# Purely electronic transport and localization in the Bose glass

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Discussions with

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Rackeve, 4th September, 2009

# What happens in a Bose insulator without any phonon bath?

- Analysis close to the SIT of preformed bosons
- Consider situation where e-phonon coupling is weak:
  Instructive Gedankenexperiment:
  No electron-phonon coupling at all!
- No long range Coulomb interactions and
  no frustration and (classical) glassiness
  to make life a bit simpler

# Outline

- The dirty superconductor-insulator transition (SIT)

- Brief review of various puzzling transport experiments in the Bose glass

- Proposed resolution:
  Study of spectral properties!
  - Transport: R(T)
  - Many-body localization and its precursors

# SI transition in thin films

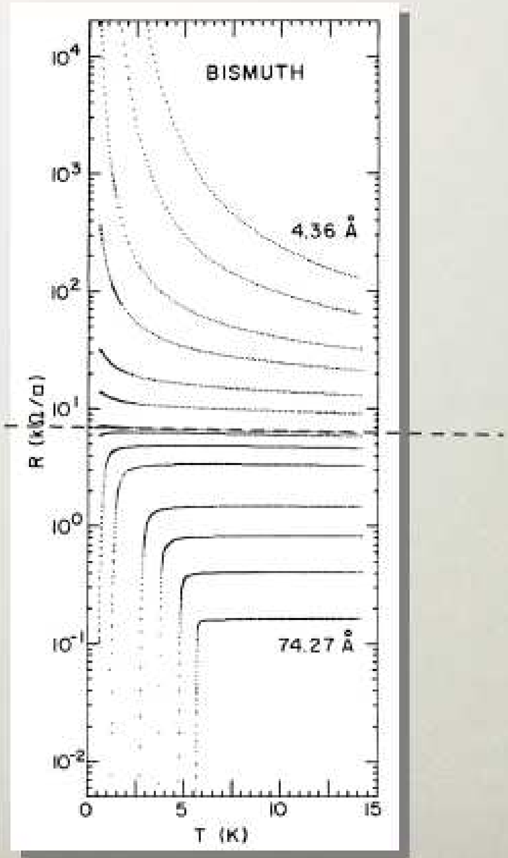
M. Strongin, et. al., Phys. Rev. **B1**, 1078 (1970).

**D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. 62, 2180 (1989)...**

2D

Thickness tuned transition

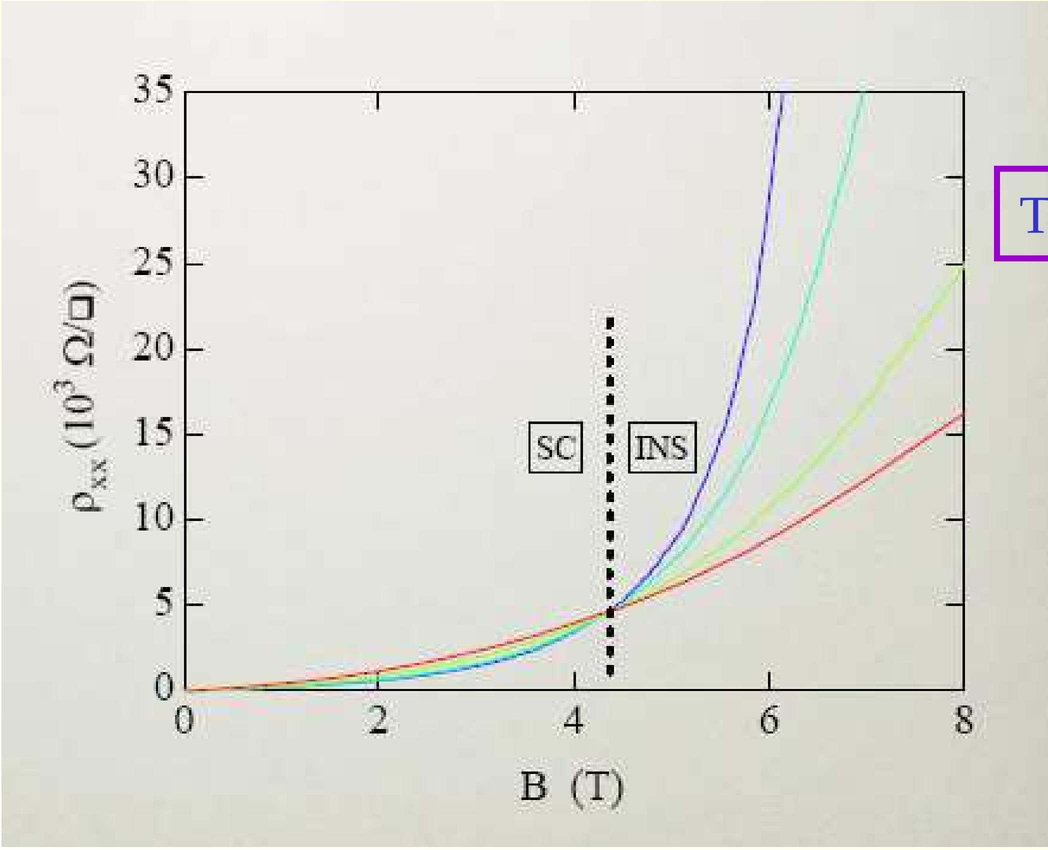
T = 0 transition




Review: Finkl'stein ('94),
Markovic and Goldman ('98).

# Field driven transition

Magnetic field destroys SC!

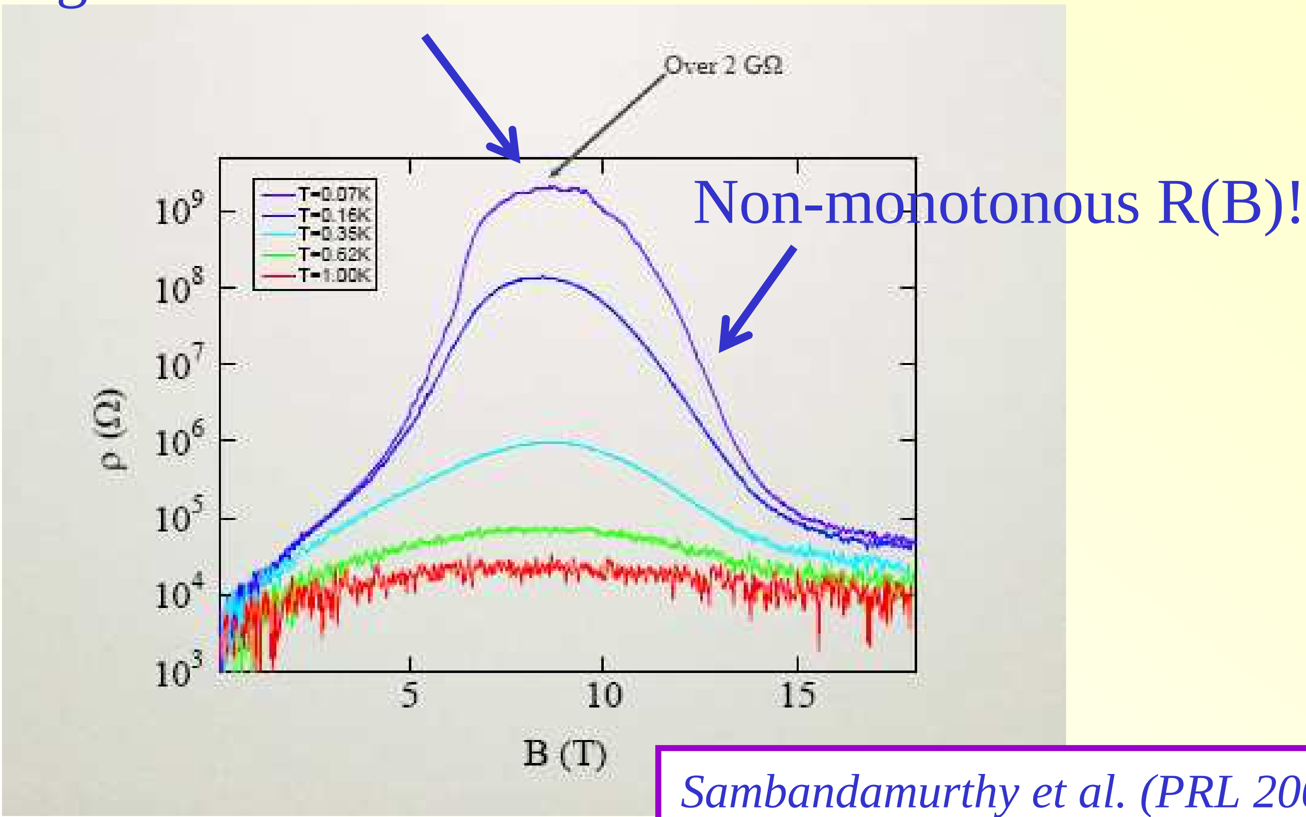



T = const.

*Gantmakher, Shahar, Kapitulnik, Goldman, Baturina*

# Insulator: Giant magnetoresistance

Giant magnetoresistance




*Sambandamurthy et al. (PRL 2005)*

Insulating behavior **enhanced** by local superconductivity!

# Insulator: Giant magnetoresistance

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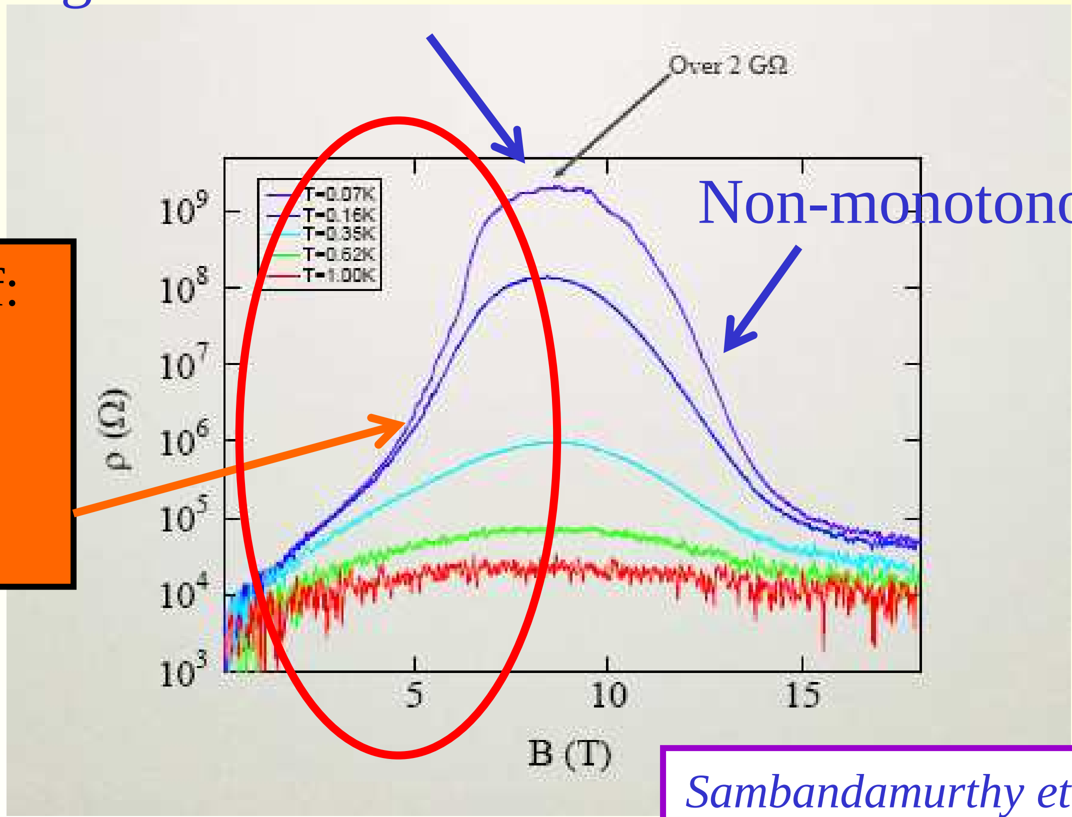



Non-monotonous R(B)!

Common belief:
Pairs (bosons) survive in the insulator:
**Bose glass**

*Sambandamurthy et al. (PRL 2005)*

Insulating behavior **enhanced** by local superconductivity!

