Establishing Definitions and Modeling Public Transport Travel Time Variability

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Abstract

Public transport travel time variability (PTTV) is essential for understanding deteriorations of travel time reliability, optimizing transit schedules and route choices. This paper establishes key definitions of PTTV in which firstly include all buses, and secondly include only a single service from a bus route. The paper then analyzes the day-to-day distribution of public transport travel time by using Transit Signal Priority data. A comprehensive approach using both parametric bootstrapping Kolmogorov-Smirnov test and Bayesian Information Creation technique is developed, recommends Lognormal distribution as the best descriptor of bus travel time on urban corridors. The probability density function of Lognormal distribution is finally used for calculating probability indicators of PTTV. The findings of this study are useful for both traffic managers and statisticians for planning and researching the transit systems.

Keywords: Public transport, travel time variability, reliability, travel time distribution, probability, indicators

INTRODUCTION

Public Transport Travel Time Variability (PTTV) is essential for transit operators. It facilitates investigating the deterioration of travel time reliability and explaining the reliability index. Knowledge of PTTV also simplifies the optimization of recovery time, which is the added time to the expected running time of a public transport schedule. Recovery time accounts for both travel time variation and a short break before the next departure. PTTV also plays an important role in traveler trip planning and route choice (*1*) since unreliable and highly variable travel time increases anxiety, stresses (*2*) and cost to the travelers (*3*). Therefore, ridership is lost when PTTV is high. A study in Oregon, US found that a 10% decrease in headway delay variation led to an increase of 0.17 passengers per trip per time-point (*4*).

Travel time variability (TTV) has been defined in the literature as the variance in travel times of vehicles travelling similar trips (*3; 5*). However, the definition is better suited for measuring private rather than public transport, as confusion arises in the definition of "similar trips". While private transport vehicles are treated as homogenous to some extent, public transport vehicles are noticeably different. By stopping at only selected stops, express routes are significantly faster than local routes, questioning the definition of "similar trips" particularly for practical purposes. Conversely, the availability of individual travel time data of each transit vehicle will provide new approaches to better define PTTV.

The empirical research on the statistics of travel time (*6*) is limited due to the limited availability of the empirical data. With the advancement of technology, advanced data sources such as Transit Signal Priority (TSP) (*7; 8*), Smart Card transactional data (*9*),bus Automatic Vehicle Location data (*10*), Bluetooth (*11; 12*) and Wi-Fi (*13; 14*) are available for research and development. This paper exploits TSP data to establish PTTV definitions and investigate its distribution and monitoring indicators. Firstly, the paper defines PTTV in consideration of all buses and a single bus route service. Secondly, the probability distribution of public transport travel time is investigated, revealing the nature and shape of travel time. Finally, the distribution probability density function (p.d.f.) is used to monitor and model PTTV. The findings of this research enable transport managers and researchers to better plan public transport systems.

TRAVEL TIME VARIABILITY IN LITERATURE

TTV has been defined in the literature as having three main types (*3; 5*):

Vehicle-to-vehicle (or inter-vehicle) variability (TTV_{v2v}) is the difference between travel times experienced by different vehicles travelling similar trips within the same time period.

Period-to-period (inter-period or within-day) (TTV_{p2p}) is the variability between the travel times of vehicles travelling similar trips at different times on the same day.

Day-to-day (or inter-day) (TTV_{d2d}) is the variability between similar trips on different days within the same time period. TTV_{d2d} is independent to the congestion effects. Within the same time period, a high demand transit system has low day-to-day TTV if congestions are recurrent.

The literature on PTTV is relatively limited. Abkowitz and Engelstein (*15*) predicted the running time and running time deviation by using linear regression analysis. Their model revealed that only the link length has significant impact on the day-to-day variability of public transport travel time. Mazloumi *et al.* (*16*) adopted the definition of variability from Noland and Polak (*3*) to explore the day-to-day PTTV in Melbourne, Australia using GPS data. The nature and pattern of variability were discussed by fitting bus travel time to Normal and Lognormal distribution, followed by a linear regression analysis to investigate the impacts of different factors to PTTV. Moghaddam *et al.* (*17*) proposed empirical models for predicting the Standard Deviation (SD) of bus travel time based on the average bus travel time, number of signalized intersection and a ratio between volume and capacity for an origin-destination path. Currie *et al.* (*18*) analyzed TTV when measuring the impacts of transit priority using Automatic Vehicle Location data. To the best of the authors' knowledge, there is no paper in the literature established public transport oriented definitions of TTV and provided a comprehensive travel time distribution analysis based on the definition.

METHODOLOGY AND DATA DESCRIPTION

The TSP sensors are operating at major corridors in Brisbane to give priority to buses at the signalized intersections. The sensors identify the unique vehicle identification number, route, timestamps and service scheduled start times of each passing bus. Service scheduled start time is the scheduled departure time from the depot, which is defined as a "service" in this paper. For instance, the 8:00 AM service includes vehicles that are scheduled to start their journey at 8:00 AM from the depot. By matching the same bus ID and service at upstream and downstream intersection, the difference between observed timestamps at upstream and downstream intersections is the travel time between the two intersections (*19*).
FIGURE 1 shows 4 major arterial corridors in Brisbane along with their operating bus routes and lengths. The Coronation Drive corridor (from High Street to Cribb Street) has been chosen as the case study site in this paper. The study site is highly congested on both morning and afternoon peak periods. The other three corridors and their bus routes are used in the final sub-section of the analysis for validation.

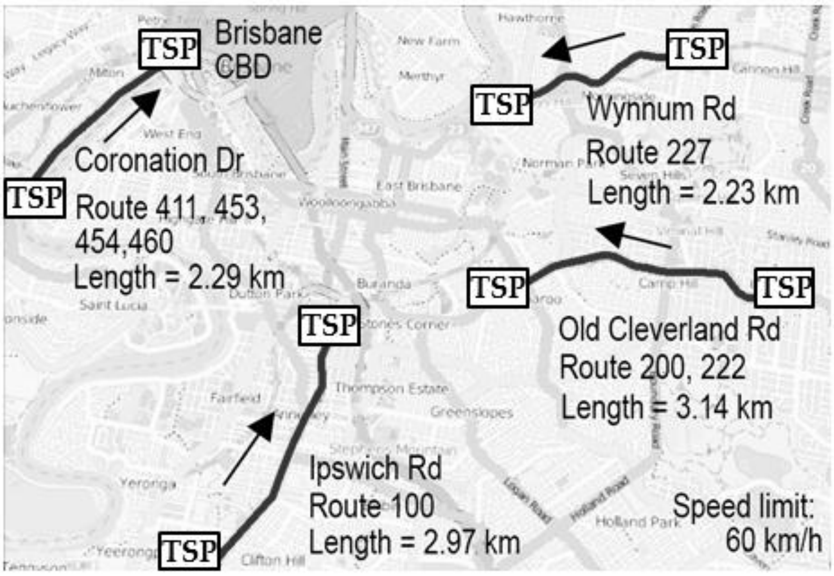


FIGURE 1 Study site

The analysis has been carried out on a year of TSP data (1st July 2011 to 30th Jun 2012) on inbound traffic. The analysis performed in this paper is on the recurrent variability of travel time of in-service buses (buses that are on operation) during working days (weekdays excluding Public Holidays and School Holidays). Public transport data is integrated with incident records to filter out travel time values during incidents. Buses that started their service earlier or later than the predetermined scheduled start time are also not considered, since different stop skipping, bus holding or priority strategies could have been applied exclusively on them.

This paper focuses on the day-to-day PTTV as it is the most advisable and practical type of TTV in public transport. From transit passengers' point of view, the variability of travel time of the same service or route on multiple days is more important than the TTV_{v2v} or TTV_{p2p}. Many transit commuters travel daily by a specific route/service at around a specific time of the day. From transit operators' point of view, day-to-day TTV provides a complete picture of transit performance on multiple days; facilitates schedule optimization and identify the sources of travel time unreliability.

