.009	-0.005	-0.037	0.018	-0.005	-0.018	0.018	-0.006	-0.018
Middle debt				-0.064 **		-0.099 **	-0.064 **	-0.149 **	-0.099 **
High debt	-0.078 **	-0.095 **	-0.151 **	-0.008	-0.095 **	-0.298 **	-0.008	-0.025	-0.298 **
Debt thresholds (%)									
Threshold 1	52.62	42.56	26.08	17.47	42.56	15.36	17.47	42.56	15.36
Threshold 2				39.13		67.04	39.13	61.98	67.04

Figure A1. **Annual real GDP growth and lagged central government debt as a % of GDP**

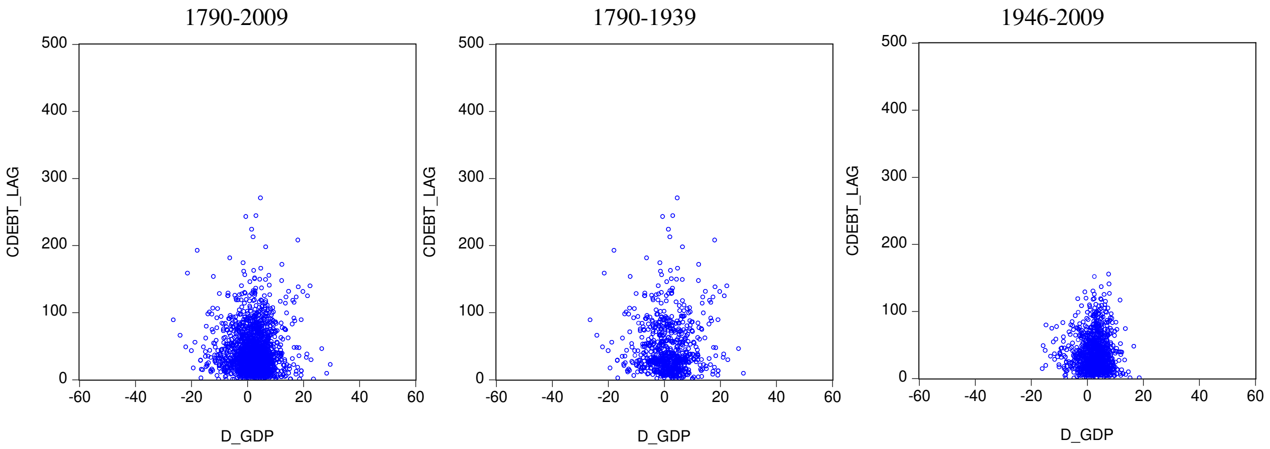
Panel A. All countries



Panel B. Developed economies



