PWM-Switch Modeling of DC-DC Converters

Edwin van Dijk, Herman J. N. Spruijt, Dermot M. O'Sullivan, and J. Ben Klaassens

Abstract—**The introduced PWM-switch modeling method is a simple method for modeling pulse-width-modulated (PWM) dc-dc converters operating in the continuous conduction mode. The main advantage of this method is its versatility and simple implementation compared to other methods. The basic idea is the replacement of the switches in the converter by their time-averaged models. These switch models have been developed in such a way that the converter model provides the same results as the state-space-averaging technique but now including nonlinear effects. Simple rules for determination of the switch models are obtained. The resulting model is a time-averaged equivalent circuit model where all branch currents and node voltages correspond to their averaged values of the corresponding original currents and voltages. The model also includes parasitics, second-order effects and nonlinearities, and can be implemented in any circuit-oriented simulation tool. The same model is used for the simulation of the steady-state and the transient behavior.**

I. Introduction

NUMEROUS papers have been written on modeling PWM dc-dc converters, proposing a variety of methods for modeling switch-mode converters. An accepted method is the state-space-averaging method as introduced in [1]. A disadvantage of this method is the necessity to accomplish a number of calculations to obtain the averaged state equations from which an equivalent circuit model is derived.

Other methods replace a part of the converter by an equivalent circuit model to obtain the converter model. The equivalent circuit model of a part of the converter is already given and can be used directly. These methods are typically restricted to a few different topologies only.

It is desirable to have a single method which makes it possible to model all PWM dc-dc converters. The model should be found by a simple inspection of the converter circuit. It should be possible to implement this model in a general-purpose simulation tool like SPICE. It should also be possible to realize various analyzes such as steady-state, transient, and small-signal analysis, on one and the same model. This paper presents such a modeling method, called PWM-switch modeling, for converters operating in the continuous conduction mode.

II. PWM-Switch

Models for the PWM-switch are introduced in [2] and [3]. The method presented in [2] is used here and slightly adapted to preserve its general character and to expand it for use in more complex converters that do not have a PWM-switch as such.

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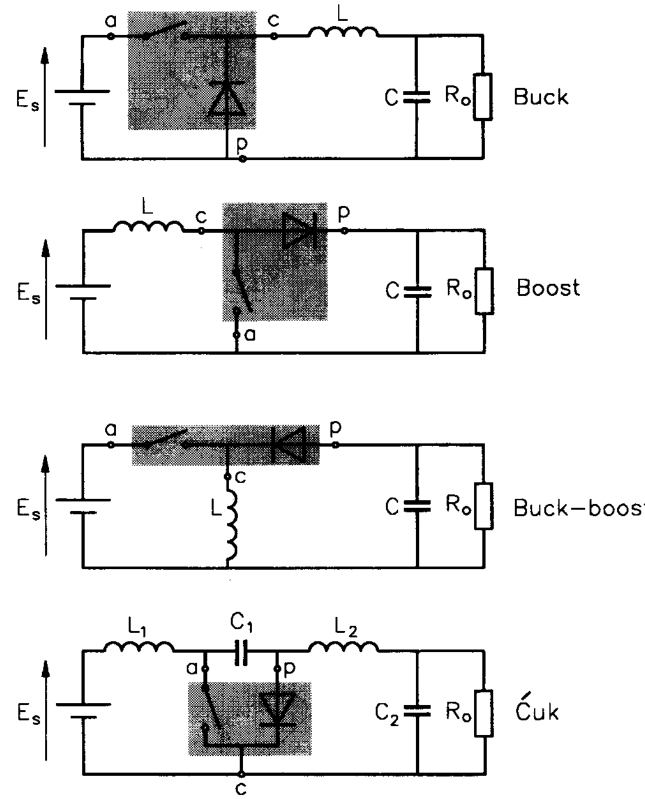


Fig. 1. Four basic converters with PWM-switches.

