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PUTTING CORONAL SEISMOLOGY ESTIMATES OF THE MAGNETIC FIELD STRENGTH TO THE TEST

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ABSTRACT

The magnetic field strength inside a model coronal loop is "estimated" using coronal seismology, to examine the reliability of magnetic field strengths derived from observed, transverse coronal loop oscillations. Three-dimensional numerical simulations of the interaction of an external pressure pulse with a coronal loop (modeled as a three-dimensional density enhancement inside a two-dimensional magnetic arcade) are analyzed and the "observed" properties of the excited transverse loop oscillations are used to derive the value of the local magnetic field strength, following the method of Nakariakov & Ofman. Due to the (unexpected) change in periodicity, the magnetic field derived from our "observed" oscillation is substantially different from the actual (input) magnetic field value (approximately 50%). Coronal seismology can derive useful information about the local magnetic field, but the combined effect of the loop curvature, the density ratio, and aspect ratio of the loop appears to be more important than previously expected.

Key words: Sun: activity – Sun: corona – Sun: magnetic fields – Sun: oscillations

1. INTRODUCTION

Coronal seismology (Uchida 1970; Roberts et al. 1984) can provide estimates of local plasma parameters that are difficult to obtain by direct measurements (see reviews by e.g., De Moortel 2005; Nakariakov & Verwichte 2005 or Banerjee et al. 2007). The recent confirmation of the ubiquitous presence of waves and oscillations in the solar atmosphere by, for example, Tomczyk et al. (2007), is likely to substantially widen the possible application of coronal seismology. One of the most promising applications so far has been the determination of the magnetic field strength from observed loop oscillations. Nakariakov et al. (1999) reported flare-induced, transverse coronal loop oscillations, which were interpreted as fast, standing, kink, magneto acoustic modes. Using this interpretation, Nakariakov & Ofman (2001) deduced the magnetic field strength of the oscillating coronal loops, by combining observed parameters and basic MHD wave properties, as suggested by Roberts et al. (1984). Assuming a density $\rho_0 = 1\times10^{-14}$–1.5×10^{-15} g cm^{-3} and a density ratio of $\rho_e/\rho_0 = 0.1$ (where ρ_0 is the density inside the coronal loop and ρ_e is the density of the external medium), the magnetic field strength was estimated to be in the range $B_0 = 4$–30 G. Following the same calculation, but for different examples of observed, transverse loop oscillations by TRACE, Aschwanden et al. (2002) and Verwichte et al. (2004) reported values of $B_0 = 3$–90 G and $B_0 = 9$–46 G, respectively. More recently, Van Doorsselaere et al. (2008) used Hinode/EIS observations of a coronal kink mode oscillation to derive a magnetic field strength of 31–47 G. Finally, Wang et al. (2007) followed essentially the same method to deduce the magnetic field strength from observed oscillations interpreted as standing, *slow* modes and found values of the order of $B_0 = 12$–61 G.

All the above authors used observed coronal loop oscillations to estimate the value of the local magnetic field strength. These estimates are essentially based on the observed periodicity, loop length, and density, combined with the interpretation of the observed oscillations as MHD modes. Pascoe et al. (2009, hereafter Paper I) presented three-dimensional numerical simulations of coronal loop oscillations, investigating the effect of the attack angle of a pressure perturbation on the induced kink mode oscillations. In Paper I, it was noted that the periodicity of the loop oscillation differed substantially from the theoretically expected value. Here, we investigate how this modified periodicity could affect coronal seismology estimates of the local magnetic field strength. We will essentially follow the analysis of Nakariakov & Ofman (2001) to "estimate" the magnetic field strength inside our model coronal loop and will then compare this value to the actual magnetic field strength in the numerical domain. This comparison will allow us to examine the reliability of the magnetic field strength derived from coronal seismology. The Letter is organized as follows: the simulations are described in Section 2 and analyzed in Section 3. A Discussion and conclusions are given in Sections 4 and 5, respectively.

