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QCD TESTS OF: G(1.6) = GLUEBALL

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ABSTRACT

QCD sum rules, low-energy theorems and current algebra for the pseudo-scalar nonet are used to support the hypothesis that the recently observed G(1.6 GeV) meson is a (relatively pure) gluonium state.

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1. - INTRODUCTION

There is increasing evidence for QCD being the correct theory of strong interactions at least up to the Fermi scale.

Although QCD describes simply short-distance phenomena, because of asymptotic freedom, the understanding of long-distance physics needs at present approximation schemes whose validity is not completely under control.

Indeed the most fundamental of such schemes, lattice QCD, is affected by uncertainties coming from limited statistics, from finite size effects and from the neglect of dynamical fermions. The latter limitation is particularly worrisome for the kind of problems we wish to address here: the possible existence of relatively "pure" gluonium states. We are thus forced to appeal to more phenomenological tools.

Among these, two have attracted considerable interest. The first one is the approach based on current algebra and effective Lagrangians: by incorporating the known symmetries of the quantum theory - including anomalies - and those assumed for the vacuum, the effective Lagrangians describe the dynamics of the low-energy states of the theory. Predictivity can be further enhanced by combining current algebra with non-perturbative expansions, such as the $1/N_{colour}$ expansion, or the quark loop ($N_{flavour}$) expansion.

The second method is the one based on QCD spectral sum rules. Here one relates the QCD expression for a given gauge invariant correlation function, evaluated at some intermediate energy, to its absorptive part parametrized in terms of experimental data (spectrum and couplings).

In this paper we shall combine these two methods in order to investigate and constrain the spectrum, mixings and couplings of 0^{++} gluonia. We shall argue that, besides the known states, a further higher mass 0^{++} meson, strongly coupled to the gluonic operator $\alpha_s G^2$ is needed in order to satisfy simultaneously two spectral sum rules. We shall then test the hypothesis that such a meson is the G(1.6 GeV) state recently observed at the GAMS spectrometer [1] in the reaction $\pi^- p \to G + n$, with decay modes $\eta\eta$, $\eta\eta'$ (but <u>not</u> $\pi\pi$ or $K\bar{K}$) and, more recently [2], also in $4\pi^0$.

2. - THE TWO SUM RULES

The two sum rules referred to above are both associated with the correlation function:

$$\psi(q^2) \equiv 16i\int d^4x\, e^{iqx} \langle 0|T[\theta_\mu^{\ \mu}(x), \theta_\nu^{\ \nu}(0)]|0\rangle \tag{1}$$

where $\theta_{\mu\nu}$ is the improved QCD energy-momentum tensor (neglecting heavy quarks) whose anomalous trace reads, in standard notation:

$$\begin{aligned} \theta_\mu^{\ \mu} &= \frac{1}{4}\beta(\alpha_s)\, G^2 + (1+\gamma_m(\alpha_s)) \sum_{i=u,d,s} m_i \bar{\psi}_i \psi_i \\ \beta &= \beta_1 \frac{\alpha_s}{\pi} + O(\alpha_s^2) \quad ; \quad \beta_1 = -\frac{11}{2} + n_F/3 \end{aligned} \tag{2}$$

From its expected asymptotic behaviour we can write a twice-subtracted dispersion relation for ψ:

$$\psi(q^2) = \psi(0) + q^4/\pi \int_0^\infty \frac{dt \ \mathrm{Im}\,\psi(t)}{t^2(t-q^2-i\epsilon)} + \psi'(0)\, q^2 \tag{3}$$

By standard techniques, one can derive from (3) and from its known QCD expression [3] two sum rules. The first [4], to be called the unsubtracted sum rule (USR), is independent of $\psi(0)$. After renormalization group improvement it reads:

$$\frac{1}{\pi}\int_0^\infty dt\, e^{-\tau t}\, \mathrm{Im}\,\psi(t) \simeq$$

$$\simeq \left(\frac{4}{\pi^2}\right)\beta_1^2 \left(\frac{\bar{\alpha}_s}{\pi}\right)\left\{\left(\frac{\bar{\alpha}_s}{\pi}\right) + \frac{1}{2}\langle g^3G^3\rangle \tau^3 + \frac{9\pi^2}{8}\langle \alpha_s G^2\rangle^2 \tau^4\right\} \tag{4a}$$

where

$$\begin{aligned} \bar{\alpha}_s/\pi &\equiv \left(\beta_1 \log \sqrt{\tau}\Lambda\right)^{-1} ; \qquad \beta_1 = -9/2 \quad (n_F = 3) \\ \langle g^3G^3\rangle &\equiv \langle g^3 f_{abc}\, G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}\rangle \end{aligned} \tag{4b}$$

As usual, the left-hand side of the sum rule is supposed to be saturated by low-energy resonances and, above a certain t_c, by a continuum identified with the discontinuity of the QCD diagrams.

