CERN-TH.4231/85

PURE SPINORS AS AUXILIARY FIELDS IN THE TEN-DIMENSIONAL SUPERSYMMETRIC YANG-MILLS THEORY

B.E.W. Nilsson*)

CERN - Geneva

A B S T R A C T

We propose a new way of introducing auxiliary fields into the ten-dimensional supersymmetric Yang-Mills theory. The auxiliary fields are commuting "pure spinors" and constitute a non-linear realization of the Lorentz group. This invalidates previous no-go theorems concerning the possibility of going off-shell in this theory. There seems to be a close relation between pure spinors and the concepts usually used in twistor theory. The non-Abelian theory can be constructed for all groups having pseudo-real representations.

---

*) On leave of absence from the Institute of Theoretical Physics, S-41296 Göteborg, Sweden.

CERN-TH.4231/85

July 1985
Revised: October 1985

It has been a long-standing problem in field theories with supersymmetry to find the auxiliary fields needed to close this symmetry off-shell. The efforts to find these fields more or less came to a halt a few years ago with the appearance of some no-go theorems[1)-3)]. The arguments used to prove these theorems showed that, e.g., in d = 4 no linear off-shell supersymmetry representations could be found for Yang-Mills in the N = 3 and N = 4 extended cases[1),3)]. Not surprisingly, this was proved to be the case also for the ten-dimensional Yang-Mills theory[1),2)]. Recently, an attempt to circumvent these no-go theorems has been made for the d = 4, N = 3 case by means of the so-called harmonic superspace[4),5)]. Unfortunately, there seems to be a serious difficulty in extending this to the N = 4 case due to the self-duality condition present in this theory[6)].

The arguments employed in obtaining the no-go theorems all rely implicitly on the assumption that all symmetries of the theory are to be realized linearly and that the fields obey canonical spin-statistics relations. A non-linear realization of supersymmetry[1)] would of course invalidate these arguments. In this note we propose, however, that one should introduce a set of fields which have non-canonical spin-statistics and transform non-linearly under the Lorentz group instead of under supersymmetry. Before doing this we will briefly review the supersymmetric Yang-Mills theory in ten dimensions following Ref. 2).

The basic field in this theory is the superspace one-form potential [note the changes in notation compared to Ref. 2)]

$$A = dz^M A_M = E^A A_A = E^\alpha A_\alpha + E^a A_a \quad . \tag{1}$$

The notation is as follows: the superspace world-index M = (μ, m) and the tangent-space index A = (α, a) where Greek and Latin indices refer to space-time and spinorial directions respectively. The spinorial co-ordinates θ^a are chiral and satisfy the Majorana condition. We use the flat metric $\eta_{\alpha\beta}$ = diag(-,+,+,...,+) and Dirac matrices satisfying

$$\{ \gamma_\alpha , \gamma_\beta \} = - 2 \eta_{\alpha\beta} \quad . \tag{2}$$

The superspace field strength is

$$F = dA \quad . \tag{3}$$

(We will only consider the Abelian case for the time being; the non-Abelian case will be commented upon below.) The components of F are

$$F_{ab}\ ,\ F_{\alpha b}\ ,\ F_{\alpha\beta}\ , \tag{4}$$

where $F_{ab} = F_{ba}$.

Before analyzing the Bianchi identities it is convenient to decompose the field strengths in (4) into irreducible tensors, i.e.,

$$F_{ab} = i(\gamma^{\alpha})_{ab} F_{\alpha} + i(\gamma^{\alpha_1 \dots \alpha_5})_{ab} \tilde{F}_{\alpha_1 \dots \alpha_5}\ , \tag{5}$$

$$F_{\alpha b} = G_{\alpha b} + (\gamma_{\alpha})_b{}^c G_c\ , \tag{6}$$

where $\tilde{F}_{\alpha_1 \dots \alpha_5}$ is self-dual due to the chiral property of the spinorial indices a and b. $G_{\alpha b}$ is γ-traceless, i.e.,

$$(\gamma^{\alpha})_b{}^c G_{\alpha c} = 0\ . \tag{7}$$

$F_{\alpha\beta}$ is the Yang-Mills field strength and is of course an irreducible tensor. Now, as in Ref. 2), we note that F_{α} in (5) can be redefined to zero by choosing the vector part of the superpotential properly. Thus

$$F_{ab} = i(\gamma^{\alpha_1 \dots \alpha_5})_{ab} \tilde{F}_{\alpha_1 \dots \alpha_5}\ . \tag{8}$$

Setting $\tilde{F}_{\alpha_1 \dots \alpha_5} = 0$, i.e., $F_{ab} = 0$[2]), will, if introduced into the Bianchi identities, put the theory on-shell. In other words, if we use $F_{ab} = 0$ in the equations

$$D_{[A} F_{BC]} + T_{[AB}{}^D F_{|D|C]} = 0\ , \tag{9}$$

where $T_{AB}{}^C$ is the torsion tensor of "flat" superspace, the field equations, $\partial^{\alpha} F_{\alpha\beta} = (\gamma^{\alpha})_a{}^b \partial_{\alpha} G_b = 0$, will follow. One also finds that $G_{\alpha b}$ vanishes.