DAY-TO-DAY PUBLIC TRANSPORT TRAVEL TIME VARIABILITY DEFINITIONS

Day-to-day TTV measures the variability between travel times of vehicles on similar trips on different days within the same time period (*3; 5*). This section establishes two key definitions of PTTV. While the first definition is an extension from the common TTV definition used for private transport, the second definition is established for measuring PTTV of each bus service.

Day-to-day PTTV definition derived from private transport TTV

TTV_{d2d} is traditionally calculated from the average travel time values of multiple days within a certain time window, or using the floating car travel time on the same study sites (*20; 21*). This research measures the variability of travel time using the Coefficient of Variation (CV) of travel time, the well-accepted measure of travel time variability in literature which measures the travel time variation as the ratio of the SD to the mean. Because CV is quantified in form of percentage, PTTV from different sites could be compared. The TTV_{d2d} can be calculated as CV_p in equation (1).

$$CV_p = \frac{\sqrt{\frac{1}{D}\sum_{d=1}^{D}(TT_{d,p}-\overline{TT}_p)^2}}{\overline{TT}_p} \qquad (1)$$

Here,
CV_p = CV of travel time (%) during time window *p* during D days,
$TT_{d,p}$ = mean travel time (s) of the vehicles traversing time window *p* on day *d,*
$\overline{TT_p}$ = the average value of all $TT_{d,p}$ (s) during time window *p* during D days, which can be expressed by the following equation.

$$\overline{TT}_p = \frac{\sum_{d=1}^{D} TT_{d,p}}{D} \quad (2)$$

The traditional definition of TTV_{d2d} can be extended to accommodate PTTV, in which PTTV is measured by the Equation (1). Each mean value $TT_{d,p}$ includes *all buses of all routes* passing the study corridor within a 30 minutes study time window on a working day. This definition of $PTTV_{d2d}$ is illustrated in FIGURE 2.

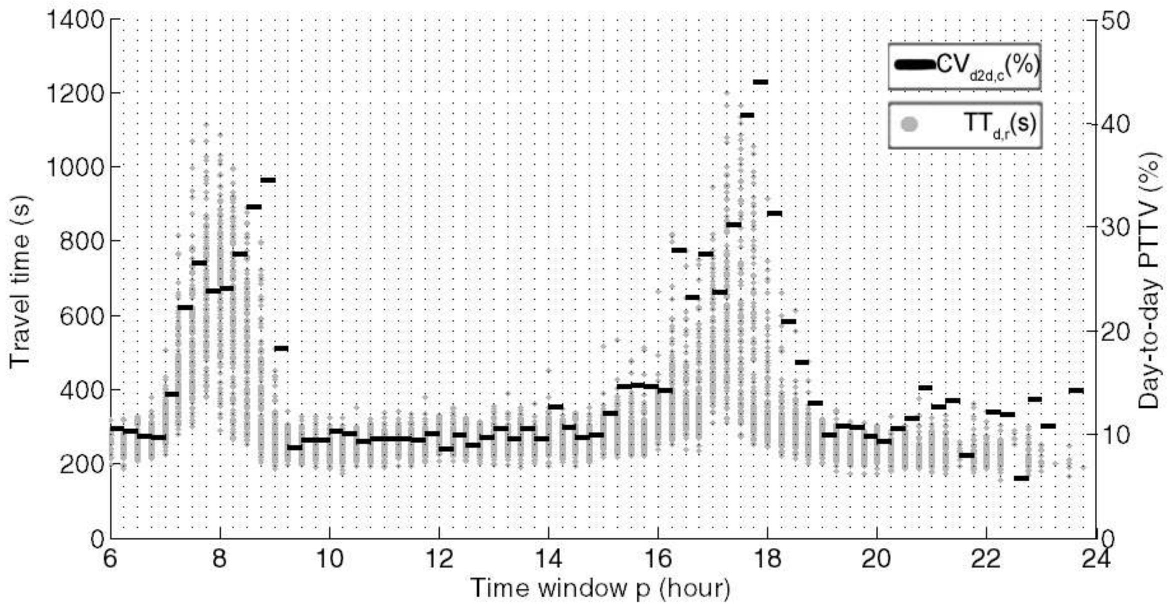


FIGURE 2 Observed $PTTV_{d2d,c}$ on Coronation Drive, Brisbane

The variability is defined in equation (1) over the mean travel time obtained from each day and period ($TT_{d,p}$). The mean travel time for each day is considered even though individual vehicle travel time is available. The reasons for this can be explained with the help of an example. Given *day1* and *day2* with *n* buses for a given period on each day, if not all travel time values are identical, there is TTV_{v2v} during that period. Assuming, travel times of individual vehicles in *day2* are exactly the same as *day1*, if all the individual vehicle travel time samples from *day1* and *day2* are used to calculate TTV_{d2d} then estimated TTV_{d2d} will be equal to TTV_{v2v}. However, in this example the TTV_{d2d} should be zero because the two days are exactly the same. If mean travel time values from the two days are used to estimate the variability, then TTV_{d2d} will be zero.

This paper terms this variability as ***day-to-day PTTV on corridor level*** ($PTTV_{d2d,c}$). This is useful for traffic managers in monitoring the day-to-day variability of bus travel time in general. Having the same method to calculate TTV enables effective comparison of the variability between different modes of transport, for instance between public and private transport.

Day-to-day PTTV definition using additional data of transit vehicles

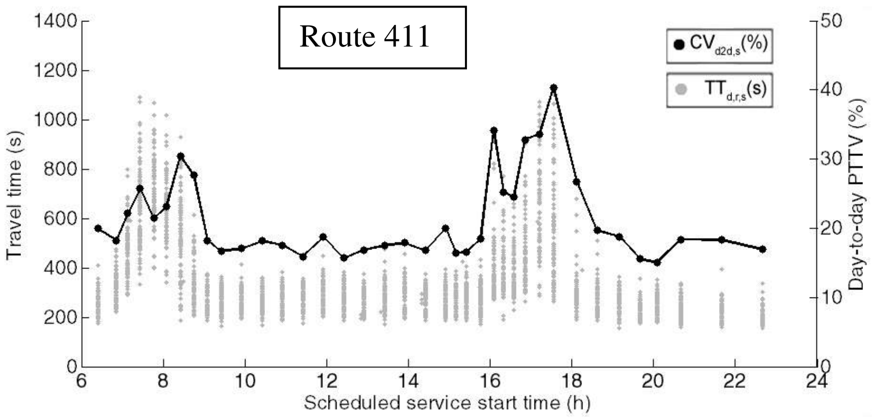
Public transport often allows tracking of each vehicle on a specific route or even a specific service. This sub-section establishes another definition of *day-to-day PTTV* to take advantage of the additional information. The definition aims for monitoring transit performance and facilitating timetable adjustments. The definition of “similar trips” now comes to the inclusion of only the buses on the *same route* and *service*, because these buses are scheduled to travel time similarly.

$$CV_{r,s} = \frac{\sqrt{\frac{1}{D}\sum_{d=1}^{D}(TT_{d,r,s} - \overline{TT_{r,s}})^2}}{\overline{TT}} \quad (3)$$

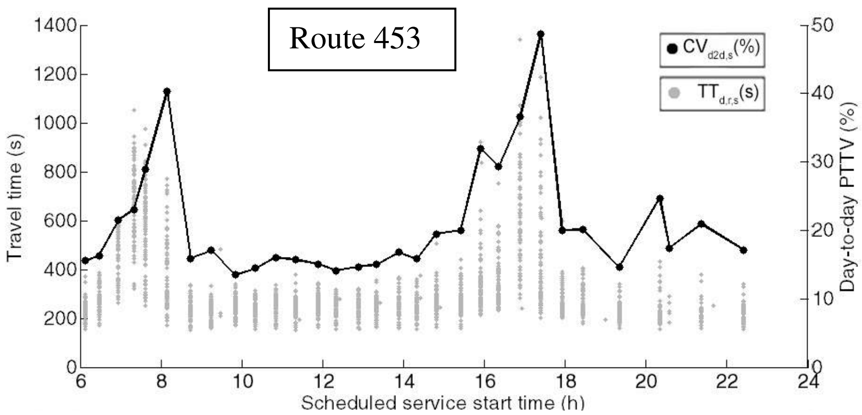
Here,
$CV_{r,s}$= CV of travel time (%) of route *r* and service *s* during D days,
$TT_{d,r,s} = d^{th}$ individual travel time sample (s) of the bus of route *r* and service *s* on day *d,*
$\overline{TT_{r,s}}$ = the average value of all $TT_{d,r,s}$ (s) of route *r* and service *s* during D days, which can be expressed by the following equation.