The second, subtracted sum rule [3] (SSR) depends crucially upon $\psi(0)$ and reads:

$$\frac{1}{\pi}\int_0^{\infty}\frac{dt}{t}\,e^{-\tau t}\,\mathrm{Im}\,\psi(t) =$$

$$= \frac{2}{\pi^2}\beta_1^2\left(\frac{\bar{\alpha}_s}{\pi}\right) + \psi(0) - \frac{4}{\pi}\beta_1^2\left(\frac{\bar{\alpha}_s}{\pi}\right)\left[\langle\alpha_s G^2\rangle + \frac{\tau}{2\pi}\langle g^3 G^3\rangle\right] \tag{5}$$

We shall discuss the two sum rules in succession.

a. The USR (4) stabilizes at large values of τ ($\tau \simeq 0.8$ GeV^{-2}) and is, consequently, dominated by the low-energy spectrum of 0^{++} mesons. Saturating the USR with the σ-contribution plus a QCD continuum above a threshold t_c, one obtains in a narrow width approximation:

$$2 f_\sigma^2 M_\sigma^4 e^{-\tau M_\sigma^2} + \frac{4}{\pi^2}\beta_1^2\left(\frac{\bar{\alpha}_s}{\pi}\right)^2 \tau^{-3} e^{-t_c\tau}\left(1 + t_c\tau + (t_c\tau)^2/2\right) \simeq$$

$$\simeq \frac{4}{\pi^2}\beta_1^2\left(\frac{\bar{\alpha}_s}{\pi}\right)\tau^{-3}\left\{\frac{\bar{\alpha}_s}{\pi} + \frac{1}{2}\langle g^3 G^3\rangle\tau^3 + \frac{9\pi^2}{8}\langle\alpha_s G^2\rangle^2\tau^4\right\} \tag{6}$$

where fermionic condensates can be neglected. Using the standard condensate values [5]:

$$\langle\alpha_s G^2\rangle \simeq 0.04\ \mathrm{GeV}^4$$

and [5a,6]:

$$\langle g^3 G^3\rangle \simeq (1\ \mathrm{GeV}^2)\langle\alpha_s G^2\rangle \tag{7}$$

$\Lambda \simeq 0.18$ GeV and $M_\sigma \simeq 1$ GeV*) as input, we can study the stability of f_σ given by (6) with respect to τ (Fig. 1a). We find a reasonable minimum at $\tau \simeq 0.8$ GeV^{-2} for $\sqrt{t_c} \geq 2.1$ GeV. In Fig. 1b, we consider the effects of $\sqrt{t_c}$ at the optimal value of τ obtained in Fig. 1a for two values of Λ. The stability of f_σ versus the changes of t_c is reached for $\sqrt{t_c} \geq 2.9$ GeV. For $M_\sigma \simeq 1$ GeV and $\Lambda \simeq 100{\sim}180$ MeV, one thus obtains:

$$f_\sigma \simeq (546 \div 677)\ \mathrm{MeV} \tag{8a}$$

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*) In the case where one has multistates with degenerate masses, one should interpret M_σ as an effective resonance.

with the definition

$$\langle 0 | 4\,\theta^{\mu}_{\mu} | \sigma \rangle = \sqrt{2}\, f_\sigma M_\sigma^2 \tag{8b}$$

The result in (8) is consistent with the one obtained from moment ratios analysis where M_σ and f_σ have been left as free parameters [4]. If, instead, one allows a deviation by a factor two [7] from the standard value of the condensates in (7), one would obtain:

$$f_\sigma \simeq (769 \div 931)\,\mathrm{Mev.} \tag{8c}$$

as shown in Fig. 1c.

b. Turning now to the SSR, Eq. (5), let us first recall that the subtraction constant $\psi(0)$ is actually known in terms of the gluon condensate via the low-energy theorem [3]

$$\psi(0) = -16\beta_1 \frac{\alpha_s}{\pi} \langle G^2 \rangle \tag{9}$$

Thus, no new input parameter is needed and the SSR reads:

$$2 \sum_{i=\sigma,\dots} f_i^2 M_i^2 e^{-\tau M_i^2} \simeq \frac{2}{\pi^2} \frac{\tau^{-2}}{\log^2 \sqrt{\tau}\Lambda} \left(1 - e^{-\tau t_c}(1+\tau t_c)\right) + \\ + \psi(0) - \frac{4\beta_1^2}{\pi^2} \bar{\alpha}_s \left(\langle \alpha_s G^2 \rangle + \frac{\tau}{2\pi} \langle g^3 G^3 \rangle \right) \tag{10}$$

A saturation of Eq. (10) by just the σ(1 GeV) gives results which are contradictory with those of the USR. We thus conclude that at least one more 0^{++} meson is needed and we shall test hereafter the hypothesis that this is just the recently observed G(1.6) state.

The sum rule (10) shows a stability region at a value of τ ($\tau \sim 0.3\ \mathrm{GeV}^{-2}$) much smaller than the one at which the USR stabilizes, due to the important contribution of $\psi(0)$ relative to the unit operator. This is consistent with the idea that the G contributes appreciably only*) to the SSR.

Saturating the SSR by the σ and the G, and using standard condensate values, one obtains (Fig. 2a,b):

$$f_G \simeq (478 \div 533)\ \mathrm{MeV} \tag{11a}$$

whilst taking twice the standard values [7]:

$$f_G \simeq (554 \div 656)\ \mathrm{MeV} \tag{11b}$$

One can double-check at this point that the G-contribution to the SSR is about the same as the σ-contribution while it is negligible in the USR. Adding also the $f_0(1.3)$ contribution, we deduce by matching the continuum of the USR with the f_0 peak:

$$f_{f_0} \lesssim (139 \sim 224)\ \mathrm{MeV} \tag{11c}$$

which affects only slightly the result in (11).

