Public Investment Financed by Seigniorage and Money Supply Control

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Abstract

This study examines the conditions necessary for the effective functioning of infrastructure management and monetary control policies in a seigniorage-dependent economy based on an overlapping generations model. Moreover, we analyze the relationship between the ratio of private capital to public capital and changes in the general price level. The results show that when the monetary growth rate that maximizes the gross domestic product (GDP) growth rate is selected, the elasticity of the ratio of private capital to public capital with respect to monetary growth depends on the private capital elasticity of GDP. If maximizing social welfare is equivalent to maximizing economic growth, the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment is zero. When the initial value of the ratio of private capital to public capital is at a sufficiently low (high) level, inflation (deflation) occurs during the transition path.

JEL classification: E52; H54; O40

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1 Introduction

Many economies in which government revenue depends on seigniorage are stagnating. What are the reasons underlying their governments' policy failures in attempting to improve macroeconomic performance? When investment in infrastructure and maintenance is implemented using revenue from seigniorage, what relationships exist between inflation and the ratio of private capital to public capital? The objective of this study is to provide theoretical answers to these questions.

There have been numerous studies on the macroeconomic effects of infrastructure development.[1] Theoretical studies often examine taxes and public bonds as sources of revenue for governments. However, it should be noted that there are countries such as Argentina, Uruguay, and Chile in which seigniorage constitutes a significant share of government revenues (Champ *et al.*, 2016). It follows that studies of macroeconomic policy in such countries should be discussed in the context of a model that incorporates seigniorage. Nevertheless, few studies have theoretically addressed infrastructure investment based on seigniorage as a financial resource (Basu, 2001; Tamai, 2008).

Previous studies that are closely related to this study include Crettez *et al.* (2002), Yakita (2008), Maebayashi (2013), and Yanagihara and Lu (2013). All of these studies are characterized by a basic framework that is based on discrete-time overlapping generations (OLG) models a la Diamond (1965). Crettez *et al.* (2002) and Yanagihara and Lu (2013) developed an argument related to public policy that uses seigniorage. However, Crettez *et al.* (2002) focused on public services that affect household utility, while Yanagihara and Lu (2013) emphasized the role of public education spending in human capital formation. Yakita (2008) considered a situation in which government revenues from income taxes are allocated to new investments in industrial infrastructure

[1] Arrow and Kurz (1970) were the first to report a study using mathematical modeling of infrastructure investment and economic growth. In particular, Irmen and Kuehnel (2009) surveyed the major theoretical studies following Barro (1990). In addition, Gramlich (1994) and Straub (2011) conducted a survey of empirical research.

and maintenance of infrastructure, and examined the selection of income tax rates to maximize the gross domestic product (GDP) growth rate and the allocation problem in relation to government spending. Maebayashi (2013) addressed the issue of social security benefits and infrastructure investment based on the model of Yakita (2008). Note that Maebayashi (2013) ignored infrastructure maintenance to simplify the analysis.

The insight of Rioja (2003), whose pioneering study examined the role of infrastructure maintenance in economic growth, is noteworthy, although the analysis is based not on an OLG model but on an infinite-horizon representative agent model. Considering the fact that many developing countries assign great importance to infrastructure maintenance, Rioja (2003) pointed out the lack of attention to this aspect in conventional studies. In addition to the remarks by Rioja (2003), the recognition of issues in this study is based on the current situation in which the theoretical examination of infrastructure improvement that relies on seigniorage has been inadequate, although seigniorage remains an important means of fundraising for the governments of some developing countries, as mentioned above.

In our model, the cash-in-advance (CIA) constraint plays an important role.[2] More specifically, it assumes a condition under which households are faced with the CIA constraint and introduces fiat money to the OLG model of Yakita (2008). In addition, seigniorage is incorporated in our model. Unlike Yakita (2008), we only examine revenue from seigniorage as a financial resource for infrastructure investment and maintenance by government, which permits control of the money supply and infrastructure management using revenue from seigniorage within a unified framework. This is a novel contribution of this study. In effect, both monetary control and infrastructure management are available as policy options, rather than governments being forced to choose one or the other. Furthermore, theoretical analysis of infrastructure management that separates infrastructure investment and maintenance is meaningful in developing

[2]The fundamental premise of the CIA constraint that transactions must be in cash, which must be held before the transactions are undertaken, was first presented by Clower (1967).

a more realistic argument when such an analysis addresses developing economies that are highly dependent on seigniorage.

The results show that when examining the effectiveness of a policy aimed at maximizing economic growth, the private capital elasticity of GDP can be regarded as an important indicator. If the monetary growth rate that maximizes social welfare is equal to that which maximizes the GDP growth rate, the elasticity of the ratio of private capital to public capital with respect to monetary growth is zero. Moreover, our model implies that the use of seigniorage does not always cause inflation.

The rest of this paper is organized as follows. In Section 2, we explain the basic structure of the model and consider the dynamic properties of the economic variables. In Section 3, we examine the conditions necessary for effective functioning of policies relating to infrastructure management and monetary control from the perspective of growth-maximizing and welfare-maximizing policies. In Section 4, we analyze the influences of the monetary growth rate and the share of expenditure on infrastructure investment on inflation. In Section 5, we summarize the main results and present our conclusions. Derivations of some equations are presented in the Appendices.

2 The Model

2.1 Households

Consider a closed economy in which the total number of individuals born at the beginning of period $t = 1, 2, \dots$ is N_t. The cohort of individuals who were born at the beginning of period t is called generation t. The individuals are unisex, and live for at most two periods, a young period and an old period. For simplicity, we follow Yakita (2008) by assuming that individuals of generation t become pregnant during period t (when young) and give birth at the beginning of period $t+1$ (when old). Moreover, we assume that individuals of generation t live for two periods with probability λ, but die immediately after being born at the beginning of period $t+1$ with probability $1-\lambda$. For each generation, the number of people N_{t+1} of generation $t+1$ when young is equal to $n_t N_t$, which is the total number of births by generation t when old. The total population of a country at the beginning of period t is expressed as $\lambda N_{t-1} + N_t$. Moreover, our model implies that the percentage of older people in the total population (that is, the rate of aging) is represented by $\lambda/(\lambda + n_{t-1})$.

Individuals supply one unit of labor inelastically while young and receive a wage-based income. They do not work when old. We assume that the lifetime utility of an individual of generation t, u_t, depends on consumption when young, c_t, consumption when old, d_{t+1}, and the number of children, n_t. Here, we normalize the time endowment of an individual in the working period to 1. Considering the time consumed by prenatal training when young, the amount of time spent on prenatal training, which is called child-rearing time in Yakita (2008), is proportional to the number of children. More specifically, if child-rearing time per child is $\theta \in (0,1)$, an individual who gives birth to $n_t \geq 1$ children must allocate θn_t to child-rearing time. Therefore, the time available for labor by an individual is $1 - \theta n_t$.