$$\overline{TT}_{r,s} = \frac{\sum_{d=1}^{D} TT_{d,r,s}}{D} \quad (4)$$

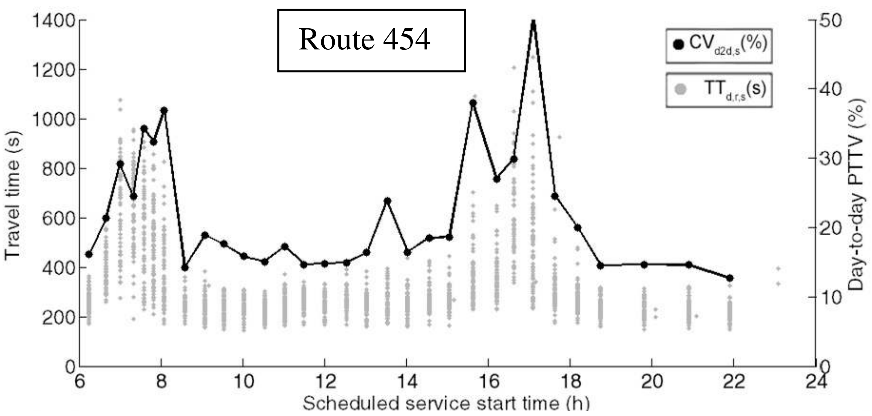
This definition is separated from the traditional measurement of private transport TTV for making use of the additional data of public transport. Each value of $TT_{d,r,s}$ includes only an individual bus of the specific service of a specific route. FIGURE 3 illustrates this definition using the four routes from Coronation Dr.



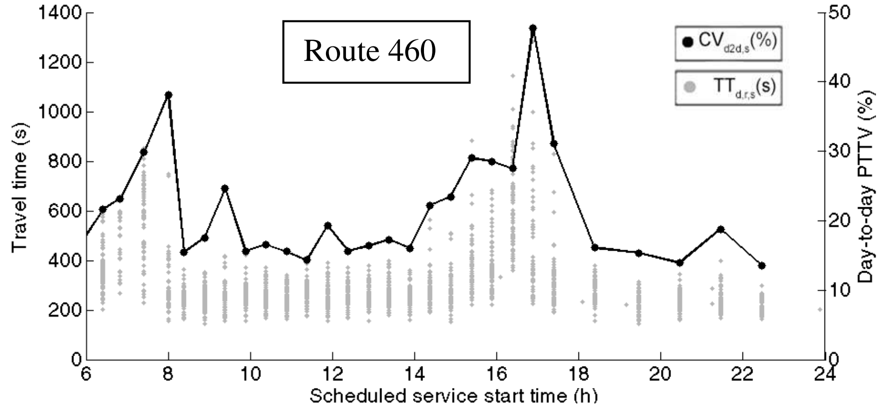
(a)



(b)



(c)



(d)

FIGURE 3 Observed $PTTV_{d2d,s}$ on Route: (a) 411, (b) 453, (c) 454 and (d) 460

FIGURE 3 shows the day-to-day PTTV of services during off-peak periods are relatively low indicating that these services are reliable. The variability follows the same pattern as the congestion increases and reduces. Afternoon congestion shows a small peak of $CV_{r,s}$ before the main peak congestion to denote the school-off time when secondary school students are traveling home. This paper terms this variability as ***day-to-day PTTV on service level*** ($PTTV_{d2d,s}$). This second established definition of PTTV is useful for transit operators in scheduling, particularly in deciding the timetable and recovery time along with discovering the multiple day reliability performance of each service because it is defined by individual bus travel time.

The aforementioned two definitions are further discussed as below:

Day-to-day PTTV on corridor level ($PTTV_{d2d,c}$) is the extension of the widely used definition of TTV in literature to public transport. The definition reflects the PTTV in general by considering all passing buses, which enables meaningful comparison with other modes of transport. For instance, PTTV provides insights on how the consistency and dependency of public transport modes are compared to private counterparts.
Day-to-day PTTV on service level ($PTTV_{d2d,s}$) measures TTV of a specified route service. The individual bus travel time samples on multiple days are used for TTV calculation. These individual buses are planned to travel on “similar trips” as they are on the same route and service. The variations in their travel times show the patterns of TTV and indicate service performance. Significantly, as it is a more focused scale compared to the first definition, the *day-to-day PTTV on service level* facilitates investigating the sources of unreliability and optimizing the timetables. The definition of *day-to-day PTTV on service level* is more useful as it provides more information on individual vehicle performances, which can be used on more transit planning purposes.

DAY-TO-DAY PUBLIC TRANSPORT TRAVEL TIME DISTRIBUTION ANALYSIS

The previous section established definitions of PTTV and identified *day-to-day PTTV on service level as* the most useful definition. This section analyzes the probability distribution of travel time to investigate *the nature and shape of* $PTTV_{d2d,s}$. For instance, a uniform distribution denotes no variability, while a long tail skewed distribution shows high and unreliable travel time. Travel time distribution is also essential in public transport planning. Resource allocations such as recovery time and timetable optimization are not often planned on the basis of average travel time, but on minimizing the opportunity that any journey would exceed the scheduled time (*17*). However, the literature on public transport travel time distribution is still limited and inconsistent, exploring only common distributions at limited time-of-the-day, and revealing symmetric types of distribution (*22*), skewed distribution (*23*) or both of them (*16*) as the descriptor of public transport travel time.
For the analyzes on $PTTV_{d2d,s}$ a comprehensive seven-step approach is proposed. The analysis aims to test all types of probability distribution which neglects only the discrete types of distribution (e.g. Binominal, Negative binominal, Poisson) as well as Uniform and limited samples distributions (Triangular, Rectangular) because the nature of travel time is continuous. The list of 23 fitted distribution types includes: Beta, Birnbaum-Saunders, Burr, Chi-Squared, Dagum, Erlang, Error, Exponential, Frechet, Gamma, Generalized Pareto, Inverse Gaussian, Levy, Logistic, Log-logistic, Lognormal, Nakagami, Normal, Rayleigh, Rician, Pareto, t location-scale and Weibull.

Seven-step approach for public transport travel time distribution analysis

Travel time samples of each service are fitted by the Maximum Likelihood Estimation (MLE) method to estimate the parameters of each distribution. Most existing studies of travel time distribution analysis performed one of the three common goodness-of-fit tests named Chi-Squared; Kolmogorov-Smirnov (KS); and Anderson-Darling to find whether the data follows the specified distribution (hypothesis H0). Any p-value larger than the significance level (α) fails to reject H0 and the distribution is considered as significantly fitted with the data. However, this method has two key drawbacks (*24*). Chi-squared requires large sample size, while the others test goodness-of-fit of distribution with predefined parameters, if estimated from the data, then original critical values of the test are not valid.
Literature offers other approaches to solve the aforementioned problems, but they also have their own disadvantages.

(1) The information creation technique such as Bayesian Information Creation (BIC) (*25*) measures the relative quality of a statistical model by trading off the complexity (by considering the number of parameters) and goodness-of-fit of the fitted distribution (by considering the maximized value of the log-Likelihood). However, the BIC statistic is difficult to interpret. The fitted distribution with the lowest BIC is the "best descriptor of the data, without a hypothesis test to validate the goodness-of-fit.
(2) The best fitted distribution could also be examined graphically by using the probability plot, histogram, stem & leaf plots, scatter plot, or box & whisker plots. This graphical approach does not provide a reference point so that multiple distributions can be compared within multiple time periods.