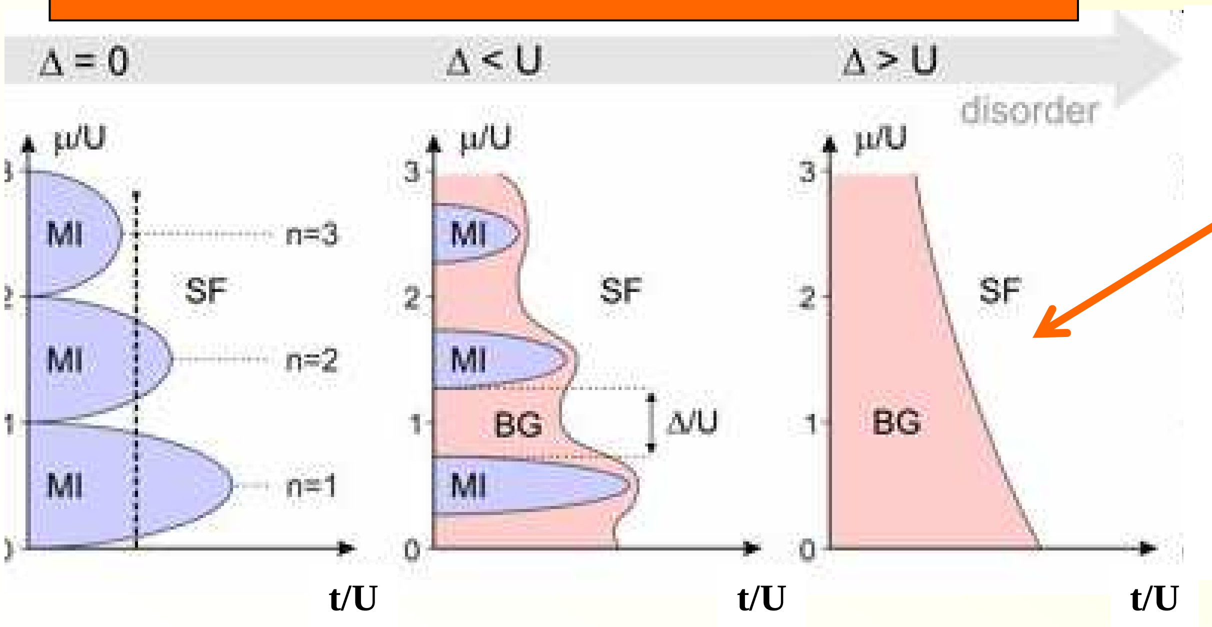
# Bose-Hubbard model and Bose glass

*Fisher et al., Phys. Rev. B 40, 546 (1989)*

- Assume "preformed Cooper pairs": bosons without global superconductivity
- Dirty boson model (Bose-Hubbard model with disorder):

$$H = t\sum_{\langle i,j\rangle} b_i^+ b_j + U\sum_i n_i(n_i - 1) + \sum_i (\varepsilon_i - \mu) n_i$$

Disorder: $\varepsilon_i \in [-\Delta, \Delta]$




Most likely scenario for experiments: Strong disorder, no Mott gap!

# Two puzzling features in transport

1. Simple activation in R(T)

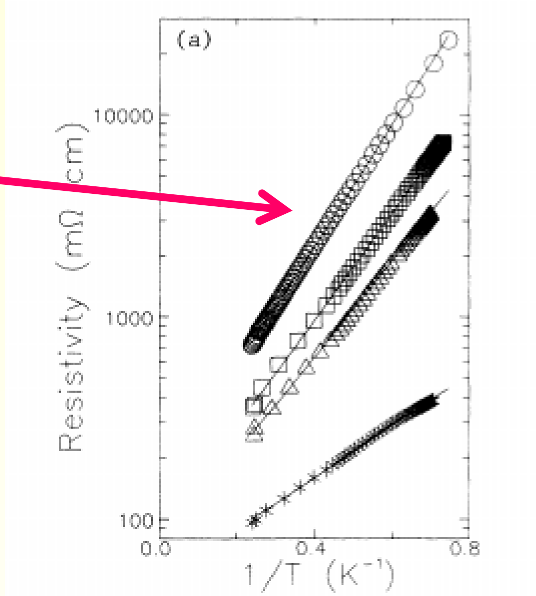
2. Evidence for purely electronic mechanism

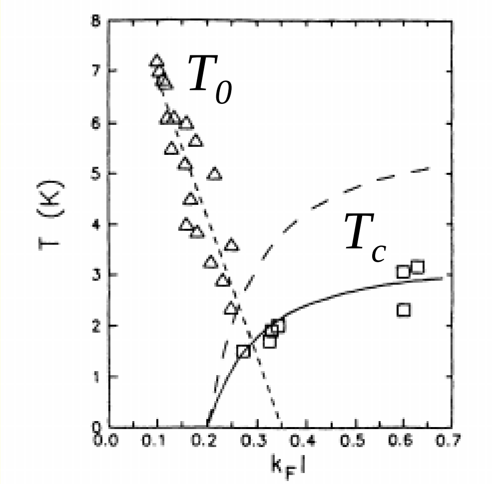
# Activated transport near the SIT

*D. Shahar, Z. Ovadyahu, PRB **46**, 10971 (1992).*

Insulating $InO_x$

Simple activation!




Activation energy increases with distance to SIT

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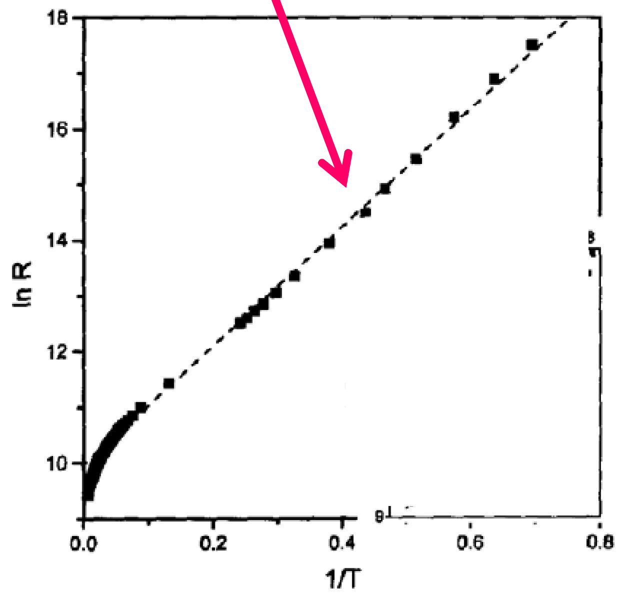


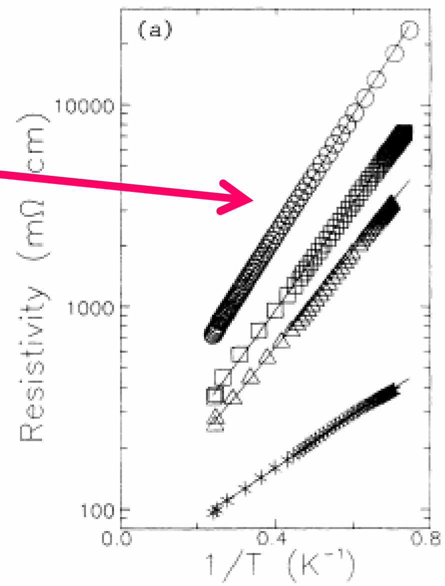


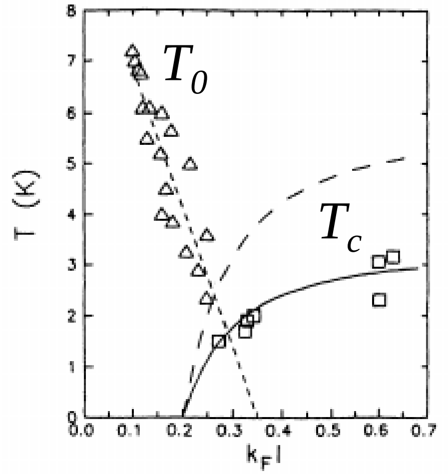


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*D. Kowal and Z. Ovadyahu, Sol. St. Comm. **90**, 783 (1994).*

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*V. F. Gantmakher, M. V. Golubkov, J. Lok, A. K. Geim, Sov. Phys. JETP, **82**, 951 (1996).*

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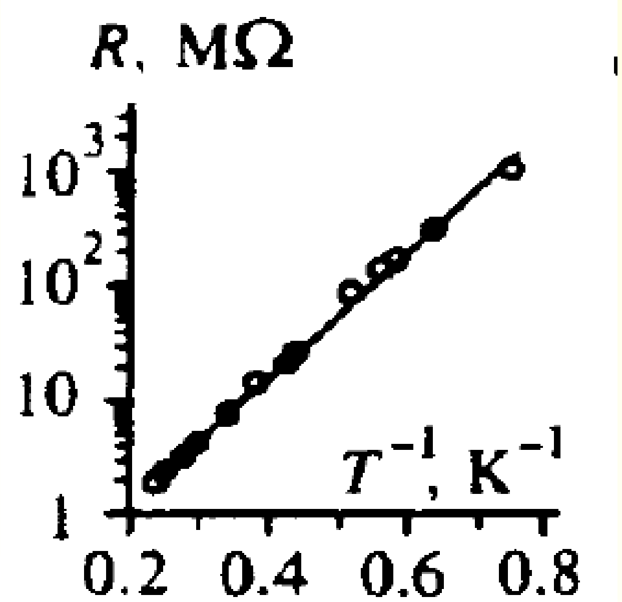


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• Gap in the density of states?
A: NO! Too disordered systems!
No Mott gap!

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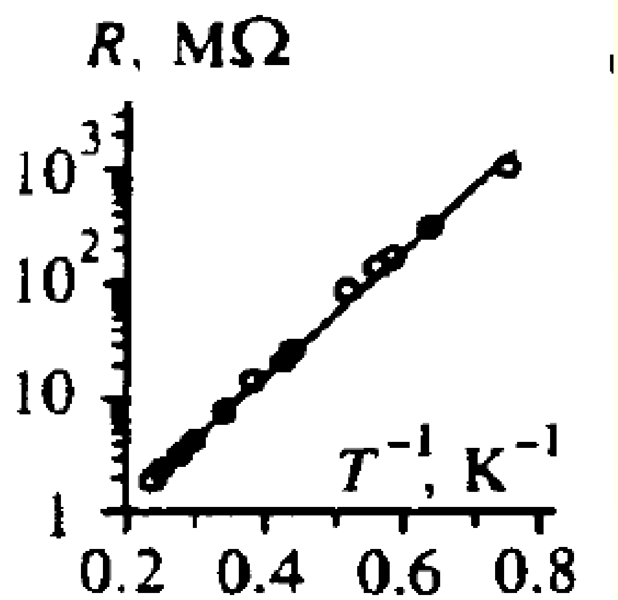


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Would give far too large prefactor.

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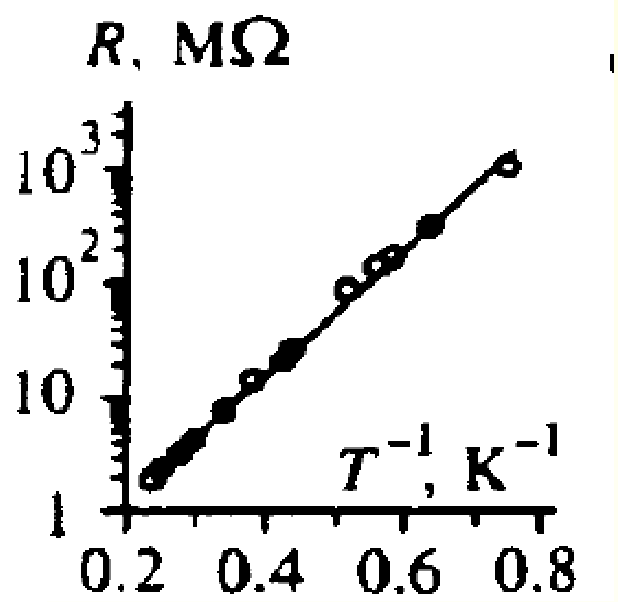


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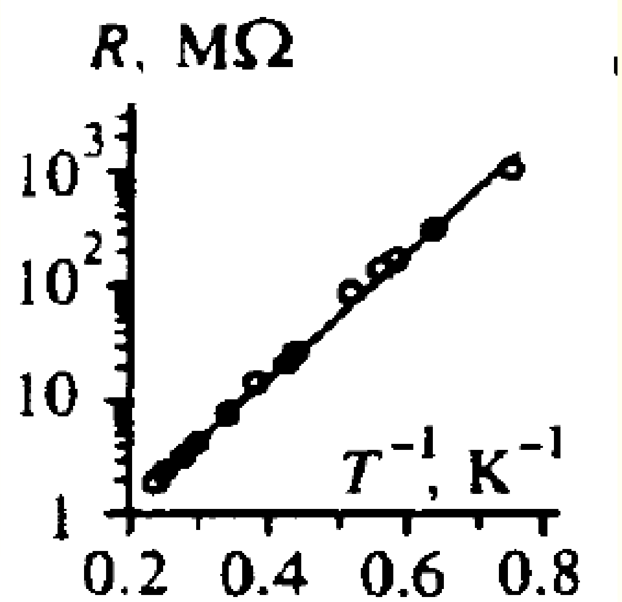


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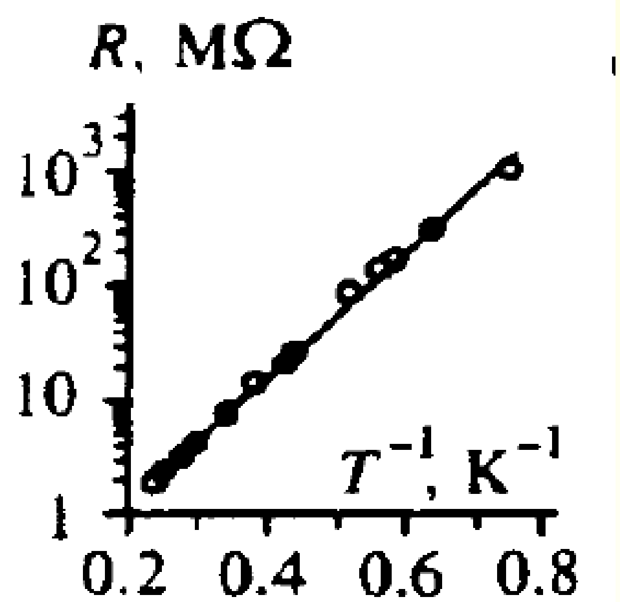


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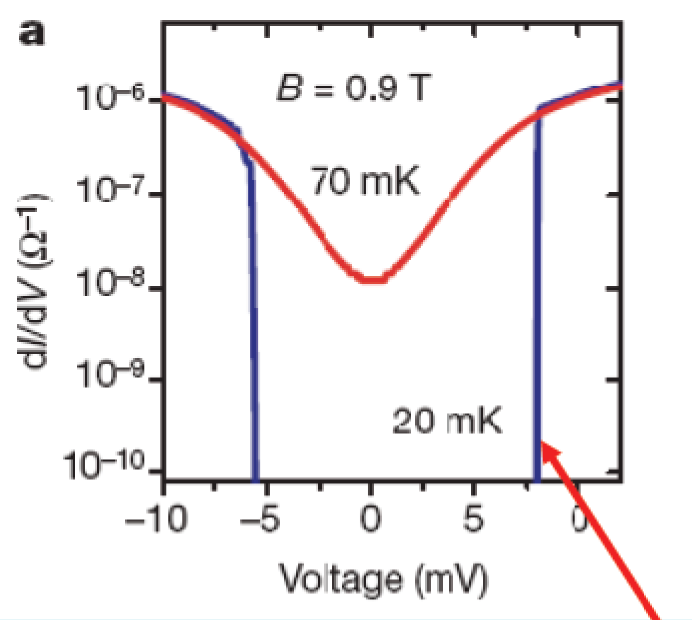
• No depairing of bosons (positive MR!)