3. - σ COUPLING TO PAIRS OF GOLDSTONE BOSONS

At this point we use vertex sum rules to obtain further constraints and/or predictions. Consider the vertex:

$$V(q^2) = \langle \pi_1 | \theta^\mu_\mu | \pi_2 \rangle \ , \quad q = p_1 - p_2 \tag{12a}$$

where

$$V(0) = O(m_\pi^2) \longrightarrow 0 \tag{12b}$$

In the chiral limit ($m_\pi^2 \simeq 0$), we have:

$$V(q^2) = q^2 \int_{4m_\pi^2}^{\infty} \frac{dt}{t - q^2 - i\epsilon} \frac{1}{\pi} \mathrm{Im}\, V(t)/t \tag{12c}$$

Using the fact that V'(0) = 1 [3], one obtains:

$$\frac{1}{4} \sum_{i=\sigma, G} g_{i\pi\pi} \sqrt{2}\, f_i / M_i^2 = 1 \tag{13}$$

Since the G coupling to $\pi\pi$ is bound from above by the GAMS data [1,2], one finds that (13) is dominated by the σ, with the result:

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*) The dominance of the σ in the USR and the appreciable contribution of the G in the SSR might explain the sparsity of previous results obtained within a one-resonance + QCD continuum parametrization of the sum rules [3,4,8].

$$g_{\sigma\pi^+\pi^-} \simeq (4.6 \pm 0.5)\ \mathrm{GeV} \tag{14a}$$

for the standard values of the condensates while:

$$g_{\sigma\pi^+\pi^-} \simeq (3.3 \pm 0.3)\ \mathrm{GeV} \tag{14b}$$

if the condensates are twice the standard values. The inclusion of the $f_0(1.3)$ contribution with the sign required by $V(0) = 0$ increases slightly the result in (14) by an amount less than 16%. Considering this effect as another source of errors, we deduce for the standard (ST) and non-standard (NST) values of the condensates:

$$\Gamma(\sigma \to \pi^+\pi^- + 2\pi^0) \simeq \begin{cases} (798 \pm 429)\ \mathrm{MeV} & : ST \\ (410 \pm 220)\ \mathrm{MeV} & : NST \end{cases} \cdot \left(\frac{M_\sigma}{1\ \mathrm{GeV}}\right)^3 \tag{15}$$

We have repeated the derivation of f_σ in (8) by taking into account finite width corrections. This leads to an increase of f_σ which is compensated by the propagator effects in the estimate of $g_{\sigma\pi\pi}$, i.e., the result in (14) remains almost unchanged.

It is interesting to compare this prediction with the known $\pi\pi$-scattering [9] and J/ψ-data [10]. Indeed the $\sigma(900)$ with a width of about 700 MeV and the $f_0(975)$ with a width of 40 MeV are good gluonia candidates. Both are revealed from a coupled channel analysis of the $I = 0$ s-wave $\pi\pi$ and $K\bar{K}$ final states and are coupled almost universally to $\pi\pi$ and $K\bar{K}$ pairs. In addition, the latter is produced in $J/\psi \to \phi\pi\pi$, $\phi K\bar{K}$, $\omega\pi\pi$ and $\gamma\pi\pi$. Our result in (15) supports the presence of gluons inside the $f_0(975)$ and $\sigma(900)$ wave functions. Their relative amounts might only be fixed after a complete mixing scheme analysis (for some attempts see, e.g., Ref. [11]).

4. - CURRENT ALGEBRA CONSTRAINTS

In order to compute couplings of G to η and η', we use the approach of Refs. [12] and [13a]. Consider the three-point function:

$$\tilde{V}_{\mu\nu}(q_1, q_2) \equiv i \int d^4x_1 \, d^4x_2 \, e^{i(q_1 x_1 + q_2 x_2)} \langle T[Q(x_1) Q(x_2) \theta_{\mu\nu}(0)] \rangle \quad (16)$$

where $\theta_{\mu\nu}$ is the energy-momentum tensor of QCD with three light quarks and

$$Q(x) = \frac{\alpha_s}{16\pi} \epsilon_{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \quad (17)$$

is the topological charge density. We recall that, from the large N_c (or the quenched) solution to the U(1) problem [12,13a], one finds:

$$\Gamma_2(q) \equiv i \int d^4x \, e^{iqx} \langle T[Q(x) Q(0)] \rangle \xrightarrow[\text{LOOPS}]{\text{NO QUARK}} \Gamma_2^{YM}(q)$$

$$\Gamma_2^{YM}(q) \underset{q \ll \Lambda}{\sim} \Gamma_2^{YM}(0) \simeq (180\ \mathrm{MeV})^4 \quad (18a)$$

while, with quark loops present [13b]:

$$\Gamma_2(q) \simeq \Gamma_2^{YM} \left(1 + \sum_{i=1}^{3} \frac{f_i(m_j)}{p^2 - m_i^2} \right) \quad (18b)$$

where m_i are the physical pseudoscalar nonet masses and the known (but complicated) constants f_i insure the vanishing of $\Gamma_2(0)$ whenever a quark mass goes to zero.

