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QoS-Based Resource Allocation and Transceiver Optimization

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Abstract

The control and reduction of multiuser interference is a fundamental problem in wireless communications. In order to increase the spectral efficiency and to provide individual quality-of-service (QoS), it is required to jointly optimize the power allocation together with possible receive and transmit strategies. This often leads to complex and difficult-to-handle problem formulations. There are many examples in the literature, where the special structure of the problem is exploited in order to solve special cases of this problem (e.g. multiuser beamforming or CDMA). So it is desirable to have a general theory, which can be applied to many practical QoS measures, like rates, delay, BER, etc. These measures can all be related to the signal-to-interference ratio (SIR) or the signal-to-interference-plus-noise ratio (SINR). This leads to the problem of SIR and SINR balancing, which is fundamental for many problems in communication theory.

In this text we derive a comprehensive theoretical framework for SIR balancing, with and without noise. The theory considers the possible use of receive strategies (e.g. interference filtering or channel assignment), which can be included in the model in an abstract way. Power allocation and receiver design are mutually interdependent, thus joint optimization strategies are derived. The main purpose of this text is to provide a better understanding of interference balancing and the characterization of the QoS feasible region. We also provide a generic algorithmic framework, which may serve as a basis for the development of new resource allocation algorithms.

We study different interference models, which are general enough to be valid for a wide range of system designs, but which are also specific enough to facilitate efficient algorithmic solutions. One important class of interference functions is based on axioms, which characterize the impact of the power allocation of the interference received by the individual users. Another class of interference functions is based on non-negative coupling matrices, which may be parameter-dependent in order to model the possible impact of receive strategies. Both models are studied with and without noise. We analyze the resulting QoS feasible region (the set of jointly achievable QoS) and discuss different allocation strategies for min-max fairness and sum-power minimization. Finally we study geometrical properties of the QoS region, which can be shown to be convex for log-convex interference functions.

Contents

1

Introduction

The wireless channel is a broadcast medium, so each communication link is possibly interfered by other users transmitting at the same resource. The traditional way of handling interference is to assign all links orthogonal resources, in time (TDMA), frequency (FDMA), or code space (CDMA). This considerably simplifies the system design since the links are no longer coupled by interference. However, reserving each link a fixed resource often comes at the cost of sacrificing spectral efficiency. The available bandwidth is generally best exploited by letting transmitted signals interfere with each other in a controlled way (see e.g. [76, 75]). Also, orthogonality may be lost because of system imperfections and the effects of the time-varying multipath channel. It can be said that interference and power constraints are the main hurdle in achieving a high per-user throughput in heavily loaded multiuser networks, as will be required in the future.

Since interference plays an important role in the optimal exploitation of the given bandwidth, it is generally not sufficient to regard the system as a collection of point-to-point communication links. The quality-of-service (QoS) of each link depends on its own transmission power, but also on the power levels of the other links, which are experienced as interference. This results in a competitive situation, where all users try to compensate interference by increasing its own transmission power, which in turn increases the overall interference in the system. A transmission strategy which neglects these interdependencies is likely to cause uncontrollable and exceeding interference, which means a waste of the overall system efficiency. Thus, it is desirable to find a suitable equilibrium that optimally exploits the available resources. This requires a joint optimization of all communication links.

Optimization can be performed with respect to various design goals, like the overall efficiency, max-min fairness, proportional fairness, network utility maximization, etc. There is no such thing as "the" optimal communication strategy. There exists a great deal of literature on resource allocation from various points of view. For example, there are network-centric strategies, which aim at finding a stable performance trade-off by bidding strategies, accounting for traffic, channel quality, and revenues. User-centric strategies, which are closely related to power control, aim at fulfilling user-specific QoS requirements. Both strategies have in common that they are determined and limited by the QoS feasible region (the set of jointly achievable QoS).