A. Definition of PWM-Switch Model

Fig. 1 shows four classical PWM converters: buck, boost, buck-boost, and Ćuk converter. These converters have one active switch and one passive switch performing the switching action in the converter. The active switch is directly controlled by an external control signal. It is usually implemented with a bipolar or a field-effect transistor. The passive switch is indirectly controlled by the state of the active switch and the circuit condition. It is usually implemented by a diode. As shown in Fig. 1, these two switches can be combined into one network with three terminals a, p, and c, which stands for active, passive, and common, respectively. This three-terminal network is called the PWM-switch. Since all other elements of the converters are supposed to be linear, the PWM-switch is the only nonlinear element and therefore responsible for the nonlinear behavior of the converters.

Fig. 2 shows the generic presentation of the PWM-switch operating in the continuous conduction mode with the terminal currents and voltages. The active and the passive switch operate like a single-pole double-throw switch. During the time interval dT_s the passive switch is off and the active switch

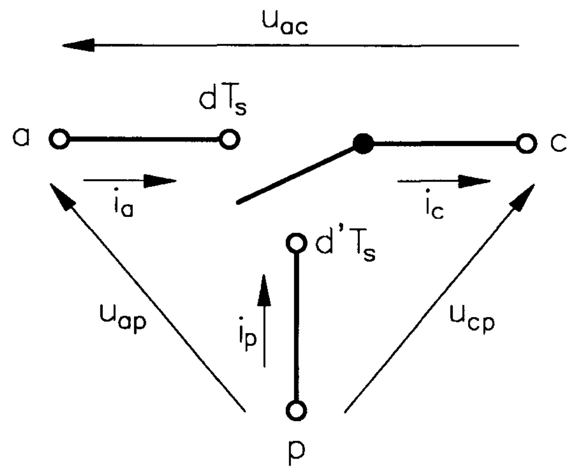


Fig. 2. PWM-switch.

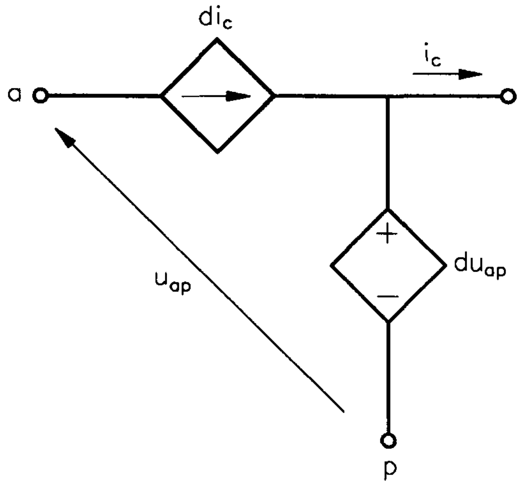


Fig. 3. Averaged model of the PWM-switch.

is on, and the active terminal is connected to the common terminal. During time interval $d'T_s$ the active switch is off and the passive switch is on, and the passive terminal is connected to the common terminal. T_s is the switching period of the active switch, d stands for the duty ratio as the ratio of the on-time of the active switch and the switching period and $d' = 1 - d$ for the continuous conduction mode.

The relations between the instantaneous terminal currents are found as

$$i_a(t) = \begin{cases} i_c(t) & \text{during } dT_s \\ 0 & \text{during } d'T_s \end{cases} \tag{1}$$

$$i_p(t) = \begin{cases} 0 & \text{during } dT_s \\ i_c(t) & \text{during } d'T_s. \end{cases} \tag{2}$$

The same applies for the instantaneous terminal voltages

$$u_{cp}(t) = \begin{cases} u_{ap}(t) & \text{during } dT_s \\ 0 & \text{during } d'T_s \end{cases} \tag{3}$$

$$u_{ac}(t) = \begin{cases} 0 & \text{during } dT_s \\ u_{ap}(t) & \text{during } d'T_s. \end{cases} \tag{4}$$

It is sufficient to inspect the averaged behavior of the PWM-switch to analyze the averaged behavior of a converter. The

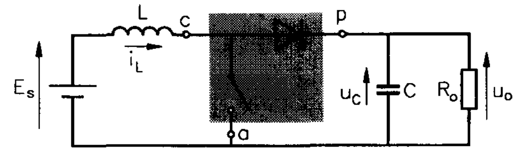


Fig. 4. The boost converter.