→ • Boson mobility edge !
(Similar to Anderson localisation)

# Purely electronic transport mechanism!

TiN films

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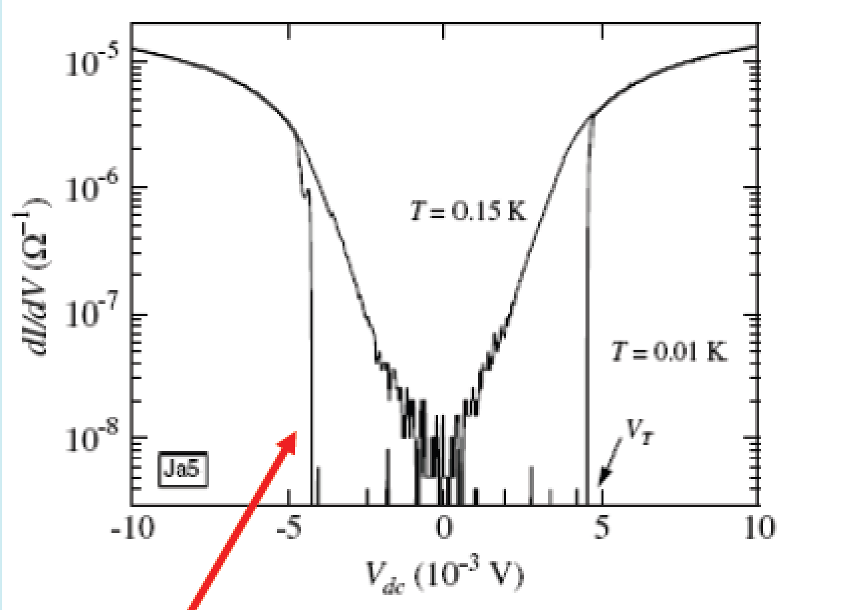


Baturina, Mironov, Vinokur, Baklanov, Strunk,'07

Sambandamurthy, Engel, Johansson, Peled, Shahar, '05

Giant jumps in resistance from kΩ to GΩ regime

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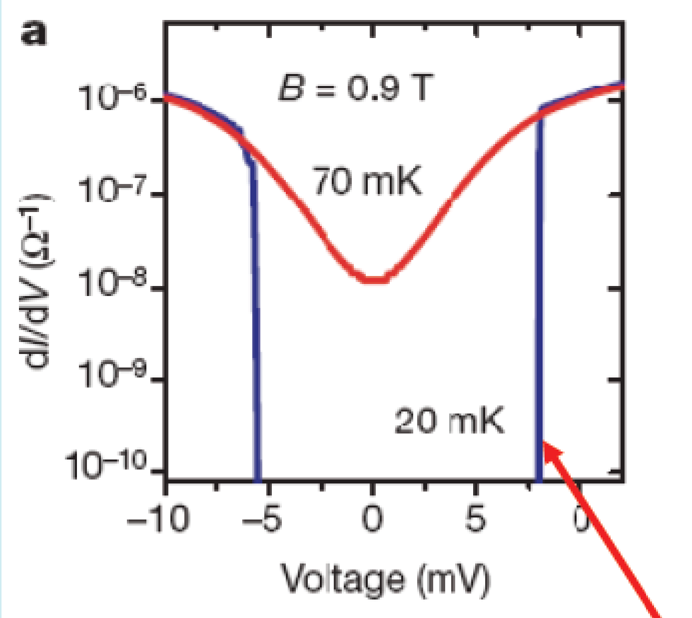


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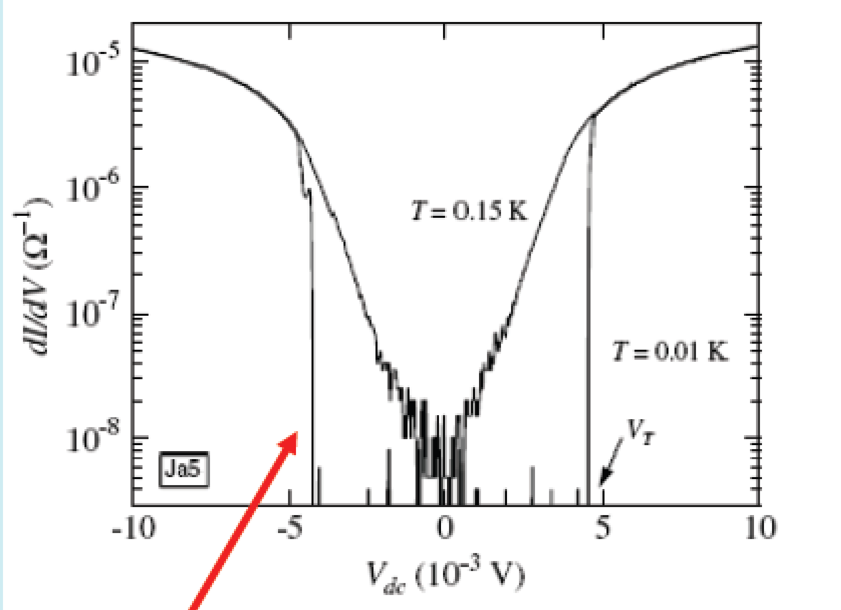


Sambandamurthy, Engel, Johansson, Peled, Shahar, '05

Giant jumps in resistance from kΩ to GΩ regime

Simple but effective explanation: bistability from low T to overheated state.

*Altshuler, Kravtsov, Lerner, Aleiner (09)*

Crucial ingredient: transport is not phonon- but electron-activated! - Mechanism???

# Summary

1. **Close to the SI transition the transport is essentially simply activated (Arrhenius):**
   **How come?**

2. **Evidence for purely electronic transport from heating instability in non-Ohmic regime:**
   **What is its origin?**

# From dirty superconductor to Bose glass

Models

$$H = t\sum_{\langle i,j\rangle} b_i^+ b_j + U\sum_i n_i(n_i - 1) + \sum_i (\varepsilon_i - \mu) n_i$$

Disorder: $\varepsilon_i \in [-\Delta, \Delta]$

Easier to think about: U = ∞ limit, i.e., hard core bosons
→ bosons equivalent to pseudospins (s=1/2)

Interactions (e.g. Coulomb)

*(Anderson, Ma+Lee, Kapitulnik+Kotliar)*

$$H = t\sum_{\langle i,j\rangle} s_i^+ s_j^- + \sum_i (\varepsilon_i - \mu) s_i^z + \sum_{\langle i,j\rangle} J_{ij} s_i^z s_j^z$$

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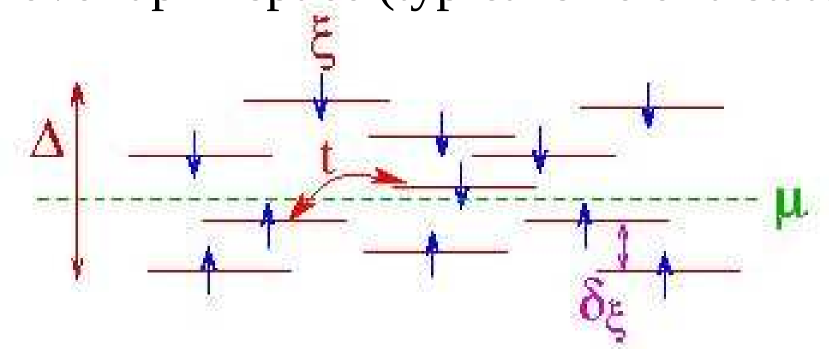
• "Sites" i: states for bosons to occupy. May overlap in space (typical size of a state: ξ)

•Relevant scale characterizing disorder:
Level spacing $\delta_\xi$ between close levels
Disorder strength: $g \equiv \delta_\xi / t$



# From dirty superconductor to Bose glass

- Superconducting phase: Bose condensation into delocalized mode

→ finite phase stiffness
→ infinite conductivity for $T < T_c$

- Bose glass: No delocalized bosonic mode anymore (otherwise condensation would occur)

 - role of disorder: no homogeneous gap, still compressible phase

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- **Note:** "Bose glass": **unfrustrated** but disordered Bose insulator)
- but: insulator, i.e. $\sigma(T \rightarrow 0) = 0$ [no Bose metal!]

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Nature of transport in the Bose glass?

# From dirty superconductor to Bose glass

Localization of the bosons?

Look at evolution of the full manybody spectrum!

*Berkovits and Shklovskii*
*Basko, Aleiner, Altshuler*
*Huse, Oganesyan*

# From dirty superconductor to Bose glass

Local spectrum at T = 0 $\rho_{O}(\omega) = \int_{0}^{\infty} \langle O(x,t) O(x,0) \rangle_{GS} e^{i\omega t}$

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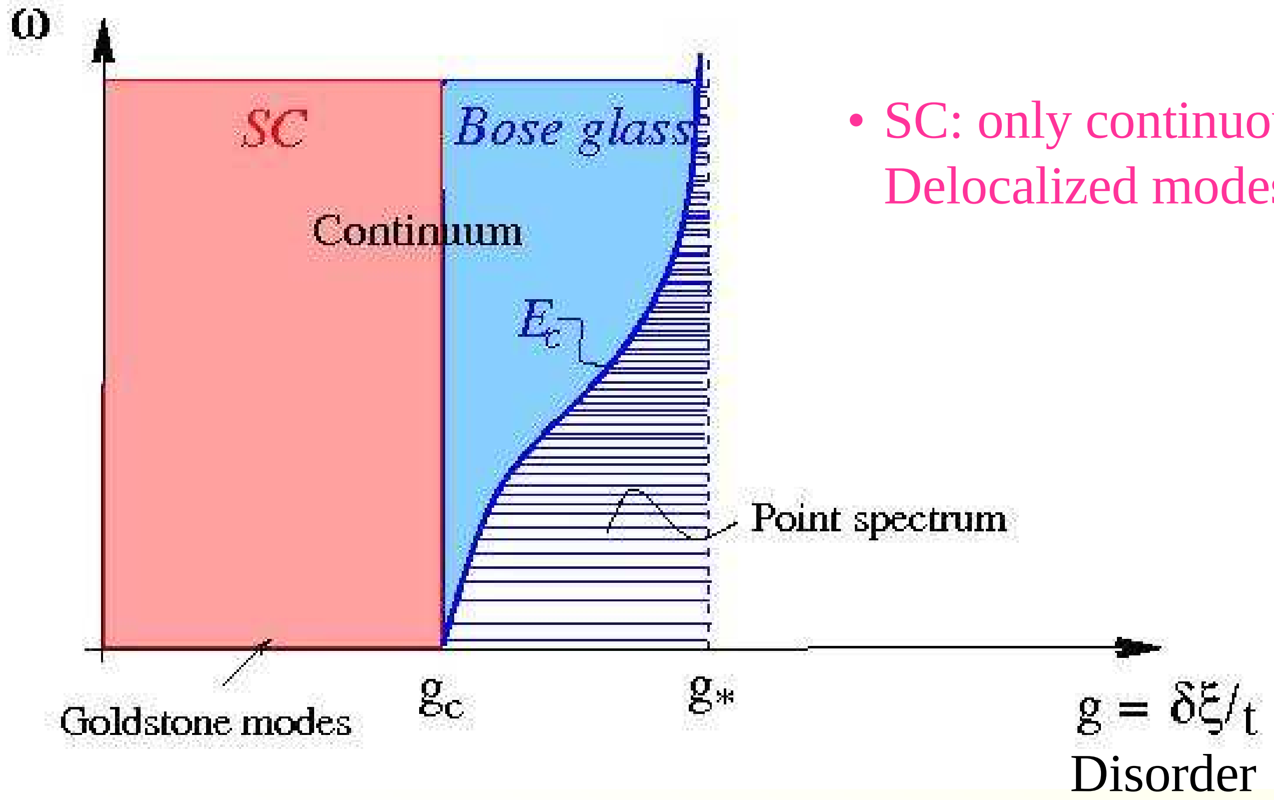