The lifetime utility of an individual of generation t, u_t, is given by

$$u_t = \log c_t + \lambda\rho \log d_{t+1} + \varepsilon \log n_t, \tag{1}$$

where $\rho \in (0,1)$ represents a subjective discount factor and $\varepsilon > 0$ is the weight of subutility resulting from the number of children. It is found that $\partial u_t/\partial n_t = \varepsilon/n_t$. This implies that when the number of children is 1, the marginal utility of having a child is equal to ε. The budget constraint of an individual of generation t in period t is given by

$$c_t + s_t + \frac{M_t}{P_t} = w_t(1 - \theta n_t), \tag{2}$$

where s_t is the non-monetary savings, M_t is the fiat money holdings, P_t is the general price level, and w_t is the wage rate. In this study, money is defined as an asset that does not generate interest. The budget constraint of an individual of generation t in period $t+1$ is given by

$$d_{t+1} = \frac{R_{t+1} s_t}{\lambda} + \frac{M_t}{\lambda P_{t+1}}. \tag{3}$$

In (3), R_{t+1} is the real gross return on investment in private capital, which is assumed to depreciate fully within a period.[3]

Assume that a part of the funds used for purchasing goods in period $t+1$ must be prepared in the form of money at the beginning of period t. More specifically, we follow Hahn and Solow (1995) by assuming a CIA constraint that individuals of generation t face as follows:

$$M_t \geq \mu P_{t+1} d_{t+1}, \tag{4}$$

where the parameter $\mu \in (0, 1)$ represents the level of the CIA constraint.[4] The closer μ is to 1, the tighter the constraint becomes. In addition, we assume that μ satisfies the condition $\mu < \lambda$. Following Yanagihara and Lu (2013), the real rate of return on money that is held is less than the rate of return on private capital, that is, $R_{t+1} \geq P_t/P_{t+1}$.

[3]Note that the probability of survival, λ, is included in the right-hand side of (3). When an individual is certain to live until they are old, i.e., when $\lambda = 1$, (3) takes the same form as the budget constraint of old individuals in the models of Crettez *et al.* (2002) and Yanagihara and Lu (2013).

[4]Conventional monetary economic models include the money-in-the-utility (MIU) model and the CIA model. The MIU model is characterized by the formulation of a utility function that incorporates a real money balance on the assumption that possession of money affects individuals' utility. Blanchard and Fischer (1989, p. 192) pointed out two shortcomings of the MIU model-based approach. The first is that the roles played by the actual transactions and money are often overlooked. The second is that it remains unclear what constraints should be placed on the objective function. Shimizu (2016, pp. 462–463) had a negative perception of the assumption that because the more money people hold, the higher their utility becomes, this might be the more realistic approach. Rather than possessing money, the minimum amount of money necessary for payments would be held, which suggests the benefits of the CIA model.

Consequently, (4) can be rewritten as

$$M_t = \mu P_{t+1} d_{t+1}. \tag{5}$$

An individual maximizes their lifetime utility as indicated in (1), subject to the conditions of (2), (3), and (5). When an individual pursues utility-maximizing behavior, we find that

$$c_t = \frac{1}{1+\lambda\rho+\varepsilon} w_t, \tag{6}$$

$$d_{t+1} = \frac{\lambda\rho}{(1+\lambda\rho+\varepsilon)\left(\dfrac{\lambda-\mu}{R_{t+1}} + \dfrac{\mu P_{t+1}}{P_t}\right)} w_t, \tag{7}$$

$$n_t = \frac{\varepsilon}{\theta(1+\lambda\rho+\varepsilon)}. \tag{8}$$

Using (6) and (7), the allocation of consumption between the period when they are young and that when they are old for an individual of generation t is expressed as

$$\frac{d_{t+1}}{c_t} = \frac{\lambda\rho}{\dfrac{\lambda-\mu}{R_{t+1}} + \dfrac{\mu P_{t+1}}{P_t}}. \tag{9}$$

From (8), the rate of aging when individuals maximize their lifetime utility, χ, can be written as

$$\chi = \frac{\lambda\theta(1+\lambda\rho+\varepsilon)}{\varepsilon + \lambda\theta(1+\lambda\rho+\varepsilon)}. \tag{10}$$

With respect to the parameters ε, θ, λ, and ρ, (10) implies that $\partial\chi/\partial\varepsilon < 0$, $\partial\chi/\partial\theta > 0$, $\partial\chi/\partial\lambda > 0$, and $\partial\chi/\partial\rho > 0$. That is, an increase in the marginal utility of having one child leads to a decrease in the rate of aging. However, an increase in the child-rearing time per child, subjective discount factor, or probability of survival over two periods leads to an increase in the rate of aging.

2.2 Firms

To produce homogeneous goods, perfectly competitive firms input private capital and labor. The production function in terms of firm $i \in [1, F_t]$ takes the Cobb–Douglas form as follows:

$$Y_{i,t} = K_{i,t}^{\alpha}(A_t L_{i,t})^{1-\alpha}, \tag{11}$$

where $Y_{i,t}$ is the output of firm i, $K_{i,t}$ is the private capital input of firm i, $L_{i,t}$ is the labor input of firm i, and A_t is a measure of labor-augmenting technology. $\alpha \in (0,1)$ is a parameter.

Let Y_t be the aggregate output, K_t the aggregate private capital, and L_t the aggregate labor. That is, we have $Y_t \equiv \sum_{i=1}^{F_t} Y_{i,t}$, $K_t \equiv \sum_{i=1}^{F_t} K_{i,t}$, and $L_t \equiv \sum_{i=1}^{F_t} L_{i,t}$. Following Kalaitzidakis and Kalyvitis (2004) and Yakita (2008), A_t is defined as

$$A_t \equiv \frac{K_t^{\beta} G_t^{1-\beta}}{L_t}, \tag{12}$$

where G_t is the aggregate infrastructure (public capital) and $\beta \in (0,1)$ is a parameter. The formulation of A_t shows that the index measure of labor-augmenting technology increases in tandem with the level of private and public capital. That relationship has a strong positive effect on the production activities of individual firms. However, the greater the total amount of labor, the smaller such a benefit becomes. For each firm, A_t is regarded as given.