A consistent formula for $\tilde{V}_{\mu\nu}$, which satisfies Ward identities as well as large N_c, low energy and small quark-mass limits turns out to be:

$$\tilde{V}_{\mu\nu}(q_1, q_2) = \Gamma_2^{YM} \left[\frac{1}{2} g_{\mu\nu} (q_1 + q_2)^2 - q_{1\mu} q_{2\nu} - q_{2\mu} q_{1\nu} \right] \cdot \sum_i \frac{f_i(m_j)}{(q_1^2 - m_i^2)(q_2^2 - m_i^2)} + \frac{1}{2} g_{\mu\nu} \left(\Gamma_2(q_1) + \Gamma_2(q_2) \right) \quad (19)$$

From this formula it is easy to argue that

$$\langle \eta_1 | \theta^\mu_\mu |_{N_f=3} | \eta_1 \rangle \simeq \frac{9}{11} \langle \eta_1 | \theta^\mu_\mu |_{Y.M.} | \eta_1 \rangle = \frac{9 \cdot 12}{11} f_\pi^{-2} \Gamma_2^{YM} \simeq 1.15\ \mathrm{GeV}^2 \quad (20)$$

Note that we have taken seriously the factor 9/11 between $\beta_1^{N_f=3}$ and $\beta_1^{N_f=0}$, although this is a higher order effect (both in $1/N_c$ and in N_f).

Saturating a dispersion relation in $q^2 = (q_1+q_2)^2$ with σ, $f_0(1.3)$ and G yields:

$$\frac{1}{4}\sum_{i=\sigma,G} g_{i\eta_1\eta_1}\sqrt{2}\, f_i = 1.15 \; Gev^2 \tag{21}$$

Picking up the singlet component in the physical η, η' via $\eta \sim \sin\theta_p \eta_1 + \ldots$, $\eta' \sim \cos\theta_p \eta_1 + \ldots$ ($\theta_p \approx -(18\pm2)^0$ being the pseudoscalar mixing angle [14] estimated from $\eta,\eta' \to 2\gamma$), we find:

$$\sum_{i=\sigma,f_0,G} g_{i\eta\eta'} f_i = \frac{8}{\sqrt{2}} M^2_{\eta_1} \sin\theta_p \to \frac{4}{\sqrt{2}}(1.15)\sin\theta_p \; Gev^2 \tag{22}$$

The $\sigma\eta\eta'$ and $f_0\eta'\eta$ couplings are limited by the observed $\pi\pi$ spectrum in $\eta' \to \eta\pi\pi$ which is known to go mainly through $a_0\pi$. Assuming that the 25% allowed by the experimental errors are due either to σ or f_0 exchanges, one deduces using a Gell-Mann, Sharp, Wagner-type model:

$$\begin{aligned} g_{\sigma\eta\eta'} &\leq 0.75 \; Gev. \\ g_{f_0\eta\eta'} &\leq 2 \; Gev. \end{aligned} \tag{23}$$

This allows us to obtain for the previous two values of the gluon condensates:

$$\begin{aligned} g_{G\eta\eta'} &\leq (3.6 \pm 0.3)\, Gev \quad : (ST) \\ g_{G\eta\eta'} &\leq (3.3 \pm 0.3)\, Gev \quad : (NST) \end{aligned} \tag{24a}$$

which becomes in terms of the width:

$$\begin{aligned} \Gamma_{G\eta'\eta}(Mev) &\leq 52 \pm 9 \quad : (ST) \\ \Gamma_{G\eta'\eta}(Mev) &\leq 44 \pm 8 \quad : (NST) \end{aligned} \tag{24b}$$

The result in (24) is in good agreement with the GAMS data [1,2]. The scheme is also known to predict:

$$r \equiv \Gamma_{G\eta\eta}/\Gamma_{G\eta\eta'} \simeq 0.26 \; ; \quad g_{G\eta\eta} \simeq \sin\theta_P \; g_{G\eta\eta'} \tag{25}$$

compared with the data [1,2] $r \approx 0.34 \pm 0.13$ and with the one in Ref. [15]. In other words, what we have found appears to be consistent with interpreting the G-meson as an almost "pure" gluonium state. The coupling of G to pseudoscalar pairs could just reflect the amount of glue in each pseudoscalar and will be maximal for the η', minimal for the pion and intermediate for the η which has a small glue component. Within our analysis, one could also interpret the tendency of the G to decay copiously into $4\pi^0$ as due to exchange of virtual σ-like pairs. That might be checked from a reconstruction of the $2\pi^0$ invariant mass.

We are thus led to conclude that, while both the σ and the G mesons couple to glue, only the latter is, to a good approximation, a pure glue state, while the σ has a large $q\bar{q}$ admixture. This fact can be related to the improving validity of the OZI rule with increasing q^2.