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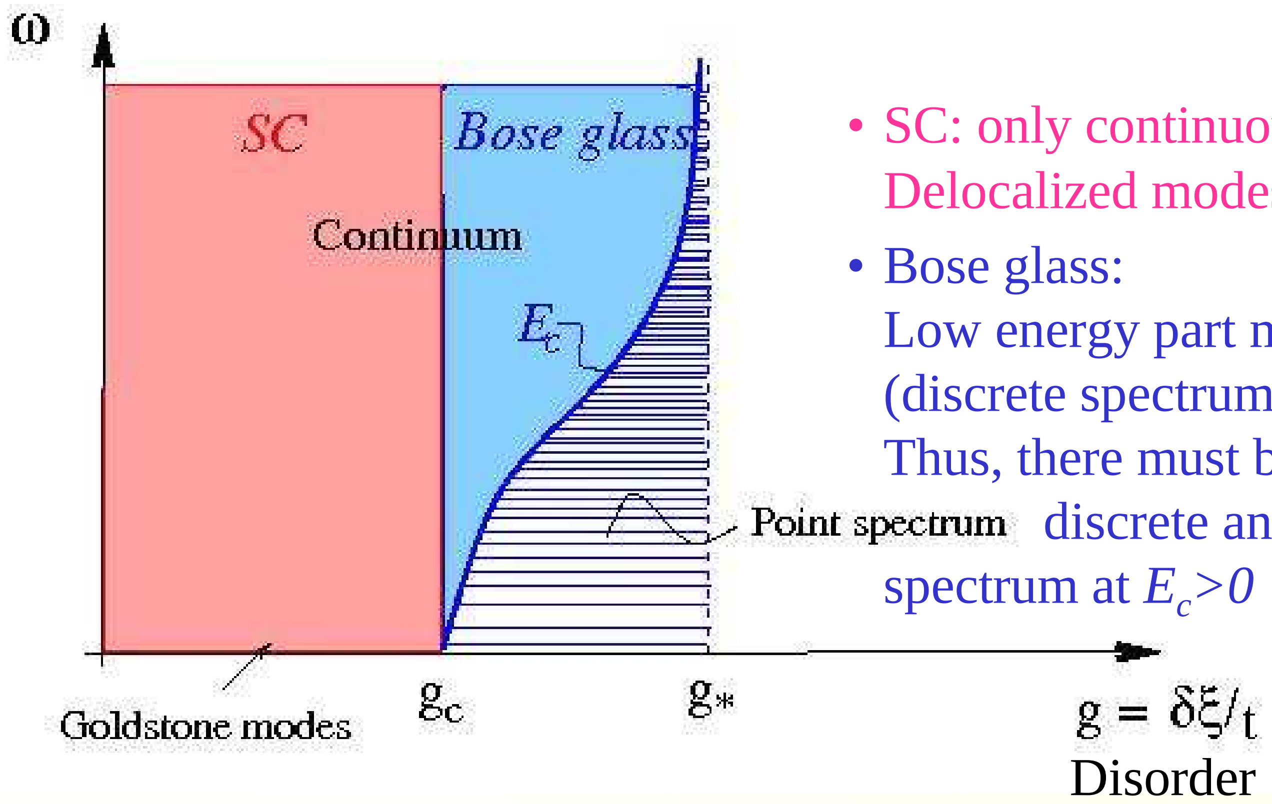
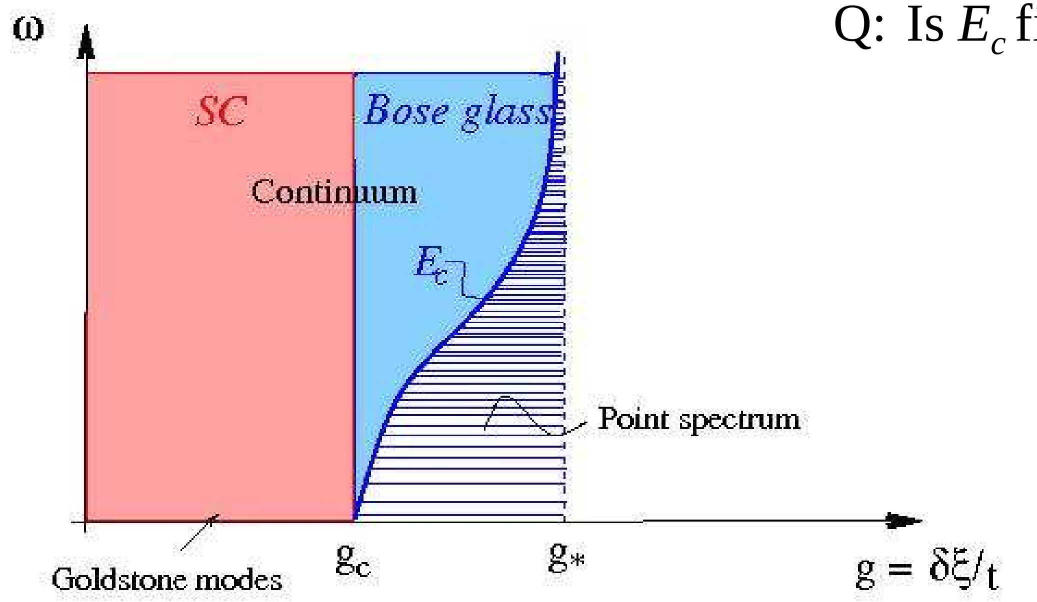



- SC: only continuous spectrum! Delocalized modes down to ω=0
- Bose glass: Low energy part must be localized (discrete spectrum). Thus, there must be a border between discrete and continuous spectrum at $E_c > 0$

# Spectral properties of the Bose glass

Many-body "mobility edge" in the Bose glass




Q: Is $E_c$ finite or extensive? (~Vol)

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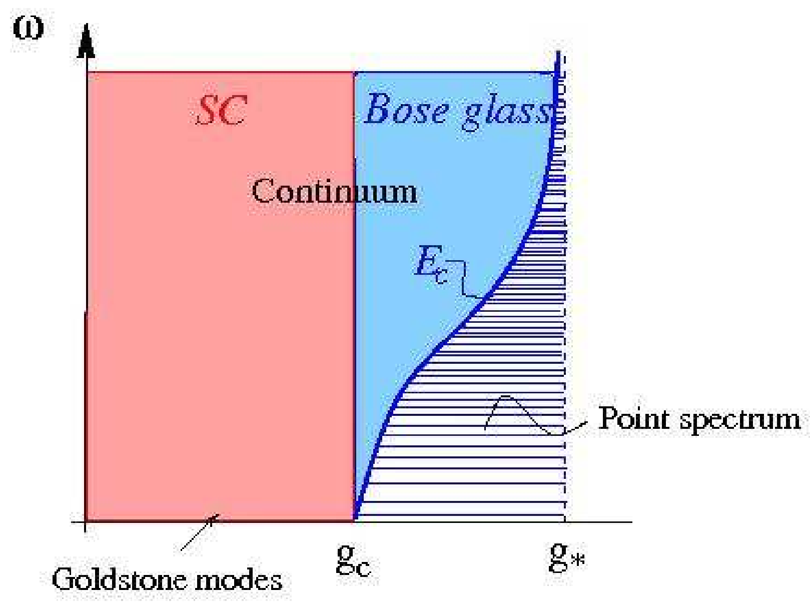
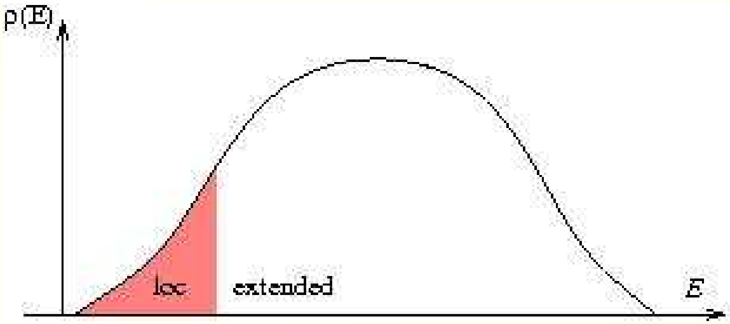



Q: Is $E_c$ finite or extensive? (~Vol)

A: Close to the SIT ($g = g_c$) $E_c$ is finite:
Single boson excitations at E-μ >> t
are delocalized → $E_c < \infty$
(while at low energies bosons localize
due to the hard core constraints)

Analogon:
Localization at band edge (Anderson)



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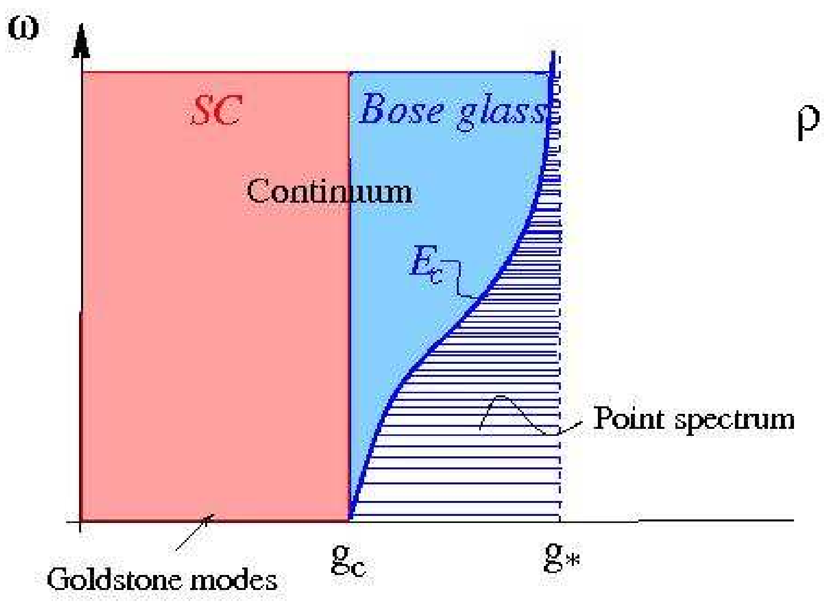
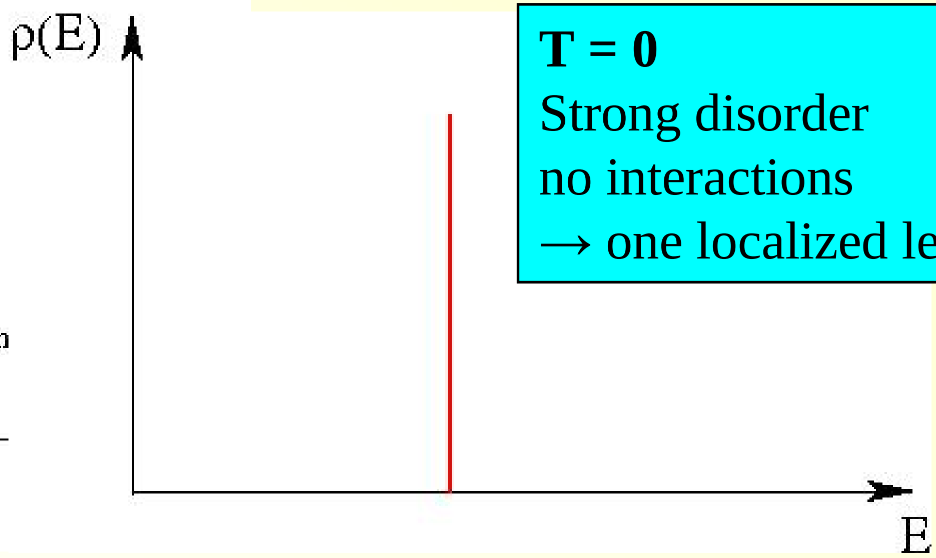



Point spectrum at T = 0




**T = 0**
Strong disorder
no interactions
→ one localized level

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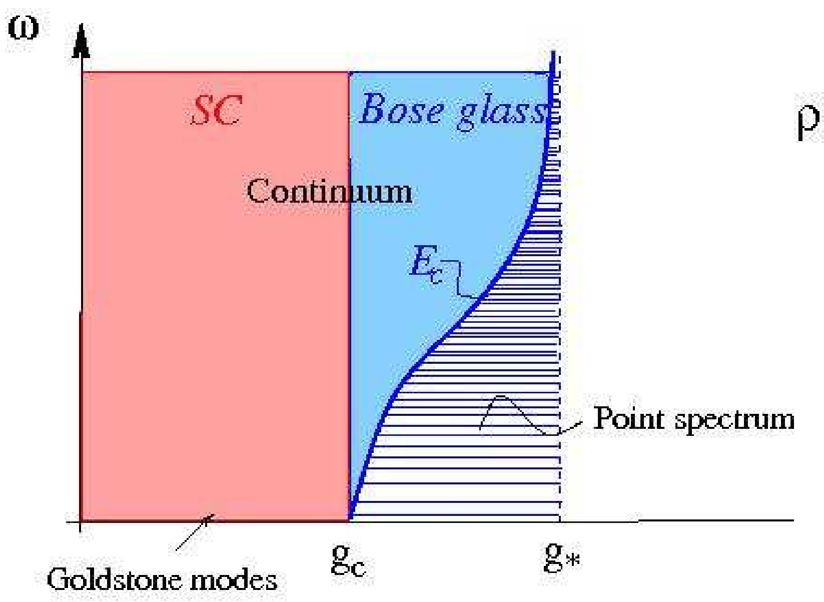