The profit of firm i, $\Pi_{i,t}$, can be written as $\Pi_{i,t} = L_{i,t}[(1-\alpha)k_{i,t}^{\alpha}A_t^{1-\alpha} - w_t]$, where $k_{i,t} \equiv K_{i,t}/L_{i,t}$. Firm i, which takes R_t and w_t as given, maximizes its profit for a given

$L_{i,t}$ by setting

$$R_t = \alpha k_{i,t}^{\alpha-1} A_t^{1-\alpha}. \tag{13}$$

In addition, in the market equilibrium, w_t equals the marginal product of labor corresponding to the value of $k_{i,t}$ that satisfies (14). That is,

$$w_t = (1-\alpha) k_{i,t}^{\alpha} A_t^{1-\alpha}. \tag{14}$$

The condition of (14) ensures that profit equals zero for any value of $L_{j,t}$.

As is clear from (13) and (14), all firms select the same amount of private capital per unit of labor in equilibrium. Considering (11), GDP, $Y_t \equiv \sum_{i=1}^{F_t} Y_{i,t}$, is equal to

$$Y_t = K_t^{\Omega} G_t^{1-\Omega}, \tag{15}$$

where $\Omega \equiv \alpha + \beta(1-\alpha)$. Using $k_{i,t} \equiv K_t/L_t$ and (12), (13) can be rewritten as

$$R_t = \alpha \left(\frac{K_t}{G_t} \right)^{\Omega-1}, \tag{16}$$

and (14) can be rewritten as

$$w_t = (1-\alpha) \left(\frac{K_t}{G_t} \right)^{\Omega} \left(\frac{G_t}{L_t} \right). \tag{17}$$

From (16) and (17), we find that the relationship $w_t/R_t = [(1-\alpha)/\alpha](K_t/L_t)$ holds. This means that the factor price ratio of labor and private capital is proportionate to the private capital per worker. Moreover, s_t is proportionate to w_t in the general equilibrium, as shown in Appendix A.

2.3 Government

Suppose an integrated government includes not only the central government per se, but also the central bank. Such an integrated government is simply called the government hereafter. Let $\overline{M}_t$ be the total money supply. Assuming that the government increases money at a rate of $\nu > 0$, the relationship between $\overline{M}_t$ and $\overline{M}_{t-1}$ is represented by

$$\overline{M}_t = (1+\nu)\overline{M}_{t-1}. \tag{18}$$

The total demand for money is written as $N_t M_t$. Therefore, in the money market equilibrium, we have

$$\begin{aligned} N_t M_t &= \overline{M}_t \\ &= (1+\nu)\overline{M}_{t-1}. \end{aligned} \tag{19}$$

For simplicity, we assume that the government's only source of revenue is seigniorage. The government maintains a balanced budget and allocates a share of revenue from seigniorage to investment in infrastructure and maintenance of infrastructure. Let B_t be the revenue from seigniorage, E_t be the investment in infrastructure, and Z_t be the expenditure on infrastructure maintenance. When the percentage of government spending on infrastructure investment (that is, the share of expenditure on infrastructure) is $\phi \in (0,1)$, the percentage of government spending on infrastructure maintenance (that is, the share of expenditure on infrastructure maintenance) is $1-\phi$. Moreover, the relationships $E_t = \phi B_t$ and $Z_t = (1-\phi)B_t$ hold. Considering (18), seigniorage is expressed as $(\nu\overline{M}_{t-1})/P_t$. Therefore, the government's budget constraint in period t is given by

$$\begin{aligned} E_t + Z_t &= B_t \\ &= \frac{\nu}{P_t}\overline{M}_{t-1}. \end{aligned} \tag{20}$$

Maintenance of infrastructure has the effect of decreasing the depreciation rate of infrastructure and increasing its durability. Infrastructure stock is accumulated according to

$$G_{t+1} = E_t + \big(1 - \delta_{G,t}\big)G_t, \tag{21}$$

where $\delta_{G,t} \in (0,1)$ is the depreciation rate of infrastructure. In addition, we assume that a relationship exists between the depreciation rate of infrastructure and the share of expenditure on maintenance as follows:

$$\delta_{G,t} = 1 - \zeta\Big(\frac{Z_t}{B_t}\Big), \tag{22}$$

where $\zeta \in (0,1)$ is a parameter. Substituting (22) into (21) yields

$$G_{t+1} = E_t + \zeta\Big(\frac{Z_t}{B_t}\Big)G_t. \tag{23}$$

In (23), given a share of expenditure on maintenance, the higher ζ becomes, the lower the depreciation rate of infrastructure becomes, which encourages the accumulation of infrastructure. However, if ζ is sufficiently small, infrastructure is not steadily accumulated. In this sense, ζ can be interpreted as an indicator of the efficiency of infrastructure maintenance (Agenor, 2013).

2.4 Dynamics

We now examine the dynamic properties of our model. As is clear from (15), GDP depends on both private capital and public capital. Therefore, we can gain an understanding of the behavior of GDP by investigating the behavior of private capital and public capital during the transition path. The dynamic equation for public capital is expressed as

$$\frac{G_{t+1}}{G_t} = \phi\nu\frac{\alpha\mu}{\lambda-\mu}\Big(\frac{K_t}{G_t}\Big)^{\Omega} + \zeta(1-\phi), \tag{24}$$

and the dynamic equation for private capital is expressed as

$$\frac{K_{t+1}}{K_t} = \frac{1}{1+\lambda\rho}\Big[\lambda\rho(1-\alpha) - (1+\nu)(1+\lambda\rho)\frac{\alpha\mu}{\lambda-\mu}\Big]\Big(\frac{K_t}{G_t}\Big)^{-(1-\Omega)}. \tag{25}$$

See Appendix B for the derivation of (24) and (25).

Here, we define $x_t \equiv K_t/G_t$. Using (24) and (25), we obtain

$$\begin{aligned} x_{t+1} &= \Big[\frac{\lambda\rho(1-\alpha)(\lambda-\mu) - \alpha\mu(1+\nu)(1+\lambda\rho)}{(1+\lambda\rho)(\lambda-\mu)}\Big]\frac{(\lambda-\mu)x_t^{\Omega}}{\phi\nu\alpha\mu x_t^{\Omega} + \zeta(1-\phi)(\lambda-\mu)} \\ &\equiv f(x_t). \end{aligned} \tag{26}$$

When all of the government revenue from seigniorage is allocated to investment in infrastructure, which means that $\phi = 1$, the ratio of private capital to public capital, x_t, is constant over time. This implies that in the case of $\phi = 1$, no transitional dynamics exist. However, because $0 < \phi < 1$ is assumed, the case in which there are no transitional dynamics is excluded.