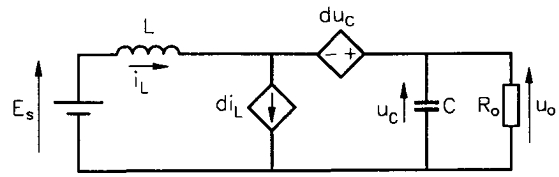


Fig. 5. Boost converter with PWM-switch replaced by the model shown in Fig. 3.

instantaneous terminal waveforms are time-averaged over one cycle T. The relations between the averaged terminal currents are found from (1) and (2) as

$$\langle i_a \rangle = d \langle i_c \rangle \tag{5}$$

$$\langle i_p \rangle = d' \langle i_c \rangle. \tag{6}$$

The relations between the averaged terminal voltages are found from (3) and (4)

$$\langle u_{cp} \rangle = d \langle u_{ap} \rangle \tag{7}$$

$$\langle u_{ac} \rangle = d' \langle u_{ap} \rangle. \tag{8}$$

With (5) through (8) it is simple to obtain a time-averaged model for the PWM-switch.

To simplify the presentation in the following, the symbols $\langle \cdot \rangle$ *indicating the time-averaged values are omitted.*

Fig. 3 shows a model applying a controlled voltage source and a controlled current source using (5) and (7). It is clear that (6) and (8) are also satisfied. The model for the PWM-switch always satisfies either (5) or (6) and (7) or (8). This means that one element should satisfy (5) or (6) and another element should satisfy (7) or (8). This explains the function of the controlled current source and the controlled voltage source. The application in the model of two voltage sources is not possible because it will leave the terminal currents undefined. The same is valid for the use of two current sources which will keep the voltages undefined.

B. Application of a PWM-Switch Model

To illustrate the use and to demonstrate the validity of the PWM-switch model, a boost converter as shown in Fig. 4 is used for the presentation. Fig. 5 presents the boost converter where the PWM-switch is replaced by the model shown in Fig. 3.

We recognize from Fig. 4 that

$$\begin{aligned} i_c &= -i_L \\ u_{ap} &= -u_C. \end{aligned} \tag{9}$$

This results in the expressions for the controlled sources in Fig. 5.

The equivalent circuit shown in Fig. 5 is a continuous, averaged circuit model of the boost converter as shown in Fig. 4. The action of the switches and the resulting ripple are removed from the circuit. However, the behavior of the circuit model is still nonlinear as demonstrated by the products of the duty ratio and a state variables in the expressions for the switch model.

The state equations are easily recovered from the circuit of Fig. 5

$$\begin{aligned} L\frac{di_L}{dt} &= -(1-d)u_C + E_s \\ C\frac{du_C}{dt} &= (1-d)i_L - \frac{u_C}{R_o}. \end{aligned} \tag{10}$$

Another method to derive the state equations is the well-known method of state-space-averaging [1]. This method averages the state equations valid for time interval dT_s with those for time interval $d'T_s$, resulting in the averaged state equations. The same equations are obtained with the PWM-switch model. Thus the use of the PWM-switch model results in a valid averaged equivalent circuit model of the boost converter.

III. PWM-Switch Model and ESR's

A boost converter with the inductor and capacitor equivalent series resistances (ESR) is shown in Fig. 6. Applying the PWM-switch model results in the equivalent circuit shown in Fig. 7 where the controlled current source is indicated by i_{cs} and the controlled voltage source by u_{cs}. The expressions for the controlled sources are:

$$\begin{aligned} i_{cs} &= di_L \\ u_{cs} &= du_o. \end{aligned} \tag{11}$$

Note that the voltage across the active and passive terminal of the PWM switch equals the output voltage u_o.