The purpose of this text is not to give a comprehensive overview on allocation strategies, but rather to provide a fundamental theoretical framework which helps to understand the underlying effects of interference coupling, and to characterize the QoS feasible region. A fundamental question in this context is: what is the region of jointly achievable QoS, and how can certain points be achieved in a spectrally efficient way? This question is closely related to the classical power allocation problem, but in this text we will go one step further in assuming that interference not only depends on the power allocation, but also on adaptive *receive strategies*, like interference filtering or channel assignment. The additional optimization of the receive strategy adds new degrees of freedom to the problem of resource allocation. Thus, new concepts and algorithms are required.

Since power allocation and receive strategies are intricately intertwined, our approach is to use abstract models, which provide a better understanding of the underlying structure of the problem at hand.

In this respect, the work can be seen as a theoretical basis, which can be applied to solve existing problems in wireless communications.

1.1 QoS-based power and resource allocation

In this section we give an overview on some aspects of QoS-Based power allocation. We start by introducing the basic model used throughout this text, which will be refined later on.

1.1.1 Interference functions

Consider a network with K communication links, whose transmission powers are collected in a power allocation vector

$$\mathbf{p} = [p_1, \dots, p_K]^T > 0 \,.$$

as illustrated in Fig. 1.1.

The interference power experienced by the kth user can be modeled by a function $\mathcal{I}_k(\mathbf{p})$. The functions $\mathcal{I}_1, \dots, \mathcal{I}_K$ describe how the links are affected by mutual cross-talk. Different definitions of $\mathcal{I}_k(\mathbf{p})$ and the resulting QoS region will be analyzed in this text.

It should be noted that the mapping $\mathcal{I}_k : \mathbb{R}_+^K \mapsto \mathbb{R}_+$ can be linear or non-linear, and it can also model the impact of adaptive receiver designs, like MMSE or interference cancellation, as well as other system aspects. A few examples are listed in the following.

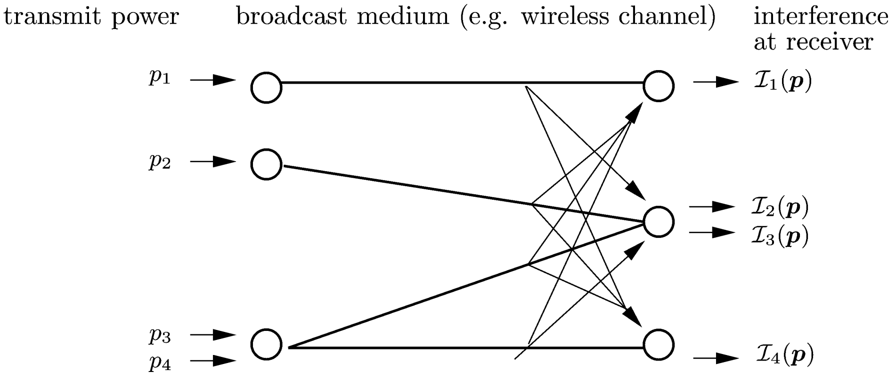

Fig. 1.1 Example of an interference-coupled multiuser system with four transmitter-receiver pairs.

- $\mathcal{I}_k(\mathbf{p}) = [\mathbf{\Psi}\,\mathbf{p}]_k$, where the positive coupling matrix $\mathbf{\Psi} > 0$ contains interference coefficients, which determine in which way the users are affected by cross-talk (interference). This is a common model in power control theory (see e.g. [96]).
- $\mathcal{I}_k(\mathbf{p}) = \min_{z\in\mathcal{Z}}[\mathbf{\Psi}(z)\,\mathbf{p}]_k$, where the adjustable receive strategy z (from a compact set of possible strategies $\mathcal{Z}$, as discussed later in Section 3.1.1) has impact on the interference structure. This specific model, which holds e.g. for multi-antenna beamforming or CDMA designs, and many more, will be studied in Sections 3 and 5.
- $\mathcal{I}_k(\mathbf{p}) = \max_c f_k(\mathbf{p},c)$, where $f_k(\mathbf{p},c)$ is the interference for a given power allocation $\mathbf{p}$ under some interference uncertainty c. This definition can be used, e.g. to model worst-case interference under imperfect channel knowledge. This model will be discussed in Section 6.