(3) Recent goodness-of-fit tests such as Lilliefors test (*26*) extends the KS test by determining the critical value by a Monte Carlo simulation, enables estimating the distribution parameters from the data. However, the critical values table supports only a few limited types of distributions, restricting the study to a few selected distributions.

To overcome the limitation of the existing approach in travel time distribution analysis, this paper extends the Liliefors test to support all types of distribution by using parametric bootstrapping for calculating KS critical value (*27; 28*). The analysis follows the following steps.

Step 1: Consider each type of distribution. MLE method is employed to estimate distribution parameter(s) from bus travel time data.

Step 2: Generates random numbers from the studied distribution using the parameter(s) from Step 1.

Step 3: Use MLE to re-estimate distribution parameter(s) from the generated data. The parameter(s) is used to build theoretical cumulative distribution function (c.d.f) $F(x)$ at each value of the generated data

Step 4: Calculate the KS statistics D_N^*, i.e., maximum difference between the empirical distribution function (e.d.f.) $S_N(x)$ from the generated data and the theoretical c.d.f. $F(x)$ at each value of the generated data.

$$D_N^* = \max |S_N(x) - F(x)| \quad (4)$$

Step 5: Repeat Step 2 to Step 4 a large number of time (say 10000) to gather the set of D_N^*. Since significance level (α) equals 0.05, the 95th percentile of the set is chosen as the critical value D_C.

Step 6: Compute the observed KS statistic D_N between the e.d.f. from the bus travel time data and the c.d.f. at each sample of the bus travel time, and compare it to the simulated critical value. If $D_N < D_C$, the test fails to reject the null hypothesis that the distribution could describe bus travel time data.

For each service, the list of accepted distribution types can be found. However, the KS test with parametric bootstrap does not provide a measure to compare the goodness-of-fit at each service if multiple distributions are accepted. A hybrid approach is then used, in which the top five distribution types in the number of passed KS test are chosen as the five candidates for the descriptor of bus travel time. The BIC statistic test finally evaluates the goodness-of-fit of each candidate to the bus travel time.

Step 7: BIC statistics are calculated for each candidate distribution from Step 6. The distribution type with lowest BIC is best fitted to the bus travel time data (*25*).

$$BIC = k \ln n - 2 \ln L_{max} \quad (5)$$

Where:

n = number of observations

k = number of parameters to be estimated

L_{max} = maximized value of the likelihood function of the estimated distribution

This seven-step approach investigates the best descriptor of public transport travel time.

Analysis results and discussion

The Step 6 of the seven-step approach reveals five candidates of bus travel time distribution: Burr, Gamma, Lognormal, Normal and Weibull. While Normal and Lognormal are commonly used in public transport studies (*16; 22; 23*), the other three are relatively new in the area. The KS test results and histogram of each distribution type, along with the lowest 2 distribution types in BIC statistics are presented in FIGURE 4. The following analyzes each aforementioned candidate's results to justify its overall goodness-of-fit to the bus travel time data.

The *Burr distribution* has been recently used in traffic engineering to model urban road travel time (*29*). Burr distribution is described as a heavy-tailed, highly-skewed distribution. FIGURE 4 shows that while the Burr distribution only passed the KS test at 18/37 services, it is the best fitted distribution where bus travel time is high left skewed and long tailed. However, this travel time pattern appears in only a few services.

The *Weibull distribution* has been widely used to represent travel time on arterial roads (*30*) and especially on duration-related studies such as traffic delay durations (*31*) and waiting time at unsignalized intersections (*32*). Weibull distribution has been described as flexible representing right-skew, left-skew and also symmetric data. The BIC results show that Weibull is almost always within the top 2 in negative skewed travel time patterns. As the services with negatively skewed distribution are few in the dataset, Weibull distribution has the lowest BIC statistic value in only 3 services.

The *Normal distribution* has been suggested as the descriptor of bus travel time in a number of studies (*16; 22*). It has a symmetric shape and its characteristics are thoroughly studied in statistics, which facilitates theoretical research. FIGURE 4 shows that Normal distribution is still a strong candidate as the descriptor of bus travel time in this study by passing the KS test in 20/37 services and having the lowest BIC statistics in 8 services, most of which are in mid-peak period.

The tests results indicate the *Gamma* and *Lognormal distributions* to be superior. The *Gamma distribution* has been long considered one of the first candidates for distribution of travel time. Polus (*33*) believed that travel time on arterial road would "closely follow" a Gamma distribution, and for this reason Dandy and McBean (*34*) suggested Gamma distribution as the descriptor for in-vehicle travel time. *Lognormal distribution* is conversely used to represent bus travel time (*16; 23; 35*) due to the flexibility and ability to accommodate skewed data.

While the Gamma distribution passed the KS test in 30/37 service, the Lognormal distribution passed in only one less services (29/37 services). Both of them are the optimal descriptors of bus travel time with moderate skewness and kurtosis (i.e. absolute value of skewness smaller than 1 and kurtosis smaller than 3). This type of travel time pattern is dominant in the dataset, making Gamma and Lognormal passed most KS tests.

Lognormal and Gamma distribution are both flexible enough to model light to heavy tailed data, but the Lognormal is better in representing higher skewed and longer tailed data, as it came with the Burr distribution in the top 2 lowest BIC statistic in several services. The BIC statistics also indicate that Lognormal is the best fitted distribution in more services than any other distribution types (14/37 services).

Another advantage of the Lognormal distribution is its mathematical characteristics facilitates TTV studies. For instance, Lognormal distribution allows direct calculation of CV from its parameter.

$$\mathrm{CV} = \sqrt{e^{\sigma^2} - 1} \qquad (6)$$

The $(p \times 100)^{\text{th}}$ percentile θ, commonly used in many variability and reliability indicators, can be computed using the lognormal quartile function as in Equation (7)

$$\theta = {F_X}^{-1}(p) = e^{\mu - \sqrt{2} erfcinv(2p)\sigma},\ 0 \leq p \leq 1 \qquad (7)$$

where $erfcinv(x)$ is the inverse complementary error function. While there is no known closed form expression, the value of $erfcinv(x)$ can be approximated to the method described in Blair et al. (*36*). Equation (7) also denotes if the data is Lognormally distributed, the Lognormal parameters μ and σ can be easily estimated from the value of two percentile values $(p_1 \times 100)$-th percentile θ_1, and the $(p_2 \times 100)$-th percentile θ_2, which means the following equations can be obtained.

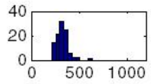
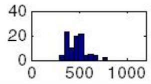
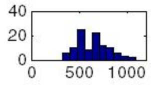
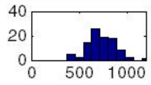
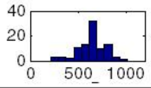
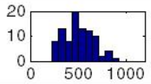
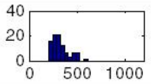
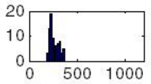
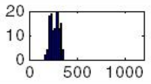
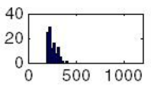
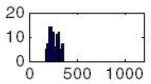
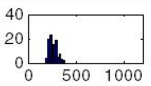
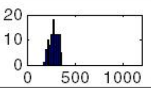
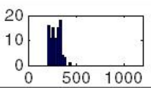
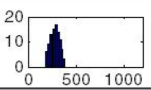
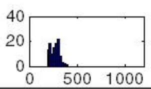
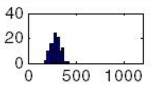
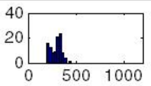
$$\begin{cases} p_1 = \frac{1}{2} erfc\left(-\frac{\ln(\theta_1) - \mu}{\sqrt{2}\sigma}\right) \\ p_2 = \frac{1}{2} erfc\left(-\frac{\ln(\theta_2) - \mu}{\sqrt{2}\sigma}\right) \end{cases} \qquad (8)$$

The parameters of Lognormal can be calculated by solving Equation (8)

$$\begin{cases} \sigma = \frac{\ln(\theta_2) - \ln(\theta_1)}{\sqrt{2}[erfcinv(2p_1) - erfcinv(2p_2)]} \\ \mu = \ln(\theta_1) + \sqrt{2} erfcinv(2p_1)\sigma \end{cases} \qquad (9)$$

Overall, Lognormal distribution provides excellent representation of the observed travel time data in this study. It is recommended as the descriptor of public transport travel time variation thanks to its high performance and the attractive mathematical characteristics that facilitate TTV studies.