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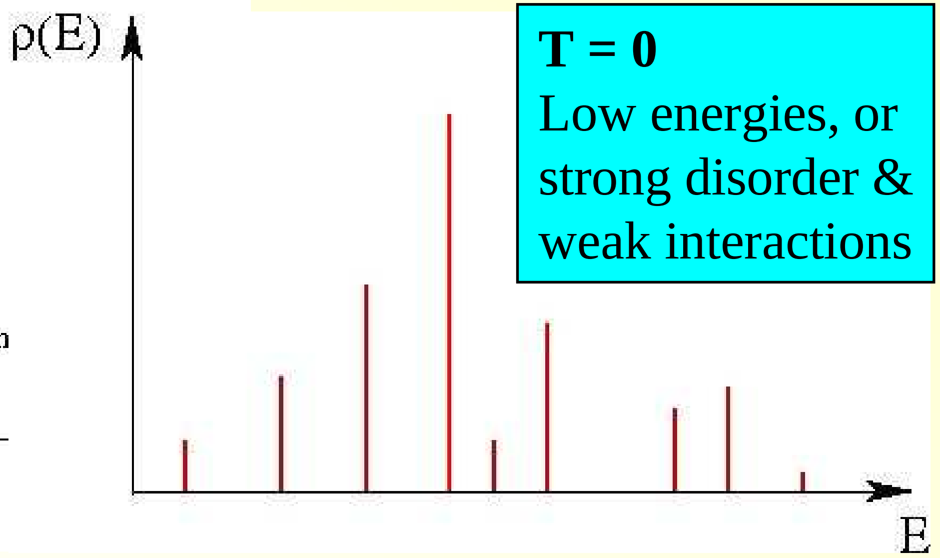


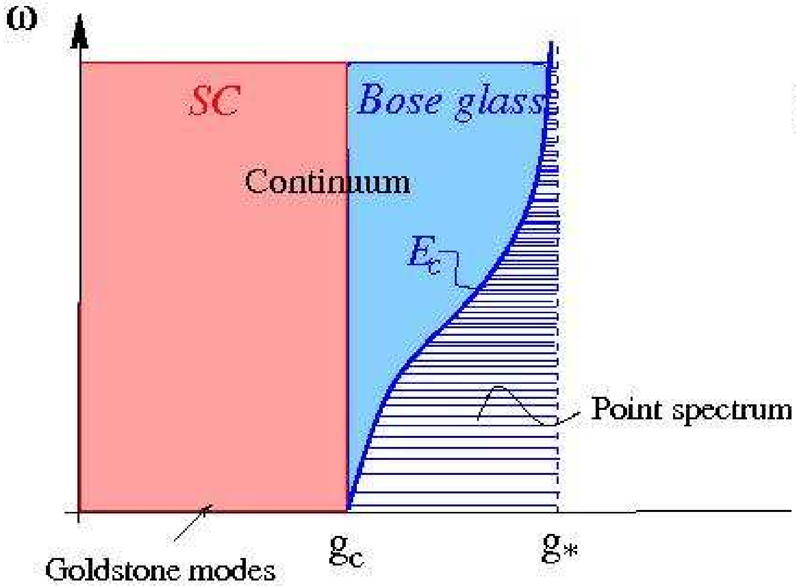

**T = 0**
Low energies, or strong disorder & weak interactions

- Discrete levels: no transport, no current!
  σ(T=0) = 0
- Genuine glass at T=0: perturbations don't relax

Reason: Transition probabilities are zero because energy conservation can never be satisfied!

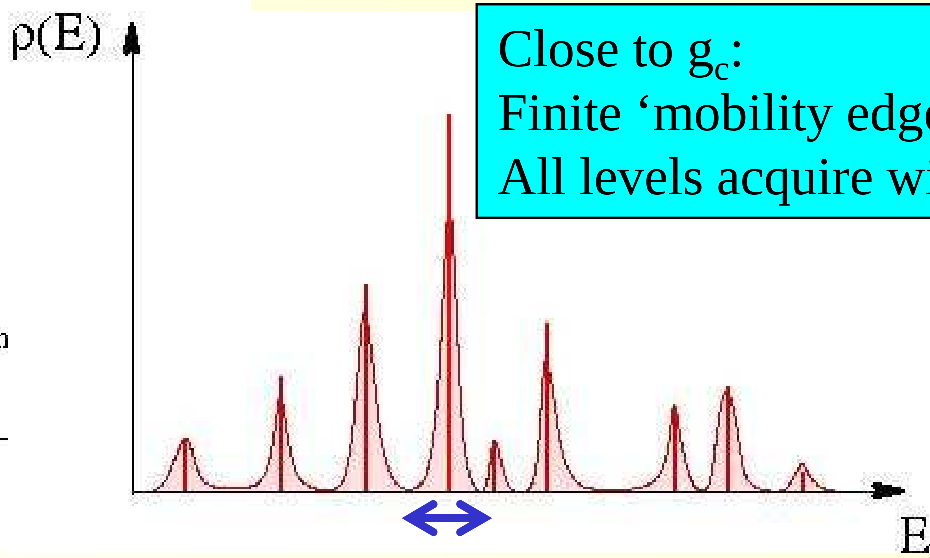
# Mobility edge

Many-body "mobility edge" in the Bose glass




Q: What happens at T > 0?

Close to $g_c$:
Finite 'mobility edge'
All levels acquire width




$\Gamma \sim \exp[-E_c/T]$

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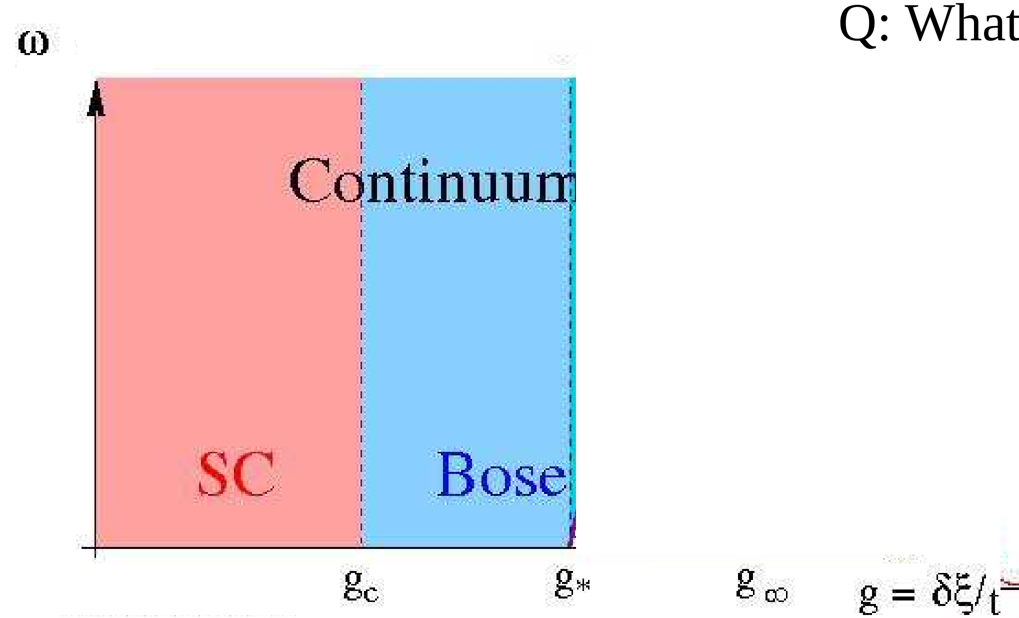


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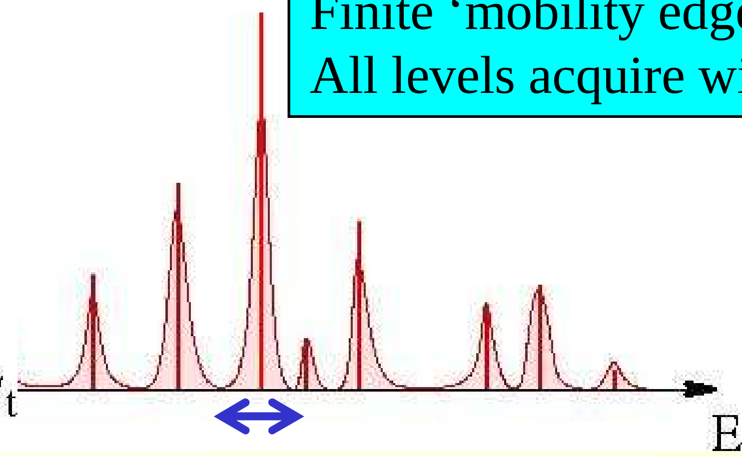
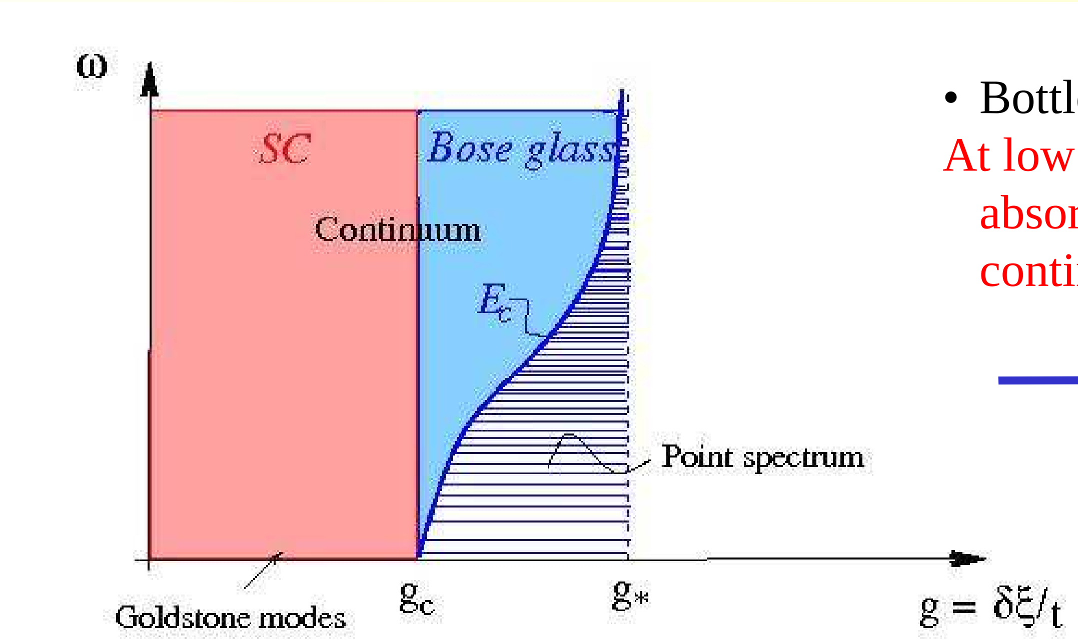



$\Gamma \sim \exp[-E_c/T]$

- Continuum everywhere! $\sigma(T > 0) \neq 0$ for $g < g_*$ where $E_c(g) < \infty$

# Electronic activated conduction

$g < g_* \;:\; E_c(g) < \infty$




- Continuum everywhere! $\sigma(T>0) \neq 0$

- Bottle neck for conduction:

At low T: Transitions allowed only due to the absorption of modes from the (T=0) continuum or diffusion above *Ec*

→ $\sigma(T) \sim \sigma_0 \exp[-E_c/T]$

Simple activation (Arrhenius) law in a compressible, gapless system! No variable range hopping $e^{T^{-\alpha}}$!

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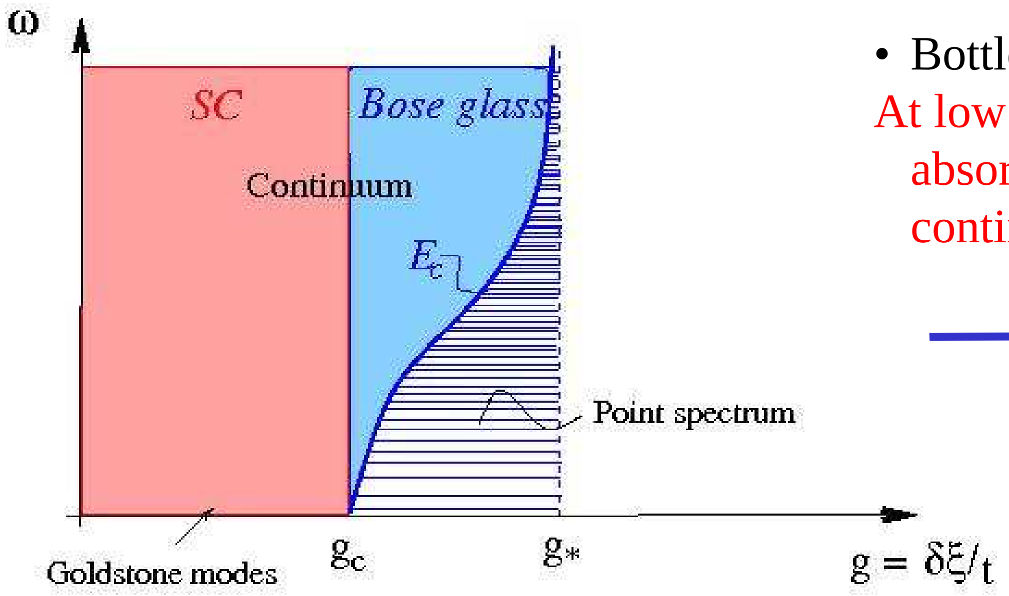


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Simple activation (Arrhenius) law in a compressible, gapless system! No variable range hopping $e^{T^{-\alpha}}$!