The state equations are determined from the equivalent circuit model of Fig. 7 and applying (11)

$$\begin{aligned} L\frac{di_L}{dt} &= -\left[R_L + (1-d)^2\frac{R_oR_C}{R_o+R_C}\right]i_L - (1-d) \\ &\quad \cdot \frac{R_o}{R_o+R_C}u_C + E_s \\ C\frac{du_C}{dt} &= (1-d)\frac{R_o}{R_o+R_C}i_L - \frac{u_C}{R_o+R_C}. \end{aligned} \tag{12}$$

To see whether the model is correct or not, the state equations are also calculated with *state-space-averaging* technique [1]. This produces the following set of state equations for the boost converter with ESR's

$$\begin{aligned} L\frac{di_L}{dt} &= -\left[R_L + (1-d)\frac{R_oR_C}{R_o+R_C}\right]i_L - (1-d) \\ &\quad \cdot \frac{R_o}{R_o+R_C}u_C + E_s \\ C\frac{du_C}{dt} &= (1-d)\frac{R_o}{R_o+R_C}i_L - \frac{u_C}{R_o+R_C}. \end{aligned} \tag{13}$$

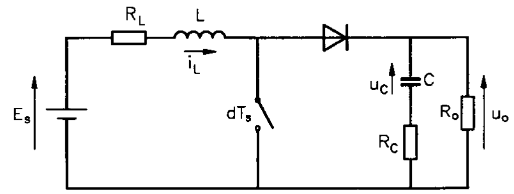


Fig. 6. Boost converter with ESR's.

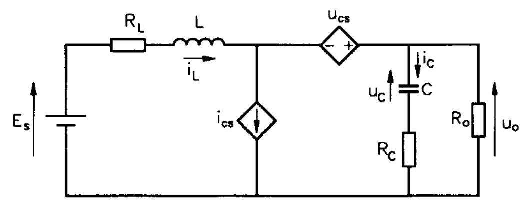


Fig. 7. Boost converter model with controlled sources and ESR's.

There is now a *difference* recognized between both sets of (12) and (13). The difference is found in the equation for the inductor current. In (12), the factor multiplied by i_L contains the term $(1-d)^2$, whereas in (13) it is $(1-d)$. It is clear that when the capacitor ESR R_C equals zero, both equations are the same.

An explanation for this discrepancy is provided in [2] and [3]. The capacitor C that is connected to the active and passive terminal of the PWM-switch, absorbs a pulsating current. When the ESR of this capacitor is zero, the instantaneous terminal voltage $u_{ap}(t) = u_C(t)$ is continuous and has only a voltage ripple, due to the switching process, which is neglected in the averaging process. When the capacitor has an ESR, the pulsating current through the ESR causes a small square wave voltage superimposed on the terminal voltage. The voltage now also depends on the value of the ESR.

We can calculate the necessary expressions for the controlled sources to obtain the same equations as obtained with State-Space-Averaging by deriving the state equation from the circuit shown in Fig. 7 and comparing them to the set of (13). This results in the expressions for i_{cs} and u_{cs}:

$$\begin{aligned} i_{cs} &= di_L \\ u_{cs} &= d\frac{R_o}{R_o+R_C}u_C. \end{aligned} \tag{14}$$

When (14) is used in the equivalent circuit model shown in Fig. 7, this model gives exactly the same results as state-space-averaging.

IV. Generalized PWM-Switch Model

It is desirable to find the expressions directly by inspection of the converter circuit. In the case of the boost converter with ESR's, we can see from (14), that i_{cs} equals the duty ratio times the current flowing through the active switch during time interval dT_s, during which the active switch is closed and the passive switch is open. This current equals the inductor current i_L. A similar approach is valid for u_{cs}. The voltage across the opened passive switch during time interval dT_s can be determined as a division of the capacitor voltage u_C across R_o and R_C. From (14) it follows that the expression for u_{cs} equals the duty ratio times this voltage across the passive switch.

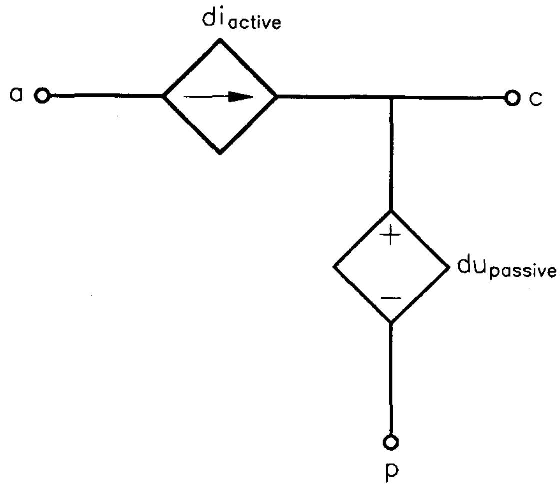


Fig. 8. Generalized PWM-switch model.