Burr | Lognormal | Normal | Gamma | Weibull

Service	Histogram	# samples	Skew-ness	Kurto-sis	KS test with bootstrap resampling (1 for Accepted, 0 for rejected)					Hartigan Dip test		Lowest BIC	2nd Lowest BIC
					Burr	Normal	Log normal	Weibull	Gamma	Dip statistic	p-value		
6:51		103	1.22	7.05	0	1	1	0	1	0.04	0.38	Lognormal	Gamma
7:08		93	0.83	3.81	1	1	1	1	1	0.06	**0.02**	Lognormal	Gamma
7:26		104	0.42	2.68	1	0	1	0	1	0.05	0.06	Gamma	Lognormal
7:46		95	0.19	4.03	0	0	0	0	0	0.03	0.97	Normal	Burr
8:05		91	-0.56	3.84	1	1	1	1	0	0.03	0.77	Weibull	Normal
8:25		91	0.20	2.57	0	1	0	1	1	0.04	0.21	Weibull	Gamma
8:45		92	1.00	3.39	1	0	1	1	1	0.03	0.7	Lognormal	Burr
9:05		79	0.58	2.25	1	1	1	1	1	0.02	0.99	Lognormal	Gamma
9:25		120	-0.01	1.94	0	1	1	1	1	0.05	**0.04**	Normal	Gamma
9:55		109	0.82	3.11	1	1	1	1	1	0.03	0.55	Burr	Lognormal
10:25		93	0.18	2.03	1	1	1	1	1	0.04	0.28	Gamma	Lognormal
10:55		109	0.53	2.56	1	0	1	1	1	0.04	0.14	Lognormal	Gamma
11:25		96	-0.23	2.18	0	0	0	0	0	0.04	0.44	Weibull	Normal
11:55		94	0.24	2.55	0	1	1	0	1	0.04	0.55	Gamma	Lognormal
12:25		108	-0.02	2.23	0	0	0	0	0	0.03	0.82	Normal	Gamma
12:55		110	0.21	2.48	1	0	1	0	1	0.03	0.46	Gamma	Lognormal
13:25		113	0.12	2.66	0	0	0	0	0	0.04	0.31	Normal	Gamma
13:55		102	-0.05	2.26	1	1	0	1	1	0.04	0.13	Normal	Gamma

Burr | Lognormal | Normal | Gamma | Weibull

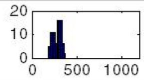
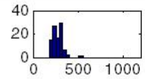
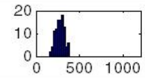
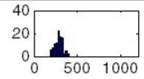
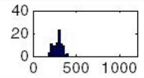
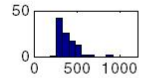
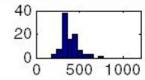
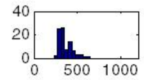
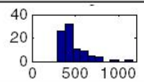
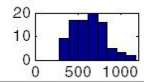
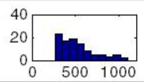
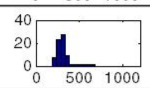
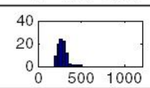
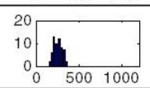
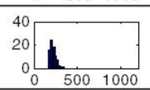
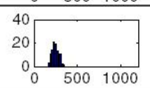
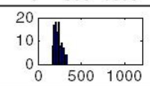
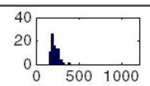
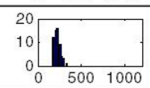
Service	Histogram	# samples	Skew-ness	Kurto-sis	KS test with bootstrap resampling (1 for Accepted, 0 for rejected)					Hartigan Dip test		Lowest BIC	2nd Lowest BIC
					Burr	Normal	Log normal	Weibull	Gamma	Dip statistic	p-value		
14:25		99	-0.22	1.98	0	1	1	1	1	0.04	0.31	Weibull	Normal
14:55		97	1.37	8.40	1	0	1	0	1	0.05	0.06	Lognormal	Gamma
15:10		102	-0.06	2.50	0	1	0	1	0	0.02	0.97	Normal	Weibull
15:25		100	0.00	2.70	0	1	0	0	1	0.03	0.82	Normal	Gamma
15:46		90	-0.06	2.61	1	1	1	1	0	0.03	0.97	Normal	Weibull
16:05		105	2.12	9.68	1	1	1	1	1	0.02	0.96	Burr	Lognormal
16:20		97	1.11	4.99	1	1	1	1	1	0.04	0.38	Burr	Lognormal
16:35		88	1.10	3.88	1	1	1	1	1	0.04	0.49	Burr	Lognormal
16:51		89	1.75	7.76	1	0	1	1	1	0.04	0.39	Lognormal	Burr
17:13		91	0.98	4.62	0	0	1	0	1	0.03	0.71	Lognormal	Gamma
17:33		101	0.99	3.21	0	0	1	1	1	0.03	0.8	Lognormal	Gamma
18:07		72	2.23	9.54	1	1	1	1	1	0.03	0.86	Burr	Lognormal
18:37		93	1.27	6.30	1	0	1	0	1	0.03	0.61	Lognormal	Burr
19:10		77	0.16	2.21	0	0	1	0	1	0.04	0.51	Gamma	Lognormal
19:40		98	0.97	4.44	0	0	1	0	1	0.03	0.95	Lognormal	Gamma
20:05		106	0.24	2.56	0	0	1	0	1	0.03	0.78	Gamma	Lognormal
20:40		105	0.55	2.65	0	0	1	0	1	0.03	0.84	Lognormal	Gamma
21:40		85	1.17	5.53	0	1	1	1	1	0.02	0.99	Lognormal	Gamma
22:40		79	0.71	3.37	0	1	1	0	1	0.04	0.72	Lognormal	Gamma

FIGURE 4 Descriptive statistic of analysis results

Hartigan Dip test for examining the bimodality

The histograms on FIGURE 4 show some signs of bimodality on two services before and after the morning peak period. Testing the bimodality is best conducted with the Hartigan Dip test. Dip statistics express the largest difference between the empirical distribution function and a unimodal distribution function that minimizes that maximum gap (*37*). If the *p-value* of the test is more than the significance value (chosen as 0.05), the data is concluded as unimodal distributed.
The results from FIGURE 4 show that although the bimodality is significant in only two services, the distributions of travel time in many services before and after the morning peak period are also nearly bimodal (*p-value* slightly larger than 0.05). The bimodality of travel time is mainly caused by a mixture of congested and uncongested population of traffic. Earliness or excessive congestion on some days, or generally the spread of congestions could be the main reason. These services are within the congestion build-up and dissipation periods, where speed could be free flow or congested depending on a day-to-day basis. The study was conducted on inbound traffic only, which means the pattern is not repeated for the afternoon.