2. MODEL SETUP

The numerical simulations on which the results presented in this Letter are based follow the same setup as described in Paper I: the three-dimensional numerical domain contains a two-dimensional arcade magnetic field ($B \sim 1/r$), in which a loop of (minor) radius a is modeled as a density enhancement, using a modified Epstein profile (see Equation (1) in Paper I). We use a density ratio $\rho_0/\rho_e = 10$. Figure 1 shows the modeled coronal loop as an isosurface of density (red), as well as a few representative magnetic field lines. The color gradient on the bottom boundary shows the vertical component of the magnetic field (B_z) in this plane.

The ideal, nonlinear MHD equations are solved using the Lare3D code (Arber et al. 2001). The numerical domain is roughly $175\times175\times90$ Mm with a resolution of $400\times400\times200$ grid points. This higher resolution (compared to Paper I) enables us to actually track the loop displacement, consistent with the analysis of observed, transverse loop oscillations (see, e.g., Nakariakov et al. 1999). As in Paper I, the open corona is approximated using damping layers to avoid reflection of any perturbations back into the numerical domain. The normalization is based on the loop oscillations studied by Nakariakov et al. (1999), so the radius of curvature r_c is chosen to give a loop length of $L = \pi r_c = 130$ Mm and a kink speed of about 1 Mm s^{-1} at the loop apex. Finally, we use a relatively

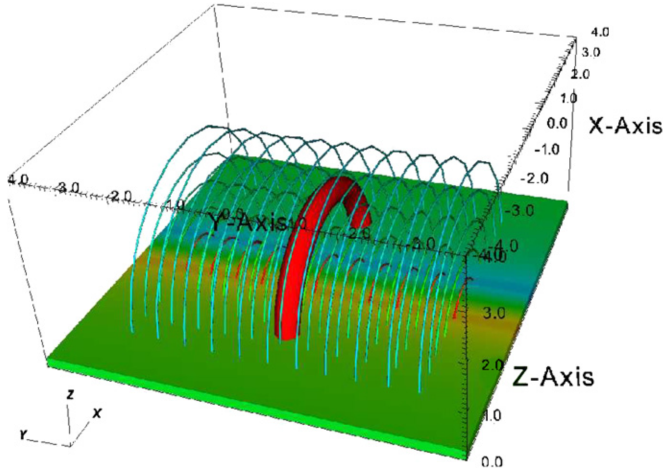

Figure 1. Three-dimensional numerical domain showing the density-loaded model coronal loop (red isosurface) as well as a few representative magnetic field lines outlining the arcade structure. The color contours at $z = 0$ represent the vertical magnetic field strength in this plane.

large value for the coronal plasma β, namely $\beta = 0.1$ at the loop apex, to avoid a loop top density enhancement caused by ponderomotive effects (Terradas & Ofman 2004).

A few initial time steps are run without a driver to ensure that any flows resulting from the nonequilibrium near $r = 0$ are negligible. Subsequently, a pressure pulse is released inside the numerical domain. The pulse has a width comparable to the loop diameter and is situated at the same height as the loop apex, at a distance of about 40 Mm away. The pressure pulse travels through the numerical domain and excites a transverse, global kink mode oscillations in the coronal loop. This process is described in detail in Paper I so we will here only repeat those aspects needed for our calculation of the magnetic field.

3. ANALYSIS OF TRANSVERSE LOOP OSCILLATIONS

Following the analysis of Nakariakov et al. (1999) and Nakariakov & Ofman (2001) we track the loop apex by determining the position of the local maximum in density. As we are using a (modified) Epstein profile for the density (Nakariakov & Roberts 1995), this maximum is clearly defined. Figure 2 shows cross sections of the density through the loop apex (and perpendicular to the plane of the loop), stacked along the y-axis as a function of time (an arbitrary constant has been added to the intensity at each time step). We can clearly see an oscillatory displacement of the loop apex, initially of the order of several grid points (red) but decaying very rapidly at later times (blue). We will here only focus on those parameters needed to determine the magnetic field strength and hence will not discuss this rapid damping further.