But instead of focusing on a particular model, this text aims at characterizing basic properties, which are a common for a wide range of interference functions. To this end, we introduce an axiomatic characterization of interference functions in Section 2. This generic model contains the above examples as special cases. The axiomatic framework will be gradually refined in the following sections. By introducing additional properties, more results can be shown.

1.1.2 The QoS feasible region

The signal-to-interference ratio (SIR) of the kth user is

$$\mathrm{SIR}_k(\mathbf{p}) = \frac{p_k}{\mathcal{I}_k(\mathbf{p})}\,, \quad 1 \le k \le K\,, \tag{1.1}$$

where p_k is the desired transmission power of the kth user. Note, that the function $\mathcal{I}_k(\mathbf{p})$ can include receiver noise. If noise is part of the assumed model (as in Sections 4 and 5), then we will emphasize this by using “SINR” instead of “SIR”. If we use SIR, then we discuss the general case where noise can be included or not. In this case, we need $\mathcal{I}_k(\mathbf{p}) > 0$ to ensure that (1.1) is well defined.

The term “QoS” is commonly used to describe the performance and reliability of a communication link. In order to keep the results as general as possible, we do not make any specific assumption on QoS, except that it is related to the SIR by a monotonic and bijective function ϕ:

$$\mathrm{QoS}_k(\mathbf{p}) = \phi\big(\mathrm{SIR}_k(\mathbf{p})\big) , \quad 1 \leq k \leq K . \tag{1.2}$$

Some examples are BER: $\phi(x) = Q(\sqrt{x})$, MMSE: $\phi(x) = 1/(1+x)$, BER-slope for α-fold diversity: $\phi(x) = x^{-\alpha}$, or capacity: $\phi(x) = \log(1+x)$.

Let γ be the inverse function of ϕ, then

$$\gamma_k = \gamma(Q_k), \quad 1 \leq k \leq K , \tag{1.3}$$

is the minimum SIR level needed by the kth user to satisfy the QoS target Q_k. Thus, the problem of achieving certain QoS requirements, carries over to the problem of achieving SIR targets $\gamma_k > 0$, $\forall k$. In the following we will also summarize the targets in a diagonal matrix

$$\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_1, \ldots, \gamma_K\} . \tag{1.4}$$

It is desirable to find a power allocation $\mathbf{p} > 0$ such that $\mathrm{SIR}_k(\mathbf{p}) \geq \gamma_k$, for all $k = 1, \ldots, K$. This can be rewritten as $\min_k \mathrm{SIR}_k(\mathbf{p})/\gamma_k \geq 1$ or equivalently as $\max_k \gamma_k \mathcal{I}_k(\mathbf{p})/p_k \leq 1$. We say that the target $\boldsymbol{\Gamma}$ is *feasible* if and only if $C(\boldsymbol{\Gamma}) \leq 1$, where

$$C(\boldsymbol{\Gamma}) = \inf_{\mathbf{p}>0} \left(\max_{1 \leq k \leq K} \frac{\gamma_k \mathcal{I}_k(\mathbf{p})}{p_k} \right) . \tag{1.5}$$

In the following we will refer to (1.5) as the "min-max balancing problem".

The optimum $C(\boldsymbol{\Gamma})$ provides a single measure for the joint feasibility of the targets $\boldsymbol{\Gamma}$. Note that the optimization is over $\mathbf{p} > 0$ to ensure that $\mathcal{I}_k(\mathbf{p})/p_k$ is always defined (see Section 2.1.1). However, this does not restrict the generality of the results since $\mathbf{p}$ can be made arbitrarily small.