PROBABILISTIC APPROACH USING LOGNORMAL DISTRIBUTION FOR INDICATING PUBLIC TRANSPORT TRAVEL TIME VARIABILITY

Lognormal has been recommended as the descriptor of the day-to-day public transport travel time in this study. This section investigates the use of *Lognormal distribution* to empirically indicate *day-to-day PTTV on service level* using a probabilistic approach.
TTV or travel time reliability is often indicated by one of the four measures (*38*): statistical range, buffer time, tardy-trips or probabilistic approach. The probabilistic approach is one of the direct measures to evaluate travel time reliability. Bell and Cassir (*39*) defined reliability as "the probability that [a] system can perform its desired function to an acceptable level of performance for some given period of time". The probabilistic approach measures the probability that travel time would be higher than a predetermined threshold under normal traffic conditions subject to day-to-day traffic flow fluctuations. The predetermined threshold is often defined as the median of travel time plus a certain amount of time, or a certain percentage of the median of travel time (*38; 40*). This section aims to use the p.d.f. of the Lognormal distribution to calculate the probabilistic indicator of PTTV, the probability that bus travel time would be larger than a certain value.

$$\Pr\left(TT_{d,r,s} \geq A\right) \quad (11)$$

Where,
A = predetermined travel time threshold to be studied, e.g. A = $\alpha \cdot T50_{r,s}$ or A = $\beta + T50_{r,s}$
$TT_{d,r,s}$= travel time of the bus of route *r* which is scheduled to start at service *s* of day *d*
$T50_{r,s}$ = median value of the set of travel time samples of route *r* and service *s*
α= threshold multiplied with the median (e.g. 1.2)
β= threshold added to the median (e.g. 10 minutes)
The p.d.f. of Lognormal distribution has the form as in Equation (12).

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\left[\frac{\ln(x)-\mu}{\sqrt{2}\sigma}\right]^2},\ x > 0 \qquad (12)$$

Where μ and σ are the two parameters of the Lognormal distribution. Mathematically, the probability $\Pr(TT_{d,r,s} \geq \alpha \cdot T50_{r,s})$ or $\Pr(TT_{d,r,s} \geq \beta + T50_{r,s})$ is the integral of the p.d.f. $f_X(x)$ between threshold value $A = \alpha \cdot T50_{r,s}$ or $A = \beta + T50_{r,s}$ and the infinity.

$$P = \int_A^{\infty} \frac{1}{x\sigma\sqrt{2\pi}} e^{-\left[\frac{\ln(x)-\mu}{\sqrt{2}\sigma}\right]^2} dx \qquad (13)$$

Substitute $y = \ln(x)$, which means $x = e^y, dy = \frac{dx}{x}$. Equation (13) becomes.

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{lnA}^{\infty} e^{-\left[\frac{y-\mu}{\sqrt{2}\sigma}\right]^2} dy \qquad (14)$$

The problem in Equation (14) can be re-written into an equation of the complementary error function.

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{lnA}^{\infty} e^{-\left[\frac{y-\mu}{\sqrt{2}\sigma}\right]^2} dy = \frac{1}{2}\left[erfc\left(\frac{lnA-\mu}{\sigma\sqrt{2}}\right)\right] \qquad (15)$$

Where erfc(x) is the complementary error function

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt \qquad (16)$$

The value of *erfc(x)* can be rationally approximated (*41*) to get the desired quantity. Equation (12)-(16) show that using the p.d.f. of the Lognormal distribution, the probability that the bus travel time exceeds a certain threshold from the median is found.

Public transport travel time variability map of some main routes in Brisbane

In this sub-section, the data of the 4 sites illustrated in FIGURE 1 along with their main bus routes are used to plot the PTTV map using the proposed definition and probabilistic indicator from previous sections. The objective is to validate the applicability of the study in monitoring PTTV of multiple routes and corridors.
The same process of collecting a year Transit Signal Priority data from specified service has been carried out on 8 routes along 4 corridors illustrated in FIGURE 1, according to the proposed definition of PTTV. Lognormal distribution is fitted to each set of data using MLE method to find the parameters σ and μ. This sub-section calculates $\Pr(TT_{d,r,s} \geq \alpha \cdot T50_{r,s})$ as a meaningful comparison of PTTV between multiple routes and multiple services.
FIGURE 5(a) and FIGURE 5(b) show the PTTV maps of 8 bus routes along the 4 study sites. While FIGURE 5(a) demonstrates PTTV in terms of CV of travel time, FIGURE 5(b) demonstrates PTTV in terms of the probability that the travel time is higher than 20% of the

median ($\Pr(TT_{d,r,s} \geq 1.2 \cdot T50_{r,s})$. The 20% is chosen to be consistent with the threshold used by Van Lint *et al.* (*38*), but any threshold can be used to calculate the probabilistic indicator. The two figures confirm that the proposed probabilistic approach captures the variability patterns on each site and indicates PTTV, and show very similar results to the traditional approach using CV of travel time. Corronation Dr's routes travel times are highly varied during both morning and afternoon peaks as the corridor is directly connected with the Brisbane CBD. The routes from other corridors are only unreliable during morning peak periods. This section validates that the study can be applied to multiple routes over multiple sites to indicate PTTV.

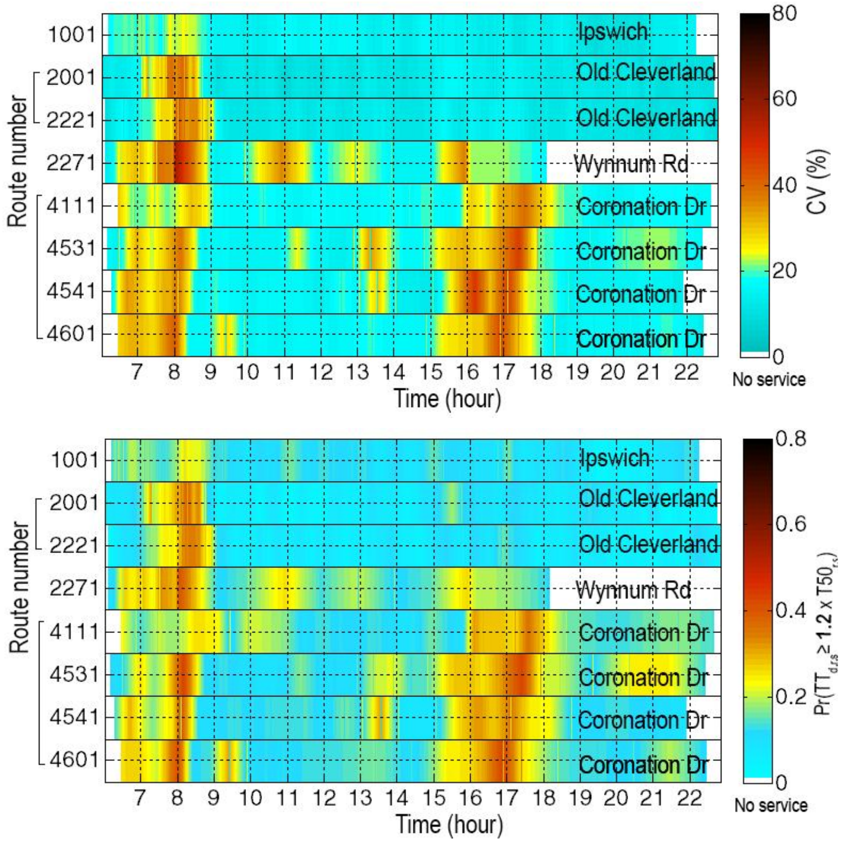


FIGURE 5 PTTV map in terms of: *(a)* CV of travel time and *(b)* $\Pr(TT_{d,r,s} \geq 1.2 \cdot T50_{r,s})$

PRACTICAL APPLICATION OF THE METHODOLOGY

The previous section confirms that the proposed probabilistic approach captures PTTV, similar to the traditional CV approach. While CV is only useful for monitoring the PTTV, the proposed probabilistic approach can evaluate the probability of bus travel time over any predefined threshold.