Let $\widehat{x}$ be the ratio of private capital to public capital when the relationship $x_t = x_{t+1}$ holds. Fig. 1 shows a graph of (26) and the 45-degree line. It can be seen from Fig. 1 that when the initial value, x_0, of the ratio of private capital to public capital is less than the steady-state value, $\widehat{x}$, the ratio of private capital to public capital increases

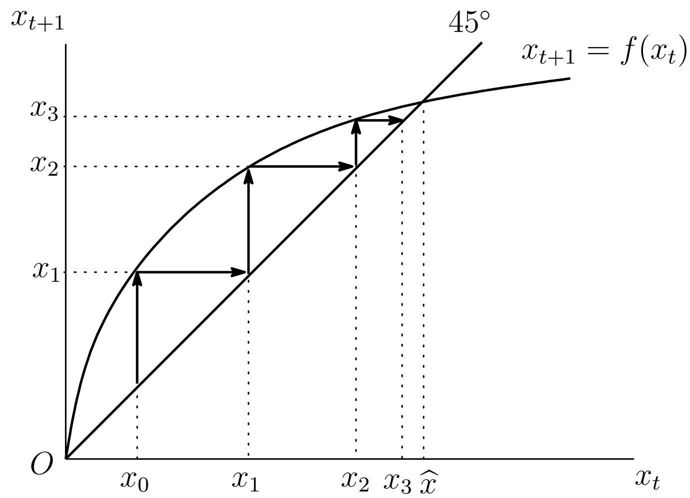

Figure 1: Behavior of the ratio of private capital to public capital

monotonously over time and converges to $\widehat{x}$. Conversely, if x_0 is greater than $\widehat{x}$, the ratio of private capital to public capital decreases monotonously over time and converges to $\widehat{x}$. Therefore, the steady growth equilibrium of the model is globally stable.

Because the relationship $K_t = \widehat{x}G_t$ holds in the steady state, the growth rates of private capital and public capital become equal. Considering (24), the gross economic growth rate, Y_{t+1}/Y_t, is represented by

$$\frac{Y_{t+1}}{Y_t} = \Big(\frac{x_{t+1}}{x_t}\Big)^{\Omega}\Big(\frac{G_{t+1}}{G_t}\Big). \tag{27}$$

When $x_t = x_{t+1} = \widehat{x}$, (27) implies that the GDP growth rate is equal to that of public capital. Consequently, the growth rates of GDP, private capital, and public capital become equal in the steady state. That is, the relationship $\gamma \equiv Y_{t+1}/Y_t = K_{t+1}/K_t = G_{t+1}/G_t \geq 1$ holds. Considering (24) and (27), we have

$$\gamma = \phi\nu\frac{\alpha\mu}{\lambda-\mu}\widehat{x}^{\Omega} + (1-\phi)\zeta. \tag{28}$$

Note that the GDP growth rate shown in (28) is expressed as a weighted average with the share of expenditure on infrastructure investment ϕ and the share of expenditure

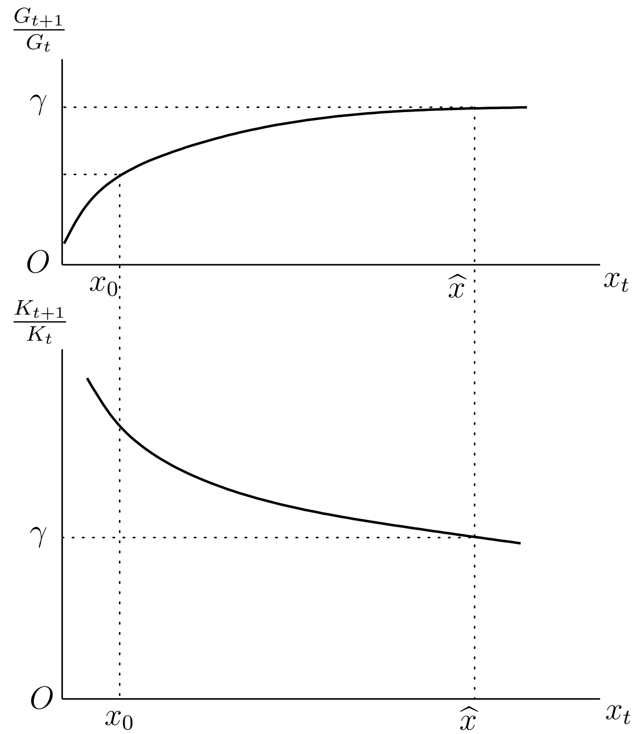

Figure 2: Growth rates of private capital and public capital

on maintenance $1 - \phi$ as the weights.

The relationship between (24) and (25) is shown in Fig. 2.

Regarding the features of these growth rates, it can be seen from Fig. 2 that although the growth rate of public capital increases monotonously along with an increase in the ratio of private capital to public capital, the growth rate of private capital declines monotonously. When the ratio of private capital to public capital increases over time, the growth rate of public capital also increases. Therefore, the GDP growth rate will increase consistently in accordance with (27). In addition, we can see from (27) that the percentage of public capital included in GDP, G_t/Y_t, decreases monotonously.

3 Policy Implications

3.1 Growth-maximizing Policy

In general, policy-makers are focused on the promotion of economic growth. This highlights the importance of implications provided by the model from the perspective of growth-promoting policies. As in the analytical procedure of Maebayashi (2013), we first consider the monetary growth rate that maximizes the GDP growth rate in the steady state (steady growth rate, hereafter) for a given share of expenditure on infrastructure investment. Subsequently, we examine the share of expenditure on infrastructure investment that maximizes the steady growth rate for a given monetary growth rate. We define $\widetilde{\nu}$ and $\widetilde{\phi}$ as the monetary growth rate and the share of expenditure on infrastructure investment, respectively, that maximize the steady growth rate. Obviously, the share of expenditure on maintenance that maximizes the steady growth rate is $1-\widetilde{\phi}$.

Let us investigate the monetary growth rate that maximizes the steady growth rate for a given share of expenditure on infrastructure investment. The derivative of (28) with respect to ν yields

$$\frac{\partial \gamma}{\partial \nu} = \frac{\phi\alpha\mu}{\lambda-\mu}\widehat{x}^{\Omega}\Big[1+\Omega\Big(\frac{\nu}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \nu}\Big)\Big]. \tag{29}$$

When $\nu=\widetilde{\nu}$, the money supply increases in accordance with the rule $\overline{M}_{t+1}=(1+\widetilde{\nu})\overline{M}_t$ under the control of policy-makers. Moreover, because the steady growth rate is maximized at $\nu=\widetilde{\nu}$, the relationship $(\partial\gamma/\partial\nu)|_{\nu=\widetilde{\nu}}=0$ holds. Therefore, (29) implies that

$$\Big(\frac{\nu}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \nu}\Big)\Big|_{\nu=\widetilde{\nu}} = -\frac{1}{\Omega}<0. \tag{30}$$

We call $(\nu/\widehat{x})\cdot(\partial\widehat{x}/\partial\nu)$ the elasticity of the ratio of private capital to public capital with respect to monetary growth. Given the share of expenditure on infrastructure investment, when the monetary growth rate that maximizes the steady growth rate is selected, the elasticity of the ratio of private capital to public capital with respect to monetary growth must be equal to $-(1/\Omega)$, from (30). The condition $0 < \Omega < 1$ implies that the absolute value of the elasticity of the ratio of private capital to public capital with respect to monetary growth is greater than 1 when the monetary growth rate maximizes the steady growth rate. Furthermore, (30) implies that the relationship $(\partial\widehat{x}/\partial\nu)|_{\nu=\widetilde{\nu}} < 0$ holds. Hence, in the steady state, the ratio of private capital to public capital decreases as the monetary growth rate increases when $\nu = \widetilde{\nu}$.