- No phonons needed! (Would anyway be very inefficient at this low T)
- Purely electronic transport mechanism
  → crucial ingredient to explain the overheating in the non-Ohmic regime
- Prefactor: $\sigma_0 \sim e^2/h\xi^{d-2}$ nearly universal in d=2, similar to experiment!
- "Conductivity at the mobility edge" more robust than for electrons:
  Relevant energy scale $t \sim T_c \sim$ few K, instead of $E_F$; no fine-tuning of $E_c$ over sample!

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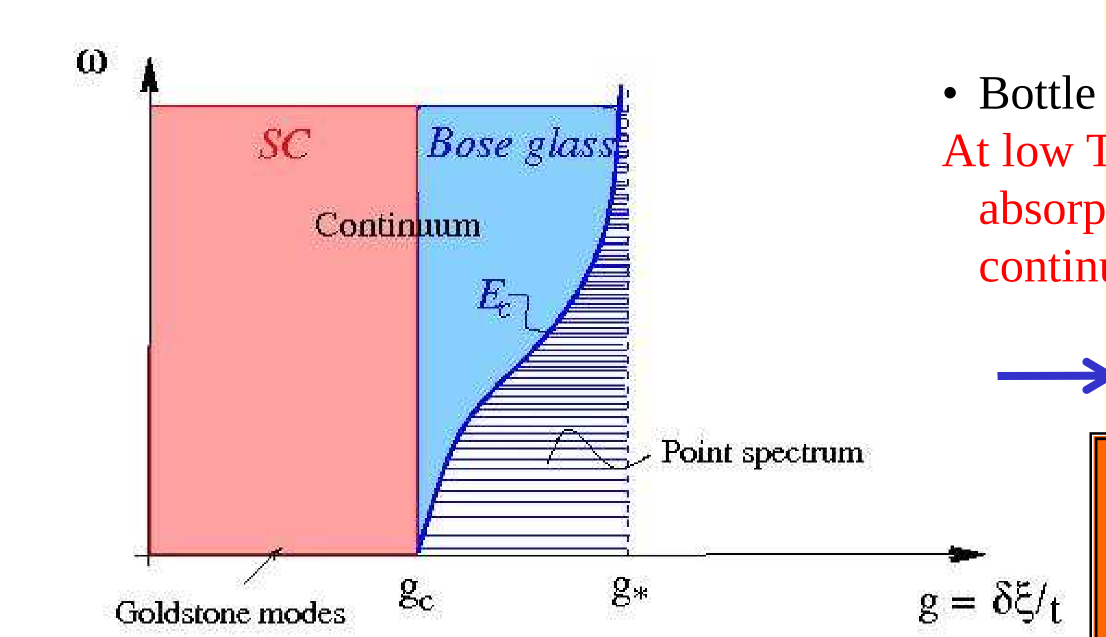


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!

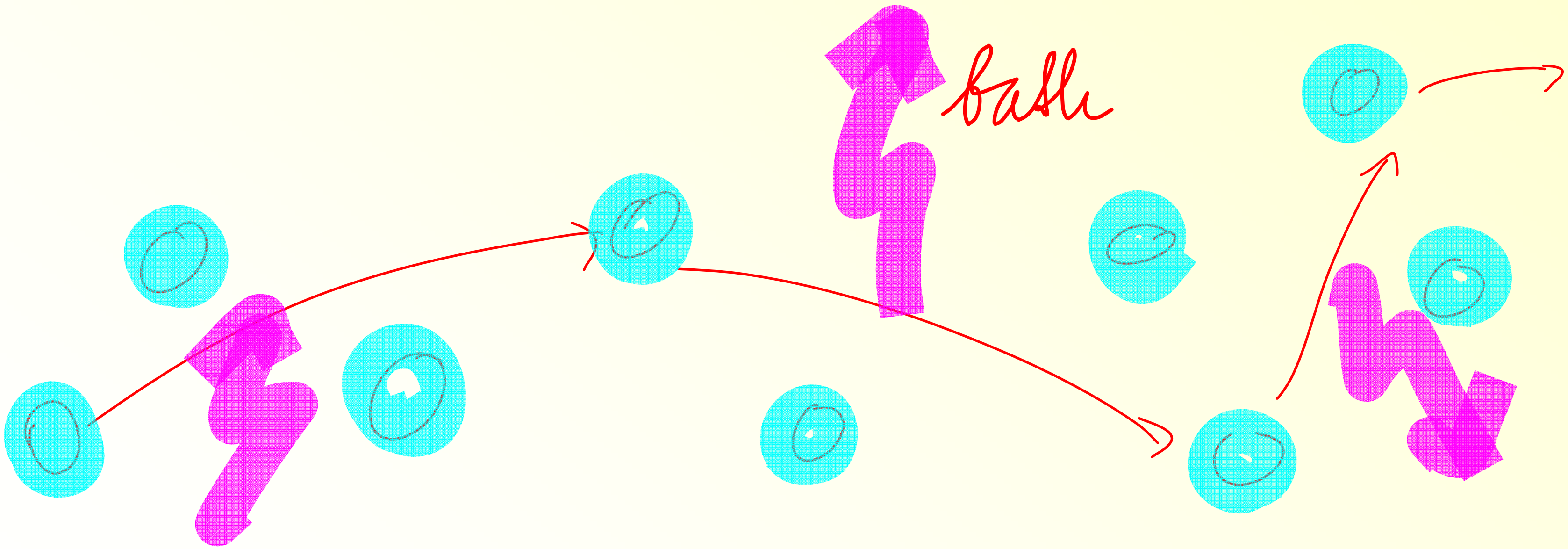
1. Arrhenius law is only asymptotic at lowest T :
Finite inelastic scattering rate at T > 0 lowers the activation energy needed to get diffusion! $\rightarrow E_{act} = E_c - \Delta E(T)$ ! → superactivation!

2. In reality: $E_{act}$ is bounded from above by depairing energy!
Bosonic description breaks down too far from SIT (or in high B field)

!

# ? How to understand that variable range hopping is not seen, but instead activation? ?

Essential ingredient into variable range hopping:
Continuous bath which activates the hops!



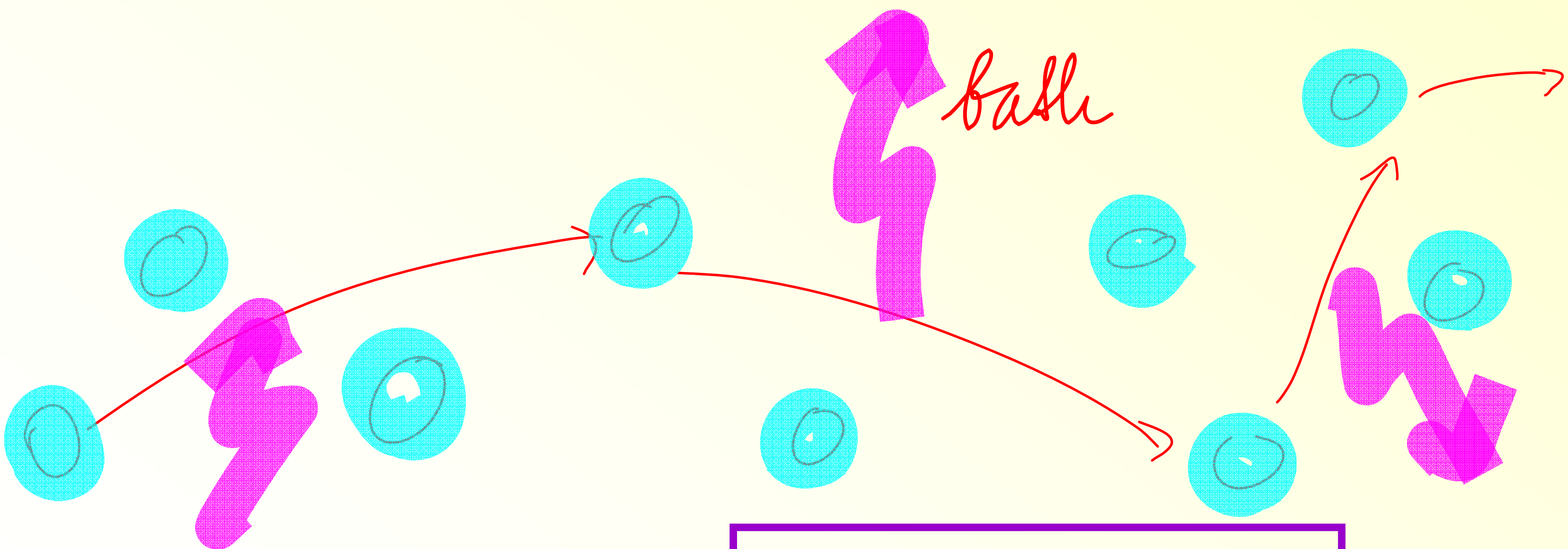

Candidates for the bath:

• Phonons: at low T for pair hopping are very inefficient!

# How to understand that variable range hopping is not seen, but instead activation?

? ?

Essential ingredient into variable range hopping:
Continuous bath which activates the hops!




Candidates for the bath:

Too weak → not considered

• ~~Phonons: at low T for pair hopping are very inefficient!~~

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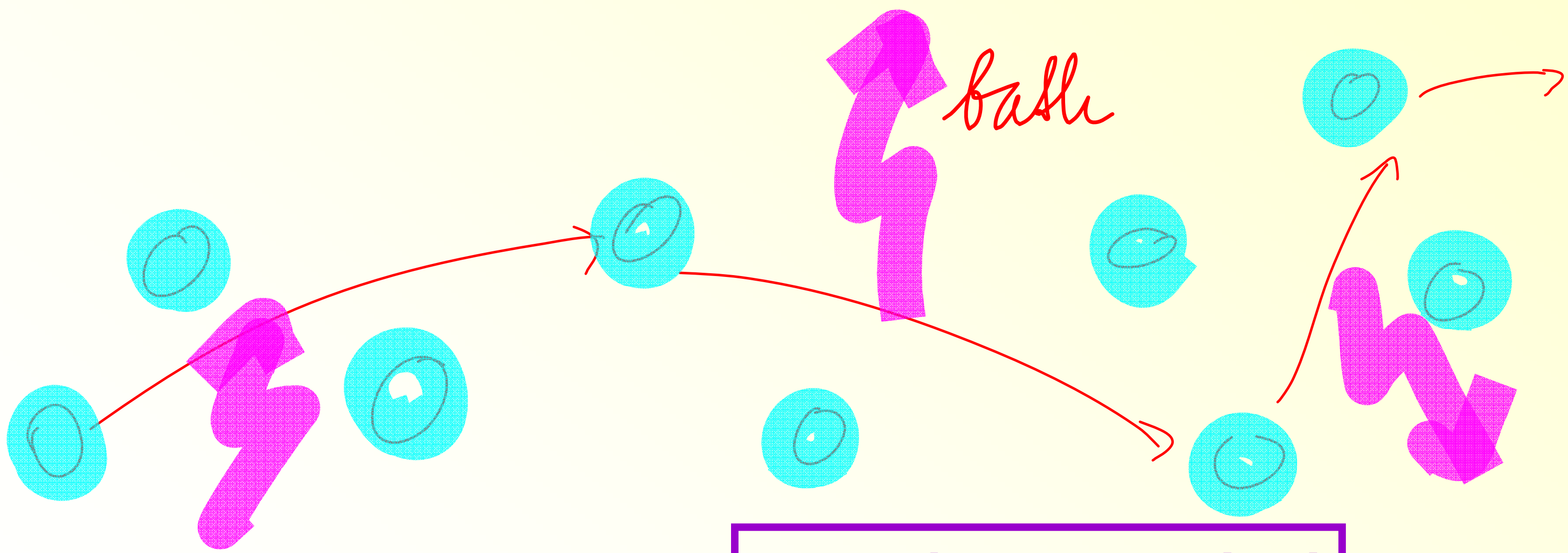


Candidates for the bath:

Too weak → not considered

- ~~Phonons: at low T for pair hopping are very inefficient!~~
- (possibly collective) pair excitations above the mobility edge

# Strong disorder

$g > g_* \ : \ E_c(g) = \infty$ (~ Volume)

- If disorder is strong ($g = \delta_\xi/t > g_*$) high energy single boson excitations above the GS (at $T = 0$) are localized as well: $E_c \to \infty$