First, the proposed method facilitates timetabling, especially in determining the recovery time. With the median value $T50_{r,s}$ between two time-points acts as the base scheduled travel time, transit operator would be interested in determining a recovery time value β to accommodate PTTV. The value of $\beta + T50_{r,s}$ is the total scheduled travel time which should allow all buses to reach the destination on time. The probability that the observed travel time would be higher than the scheduled travel time is calculated using the proposed probabilistic approach.

$$\Pr\left(TT_{d,r,s} \geq \beta + T50_{r,s}\right) \qquad (17)$$

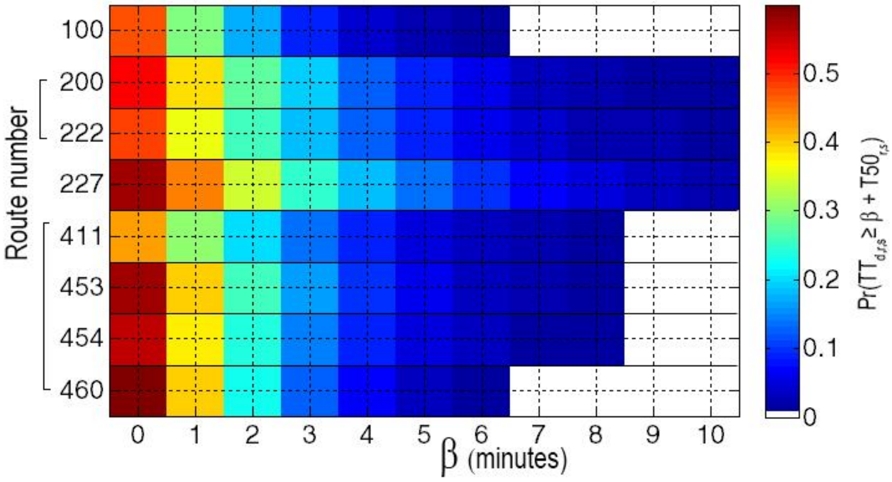
Where:

$T50_{r,s}$= median value of the set of travel time samples of route *r* and service *s*, set as the base scheduled travel time

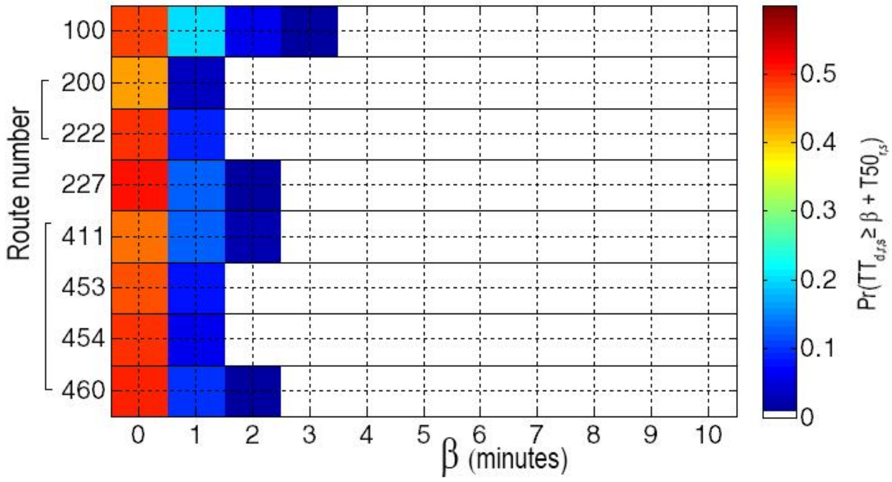
β= recovery time

$\beta + T50_{r,s}$= total scheduled travel time

The recovery time β is chosen so that $\Pr\left(TT_{d,r,s} \geq \beta + T50_{r,s}\right)$ is minimized, which means the number of late buses is minimized. FIGURE 6 illustrates the value of $\Pr\left(TT_{d,r,s} \geq \beta + T50_{r,s}\right)$ when β varies from 0 to 10 for the different study routes where FIGURE 6(a) is for 8:00 am and FIGURE 6(b) is for 12:00pm. The figure clearly indicates that recovery time is dynamic over both route and time. A static constant recovery time for all the routes may not be optimal. For the study site, if transit operators aims for 90% of buses for on-time, then recovery time for morning period (8:00 am, FIGURE 6(a)) should be around 3 to 7 minutes depending on the route. For instance, 7 minutes for route 411 and 3 minutes for route 100. Similarly, for afternoon non-peak period it should be around 1 to 2 minutes. While most transit operators currently set a fixed scheduled travel time for all time-of-the-day, the information in FIGURE 6 facilitates a better timetabling to make sure that all passengers would be served on-time. While adding more recovery time would also reduce commercial speeds, the proposed method enables analytical calculation to balance between high commercial speed and reliable travel time.



(a)



(b)

FIGURE 6 Value of $\Pr\left(TT_{d,r,s} \geq \beta + T50_{r,s}\right)$ with varied β at: (a) 8:00 and (b) 12:00

Moreover, the probability distribution of travel time is needed for any statistical studies related to travel time. For instance, dynamic and stochastic traffic assignment usually assumes travel time as random variable and models travel time as a stochastic process follows a probability density function (*42; 43*). The distribution also shows the probability of excessive travel time, which is importance in route choice modeling (*44*). The method for exploring travel time probability distribution in this paper facilitates these studies.

CONCLUSION

This paper established the public transport-oriented definitions of day-to-day TTV and modeled PTTV for transit planning purposes. The first, corridor-level PTTV definition is an extension of the commonly used definition of TTV to include all buses that passing an urban intersection to provide the information of variability of buses in general. This is useful to compare between multiple modes of transport. The second, service-level PTTV definition includes only a specific bus route service, which can be used for performance measurement, and optimizing recovery time. The second definition on service level is the most useful as it enables service monitoring and recovery time planning.

For the investigation of public transport travel time probability distribution this paper has proposed a comprehensive seven-step approach which allows fitting most of continuous probability distributions to all services. Each type of distribution is tested by both KS test with parametric bootstrapping and BIC method, identifying Lognormal distribution as the descriptor of day-to-day public transport travel time.

Using the Lognormal distribution p.d.f. to calculate probabilistic indicators of $PTTV_{d2d,s}$ is useful in PTTV monitoring and recovery time optimization. In fact, data from 8 bus routes along 4 corridors in Brisbane confirmed the applicability of the proposed probabilistic method for PTTV indicators.

The definitions and modeling methods presented in this paper established a strong basis for future researches. Statistical analysis, especially the ones using the p.d.f. of Lognormal distribution, can be further investigated.

For this current research only Transit Signal Priority data was available, which provide only the timestamp at each signalized intersection but not dwell time at stops. The factors that causing the long tail of the public transport travel time distribution or high travel time variability would be explored in future research, where advanced data sources such as AVL data should be used to investigate all sources of travel time variability. The possible variables that contribute to the PTTV are dwell time, road congestion and any online tactics (public transport priority systems or bus bunching prevention strategies). In the meantime, the findings of this paper are best suited for PTTV monitoring, recovery time optimization and statistical analysis of public transport travel time.

REFERENCE

[1] Abdel-Aty, M. A., R. Kitamura, and P. P. Jovanis. Investigating effect of travel time variability on route choice using repeated-measurement stated preference data. *Transportation Research Record*, No. 1493, 1995, pp. 39-45.

[2] Bates, J., J. Polak, P. Jones, and A. Cook. The valuation of reliability for personal travel. *Transportation Research Part E: Logistics and Transportation Review,* Vol. 37, No. 2–3, 2001, pp. 191-229.

[3] Noland, R. B., and J. W. Polak. Travel time variability: a review of theoretical and empirical issues. *Transport Reviews,* Vol. 22, No. 1, 2002, pp. 39-54.

[4] Kimpel, Strathman, Dueker, Griffin, Gerhart, and Turner. Time Point-Level Analysis of Passenger Demand and Transit Service Reliability.In *Report TNW2000-03, TransNow, Seattle, WA (July 2000)*, 2000.

[5] Bates, J., M. Dix, and A. D. May. Travel time variability and its effect on time of day choice for the journey to work. *PLANNING AND TRANSPORT RESEARCH AND COMPUTATION*, 1987.

[6] Bhaskar, A., E. Chung, and A.-G. Dumont. Fusing Loop Detector and Probe Vehicle Data to Estimate Travel Time Statistics on Signalized Urban Networks. *Computer-Aided Civil and Infrastructure Engineering,* Vol. 26, No. 6, 2011, pp. 433-450.