Regarding the case in which the share of expenditure on infrastructure investment maximizes the steady growth rate for a given monetary growth rate, (28) leads to

$$\frac{\partial\gamma}{\partial\phi} = \frac{\nu\alpha\mu}{\lambda-\mu}\widehat{x}^{\Omega}\Big[1+\Omega\Big(\frac{\phi}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\phi}\Big)\Big] - \zeta. \tag{31}$$

Note that $(\partial\gamma/\partial\phi)|_{\phi=\widetilde{\phi}} = 0$. Therefore, from (31), we obtain

$$\Big(\frac{\phi}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\phi}\Big)\Big|_{\phi=\widetilde{\phi}} = -\frac{1}{\Omega}\Big[1-\frac{\zeta(\lambda-\mu)}{\widehat{x}^{\Omega}\nu\alpha\mu}\Big]. \tag{32}$$

Moreover, (30) and (32) imply that

$$\Big(\frac{\phi}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\phi}\Big)\Big|_{\phi=\widetilde{\phi}} > \Big(\frac{\nu}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\nu}\Big)\Big|_{\nu=\widetilde{\nu}} = -\frac{1}{\Omega}.$$

Hence, the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment is greater than the elasticity of the ratio of private capital to public capital with respect to monetary growth.

3.2 Welfare-maximizing Policy

Suppose that a benevolent government maximizes social welfare. The policy instruments of the government are designed to control monetary growth and the share of expenditure on infrastructure investment (or maintenance). The monetary growth rate and the share of expenditure on infrastructure investment that maximize social welfare are called the optimal monetary growth rate and the optimal share of infrastructure investment, respectively.

Social welfare, U, is given by

$$U = \sum_{t=0}^{\infty} \sigma^t \Big(\log c_t + \frac{\lambda\rho}{\sigma} \log d_t + \varepsilon \log n_t \Big), \tag{33}$$

where $\sigma \in (0,1)$ is the social discount factor.[5] We express values of c_t, d_t, and n_t in the steady state as $\widehat{c}_t$, $\widehat{d}_t$, and $\widehat{n}$, respectively. In the steady state, (33) can be rewritten as

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \Big(\sigma^t \log \widehat{c}_t \Big) + \sum_{t=0}^{\infty} \Big(\sigma^t \frac{\lambda\rho}{\sigma} \log \widehat{d}_t \Big) + \Big(\sum_{t=0}^{\infty} \sigma^t \varepsilon \log \widehat{n} \Big) \\ &= \frac{1}{1-\sigma} \log \Big[\frac{1-\alpha}{1+\lambda\rho} \widehat{x}^{\Omega} \Big(\frac{G_0}{N_0} \Big) \Big(\frac{\gamma}{\widehat{n}} \Big)^{\frac{\sigma}{1-\sigma}} \Big] \\ &\quad + \frac{\lambda\rho}{\sigma(1-\sigma)} \log \Big[\frac{\alpha}{\lambda-\mu} \widehat{x}^{\Omega} \Big(\frac{G_0}{N_0} \Big) \widehat{n} \Big(\frac{\gamma}{\widehat{n}} \Big)^{\frac{\sigma}{1-\sigma}} \Big] \\ &\quad + \frac{\varepsilon}{1-\sigma} \log \widehat{n}. \end{aligned} \tag{34}$$

See Appendix C for the derivation of (34). Differentiation of (34) with respect to ν yields

$$\frac{\partial U}{\partial \nu} = \frac{1}{1-\sigma} \Big(1 + \frac{\lambda\rho}{\sigma} \Big) \Big[\Big(\frac{\Omega}{\widehat{x}} \frac{\partial \widehat{x}}{\partial \gamma} \Big) + \frac{\sigma}{1-\sigma} \Big(\frac{1}{\gamma} \frac{\partial \gamma}{\partial \nu} \Big) \Big]. \tag{35}$$

[5] See de la Croix and Michel (2002) for the theoretical basis of the social welfare function in an overlapping generations model.

Considering the relationship $(\partial U/\partial \nu)|_{\nu=\widetilde{\nu}} = 0$, (35) implies that

$$\Big(\frac{\nu}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \nu}\Big)\Big|_{\nu=\nu^*} = -\frac{\sigma}{\Omega(1-\sigma)}\Big(\frac{\nu}{\gamma}\frac{\partial \gamma}{\partial \nu}\Big)\Big|_{\nu=\nu^*}. \tag{36}$$

In (36), the term $(\nu/\widehat{x}) \cdot (\partial \widehat{x}/\partial \nu)$ represents the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment and the term $(\nu/\gamma) \cdot (\partial \gamma/\partial \nu)$ represents the elasticity of economic growth with respect to the share of expenditure on infrastructure investment. Because $\sigma/[(1-\sigma)\Omega] > 0$, if the sign of the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment is positive, the sign of the elasticity of economic growth with respect to the share of expenditure on infrastructure investment is negative. Furthermore, we find from (36) that

$$\frac{\partial \gamma}{\partial \nu}\Big|_{\nu=\nu^*} \lesseqqgtr 0 \Longrightarrow \Big(\frac{\nu}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \nu}\Big)\Big|_{\nu=\nu^*} \gtreqqless 0. \tag{37}$$

We can interpret (37) as follows. If the optimal monetary growth rate and the monetary growth rate that maximizes the steady growth rate are equal, the elasticity of the ratio of private capital to public capital with respect to monetary growth is zero.

However, when the elasticity of the ratio of private capital to public capital with respect to monetary growth is not zero, the optimal monetary growth rate is not equal to the monetary growth rate that maximizes the steady growth rate. Thus, the optimal monetary growth rate and the monetary growth rate that maximizes the steady growth rate are the same only when the condition that the elasticity of the ratio of private capital to public capital with respect to monetary growth is zero is satisfied.