5. - COUPLINGS TO PHOTONS AND TO HEAVY QUARKONIA

Let us now study the $\gamma\gamma$ widths of these gluonium candidates and their productions in J/ψ and Υ radiative decays. As usual, we start from the Euler-Heisenberg Lagrangian which controls the low-energy behaviour of the $\gamma\gamma gg$ box diagrams in Fig. 3:

$$\mathcal{L}_{\gamma g} = \frac{\alpha\,\alpha_s Q_H^2}{180 M_H^4}\Big\{ 28\, F_{\mu\nu} F_{\nu\lambda} G_{\lambda\sigma} G_{\sigma\mu} + 14\, F_{\mu\nu} G_{\nu\lambda} F_{\lambda\sigma} G_{\sigma\mu} - 10\, F_{\mu\nu} G_{\mu\nu} F_{\alpha\beta} G_{\alpha\beta} - 5\, F_{\mu\nu} F_{\mu\nu} G_{\alpha\beta} G_{\alpha\beta} \Big\} \tag{26}$$

where M_H is the mass of the heavy quark ($M_c \simeq 1.46$ GeV; $M_b \simeq 4.6$ GeV for $\Lambda \simeq 0.1$-0.18 GeV) [16]. The $J/\psi \to \gamma X$ ($X \equiv \sigma, f_0, G$) process can be estimated using standard dispersion relation techniques [3] where a spectral function is saturated by the J/ψ plus a continuum. The glue part of the amplitude can be converted into a physical matrix element in terms of $\langle 0|\alpha_s G^2|X\rangle$ [3] which is known from our previous analysis. If the continuum contribution is small, as argued in Ref. [3], we get:

$$\Gamma(J/\psi \to \gamma X) \simeq \frac{\alpha^3 \pi}{\beta_1^2\, 656100} \left(\frac{M_{J/\psi}}{M_H}\right)^4 \left(\frac{M_X}{M_H}\right)^4 \frac{f_X^2 \left(1 - M_X^2/M_{J/\psi}^2\right)^3}{\Gamma(J/\psi \to e^+e^-)} \tag{27a}$$

where we take $-\beta_1 = 7/2$ for six flavours. This leads to the rough estimates:

$$\begin{aligned} &B(J/\psi \to \gamma\sigma)\cdot B(\sigma \to \pi\pi) \simeq (6 \div 16)\,10^{-4} \\ &B(J/\psi \to \gamma f_0)\; B(f_0 \to \pi\pi) \simeq (0.7 \div 1.9)\,10^{-4} \\ &B(J/\psi \to \gamma G)\left[B(G \to \eta'\eta) + B(G \to 4\pi^0)\right] \simeq (13 \div 25)\,10^{-4} \end{aligned} \tag{27b}$$

These branching ratios can be compared with the observed $B(J/\psi \to \eta'\gamma)$ and $B(J/\psi \to f_2\gamma)$ ones which are $4\cdot10^{-3}$ and $1.6\cdot10^{-3}$. The extension of this analysis to the Υ is straightforward. One gets:

$$\begin{aligned} &B(\Upsilon \to \gamma\sigma)\;\; B(\sigma \to \pi\pi) \simeq (2 - 6)\,10^{-6} \\ &B(\Upsilon \to \gamma f_0)\;\; B(f_0 \to \pi\pi) \simeq (0.5 \sim 1)\,10^{-6} \\ &B(\Upsilon \to \gamma G)\;\; B(G \to \eta\eta' + 4\pi^0) \simeq (9.5 - 17.5)\,10^{-6} \end{aligned} \tag{28}$$

The two-photon widths of these gluonia can be estimated from the identification of the scalar $\gamma\gamma$ Lagrangian:

$$\mathcal{L}_{\sigma\gamma\gamma} = g_{\sigma\gamma\gamma}\,\sigma(x)\,F^{(1)}_{\mu\nu}F^{(2)}_{\mu\nu} \tag{29}$$

with the previous one in (26) where light quarks have to be added. This gives:

$$g_{\sigma\gamma\gamma} \simeq \frac{\alpha}{60}\sqrt{2}\, f_\sigma M_\sigma^2 \left(\frac{\pi}{-\beta_1}\right) \sum_{i=u,d,s} Q_i^2/m_i^4 \tag{30a}$$

where Q_i is the quark charge and m_i the so-called constituent light quark masses which we take to be:

$$m_u \simeq m_d \simeq M_\rho/2 \quad , \quad m_s \simeq M_\phi/2 \tag{30b}$$

Therefore, we obtain

$$\begin{aligned} &\Gamma(\sigma \to \gamma\gamma) \simeq (0.03 - 0.08)\,\mathrm{Kev} \\ &\Gamma(f_0 \to \gamma\gamma) \simeq (0.01 \sim 0.03)\,\mathrm{Kev} \\ &\Gamma(G \to \gamma\gamma) \simeq (0.3 \sim 0.6)\,\mathrm{Kev} \end{aligned} \tag{31}$$