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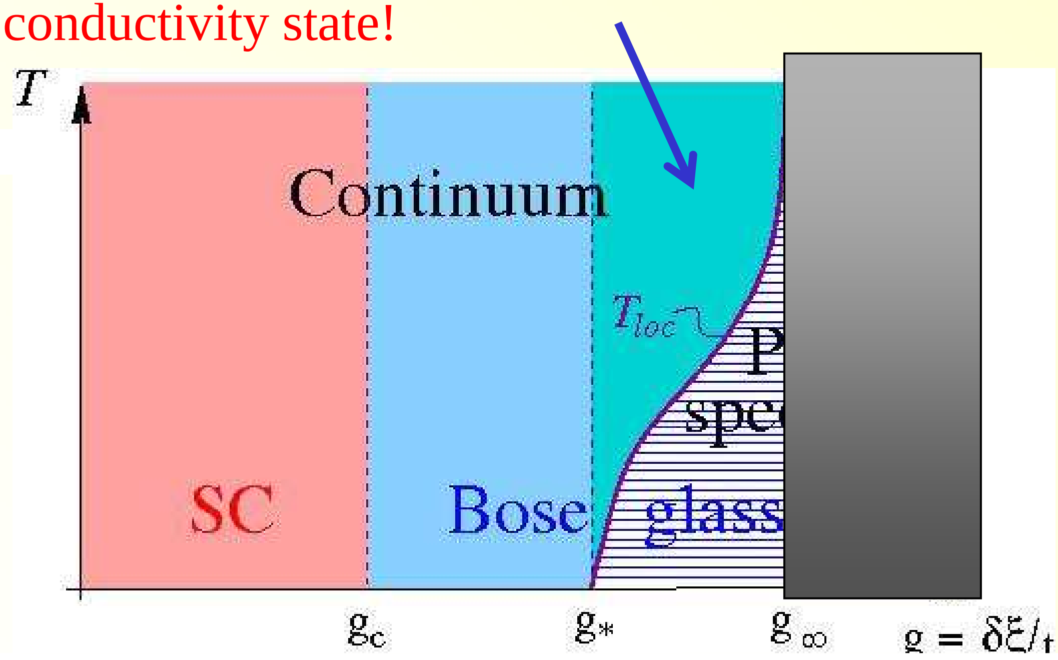
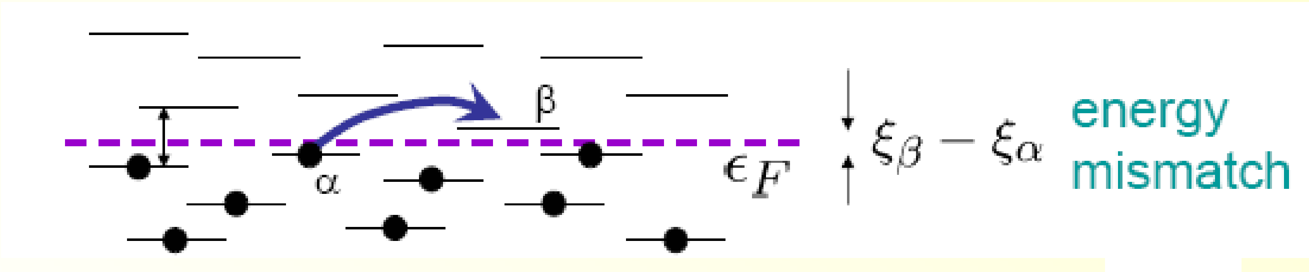
  → Finite T transition to zero conductivity state!



# Localization despite interactions?

*Fleishman, Anderson, Licciardello (1980, 1982)*
*Basko et al., Gornyi et al. (2005, 2006)*

Is there many-body localization (localization in Fock space) ↔ absence of diffusion; even at finite T?



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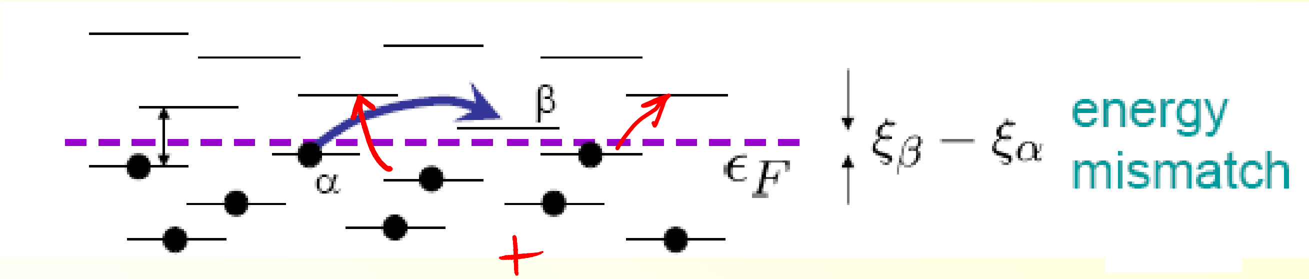


Can multi-particle arrangements bridge the energy mismatch?

NO: not if interactions are too weak!

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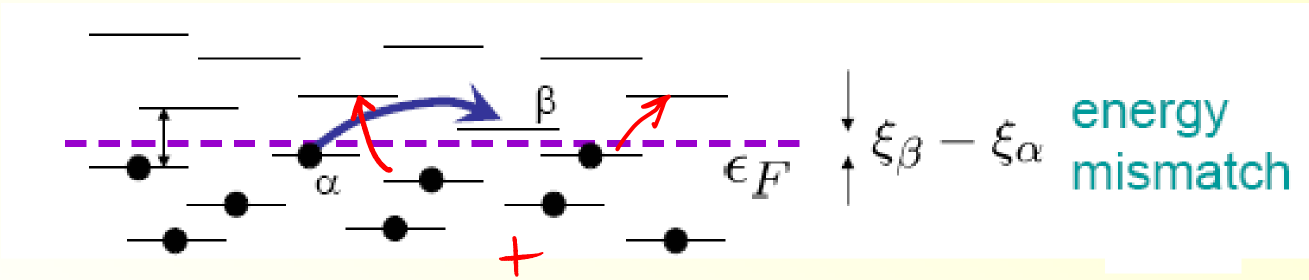


Answer: For $T < \delta_\xi / \lambda$ ( $\lambda << 1$: interaction parameter)

- Energy conservation impossible: electrons do not constitute a continuous bath!
- All many body excitations remain discrete in energy!
- Conductivity = 0 even at finite T – and no thermal equilibration either!

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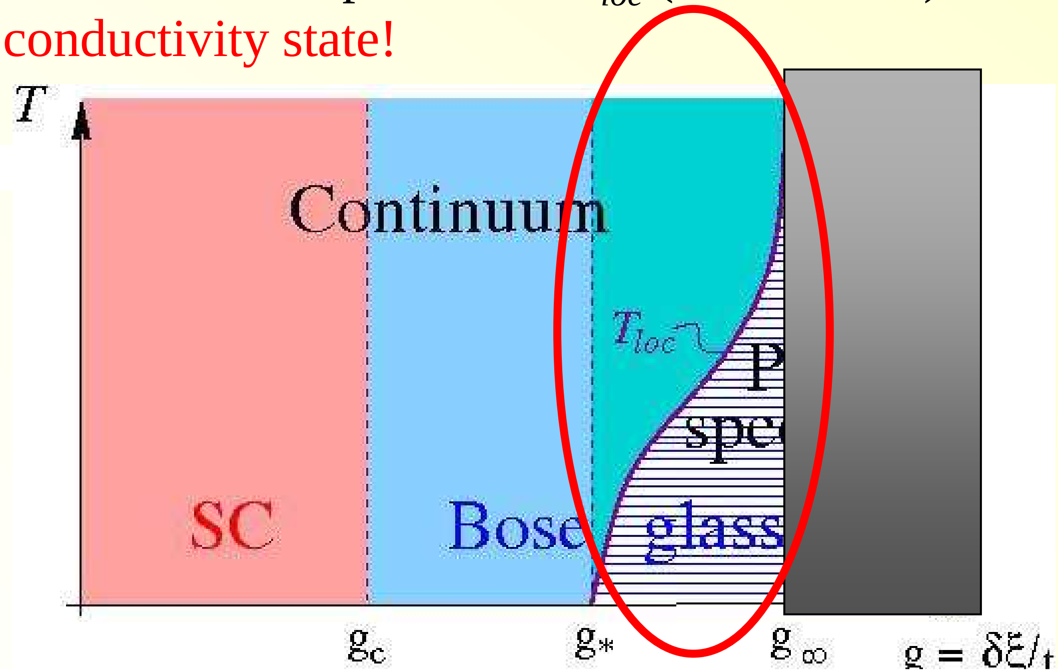

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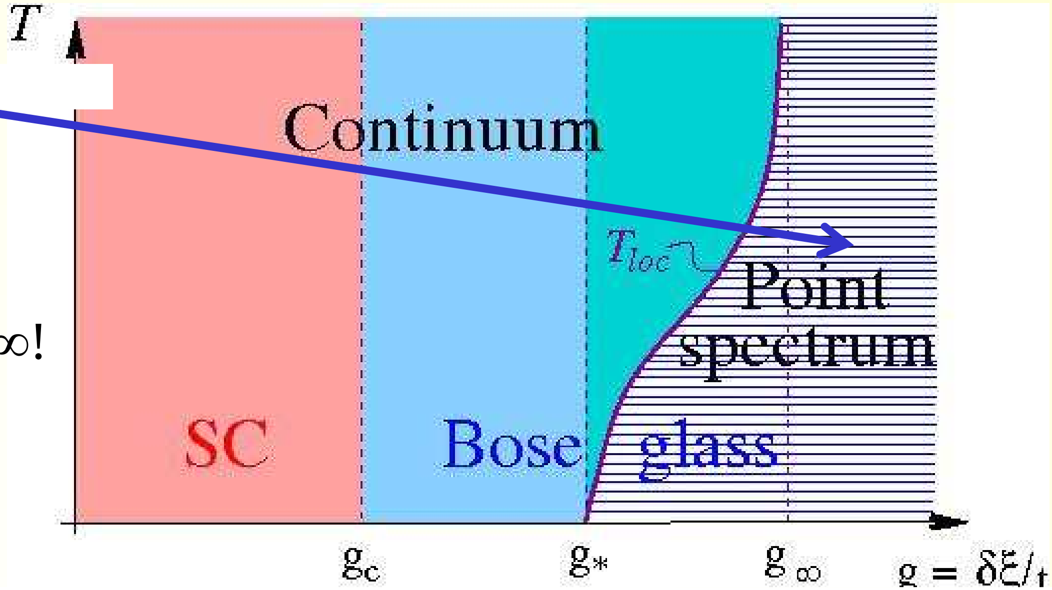
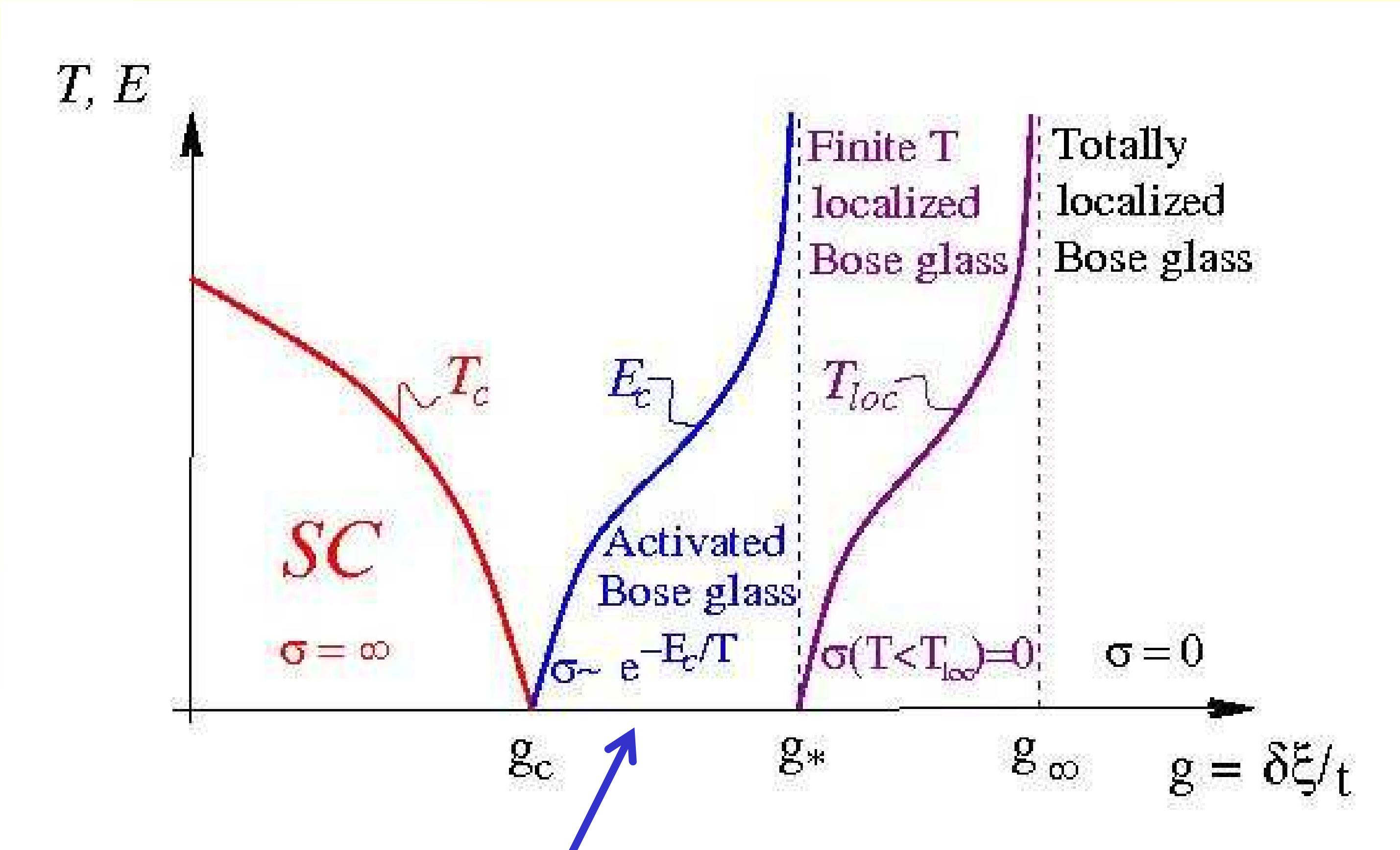
  → Finite T transition to zero conductivity state!
- At biggest $g > g_\infty$:
  If energy range Δ is finite →
  maximal scattering rate →
  complete localization in very strong disorder when $T_{loc} \to \infty$!