For the effect of the share of expenditure on infrastructure investment on social welfare, we obtain

$$\frac{\partial U}{\partial \phi} = \frac{1}{1-\sigma}\Big(1+\frac{\lambda\rho}{\sigma}\Big)\Big[\Big(\frac{\Omega}{\widehat{x}}\frac{\partial \widehat{x}}{\partial \phi}\Big) + \frac{\sigma}{1-\sigma}\Big(\frac{1}{\gamma}\frac{\partial \gamma}{\partial \phi}\Big)\Big]. \tag{38}$$

Let ϕ^* represent the optimal share of expenditure on infrastructure investment, and let $1-\phi^*$ represent the optimal share of expenditure on maintenance. When $\partial U/\partial\phi = 0$, (38) leads to

$$\left(\frac{\phi}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\phi}\right)\bigg|_{\phi=\phi^*} = -\frac{\sigma}{\Omega(1-\sigma)}\left(\frac{\phi}{\gamma}\frac{\partial\gamma}{\partial\phi}\right)\bigg|_{\phi=\phi^*}, \tag{39}$$

where the term $(\phi/\widehat{x})\cdot(\partial\widehat{x}/\partial\phi)$ represents the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment and the term $(\phi/\gamma)\cdot(\partial\gamma/\partial\phi)$ represents the elasticity of economic growth with respect to the share of expenditure on infrastructure investment. (39) implies that

$$\frac{\partial\gamma}{\partial\phi}\bigg|_{\phi=\phi^*} \lesseqgtr 0 \Longrightarrow \left(\frac{\phi}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\phi}\right)\bigg|_{\phi=\phi^*} \gtreqless 0. \tag{40}$$

Based on (40), if the optimal share of infrastructure investment and the share of infrastructure investment that maximizes economic growth are equal, the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment is zero. However, if the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment is not zero, the optimal share of infrastructure investment and the share of expenditure on infrastructure investment that maximizes the steady growth rate do not match. That is, the optimal share of infrastructure investment and the share of expenditure on infrastructure investment that maximizes the steady growth rate are the same only when the condition under which the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment is zero is satisfied.

4 Inflation

What influence will changes in the monetary growth rate and the share of expenditure on infrastructure investment have on changes in the inflation rate? Analyzing this issue is conducive to understanding whether infrastructure management and monetary control in a seigniorage-dependent economy lead to an increase in the inflation rate. In our model, P_{t+1}/P_t can be written as

$$\frac{P_{t+1}}{P_t} = \Big[\frac{(1+\nu)(1+\lambda\rho)(\lambda-\mu)}{\lambda\rho(1-\alpha)(\lambda-\mu)-\alpha\mu(1+\nu)(1+\lambda\rho)}\Big] x_t^{1-\Omega}. \tag{41}$$

See Appendix D for the derivation of (41).

As can be seen from Fig. 2, when the initial value of the ratio of private capital to public capital, x_0, is sufficiently small, x_t increases over time. Therefore, (41) implies that if x_0 is sufficiently small, P_{t+1}/P_t increases over time. This means that a sustained increase in the general price level, that is, inflation, occurs during the transition path. However, as it approaches the steady state, the rate of increase of the ratio of private capital to public capital gradually falls; that is, the inflation rate declines. Conversely, when the initial value x_0 is higher than the steady-state value $\widehat{x}$, x_t decreases over time, which results in deflation during the transition path.

We now focus on the steady state and signify the inflation rate in the steady state as $\xi \equiv P_{t+1}/P_t$. For a given share of expenditure on infrastructure investment that maximizes the GDP growth rate, when the monetary growth rate that maximizes the GDP growth rate is selected, we have

$$\begin{aligned}\left.\frac{\partial \xi}{\partial \nu}\right|_{\nu=\widetilde{\nu}} &= \frac{(1+\lambda\rho)(\lambda-\mu)[\Psi+\alpha\mu(1+\widetilde{\nu})(1+\lambda\rho)]}{\Psi^2}(\widehat{x}|_{\nu=\widetilde{\nu}})^{1-\Omega} \\ &\quad + \frac{(1+\widetilde{\nu})(1+\lambda\rho)(\lambda-\mu)}{\Psi}(1-\Omega)(\widehat{x}|_{\nu=\widetilde{\nu}})^{-\Omega}\left.\frac{\partial \widehat{x}}{\partial \nu}\right|_{\nu=\widetilde{\nu}},\end{aligned} \tag{42}$$

where $\Psi \equiv \lambda\rho(1-\alpha)(\lambda-\mu)-\alpha\mu(1+\widetilde{\nu})(1+\lambda\rho)$. The sign of the first term on the right-hand side of (42) is positive, while the sign of the second term on the right-hand side is negative because $(\partial\widehat{x}/\partial\nu)|_{\nu=\widetilde{\nu}} < 0$. Therefore, we find that

$$\frac{\widetilde{\nu}[\Psi + \alpha\mu(1+\widetilde{\nu})(1+\lambda\rho)]}{\Psi(1+\widetilde{\nu})(1-\Omega)} \gtreqless -\left(\frac{\nu}{\widehat{x}}\frac{\partial\widehat{x}}{\partial\nu}\right)\bigg|_{\nu=\widetilde{\nu}} \Longrightarrow \frac{\partial\xi}{\partial\nu}\bigg|_{\nu=\widetilde{\nu}} \gtreqless 0. \tag{43}$$

The direction of the change in the inflation rate caused by an increase in the monetary growth rate is dependent on the elasticity of the ratio of private capital to public capital with respect to monetary growth. More specifically, (43) implies that if the absolute value of the elasticity of the ratio of private capital to public capital with respect to monetary growth is sufficiently small (large), an increase in the monetary growth rate leads to an increase (decrease) in the inflation rate at the monetary growth rate that maximizes the GDP growth rate. This suggests that an increase in the monetary growth rate in the steady state does not necessarily accelerate inflation.

Considering the effects of the share of expenditure on infrastructure investment on the change in the general price level, we obtain

$$\frac{\partial\xi}{\partial\phi}\bigg|_{\phi=\widetilde{\phi}} = \frac{(1+\nu)(1+\lambda\rho)(\lambda-\mu)(1-\Omega)}{\Psi}(\widehat{x}|_{\phi=\widetilde{\phi}})^{-\Omega}\,\frac{\partial\widehat{x}}{\partial\phi}\bigg|_{\phi=\widetilde{\phi}}. \tag{44}$$

As a result, (44) implies that

$$\frac{\partial\widehat{x}}{\partial\phi}\bigg|_{\phi=\widetilde{\phi}} \gtreqless 0 \Longrightarrow \frac{\partial\xi}{\partial\phi}\bigg|_{\phi=\widetilde{\phi}} \gtreqless 0. \tag{45}$$

For example, we find from (45) that when the ratio of private capital to public capital increases as the share of expenditure on infrastructure investment increases, the inflation rate increases in line with the increase in the share of expenditure on infrastructure investment.