The min-max optimum $C(\boldsymbol{\Gamma})$ can be used to characterize the QoS feasible region:

$$\mathcal{Q} = \big\{ [\phi(\gamma_1), \ldots, \phi(\gamma_K)] : C(\boldsymbol{\Gamma}) \leq 1 \big\} . \tag{1.6}$$

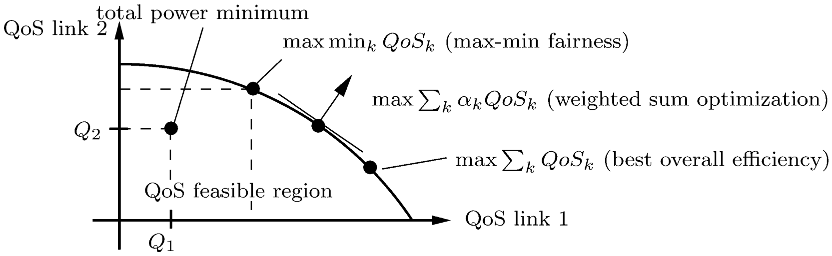

Fig. 1.2 QoS-based resource allocation strategies, illustrated for two users with QoS requirements Q_1 and Q_2

A fundamental problem in resource allocation theory is to find a feasible point $[Q_1,\ldots,Q_K] \in \mathcal{Q}$ according to certain design criteria, like network efficiency, stability, or fairness. The optimization strategy can depend on many parameters, like operator revenue, user requests, queuing lengths, individual link priorities, etc. Examples for different points of interest are depicted in Fig. 1.2. But there exists no joint optimization framework. They actual problem structure strongly depends on the geometry of $\mathcal{Q}$ and on the definition of the underlying interference function.

So the purpose of this text is not to give a comprehensive overview on allocation strategies, but rather to provide a theoretical framework which helps to understand underlying principles. Most of the optimization problems illustrated in Fig. 1.2 are directly connected with the min-max balancing problem (1.5) and the associated QoS feasible region $\mathcal{Q}$.

In the following we will study the QoS (resp. SIR) feasible region for different interference functions $\mathcal{I}_k(\mathbf{p})$, including adaptive receive strategies and worst-case designs. But before we start with the most basic (axiomatic) interference model in Section 2, we provide some additional motivation by discussing the relationship of the generic interference model with problems in wireless communications.

1.2 Related results in wireless communications

A few examples for possible definitions of the interference function $\mathcal{I}_k(\mathbf{p})$ have already been given in Section 1.1.2. We will now discuss the SIR balancing problem in the context of previous work.

The linear function $\mathcal{I}_k(\mathbf{p}) = [\boldsymbol{\Psi}\,\mathbf{p}]_k$ is a classical model, which is used, e.g. in power control theory. The square matrix $\boldsymbol{\Psi} \geq 0$ models the link gains between all receiver/transmitter pairs. The min-max problem (1.5) for this case was already studied in [1] in the context of power balancing for satellite communication systems employing frequency reuse. Under the assumption that $\boldsymbol{\Psi}$ is non-negative and irreducible (see Section 3.1.4 for a definition), it was shown that the min-max-optimal power allocation is given as the principal eigenvector of $\boldsymbol{\Psi}$, and the optimum is the maximal eigenvalue (Perron root). This work was later extended by [42, 2, 46, 94, 95, 93, 33, 34]. An overview is given in [96, 38].

The above model can be extended to include AWG receiver noise, i.e., $\mathcal{I}_k(\mathbf{p}) = [\boldsymbol{\Psi}\,\mathbf{p}]_k + \sigma^2$. The presence of noise results in a situation where possible constraints on the transmission power do matter. Thus, the power allocation problem can be formulated so as to minimize the total power while maintaining certain SINR levels at the receiver. The optimal power allocation is obtained as the solution of a system of linear equations. Iterative solutions were proposed in [26, 43, 31, 4, 3, 7, 87].