Now, let us check the approximate validity of the results in (31). We can alternatively estimate the $\gamma\gamma$-width from the trace anomaly (Fig. 3b):

$$\langle 0 | \frac{1}{4}\beta(\alpha_s) G^2 | \gamma_1 \gamma_2 \rangle = - \langle 0 | \frac{\alpha R}{3\pi} F_1^{\mu\nu} F_2^{\mu\nu} | \gamma_1 \gamma_2 \rangle \tag{32}$$

where $F^{\mu\nu}$ is the photon field strength and $R \equiv 3\Sigma Q_i^2$. The fact that the left-hand side of (32) is $O(k^4)$ whilst its right-hand side is $O(k^2)$ implies the sum rule [17,3]:

$$\langle 0 | \theta_\mu^\mu | \gamma_1 \gamma_2 \rangle = \langle 0 | \frac{1}{4}\beta(\alpha_s) G^2 + \frac{\alpha R}{3\pi} F_1^{\mu\nu} F_2^{\mu\nu} | \gamma_1 \gamma_2 \rangle \tag{33}$$

One can deduce from (33) the couplings:

$$\frac{\sqrt{2}}{4} \sum_{i=\sigma, G} f_i \, g_{i\gamma\gamma} \simeq \frac{\alpha R}{3\pi} \tag{34}$$

Using the σ and f_0 couplings from Eq. (30), we deduce from (34):

$$\Gamma_{G \to \gamma\gamma} \simeq (1-6)\,\mathrm{keV} \tag{35}$$

which is too high compared to (31). This appreciable G-contribution to the anomaly constraint in (34) might explain why the authors in Ref. [18] obtain an unexpectedly high $\sigma \to \gamma\gamma$ width by using a σ-dominance in (34) or might indicate that the anomaly approach in (34) is a very rough approximation for the estimate of the $\gamma\gamma$ gluonia width. However, we expect that the approximate uses of the single mesons dominance in order to get Eqs. (31) and (35) is lesser accurate for higher mass mesons like the G(1.6) due to the nearby presence of the radial excitations in this energy region. Therefore, Eqs. (31) and (35) might give an overestimate of the real value of the $\gamma\gamma$ width.

The $\gamma\gamma$ widths in (31) can be compared with the well-known quarkonia widths: $\Gamma_{\eta\to\gamma\gamma} \simeq 0.56$ keV and $\Gamma_{f_2 \to \gamma\gamma} \simeq 2.64$ keV.

The conclusions stemming from the above analysis of the couplings of 0^{++} states to heavy quarkonia + γ and $\gamma\gamma$ are as follows:

i) The absence of a G signal in $J/\psi \to \gamma\pi\pi$ is due mainly to its weak coupling to $\pi\pi$.

ii) One should stand a better chance of observing the G in the $\gamma 4\pi^0$ or $\gamma\eta(\eta')\eta$ decay modes of J/ψ.

iii) The fact that the f(975) has been seen in J/ψ decays and the $\sigma(0.9)$ has not, may be due to the large width of the latter.

From the alternative way of detecting gluonium candidates by the study (19) of inclusive ($J/\psi \to \gamma X$) or exclusive ($J/\psi \to \gamma\gamma\gamma$) γ-spectra we conclude that:

iv) Effects of gluonia in the 1 GeV region are difficult to isolate, being swamped by the η' contribution (which is one to two orders of magnitude wider than σ in the $\gamma\gamma$ mode).

v) A G(1.6) signal is also difficult to disentangle from so many other candidates in this energy region.

Finally, as far as $\gamma\gamma$ scattering data are concerned:

vi) The weak couplings of the σ and f_0 to $\gamma\gamma$ could explain their absence there

vii) $\gamma\gamma$ production of G(1.6) selecting $\eta'\eta$ or $4\pi^0$ final states can provide a good way to confirm its properties.

6. - CONCLUSIONS

The G(1.6) meson passes well all our QCD tests for being identified with a (relatively pure) gluonium state, within experimental and theoretical errors. Even more, the existence of an object with the observed properties is welcome for fulfilling QCD sum rules in the 0^{++} channel. Of course, more experimental and theoretical work is needed in order to confirm the above interpretation of G(1.6). Experimentally, its isolation in inclusive γ spectra and in the $\gamma\eta\eta(\eta')$, $\gamma 4\pi^0$ channels of J/ψ and Υ decays, as well as its formation in $\gamma\gamma$ scattering, looks like necessary complements to its confirmation in hadronic reactions. Theoretically, a mixing scheme for the σ, the f_0, the G and other 0^{++} states explaining simultaneously the rich body of data for all mesons in this channel should be worked out.

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FIGURE CAPTIONS

Fig. 1 : a) Behaviour of f_σ versus the sum rule scale τ for a given value $\Lambda = 0.18$ GeV and $\alpha_s\langle G^2\rangle \simeq 0.04$ GeV^4.