Focusing on (9), inflation has an impact on the allocation of consumption between the period when individuals are young and that when they are old. In consideration of the relationships expressed by (9), (17), and (41), we can confirm that d_{t+1}/c_t in the steady state depends on $\widehat{x}$. Recall from (30) that $(\partial\widehat{x}/\partial\nu)|_{\nu=\widetilde{\nu}} < 0$. Therefore, when the monetary growth rate that maximizes the steady growth rate is selected, an increase in the monetary growth rate leads to a decrease in the ratio of private capital to public capital, resulting in an increase in the ratio of consumption when old to consumption when young. However, the influence of the share of expenditure on infrastructure investment (or maintenance) is ambiguous. For example, based on (45), when the share of expenditure on infrastructure investment that maximizes the steady growth rate is selected, if an increase in the share of expenditure on infrastructure investment leads to an increase in the inflation rate, the ratio of private capital to public capital increases and the ratio of consumption when old to consumption when young decreases.

5 Conclusion

We examined the conditions that must be satisfied when attempting to maximize economic growth and social welfare in an economy in which the government is dependent on seigniorage for revenue. In addition, we analyzed the conditions under which infrastructure management and monetary control in a seigniorage-dependent economy lead to an increase in the inflation rate. Therefore, a key contribution of this study is a clarification of such conditions within the context of a simple OLG model.

The main results are summarized as follows. If the monetary growth rate that maximizes the GDP growth rate is selected, the absolute value of elasticity of the ratio of private capital to public capital with respect to monetary growth is equal to the reciprocal of the private capital elasticity of GDP. Moreover, the elasticity of the ratio of private capital to public capital, with respect to the share of expenditure on infrastructure investment when the share of expenditure on infrastructure investment that maximizes the GDP growth rate is selected, is higher than the elasticity of the ratio of private capital to public capital, with respect to monetary growth when the monetary growth rate that maximizes the GDP growth rate is selected. Therefore, when examining the effectiveness of a policy aimed at maximizing economic growth, the private capital elasticity of GDP can be regarded as an important indicator.

Regarding the welfare implications of our model, if the monetary growth rate that maximizes social welfare is equal to that which maximizes the GDP growth rate, the elasticity of the ratio of private capital to public capital with respect to monetary growth is zero. Similarly, if the share of expenditure on infrastructure investment that maximizes social welfare is equal to that which maximizes the GDP growth rate, the elasticity of the ratio of private capital to public capital with respect to the share of expenditure on infrastructure investment is zero. These results suggest that simultaneous maximization of both economic growth and welfare is extremely difficult. For example, even when a country's social welfare is maximized, economic growth might not be maximized. Regarding the patterns of changes in the general price level that are attributable to the accumulation of private and public capital, inflation occurs during the transition path when the initial value of the ratio of private capital to public capital is at a sufficiently low level. Conversely, when the initial value of the ratio of private capital to public capital is at a sufficiently high level, deflation occurs during the transition path. Therefore, the results show that the use of seigniorage does not always cause inflation. Moreover, our model implies that an increase in the monetary growth rate or the share of expenditure on infrastructure investment in the steady state does not necessarily accelerate inflation.

Conventionally, political instability and institutional factors have attracted considerable attention as the main causes of economic stagnation in developing countries that are strongly dependent on seigniorage (Cukierman *et al.*, 1992; Click, 1998; Aisen and Veiga, 2008). Thus, there have been few theoretical studies focusing on economic growth and social welfare in a seigniorage-dependent economy. Our results suggest that the primary cause of stagnation might not be seigniorage per se, but the fact that it leads to a situation whereby the necessary conditions for maximizing economic growth and social welfare are not satisfied. This study shows that even when political instability is absent and institutional problems are solved, it is still possible that macroeconomic policies aimed at promoting economic growth and enhancing social welfare fail to function effectively.

Appendix A: Relationship between non-monetary savings and wages

We prove that there is a proportional relationship between the non-monetary savings of households and the wage income that households receive in the general equilibrium. Recall that the depreciation rate of private capital is assumed to be 1. Consequently, in the equilibrium private capital market, aggregate private capital at the beginning of period $t + 1$ is equal to aggregate non-monetary savings in period t. That is,

$$K_{t+1} = N_t s_t. \tag{A1}$$

Moreover, in the labor market equilibrium, we have

$$L_t = (1 - \theta n_t)N_t. \tag{A2}$$

Considering the CIA constraint in (5), (19) can be rewritten as

$$\overline{M}_{t-1} = N_{t-1}\mu P_t d_t, \tag{A3}$$

and (3) can be rewritten as

$$d_t = \frac{1}{\lambda - \mu} R_t s_{t-1}. \tag{A4}$$

Substituting (A4) into (A3) yields

$$\overline{M}_{t-1} = N_{t-1} \frac{\mu}{\lambda - \mu} P_t R_t s_{t-1}. \tag{A5}$$

From (16), (21), (A1), and (A5), we obtain

$$m_t = (1+\nu) \frac{\alpha\mu}{\lambda - \mu} \Big(\frac{K_t}{G_t}\Big)^{\Omega-1} \Big(\frac{K_t}{N_t}\Big), \tag{A6}$$

where $m_t \equiv M_t/P_t$. Using (A2), (A6) can be rewritten as

$$m_t = (1+\nu) \frac{\alpha\mu}{\lambda - \mu} \Big(\frac{K_t}{G_t}\Big)^{\Omega-1} (1 - \theta n_t) \Big(\frac{K_t}{L_t}\Big). \tag{A7}$$

In addition, (A7) and (8) lead to

$$m_t = (1+\nu) \frac{\alpha\mu}{\lambda - \mu} \Big(\frac{K_t}{G_t}\Big)^{\Omega-1} \Big(\frac{K_t}{L_t}\Big) \frac{1+\lambda\rho}{1+\lambda\rho+\varepsilon}. \tag{A8}$$

Furthermore, (2), (6), and (8) yield

$$s_t + m_t = \frac{\lambda\rho}{1+\lambda\rho+\varepsilon} w_t. \tag{A9}$$

Combining (17), (A8), and (A9), we obtain

$$s_t = \frac{1}{(1-\alpha)(1+\lambda\rho+\varepsilon)} \Big[\lambda\rho(1-\alpha) - (1+\nu)(1+\lambda\rho)\frac{\alpha\mu}{\lambda - \mu}\Big] w_t. \tag{A10}$$

From (A10), we can confirm that non-monetary savings change at the same rate as wages.

Appendix B: Derivation of (24) and (25)

First, we derive the dynamic equation for public capital. Dividing both sides of (23) by G_t implies that

$$\frac{G_{t+1}}{G_t} = \frac{E_t}{G_t} + \zeta\Big(\frac{Z_t}{B_t}\Big). \tag{B1}$$

Considering the relationships $E_t = \phi B_t$, $Z_t = (1-\phi)B_t$, and (20), (B1) can be rewritten as

$$\frac{G_{t+1}}{G_t} = \frac{\phi}{G_t}\frac{\nu}{P_t}\overline{M}_{t-1} + \zeta(1-\phi). \tag{B2}$$

Substituting (16), (A1), and (A5) into (B2) gives (24).