Fig. 8 shows the PWM-switch model of Fig. 3, where di_c and du_{ap} in the expressions for the controlled sources are replaced with di_{active} and du_{passive}, respectively.

The expressions for i_{active} and u_{passive} are determined from the converter topology by inspection. i_{active} equals the current through the closed active switch (from terminal a to c), during time interval dT_s, written as a function of the state variables and input variables. The state variables are the inductor currents and the capacitor voltages. The input variables can be voltage or current sources. Similarly, u_{passive} equals the voltage across the opened passive switch (across terminals c and p), during time interval dT_s, written as a function of the state variables and input variables.

The State-Space-Averaging method averages the entire converter. This is realized by presenting the state equations for both time intervals dT_s and $d'T_s$ and evaluating the averaged equations. The waveforms and duty ratio signal are divided in two parts: A constant (dc) term and a varying (ac) term. The cross-products of two ac terms are neglected assuming small-signal variations which results in linearized small-signal equations. An equivalent circuit model is obtained from these equations. The model is limited to small-signal variations from which only input and output variables are usually obtained.

The PWM-Switch modeling method averages the PWM-switches of the converter. This is realized by calculating the expressions for the controlled sources of the PWM-switch model from inspection of the circuit as explained before. The PWM-switch model simply replaces a PWM-switch in the circuit avoiding small-signal assumptions. Therefore, the resulting equivalent circuit model is valid for large-signal variations. All internal branch currents and node voltages are available from the circuit model. Small-signal transfer functions can be calculated from the model.

The method is only valid for the continuous conduction mode. When a mosfet is used for the passive switch, instead of a diode, and it is controlled by the duty ratio opposite to the active switch, the converter can operate in both directions and will always operate in the continuous conduction mode. The PWM-Switch modeling method can be applied to this type of bidirectional converters.

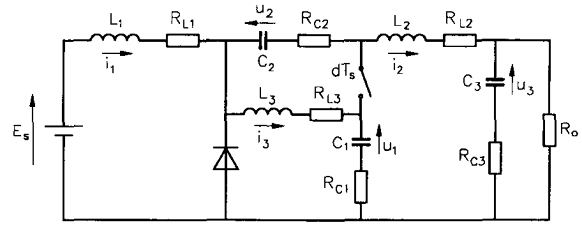


Fig. 9. Topology with separated PWM-switch [4].

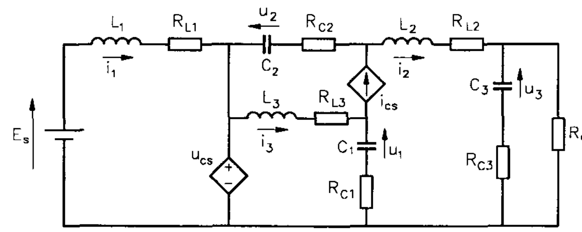


Fig. 10. Model of converter shown in Fig. 9 with active and passive switch replaced by a controlled current and a controlled voltage source respectively.

V. Separated or Multiple PWM-Switches

Each PWM dc-dc converter with a single PWM-switch can be modeled with the PWM-switch model. There are also PWM dc-dc converters that do not have a PWM-switch as such, or have more than one switch. It is possible to model these converters as well.