In Paper I, we interpreted the loop oscillations as global kink mode oscillations and hence, the phase speed is given by

$$\frac{2L}{P} = c_k \approx v_{A0}\left(\frac{2}{1+\rho_e/\rho_0}\right)^{1/2}, \tag{1}$$

where P is the period of the oscillation, L the loop length, v_{A0} the Alfvén speed inside the loop, and c_k the kink speed. Note that this approximation is based on the assumption that $B_0 = B_e$. In our magnetic arcade model, the field strength of the

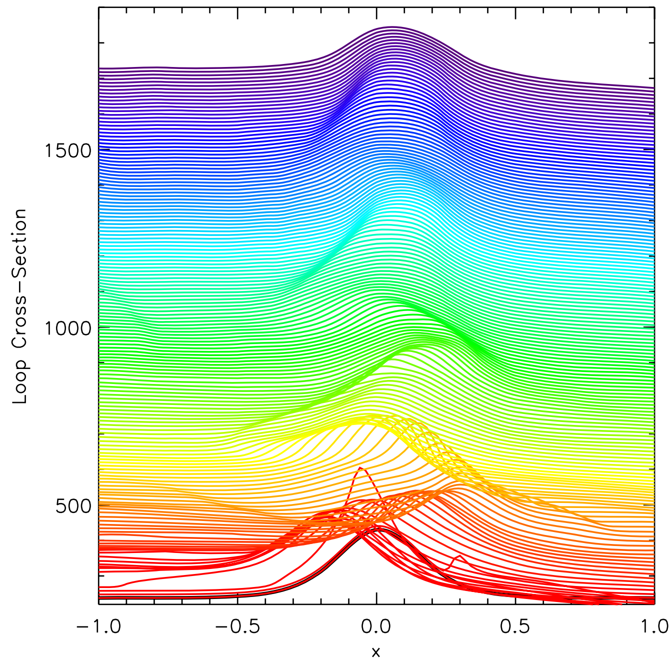

Figure 2. Cross sections of the density around the loop apex (perpendicular to the plane of the loop) for a strong initial pressure perturbation ($\delta p/p \approx 4000$). The cross sections are stacked along the y-axis as a function of time by adding an arbitrary constant to the density at each time step (red corresponds to earlier times, blue to later times).

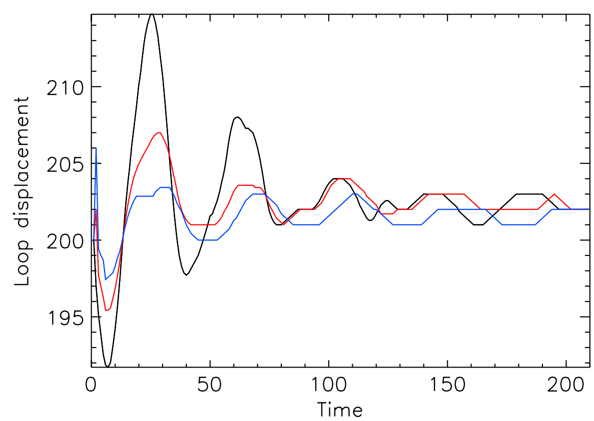

Figure 3. Displacement of the loop apex as a function of (nondimensional) time for three different amplitudes of the initial pressure pulse (black: $\delta p/p \approx 4000$; red: $\delta p/p \approx 2000$; blue: $\delta p/p \approx 1000$).

external magnetic field (B_e) varies but the assumption $B_0 = B_e$ is still true in the plane of the (transverse) kink oscillation. The Alfvén speed is given by $v_{A0} = B_0/\sqrt{\rho_0}$ (as the normalization of LARE3D implies that $\mu = 1$) and hence we can express the magnetic field (in nondimensional units) inside the coronal loop as

$$B_0 = \frac{\sqrt{2}L}{P}\sqrt{\rho_0(1+\rho_e/\rho_0)}\,. \tag{2}$$

From this expression we can see that the estimate of the magnetic field strength depends on the loop length, the "observed" period, the internal density, and the density ratio.