In trying to derive an off-shell formulation of this theory using the above formalism one must obviously relax the condition $\tilde{F}_{\alpha_1 \dots \alpha_5} = 0$. However, since this tensor is irreducible one is then not imposing any condition on the superfield strength. As a consequence the Bianchi identities are still proper identities and strictly speaking void of information. This situation was fully analyzed in Ref. 2), where it was found that the off-shell fields in x-space could not in any simple way be used to construct a supersymmetric action.

We now propose a quite different approach to define the theory off-shell. By introducing a complex <u>commuting</u> auxiliary superfield ψ_a we can put [recall that $(\gamma_{\alpha_1}\cdots\gamma_{\alpha_5})_{ab}$ is symmetric in a and b]:

$$\tilde{F}_{\alpha_1\ldots\alpha_5} \sim \psi_a (\gamma_{\alpha_1\ldots\alpha_5})^{ab}\psi_b + c.c. \quad . \tag{10}$$

The self-duality of $\tilde{F}_{\alpha_1\ldots\alpha_5}$ follows from the chirality property of the index a on ψ_a. However, we will go a step further and impose

$$\psi_a (\gamma_\alpha)^{ab}\psi_b = 0 \tag{11}$$

in which case Eq. (8) can be written

$$F_{ab} = -2i(\psi_a\psi_b + c.c.) \quad . \tag{12}$$

Complex commuting spinors satisfying the bilinear constraint (11) are called pure. Pure spinors were introduced by Cartan some decades ago [see Ref. 7)] and were seen, in the case of a complex space-time, to be in a one-one correspondence with d/2-dimensional null-planes (called isotropic by Cartan). In ten-dimensional Minkowski space[8)], such spinors have 22 independent components which constitute a non-linear realization of the Lorentz group. [Note that it is impossible to impose the Majorana condition on ψ_a because (11) would then imply $\psi_a = 0$.] Thus we have reduced the auxiliary (super)-field content of the theory from $\tilde{F}_{\alpha_1\ldots\alpha_5}(z)$, which is a <u>126</u> of SO(1,9), to $\psi_a(z)$ having 22 complex components.

Unfortunately, to obtain a complete solution of the Bianchi identities using the constraint (12) seems quite difficult. Here we will only derive the expression that keeps the theory off-shell in the case of the physical spin-½ field. This can easily be done using the results of Ref. 2). We start by giving the explicit form of the Bianchi identities (9):

$$D_{(a}F_{bc)} + i(\gamma^\alpha)_{(ab}F_{c)\alpha} = 0 \ , \tag{13}$$

$$\partial_\alpha F_{bc} + 2 D_{(b} F_{c)\alpha} + i(\gamma^\beta)_{bc} F_{\alpha\beta} = 0 \quad , \tag{14}$$

$$2\, \partial_{[\alpha} F_{\beta]c} + D_c F_{\alpha\beta} = 0 \quad , \tag{15}$$

$$\partial_{[\alpha} F_{\beta\gamma]} = 0 \quad , \tag{16}$$

where we have used the background torsion which has as its only non-zero component:

$$T_{ab}{}^{\gamma} = -i\,(\gamma^\gamma)_{ab} \quad . \tag{17}$$

Making use of the facts that $(\gamma_\beta)^{ab}F_{ab} = (\gamma^\alpha)_a{}^b G_{\alpha b} = 0$, we find from (14) that

$$\begin{aligned} D_a G_b = &-\tfrac{1}{4} i (\gamma^{\alpha\beta})_{ab} F_{\alpha\beta} - \tfrac{1}{32}(\gamma^{\alpha\beta})_{ab}(\gamma_\alpha)^{cd} D_c G_{\beta d} \\ &- \tfrac{1}{32\cdot 144}(\gamma^{\alpha_1\ldots\alpha_4})_{ab}\left[(\gamma_{\alpha_1\ldots\alpha_4}{}^{\alpha})^{cd}\partial_\alpha F_{cd} - 8(\gamma_{\alpha_1\ldots\alpha_3})^{cd} D_c G_{\alpha_4 d}\right]; \end{aligned} \tag{18}$$