A. Converters with Separated PWM-Switch

A classification of PWM converter topologies with one active switch and one passive switch has been presented in [4]. The presented topologies are derived from five basic topologies leading to a number of new converter topologies. In a number of topologies the active and the passive switch are not connected to each other. The PWM-switch as such cannot easily be distinguished in these topologies. Such a topology is shown in Fig. 9. This topology is referred to as topology 5.1 in [4]. The inductor and capacitor ESRs are also included in Fig. 9. It is a buck type topology with an ideal voltage gain of $(2d-1)/d$, were $0.5 < d < 1$. It is clear that the two switches are separated from each other. These two switches will be referred to as a *separated PWM-switch.*

The examination of the boost converter in Fig. 6 and its model in Fig. 7 reveals that when the PWM-switch is replaced by its model, in fact, the active switch is replaced by the controlled current source and the passive switch is replaced by the controlled voltage source. This can also be accomplished with a *separated PWM-switch.* Fig. 10 presents the converter shown in Fig. 9 with the active and passive switch of the separated PWM-switch replaced by their respective controlled sources.

Comparison of the state equations obtained from Fig. 10, with the state equations that are calculated with the state-space-averaging technique, will result in the expressions for the controlled sources

$$\begin{aligned} i_{cs} &= d(i_2 + i_3 - i_1) \\ u_{cs} &= d[u_1 + R_{C1}(i_1 - i_2) + u_2 + R_{C2}(i_1 - i_3)]. \end{aligned} \quad (15)$$

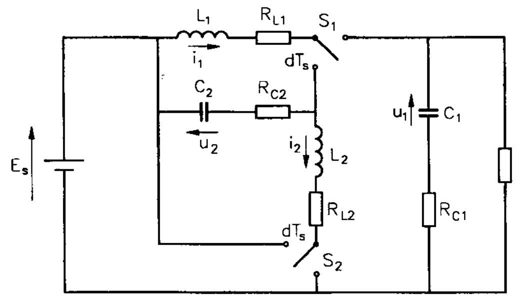


Fig. 11. Converter topology with two PWM-switches [5].

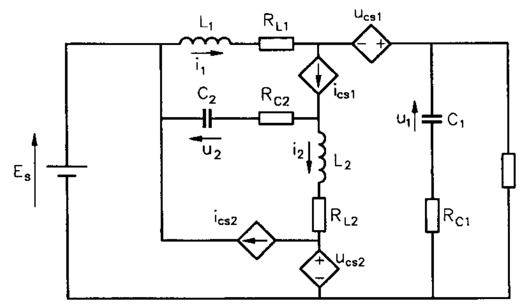


Fig. 12. Converter as shown in Fig. 11 with switches replaced by controlled sources.

Equation (15) is also obtained when the method, described in section "Generalized PWM-Switch Model," is used to determine the expressions for the controlled sources. The PWM-Switch modeling method cannot only be used for converters with a PWM-switch, but also for any PWM dc-dc converter that has one active/passive switch pair, not necessarily connected to each other and operating in the continuous conduction mode.

B. Converters with Multiple PWM-Switches

A classification of converters with one and two PWM-switches has been presented in [5]. A converter with two PWM-switches is illustrated in Fig. 11. The ESR's are also indicated. Both active switches are controlled with the same duty ratio signal. The ideal voltage gain equals $1 + d$.

As shown in Fig. 12 the PWM-switches are replaced by the model given in Fig. 8. Comparison of the state equations obtained from Fig. 12, with the state equations that are calculated with the state-space-averaging technique, results in the following set of expressions:

$$\begin{aligned} i_{cs1} &= di_1 \\ u_{cs1} &= d\left(u_2 - E_s + R_{C2}(i_2 - i_1) + \frac{R_o}{R_o + R_{C1}} u_1\right) \\ u_{cs2} &= dE_s \end{aligned} \tag{16}$$

To find the missing expression for i_{cs2}, we assume that the PWM-switch S2 is ideal, which means that its switch model has no losses. The following expression is derived from Fig. 12:

$$u_{cs2}(i_2 - i_{cs2}) + i_{cs2}(u_{cs2} - E_s) = 0. \tag{17}$$

Using the expression for u_{cs2} from (16) in (17) results in an expression for i_{cs2}:

$$i_{cs2} = di_2. \tag{18}$$

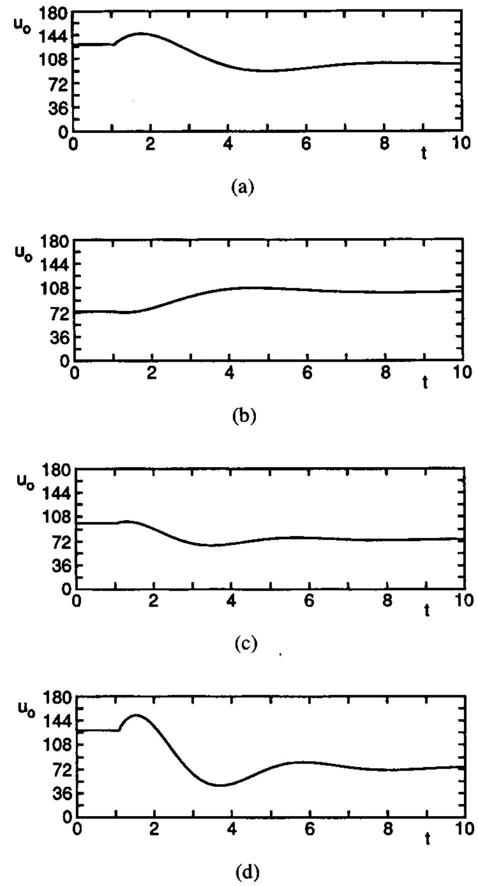


Fig. 13. Output voltage waveforms for different steps in the duty ratio: (a) $0.75 \Rightarrow 0.50$, (b) $0.25 \Rightarrow 0.50$, (c) $0.50 \Rightarrow 0.25$, (d) $0.75 \Rightarrow 0.25$.

The expressions of (16) and (18) for the switch model of both PWM-switches are also obtained, when the method for deriving the PWM switch model, as it is described in the section "Generalized PWM-Switch Model" in this paper, is used.

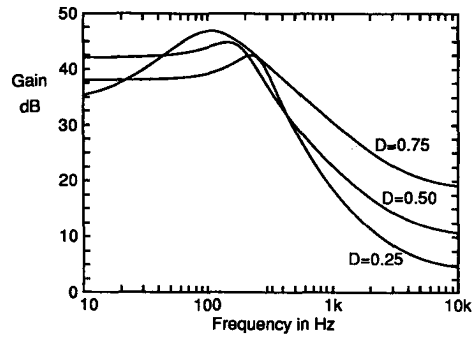
VI. Demonstration of PWM-Switch Modeling

A boost converter with inductor and capacitor ESR's, as illustrated in Fig. 6, is taken from [6] as an example. The circuit model shown in Fig. 7 using (14) is implemented in the simulation program Micro-Cap III. The data for the circuit is

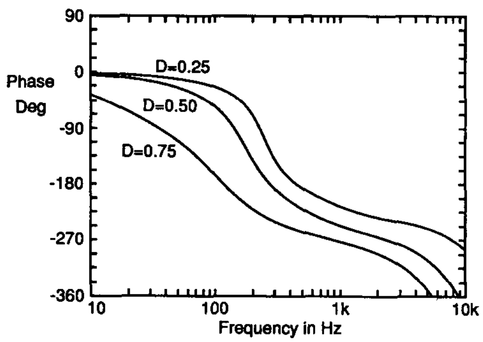
$$\begin{aligned} E_s &= 60 \text{ V} & R_o &= 60\ \Omega \\ L &= 6 \text{ mH} & R_L &= 3\ \Omega \\ C &= 1 \text{ mF} & R_C &= 1\ \Omega \\ T_s &= 100\ \mu\text{s}. \end{aligned}$$

Results in the time domain results are presented in Fig. 13. The output voltage waveforms are shown for different steps in the duty ratio applied at $t = 1$ ms. The effect of the pulse width modulator on the transients is very small.

The same model is used to predict the transfer functions in the frequency domain. The shown function in Fig. 14 is the small-signal transfer function from the duty ratio to the output voltage for different steady-state duty ratios. Here, the effect of the pulse width modulator creates an extra phase lag which increases linearly with the frequency.



(a)



(b)

Fig. 14. Control to output frequency responses for different duty ratios D: (a) gain, (b) phase.