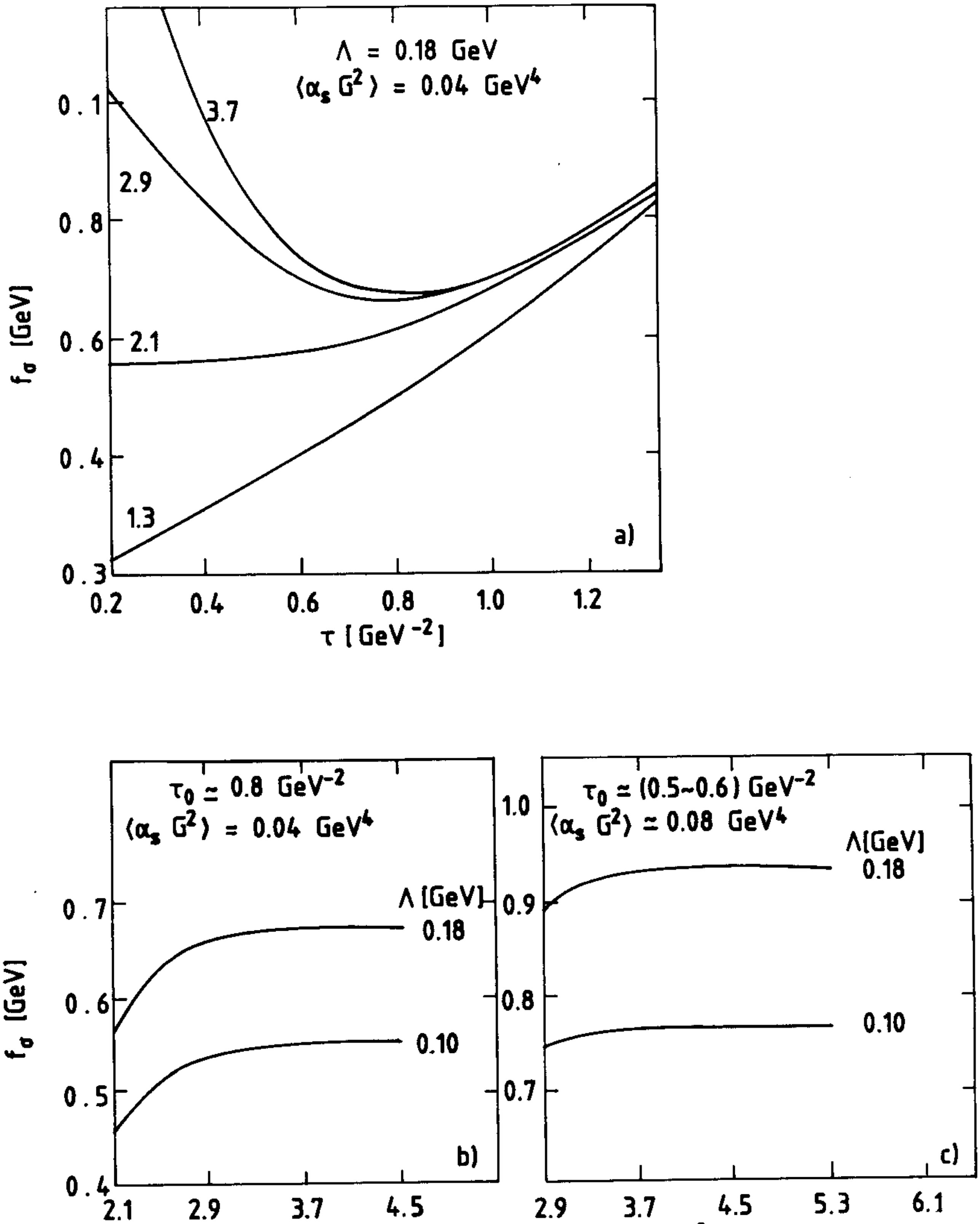
b) Influence of $\sqrt{t_c}$ and Λ on the optimal value of f_σ obtained from a).

c) The same as b) but for $\alpha_s\langle G^2\rangle = 0.08$ GeV^4.

Fig. 2 : The same as Fig. 1 but for f_G.

Fig. 3 : a) $\gamma\gamma gg$ box diagram controlling the processes $J/\psi \to \gamma X$ and $X \to \gamma\gamma$.

b) Anomaly diagram for $X \to \gamma\gamma$ decays.



Λ = 0.18 GeV
⟨αs G²⟩ = 0.04 GeV⁴
3.7
2.9
2.1
1.3
a)
0.1
0.8
0.6
0.4
0.3
0.2
0.4
0.6
0.8
1.0
1.2
τ [GeV⁻²]
fσ [GeV]
τ0 ≃ 0.8 GeV⁻²
⟨αs G²⟩ = 0.04 GeV⁴
Λ [GeV]
0.18
0.10
b)
0.7
0.6
0.5
0.4
2.1
2.9
3.7
4.5
√tc [GeV]
τ0 ≃ (0.5~0.6) GeV⁻²
⟨αs G²⟩ ≃ 0.08 GeV⁴
Λ[GeV]
0.18
0.10
c)
1.0
0.9
0.8
0.7
2.9
3.7
4.5
5.3
6.1
√tc [GeV]