In Figure 3, the position of the loop apex is plotted as a function of time. The different colors correspond to three different amplitudes of the initial pressure perturbation: $\delta p/p \approx 1000$ (blue), $\delta p/p \approx 2000$ (red), and $\delta p/p \approx 4000$ (black). The

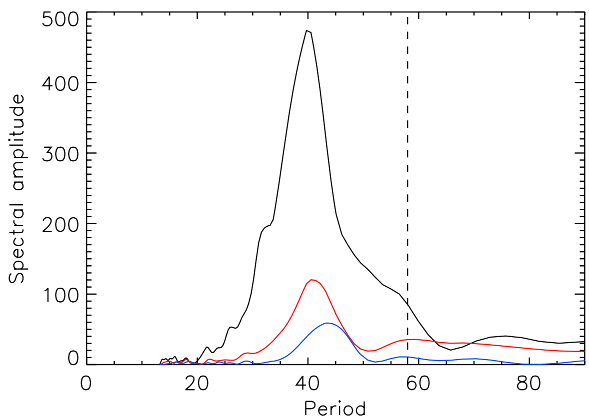

Figure 4. Periodogram of the oscillatory loop displacements shown in Figure 3. Note that the period on the horizontal axis is given in nondimensional units (multiplying by 4.3 gives the corresponding periods in seconds). The dashed vertical line represents the theoretically predicted period.

displacement of the loop apex is clearly oscillatory. Naturally, the amplitude of the displacement is higher for a stronger pressure pulse but the period of the oscillation is roughly the same in all three cases. This is confirmed in the corresponding periodogram (Figure 4) and the actual "observed" periods for the three different examples are $P_{\text{obs}} = 39.7$ (high pulse), $P_{\text{obs}} = 40.5$ (medium pulse), and $P_{\text{obs}} = 43.1$ (low pulse). In dimensional units, using $\tilde{t} = 4.3$ s, these periods correspond to 171, 174, and 185 s, respectively.

Using these periods, as well as a (nondimensional) loop length $L = 5.8$, a loop density $\rho_0 = 10$ and density ratio $\rho_e/\rho_0 = 0.1$, we can now estimate the local magnetic field strength from Equation (2). We find values of $B_{\text{obs}} = 0.69$ (high pulse), $B_{\text{obs}} = 0.67$ (med pulse), and $B_{\text{obs}} = 0.63$ (low pulse). In our subsequent discussion, we will use the average value of $B_{\text{obs}} = 0.66$. From the initial snapshot, the actual input value of the magnetic field strength at the loop apex, in nondimensional units, is given by $B_0 = 0.46$ and hence, the estimates obtained from coronal seismology (of our numerical oscillations) are substantially different.

To convert the values of the magnetic field strength to dimensional quantities, we choose a loop density $\rho_0 = 10^{9.3\pm0.3}$ cm^{-3} (see Nakariakov & Ofman 2001). Using this value, the normalizing constant for the magnetic field strength in our simulations is 22–44 G. Hence, the value of the derived magnetic field is about 15–30 G, compared to an input magnetic field of 10–20 G.

4. DISCUSSION

The value for the magnetic field strength obtained from our coronal seismology estimate differs from the actual (input) magnetic field strength by almost 50%. This substantial difference is caused by the modification of the period of the loop oscillations. In Paper I, we showed that, qualitatively, the (numerical) loop oscillations correspond to a global, transverse kink mode; the amplitude of the displacement has a single maximum at the loop apex and is zero at both loop footpoints. Theoretically, for a global kink mode, the given loop length ($L = 5.8$) and kink speed ($c_k = 0.2$) would result in a global kink mode period of 58 (using $P \approx 2L/c_k$, see vertical dashed line in Figure 4). The actual periods of our loop oscillation are of the order of 40–43, much smaller than theory predicts. This reduction in period was already noted in Paper I and was previously described by Miyagoshi et al. (2004). It is also present (but not noted) in McLaughlin & Ofman (2008). Miyagoshi et al. (2004) found (empirically) that in a curved geometry, the period scales as $P \sim \rho_0^{0.33}$, whereas theoretically (in a straight cylinder), one would expect $P \sim \rho_0^{0.5}$. According to this scaling and using $\rho_0 = 10$, our "expected" period of 58 would reduce to 39, which is very close to the actual periods of the loop oscillations.