The waveforms and functions shown in Figs. 13 and 14 are identical to the experimental results presented in [6].

VII. Conclusions

The PWM-switch modeling method is a simple method to model PWM dc-dc converters operating in the continuous conduction mode. The PWM-switch model has been developed such that the model gives the same results as the state-space-averaging technique [1]. This resulted in a general method for deriving the PWM-switch model. The state-space-averaging method averages the complete converter while the PWM-switch modeling method only averages the switches. The averaged models of the switches are simply obtained by inspection of the converter circuit. It is a simple and fast method to obtain the model.

Another advantage of the PWM-switch modeling method is that the model corresponds directly to the original converter circuit. Only the switches are replaced by their models. All branch currents and node voltages of the converter are directly available from the model as averaged quantities.

The model developed with the PWM-switch modeling method can be implemented in any circuit-oriented simulation tool. It is a nonlinear, large-signal, averaged model. No small-signal assumptions have been made. The nonlinearity due to the switching process in a converter is included in the model. Other nonlinearities, such as the duty ratio limitation, can easily be modeled. Parasitic elements and other second-order effects can also be added to the model. The same model can also be used for frequency domain simulations. All important transfer functions such as input/output impedances, susceptibility, or loop gain can easily be predicted.

The PWM-switch modeling method has the same limitation as state-space-averaging method. It is assumed that the switching period of the switches is small compared to the time constants of the converter, which is usually the case.

The PWM-switch modeling method is only valid for the continuous conduction mode. It can be applied to bidirectional converters which always operate in the continuous conduction mode.

An interesting feature is the possibility that the method presented can be used to model other types of converters than PWM dc-dc converters. Equivalent circuit models for resonant switches in quasi-resonant converters are presented in [7]. The equivalent circuits for the resonant switches show similarities to those for the PWM-switches. This could imply that it is possible to join them into one general model.

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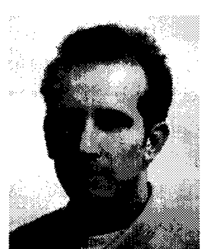

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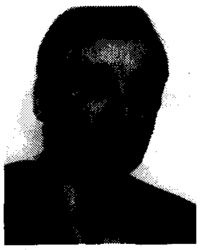
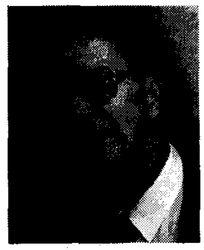
He has worked at the European Space Agency in Noordwijk, The Netherlands, since 1966, the first six years as a satellite project engineer dealing with the HEOS-1, HEOS-2, GEOS-1, and GEOS-2 scientific satellites, and the last 23 years as a power subsystem engineer in the Power & Energy conversion division. He was also responsible for the execution of several research and development contracts with the European industry. He is the holder of two ESA patents, one on the battery charge termination using the TDT (temperature derivative termination), the other on 16 new dc/dc PWM converter topologies. He is presently involved in the support of the ESA moon program.



Dermot M. O'Sullivan was born on May 13, 1943, in Dublin, Ireland. He received the B.E. degree in electrical engineering from the University College Dublin in 1965.

In 1965, he joined Hawker Siddeley Dynamics as a Power System Engineer. From 1969 to the present, he has been a member of the European Space Agency in the Technical Center (ESTEC) of Noordwijk, The Netherlands. He is presently Head of the Power and Energy Conversion Division and has applied for several patents in this area.



J. Ben Klaassens was born in Assen, The Netherlands, in 1942. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from the Delft University of Technology, Delft, The Netherlands.

He is currently an associate professor at the Delft University of Technology. His work has been concerned with inverter circuits, pulse width modulation, and the control of electrical machinery. His research work and professional publications are in the area of converter systems with high internal pulse frequencies for sub-megawatt power levels employing thyristors, power transistors, and IGBT's. His current interest is in the area of control of converters and electrical drives. He has published a variety of papers on series-resonant converters for low and high power applications. He has designed and built prototypes of the early dc-dc to the recent ac-ac series-resonant converters for a wide variety of applications such as electric motors and generators, communication power supplies, radar signal generators, arc welders, and space applications.