FIGURE 1

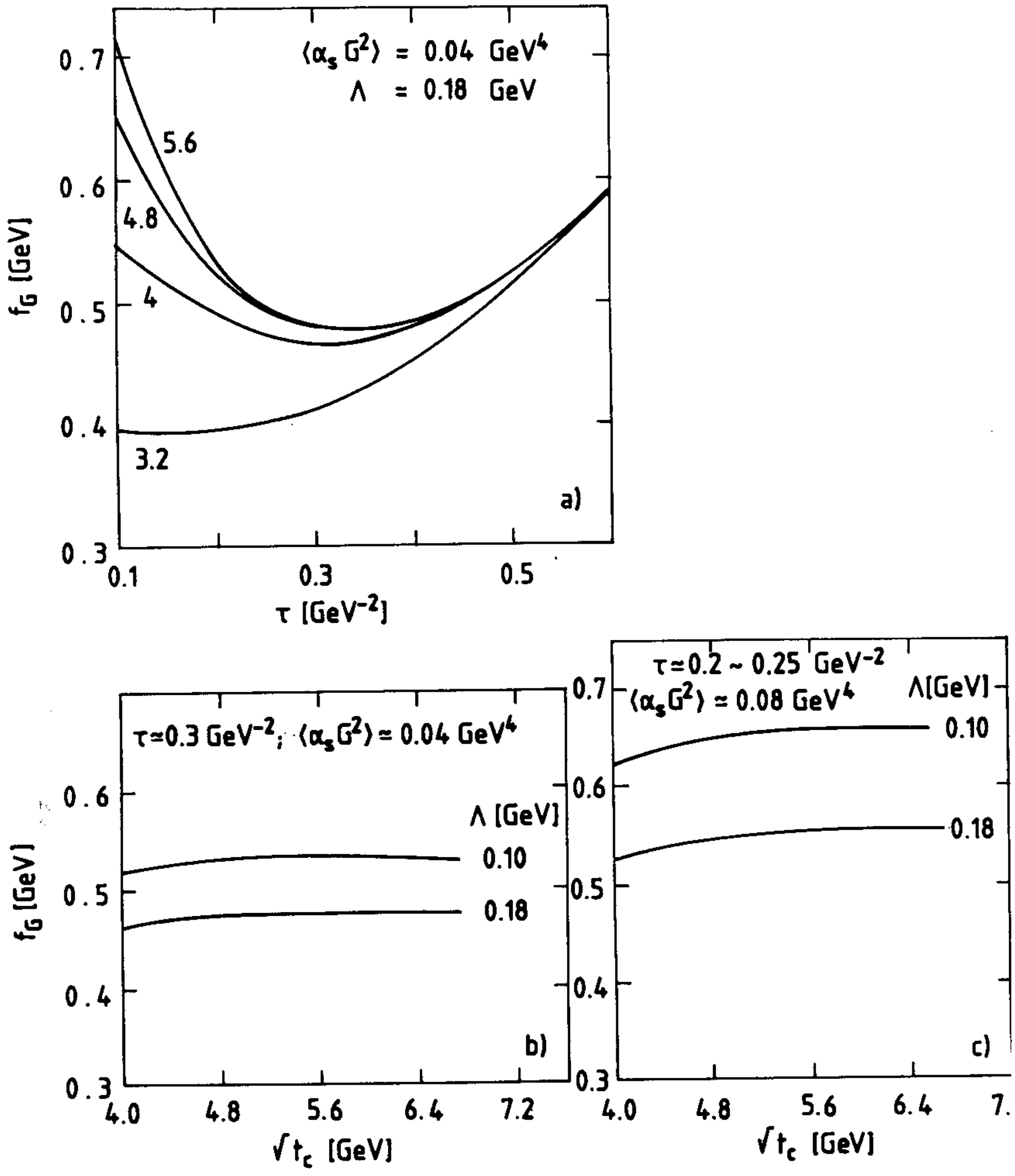

$\langle \alpha_s G^2 \rangle = 0.04\ GeV^4$
$\Lambda = 0.18\ GeV$
5.6
4.8
4
3.2
a)
f_G [GeV]
τ [GeV^{-2}]
$\tau \simeq 0.3\ GeV^{-2}$; $\langle \alpha_s G^2 \rangle \simeq 0.04\ GeV^4$
Λ [GeV]
0.10
0.18
b)
$\sqrt{t_c}$ [GeV]
$\tau \simeq 0.2 \sim 0.25\ GeV^{-2}$
$\langle \alpha_s G^2 \rangle \simeq 0.08\ GeV^4$
Λ[GeV]
0.10
0.18
c)
$\sqrt{t_c}$ [GeV]

FIGURE 2

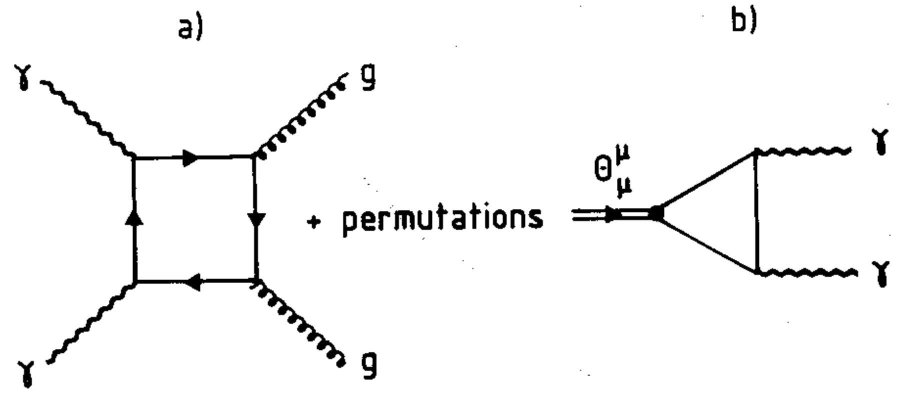


FIGURE 3