Next, we derive the dynamic equation for private capital. (A1) and (A10) imply that

$$K_{t+1} = N_t\frac{1}{1+\lambda\rho+\varepsilon}\Big[\lambda\rho(1-\alpha) - (1+\nu)(1+\lambda\rho)\frac{\alpha\mu}{\lambda-\mu}\Big]\Big(\frac{K_t}{G_t}\Big)^{\Omega}\Big(\frac{G_t}{L_t}\Big). \tag{B3}$$

Substituting (A2) into (B3) yields

$$K_{t+1} = \frac{1}{1+\lambda\rho}\Big[\lambda\rho(1-\alpha) - (1+\nu)(1+\lambda\rho)\frac{\alpha\mu}{\lambda-\mu}\Big]\Big(\frac{K_t}{G_t}\Big)^{\Omega}G_t. \tag{B4}$$

Furthermore, (B4) can be rewritten to obtain (25).

Appendix C: Derivation of (34)

Using (6), (8), and (17), we have

$$\widehat{c}_t = \frac{1}{1+\lambda\rho}(1-\alpha)\widehat{x}^{\Omega}\Big(\frac{G_t}{N_t}\Big). \tag{C1}$$

Because the relationships $G_t = \gamma^{\,t} G_0$ and $N_t = \widehat{n}^{\,t} N_0$ hold in the steady state, (C1) can be rewritten as

$$\widehat{c}_t = \frac{1-\alpha}{1+\lambda\rho}\widehat{x}^{\Omega}\left(\frac{\gamma}{\widehat{n}}\right)^t\left(\frac{G_0}{N_0}\right). \tag{C2}$$

From (16), (A1), and (A4), we obtain

$$\widehat{d}_t = \frac{\alpha}{\lambda-\mu}\widehat{x}^{\Omega}\left(\frac{\gamma}{\widehat{n}}\right)^t\left(\frac{G_0}{N_0}\right)\widehat{n}. \tag{C3}$$

Substituting (8), (C2), and (C3) into (33) gives (34).

Appendix D: Derivation of (41)

From (5), (A4), and (A9), we obtain

$$\frac{\lambda\rho}{1+\lambda\rho\varepsilon}w_t - s_t = \frac{\mu}{\lambda-\mu}\left(\frac{P_{t+1}}{P_t}\right)R_{t+1}s_t. \tag{D1}$$

Using (16), (17), and (A1), (D1) can be rewritten as

$$\frac{\lambda\rho}{1+\lambda\rho+\varepsilon}(1-\alpha)x_t^{\Omega}\frac{G_t}{L_t} - \frac{K_{t+1}}{N_t} = \frac{\mu}{\lambda-\mu}\left(\frac{P_{t+1}}{P_t}\right)\alpha x_t^{\Omega-1}\frac{K_{t+1}}{N_t}. \tag{D2}$$

Substituting (A2) and (25) into (D2) gives (41).

References

[1] Agénor, P.-R., 2013. Public Capital, Growth and Welfare: Analytical Foundations for Public Policy. Princeton: Princeton University Press, Princeton (US).

[2] Aisen, A., Veiga, F.J., 2008. The political economy of seigniorage. Journal of Development Economics 87(1), 29-50.

[3] Arrow, K. J., Kurz, M., 1970. Public Investment, the Rate of Return, and Optimal Fiscal Policy. Johns Hopkins Press, New York (US).

[4] Barro, R. J., 1990. Government spending in a simple model of endogenous growth. Journal of Political Economy 98(5), S103-S126.

[5] Basu, P., 2001. Reserve ratio, seigniorage and growth. Journal of Macroeconomics 23(3), 397-416.

[6] Blanchard, O.J., Fischer, S., 1989. Lectures on macroeconomics, The MIT Press, Cambridge (UK)

[7] Champ. B., Freeman, S., Haslag, J., 2016. Modeling monetary economies, Fourth Edition, Cambridge University Press, Cambridge (UK)

[8] Click, R.W., 1998. Seigniorage in a cross-section of countries. Journal of Money, Credit and Banking 30(2), 154-171.

[9] Clower, R.W., 1967. A reconsideration of the microfoundations of monetary theory. Western Economic Journal 6(1), 1-8.

[10] Crettez, B., Michel, P., Wigniolle, B., 2002. Seigniorage and public good in an OLG model with cash-in-advance constraints. Research in Economics 56(4), 333-364.

[11] Cukierman, A., Edwards, S., Tabellini, G., 1992. Seigniorage and political instability. American Economic Review 82(3), 537-555.

[12] de la Croix, D., Michel, P., 2002. A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations. Cambridge University Press, Cambridge (UK).

[13] Diamond, P.A., 1965. National debt in a neoclassical growth model. American Economic Review 55(5), 1126-1150.

[14] Gramlich, E.M. 1994. Infrastructure investment: A review essay. Journal of Economic Literature 32(3), 1176-1196.

[15] Hahn, F., Solow, R.M., 1995. A Critical Essay on Modern Macroeconomic Theory. Basil Blackwell, Oxford (UK).

[16] Irmen, A., Kuehnel, J. 2009. Productive government expenditure and economic growth. Journal of Economic Surveys 23(4), 692-733.

[17] Kalaitzidakis, P., Kalyvitis, S., 2004. On the macroeconomic implications of maintenance in public capital. Journal of Public Economics 88(3-4), 695-712.

[18] Maebayashi, N., 2013. Public capital, public pension, and growth. International Tax and Public Finance 20(1), 89-104.

[19] Rioja, F.K., 2003. Filling potholes: Macroeconomic effects of maintenance versus new investments in public infrastructure. Journal of Public Economics 87(9-10), 2281-2304.

[20] Shimizu, K., 2016. Advanced Theory of Banking and Finance. University of Tokyo Press, Tokyo (JP) (in Japanese).

[21] Straub, S., 2011. Infrastructure and development: A critical appraisal of the macro-level literature. Journal of Development Studies 47(5), 683-708.

[22] Tamai, T., 2008. Fiscal and monetary policy in an endogenous growth model with public capital. FinanzArchiv: Public Finance Analysis 64(4), 403-421.

[23] Yakita, A., 2008. Ageing and public capital accumulation. International Tax and Public Finance 15(5), 582-598.

[24] Yanagihara, M., Lu, C., 2013. Cash-in-advance constraint, optimal monetary policy, and human capital accumulation. Research in Economics 67(3), 278-288.