# Summary: Bose-Hubbard model and Bose glass




Purely electronic transport at low T: **Asymptotically** Arrhenius law!

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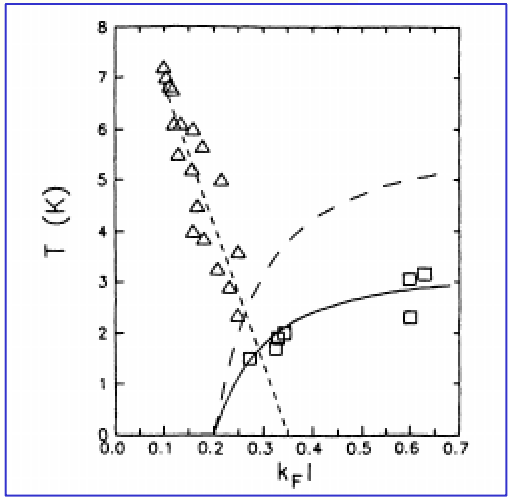
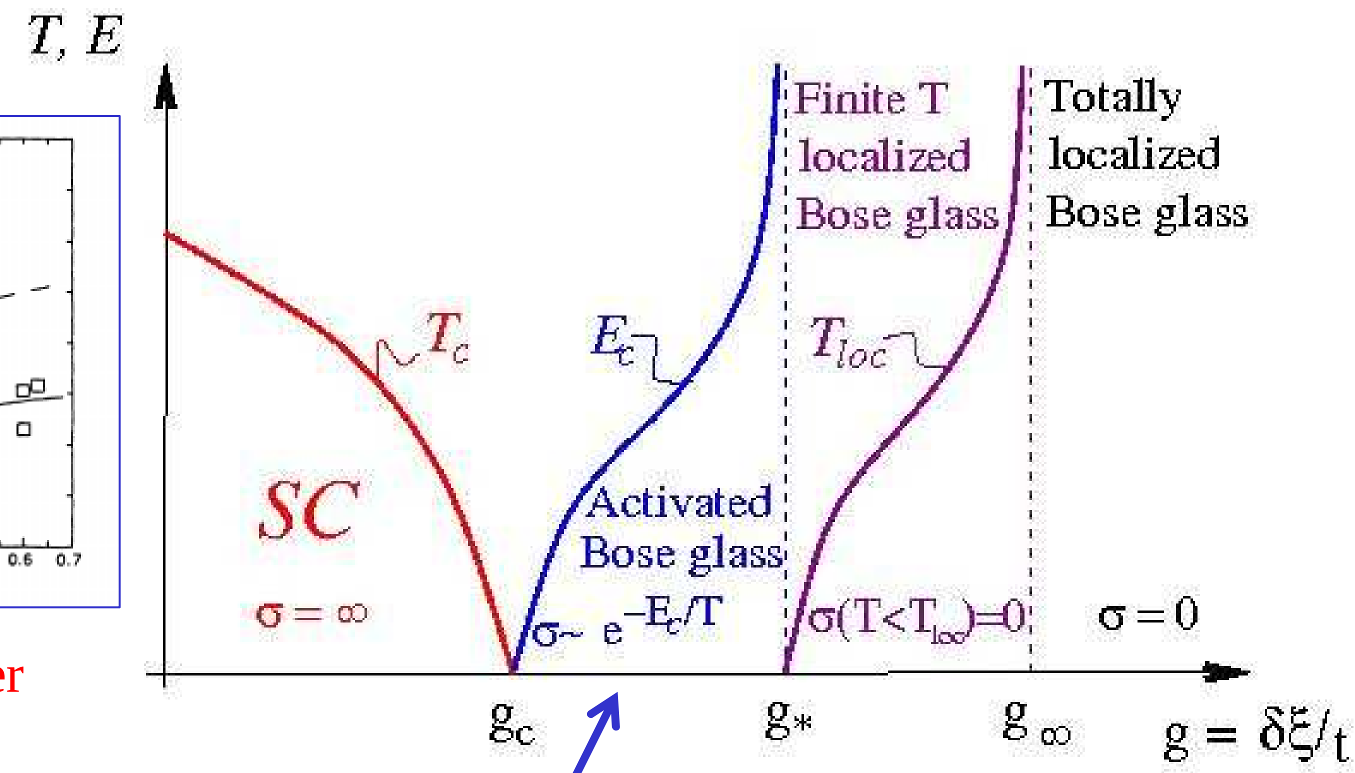



1/Disorder

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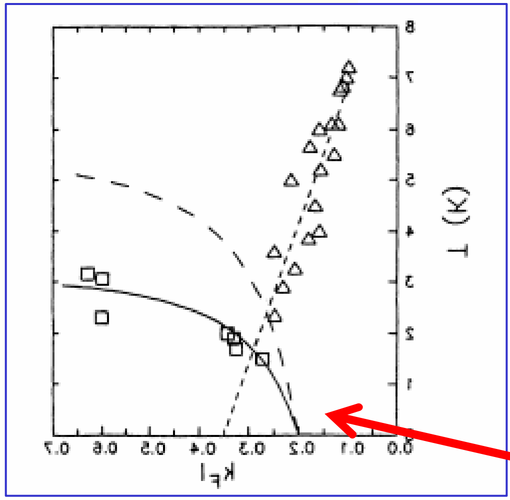

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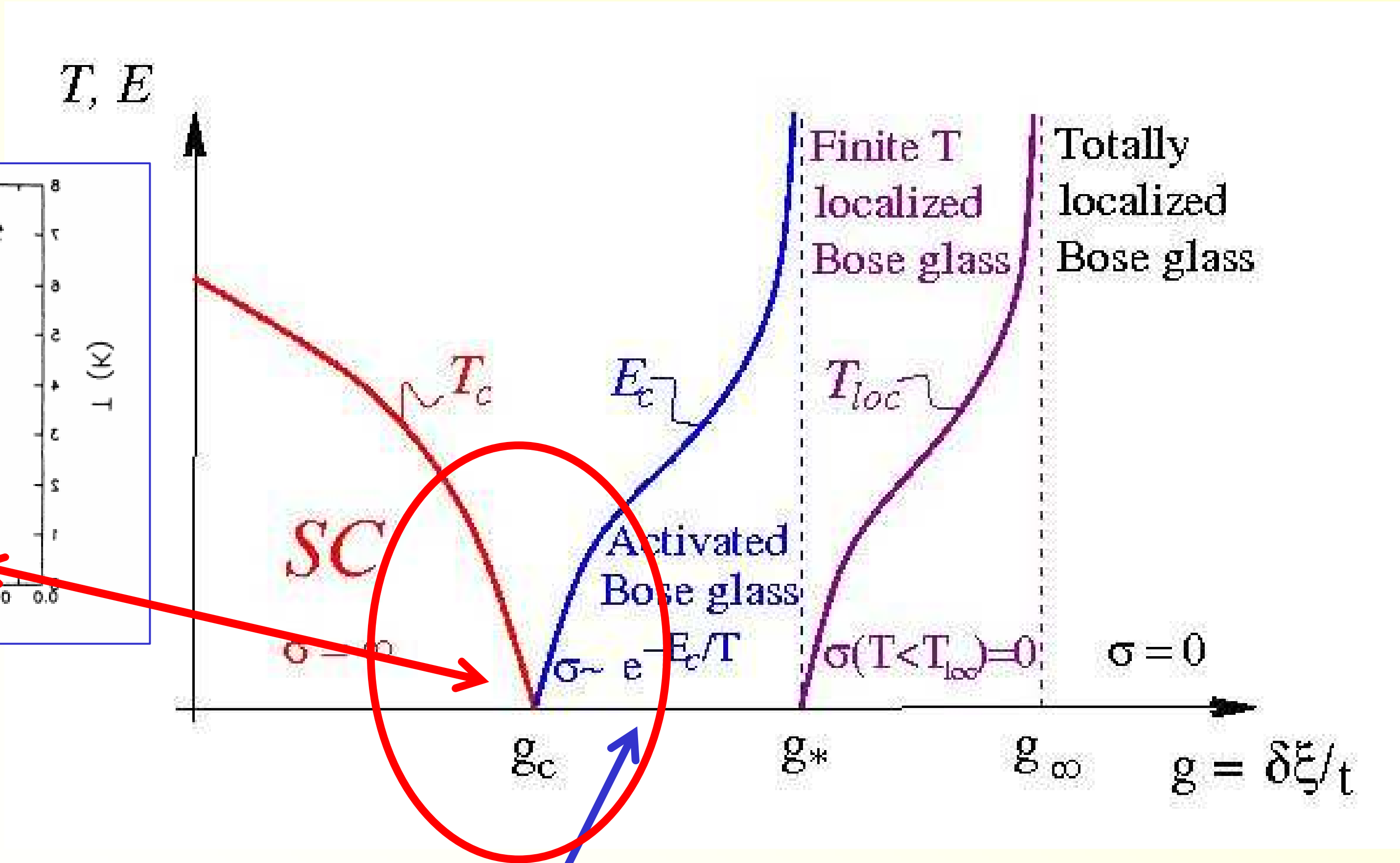


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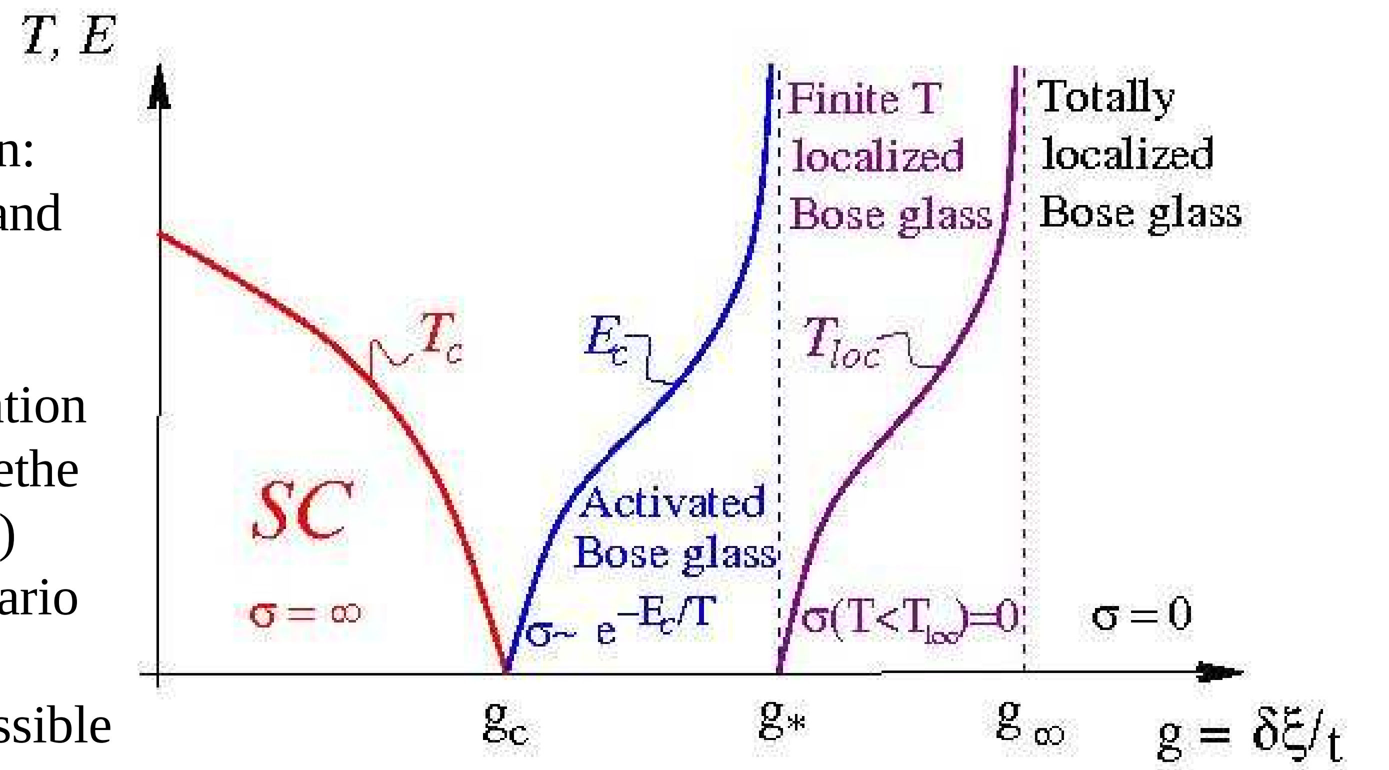
# Summary: Bose-Hubbard model and Bose glass

Can this scenario be proved?

• $T_{loc}$ & total localization: similar to Mirlin et al. and Basko et al.

• Controlled approximation on high connectivity Bethe lattice (Ioffe & Mézard) in agreement with scenario

• Total localization: possible that it can be proved soon. Work in progress.



# Conclusion

• Transport in the Bose glass (without phonons) is a very rich problem due to various localization phenomena

• Phase diagram is generic for disorder-driven delocalization transitions quantum phase transitions. Similar features close to the Metal-Insulator transition with interactions (such as e-assisted transport)