[7] Kieu, L. M., A. Bhaskar, and E. Chung. Empirical Evaluation of Public Transport Travel time Variability.In *Australasian Transport Research Forum*, 2-4 October, Brisbane Australia, 2013.

[8] ---. Bus and car travel time on urban networks: integrating bluetooth and bus vehicle identification data.In *25th ARRB Conference 2012*, Perth, Australia, 2012.

[9] ---. Transit passenger segmentation using travel regularity mined from Smart Card transactions data.In *Transportation Research Board 93rd Annual Meeting*, 2014.

[10] Widanapathiranage, R., J. M. Bunker, and A. Bhaskar. Modelling Busway Station Dwell Time Using Smart Cards.In *36th Australasian Transport Research Forum (ATRF)*, Brisbane, Australia, 2013.
[11] Bhaskar, A., K. Le Minh, M. QU, A. Nantes, M. Marc, and E. Chung. On the use of Bluetooth MAC Scanners for live reporting of the transport network.In *10th International Conference of Eastern Asia Society for Transportation Studies*, Taipei, Taiwan, 2013.
[12] Bhaskar, A., and E. Chung. Fundamental understanding on the use of Bluetooth scanner as a complementary transport data. *Transportation Research Part C: Emerging Technologies,* Vol. 37, No. 0, 2013, pp. 42-72.
[13] Abedi, N., A. Bhaskar, and E. Chung. Bluetooth and Wi-Fi MAC Address Based Crowd Data Collection and Monitoring: Benefits, Challenges and Enhancement.In *36th Australasian Transport Research Forum (ATRF)*, Brisbane, Australia, 2013.
[14] Abbott-Jard, M., H. Shah, and A. Bhaskar. Empirical evaluation of Bluetooth and Wifi scanning for road transport In *36th Australasian Transport Research Forum (ATRF)*, Brisbane, Australia, 2013.
[15] Abkowitz, M. D., and I. Engelstein. Factors affecting running time on transit routes. *Transportation Research Part A: General,* Vol. 17, No. 2, 1983, pp. 107-113.
[16] Mazloumi, E., G. Currie, and G. Rose. Using GPS Data to Gain Insight into Public Transport Travel Time Variability. *Journal of Transportation Engineering,* Vol. 136, No. 7, 2010, pp. 623-631.
[17] Moghaddam, S., R. Noroozi, J. Casello, and B. Hellinga. Predicting the Mean and Variance of Transit Segment and Route Travel Times. *Transportation Research Record: Journal of the Transportation Research Board,* Vol. 2217, No. -1, 2011, pp. 30-37.
[18] Currie, G., K. Goh, and M. Sarvi. An Analytical Approach to Measuring the Impacts of Transit Priority.In *92th Annual Meeting of Transportation Research Board*, Washinton D.C, 2013.
[19] Bhaskar, A., L. M. Kieu, M. Qu, A. Nantes, M. Miska, and E. Chung. On the use of Bluetooth MAC Scanners for live reporting of the transport network.In *10th International Conference of Eastern Asia Society for Transportation Studies, 9-12 September 2013, Taipei, Taiwan.*, 2013.
[20] Chien, S., and X. Liu. An Investigation of Measurement for Travel Time Variability. *Intelligent Transportation Systems*, 2012.
[21] Oh, J.-S., and Y. Chung. Calculation of Travel Time Variability from Loop Detector Data. *Transportation Research Record: Journal of the Transportation Research Board,* Vol. 1945, No. -1, 2006, pp. 12-23.
[22] Taylor, M. Travel time variability—the case of two public modes. *Transportation Science,* Vol. 16, No. 4, 1982, pp. 507-521.
[23] Andersson, P.-Å., Å. Hermansson, E. Tengvald, and G.-P. Scalia-Tomba. Analysis and simulation of an urban bus route. *Transportation Research Part A: General,* Vol. 13, No. 6, 1979, pp. 439-466.
[24] Durbin, J. *Distribution theory for tests based on sample distribution function*. Siam, 1973.
[25] Schwarz, G. Estimating the dimension of a model. *The Annals of Statistics,* Vol. 6, No. 2, 1978, pp. 461-464.
[26] Lilliefors, H. W. On the Kolmogorov-Smirnov test for normality with mean and variance unknown. *Journal of the American Statistical Association,* Vol. 62, No. 318, 1967, pp. 399-402.
[27] D'Agostino, R. B., and M. A. Stephens. *Goodness-of-fit Technicques*. CRC press, 1986.
[28] Babu, G. J., and C. Rao. Goodness-of-fit tests when parameters are estimated. *Sankhyā: The Indian Journal of Statistics*, 2004, pp. 63-74.

[29] Susilawati, S., M. A. P. Taylor, and S. V. C. Somenahalli. Distributions of travel time variability on urban roads. *Journal of Advanced Transportation*, 2011, pp. n/a-n/a.

[30] Al-Deek, H., and E. Emam. Computing Travel Time Reliability in Transportation Networks with Multistates and Dependent Link Failures. *Journal of Computing in Civil Engineering,* Vol. 20, No. 5, 2006, pp. 317-327.

[31] Mannering, F., S.-G. Kim, W. Barfield, and L. Ng. Statistical analysis of commuters' route, mode, and departure time flexibility. *Transportation Research Part C: Emerging Technologies,* Vol. 2, No. 1, 1994, pp. 35-47.

[32] Hamed, M., S. Easa, and R. Batayneh. Disaggregate gap-acceptance model for unsignalized T-intersections. *Journal of Transportation Engineering,* Vol. 123, No. 1, 1997, pp. 36-42.

[33] Polus, A. A study of travel time and reliability on arterial routes. *Transportation,* Vol. 8, No. 2, 1979, pp. 141-151.

[34] Dandy, G., and E. McBean. Variability of Individual Travel Time Components. *Journal of Transportation Engineering,* Vol. 110, No. 3, 1984, pp. 340-356.

[35] Ma, Z., L. Ferreira, and M. Mesbah. A Framework for the Development of Bus Service Reliability Measures.

[36] Blair, J., C. Edwards, and J. Johnson. Rational Chebyshev approximations for the inverse of the error function. *Mathematics of Computation,* Vol. 30, No. 136, 1976, pp. 827-830.

[37] Hartigan, J. A., and P. Hartigan. The dip test of unimodality. *The Annals of Statistics*, 1985, pp. 70-84.

[38] van Lint, J. W. C., H. J. van Zuylen, and H. Tu. Travel time unreliability on freeways: Why measures based on variance tell only half the story. *Transportation Research Part A: Policy and Practice,* Vol. 42, No. 1, 2008, pp. 258-277.

[39] Bell, M. G. H., and C. Cassir. *Reliability of transport networks.* . Baldock, Hertfordshire, England: Research Studies Press., 2000.

[40] Asakura, Y. Reliability measures of an origin and destination pair in a deteriorated road network with variable flows.In *Transportation Networks: Recent Methodological Advances. Selected Proceedings of the 4th EURO Transportation Meeting*, 1999.

[41] Cody, W. J. Rational Chebyshev approximations for the error function. *Mathematics of Computation,* Vol. 23, No. 107, 1969, pp. 631-637.

[42] Liu, H. X., X. Ban, B. Ran, and P. Mirchandani. Analytical dynamic traffic assignment model with probabilistic travel times and perceptions. *Transportation Research Record: Journal of the Transportation Research Board,* Vol. 1783, No. 1, 2002, pp. 125-133.

[43] Mirchandani, P., and H. Soroush. Generalized traffic equilibrium with probabilistic travel times and perceptions. *Transportation Science,* Vol. 21, No. 3, 1987, pp. 133-152.

[44] Watling, D. User equilibrium traffic network assignment with stochastic travel times and late arrival penalty. *European journal of operational research,* Vol. 175, No. 3, 2006, pp. 1539-1556.