The same power minimization problem was considered in [91, 39], where $\mathcal{I}_k(\mathbf{p})$ was not defined by a coupling matrix, but by using an axiomatic framework, equivalent to the one used in Section 4.1.

Since the mid-nineties, there has been a series of publications on *multiuser beamforming* for the downlink channel. In analogy to the power control problem, it was first proposed in [28, 29], to maximize the minimum SIR, assuming that the SIR not only depends on the power allocation, but also on a set of transmit beamformers $\mathbf{u}_1, \ldots, \mathbf{u}_K \in \mathbb{C}^M$, which can be seen as a bank of linear unity-norm filters, which distribute all K signals across the M elements of an antenna array. Given $M \times M$ array covariance matrices $\mathbf{R}_1, \ldots, \mathbf{R}_K$, the interference experienced by the kth receiver is $\sum_{l \neq k} p_l \mathbf{u}_l^* \mathbf{R}_k \mathbf{u}_l$. This is illustrated in Fig. 1.3.

The resulting min-max balancing problem is

$$\inf_{\mathbf{p}>0, \mathbf{u}_1, \ldots, \mathbf{u}_K} \left(\max_{1 \leq k \leq K} \frac{\sum_{l \neq k} p_l \mathbf{u}_l^* \mathbf{R}_k \mathbf{u}_l}{p_k \mathbf{u}_k^* \mathbf{R}_k \mathbf{u}_k} \right) \quad \text{s.t.} \quad \|\mathbf{u}_k\|_2 = 1 \,. \tag{1.7}$$

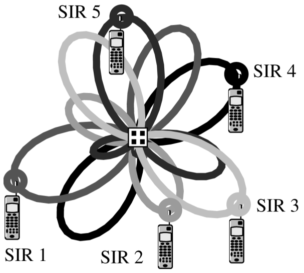

Fig. 1.3 Crosstalk is caused by non-orthogonal beams in a cellular system with multiuser beamforming, where a base station (BS) is simultaneously connected with K mobiles.

It can be observed that the interference in the numerator is not only affected by the powers, but also by the beamformers, thus beamforming adds an additional degree of freedom to the optimization. Problem (1.7) is difficult to handle in its direct form, since all the interference terms are coupled by the transmit beamformers $\mathbf{u}_1,\ldots,\mathbf{u}_K$. The kth beamformer $\mathbf{u}_k$ can be adjusted such that the desired power $\mathbf{u}_k^*\mathbf{R}_k\mathbf{u}_k$ becomes maximal. However, this strategy is generally not optimal for the other users, which are affected by the interference caused by $\mathbf{u}_k$. There is no obvious way how to obtain a good tradeoff between desired power and interference.

It was recognized in [44] that problem (1.7) can be reformulated as an eigenvalue optimization problem, which can be solved by an iterative algorithm. This work was further extended by [10, 13], where it was shown that this algorithm is closely connected with an equivalent uplink channel (see also the discussion in Sections 5.6.4, 3.5.4 and 5.7). By optimizing the uplink interference functions

$$\mathcal{I}_k(\mathbf{p}) = \min_{\mathbf{u}_k} \frac{\mathbf{u}_k^*\left(\sum_{l\neq k} p_l \mathbf{R}_l\right)\mathbf{u}_k}{\mathbf{u}_k^*\mathbf{R}_k\mathbf{u}_k} \quad \text{s.t.} \quad \|\mathbf{u}_k\|_2 = 1\ , \tag{1.8}$$

the optimal downlink beamformers can be found. Note that the beamformer $\mathbf{u}_k$ in (1.8) is adaptively adjusted for each power allocation $\mathbf{p}$. This results in a non-linear dependency between the powers and the experienced interference. Nevertheless, the min-max SIR balancing problem (1.5) can be solved efficiently for the special choice of interference functions (1.8).