The next step is to recall that

$$\tfrac{1}{2} i\, (\gamma^\alpha)_{ab}\, \partial_\alpha G_c = D_{(a} D_{b)} G_c \quad , \tag{19}$$

and substitute (18) together with $D_c F_{\alpha\beta}$ as obtained from (15) into the right-hand side of Eq. (19). Contracting two free spinorial indices then gives

$$\begin{aligned} 7(\gamma^\alpha \partial_\alpha G)_a = \;& \partial^\alpha G_{\alpha a} + \tfrac{1}{16} i (\gamma^{\alpha\beta})_a{}^b D_b (\gamma_\alpha)^{cd} D_c G_{\beta d} \\ &+ \tfrac{i}{16\cdot 144}(\gamma^{\alpha_1\ldots\alpha_4})_a{}^b D_b (\gamma_{\alpha_1\ldots\alpha_4}{}^{\alpha})^{cd}\partial_\alpha F_{cd} \\ &- \tfrac{i}{16\cdot 18}(\gamma^{\alpha_1\ldots\alpha_4})_a{}^b D_b (\gamma_{\alpha_1\ldots\alpha_3})^{cd} D_c G_{\alpha_4 d} \quad . \end{aligned} \tag{20}$$

From Ref. 2) we know that the general solution to (13) is

$$D_a \tilde{F}_{\alpha_1\ldots\alpha_5} = 10(\gamma_{[\alpha_1\alpha_2})_a{}^b B_{b\alpha_3\alpha_4\alpha_5]} + \tfrac{1}{336}(\gamma_{[\alpha_1\ldots\alpha_4})_a{}^b G_{b\alpha_5]} \; , \tag{21}$$

$$F_{a\beta} = G_{a\beta} + (\gamma_\beta)_a{}^c G_c \quad . \tag{22}$$

By using (21) in (20) one discovers that the tensor $B_{b\alpha\beta\gamma}$ [defined by (21)] drops out in (20). Thus (20) becomes

$$(\gamma^\alpha)_a{}^b \partial_\alpha G_b \sim \partial^\alpha G_{\alpha a} \tag{23}$$

or, in terms of the auxiliary pure spinor ψ_a:

$$(\gamma^\alpha)_a{}^b \partial_\alpha G_b \sim (\gamma^\alpha)^{bc} (\gamma_\alpha)^{de} D_b D_c D_d (\psi_e \psi_a + c.c.), \tag{24}$$

which follows directly from (23) using (21) and (8). One consequence of (24) is that the equation of motion for this theory is simply

$$\psi_a = 0 \ . \tag{25}$$

To summarize, we have introduced an auxiliary field ψ_a which is a complex commuting pure spinor. This may circumvent previous no-go theorems about the non-existence of auxiliary fields in this theory. Although the transformation rules in x-space are easily generalized to include ψ_a, it is harder to see if a Lagrangian formulation can be found. However, even if the answer to this is not in the affirmative, we believe the approach proposed here might be of some importance for the following reasons. The pure spinor ψ_a seems to be related to concepts which have been used extensively in the theory of twistors in four dimensions[9)]. For example, in a complexified space-time ψ_a defines by (10) a five-dimensional null-plane. The intersection between two null-planes defined by ψ_a and a pure spinor ϕ_a of opposite chirality is a <u>point</u> in space-time provided $\phi^a\psi_a = 0$[7)]. This is analogous to d = 4 twistors defining light-like lines and their intersections describing points in complexified d = 4 space-time. Twistors in ten dimensions have been discussed recently by Witten[10)] but seemingly for completely different reasons.

It is of course important to generalize our approach to the non-Abelian case. From (12) it is clear that in the case of simple groups this will be possible only when the group has pseudoreal representations[11)], i.e., representations which contain the adjoint representation in their <u>symmetric</u> product. Note that not all groups have such representations [see Table 12 of Ref. 11)].

Finally, we recall that self-dual five-forms appear also in various d = 10 supergravity theories. It would be interesting to see what effect an introduction of pure spinors would have in these theories. It is also conceivable that pure spinors are related to the auxiliary fields of the harmonic superspace approach[4),5)], or a generalization thereof applicable to Yang-Mills in d = 10 and N = 4 in d = 4.

ACKNOWLEDGEMENTS

I am very grateful to I. Bengtsson, G. Horowitz, K. Stelle and B. Zumino for stimulating discussions.

REFERENCES

1) W. Siegel and M. Rocek - Phys.Lett. 105B (1981) 275.

2) B.E.W. Nilsson - Göteborg Preprint 81-6 (1981).

3) V.O. Rivelles and J.-G. Taylor - Phys.Lett. 121B (1983) 37.

4) A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev - Class.Quantum Grav. 1 (1984) 469 and 2 (1985) 155.

5) R.E. Kallosh - CERN Preprint TH. 4188 (1985).

6) R.E. Kallosh and E. Sokatchev - Private communication.

7) E. Cartan - The Theory of Spinors, Hermann, Paris (1966).

8) D. Furlan and R. Raczka - "Non-Linear Spinor Representations", Triest University Preprint 17/85/EP (1985).

9) R. Penrose - in Quantum Gravity, Eds. C.J. Isham, R. Penrose and D.W. Sciama, Oxford University Press (1975);
Some more recent references may be found in Ref. 10).

10) E. Witten - "Twistor-like transform in ten dimensions", Princeton University Preprint (1985).

11) R. Slansky - Physics Reports 79 (1981) 1.