As stated above, the modification of the period and the possible scaling with the loop density ρ_0 has so far only been determined empirically and hence, more work is needed to investigate in which circumstances this reduction of the period will occur. Curvature and loop density seem to be important ingredients but the loop aspect ratio (ratio of the loop length and loop (minor) radius) probably also plays a role. Indeed, Van Doorsselaere et al. (2004) found that in the limit of thin coronal loops, the period of oscillation was only marginally affected by the curvature whereas the numerical simulations tend to model loops with relatively large aspect ratios, due to restriction in the number of grid points. The aspect ratio used in our numerical simulation is 0.035, compared to the observational value of 0.013–0.018 (corresponding to a loop radius of 2 ± 0.36 Mm) quoted by Nakariakov et al. (1999). For larger aspect ratios (i.e., a larger loop radius a), Edwin & Roberts (1983) expected a lower phase speed, and hence, a higher period. It is not clear why our simulations, as well as those by Miyagoshi et al. (2004) and McLaughlin & Ofman (2008), do not follow this trend. We speculate it is due to the combination of a relatively large aspect ratio and density contrast and the loop curvature but this aspect will be investigated in detail in a forthcoming paper.

The loop length and density ratio in our numerical simulations are based on the observational values also used in the magnetic field estimate by Nakariakov & Ofman (2001). Our numerical aspect ratio is somewhat larger than the observational value but only by a factor of 2–3. Hence, the modification of the period, and, as a consequence, the substantial overestimate of the magnetic field, could be a realistic problem for coronal seismology. In their original paper, Nakariakov & Ofman (2001) acknowledged that there is significant uncertainty in the value of the density ratio ρ_e/ρ_0 but pointed out that this does not affect the estimate of the magnetic field strength substantially. However, the modification of the period has a much larger effect on the determination of the magnetic field strength. Nakariakov & Ofman (2001) estimate the uncertainty in the value of B as

$$\delta B = \sqrt{(\delta L)^2 + (\delta P)^2 + (\delta\rho_0/2)^2}, \tag{3}$$

where δL, δP, and $\delta\rho_0$ are the relative errors in the loop length, oscillation period, and loop density, respectively. Using Equation (3) and $\delta L \approx 10\%$, $\delta P \approx 3\%$, and $\delta\rho_0 \approx 50\%$ (the values quoted by Nakariakov & Ofman (2001)), the error in the magnetic field strength would be of the order of 30%. However, if the uncertainty associated with the period (compared to the theoretically expected value) is of the order of $\delta P \approx 35\%$, the relative error in B increases to about 45% (which is in good agreement with the difference between the derived and theoretically predicted values in our numerical simulations).

5. CONCLUSIONS

In this study, we used three-dimensional numerical simulations of the interaction between a model coronal loop and an initial pressure pulse to examine impulsively excited coronal loop oscillations. Following the analysis of Nakariakov & Ofman (2001), we used the properties of the oscillations to derive an estimate for the magnetic field strength of our model coronal loop and found that this "coronal seismology" estimate differs substantially from the actual input magnetic field. The error is of the order of 50% and is caused by a reduction in the oscillation period compared to the theoretically predicted value. Although this modification of the period has been noted before in Paper I (as well as in Miyagoshi et al. 2004), a large-scale parameter study of the combined effects of curvature, loop density, and aspect ratio is needed to assess the impact and occurrence of this modified periodicity. We plan to verify the possible $P \sim \rho_0^{0.33}$ scaling both numerically and analytically in a forthcoming paper. It is likely that at least in some cases, the uncertainty associated with the value of the magnetic field strength derived from coronal seismology will be larger than previously expected. The results presented in this Letter are based on a very simple arcade model but clearly highlight the need to develop three-dimensional numerical simulations of realistic active region environments, to assess the reliability of plasma parameters derived from coronal seismology.

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