Downlink beamforming was also studied under the assumption of additional receiver noise [97, 21, 22, 70, 30, 89, 50, 78, 73, 6, 57, 58, 85]. Similar to the noiseless case, the uplink/downlink duality can be exploited in order to develop iterative algorithmic solutions. In [50, 78], an optimal algorithm was proposed, which consists of an iterative optimization of powers and beamformers for a "virtual uplink" problem. In retrospective, this algorithm can also be understood as a special case of the axiomatic interference model proposed in [91]. An equivalent axiomatic model will be studied in detail in Section 4. Another iterative solution was proposed in [57, 58, 9], where techniques from the theory of non-negative matrices were used to prove monotonicity and convergence. This was extended in [52], where it was shown that additional constraints on the beamformers can be added without affecting the convergence. This already points to the existence of a more general framework for interference balancing which will be introduced in Sections 4 and 5. Many of the results in [57, 58, 9, 52] can also be understood in the context of this general theory.

Besides beamforming, there are other examples for joint power allocation and receiver/transmitter optimization. This includes results on CDMA equalization [76, 71, 79, 72], multi-antenna MMSE filtering [88, 53, 51, 54, 20, 36, 37, 61], as well as recent progress on transceiver optimization for point-to-point MIMO systems [47]. Information-theoretical aspects of MIMO communication have been studied, e.g. in [25, 69, 86, 74, 77, 80, 82, 92]

All these results all have in common that they aim at a better understanding of the joint optimization of interference-coupled links in a network. While the discussed examples are focused on particular scenarios, it is desirable to have a general theory for resource allocation over the QoS region, where QoS can stand for different performance measures, like SINR, MMSE, or capacity. So the motivation behind this text is to find general principles behind interference balancing, which include some of the discussed results as special cases.

1.3 Outline

The sections of this text build on each other. Starting with the most general case, we successively add specifying assumptions, which sometimes restrict the generality, but also allow to show more specific properties. We will conclude each section with a short summary of the main results.

We start in Section 2 with an axiomatic interference model, which describes an interference situation in a most abstract and general way. The properties shown here can be regarded as the most common basis for interference balancing.

Section 3 focuses on the practically relevant case where interference can be modeled with a non-negative coupling matrix. This is known in the literature as the "SIR Balancing Problem". But unlike classical power control theory, we assume that the powers are optimized jointly with an adaptive receiver design. This generalizes known results and algorithms from the aforementioned beamforming example [28, 29, 44, 10, 13]. The impact of the receiver design on the interference is modeled by a parameter-dependent coupling matrix. This adds an additional degree of freedom, so classical results and concepts need to be reconsidered.

From Section 4 on, we study the impact of an additional noise component, which leads to the problem of SINR balancing. Section 4 starts with an axiomatic model, which extends the model of Section 2 by an additional axiom which requires that the interference function is strictly monotone with respect to noise. This constant power level can also be regarded as a fixed interferer, thus the model can be seen as a special case of the more general model used in Section 2, where all interferers are assumed to be varying.

Section 5 further specifies the interference functions. As for the SIR balancing case, we use a parameter-dependent coupling matrix in order to model the impact of interference and noise. The assumption of a fixed noise component leads to additional properties. We study the problems of SINR-constrained power minimization and power-constrained SINR balancing.

Section 6 investigates the QoS feasible region under the assumption of log-convex interference functions. In this case, it can be shown that the resulting QoS region is convex. This useful property is the basis for the development of fast-convergent algorithms for resource allocation and scheduling.

Notation

Some general notational conventions are: matrices and vectors are set in boldface. Let $\mathbf{y}$ be a vector, then $\mathbf{y}_l := [\mathbf{y}]_l$ is the lth component. We use $:=$ for definitions. Finally, $\mathbf{y} \geq 0$ means component-wise inequality, i.e., $\mathbf{y}_l \geq 0$ for all indices l. The set $\mathbb{R}_+$ does include the zero element, while $\mathbb{R}_{++}$ only contains strictly positive elements.

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