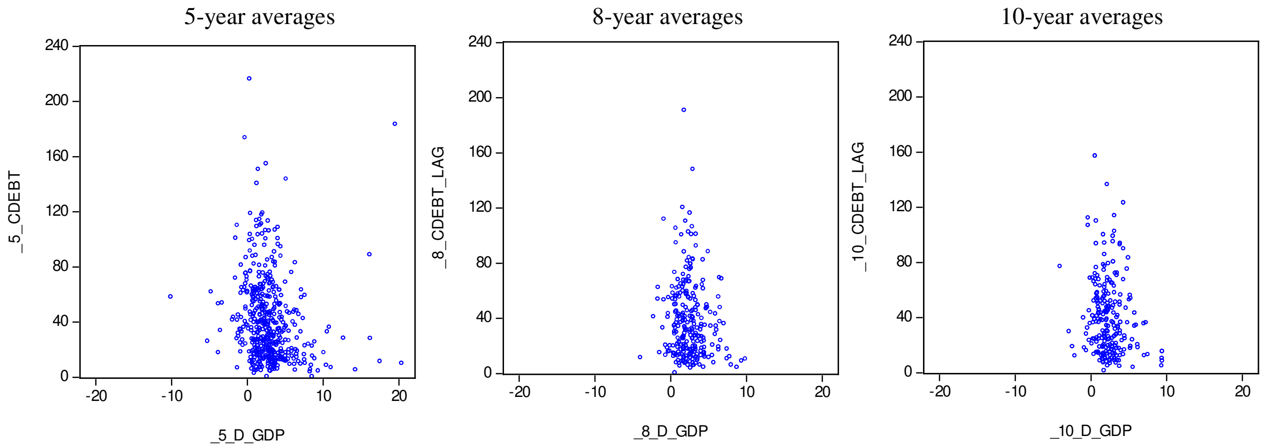
Panel C. 21 emerging market economies



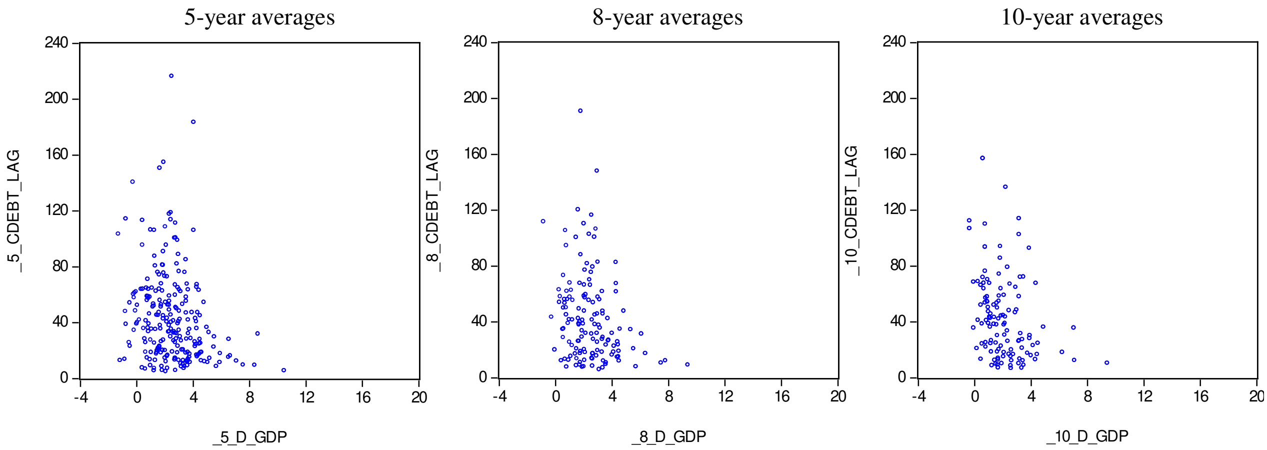
Source: Author's calculations.

Figure A2. **Non-overlapping multi-year real GDP growth and lagged central government debt as a % of GDP, 1946-2009**

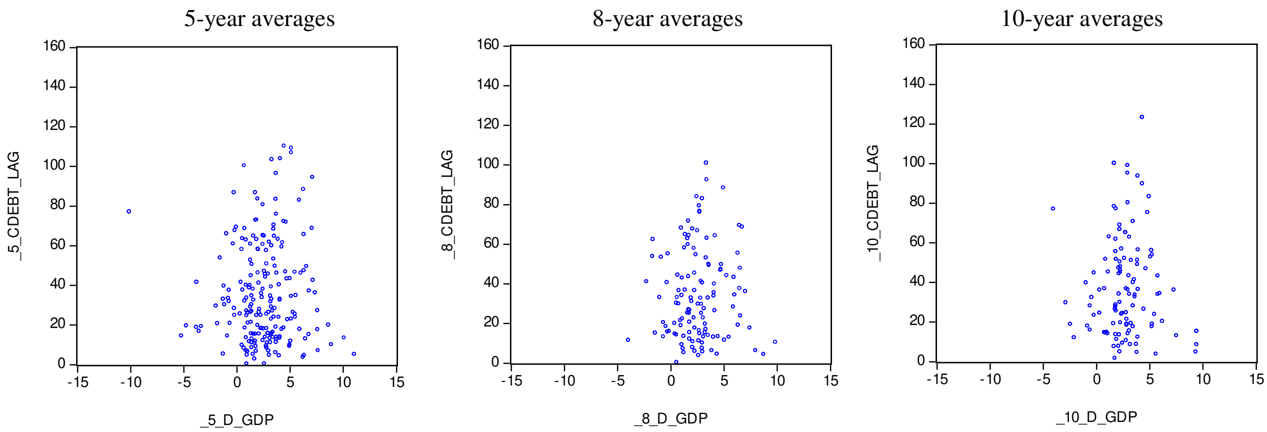
Panel A. Developed and emerging market economies



Panel B. Developed economies



Panel C. 21 emerging market economies



Source: Author's calculations.

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