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QPOs during magnetar flares are not driven by mechanical normal modes of the crust

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ABSTRACT

QPOs have been observed during three powerful magnetar flares, from SGR 0526−66, SGR 1806−20 and SGR 1900+14. These QPOs have been commonly interpreted as being driven by the mechanical modes of the magnetar's solid crust which are excited during the flare. Here we show that this interpretation is in sharp contradiction with the conventional magnetar model. Firstly, we show that a magnetar crustal mode decays on the time-scale of at most 1 s due to the emission of Alfvén waves into the neutron star interior. A possible modification is then to assume that the QPOs are associated with the magnetars' *global* modes. However, we argue that at the frequencies of the observed QPOs, the neutron star core is likely to support a continuum of magnetohydrodynamic normal modes. We demonstrate this on a completely solvable toy model which captures the essential physics of the system. We then show that the frequency of the global mode of the whole star is likely to have a significant imaginary component, and its amplitude is likely to decay on a short time-scale. This is not observed. Thus we conclude that either (i) the origin of the QPO is in the magnetar's magnetosphere, or (ii) the magnetic field has a special configuration: either it is expelled from the magenta's core prior to the flares, or its poloidal component has very small coherence length.

Key words: stars: magnetic fields – stars: neutron – stars: oscillations – quasars: general.

1 INTRODUCTION

In a prophetic paper, Duncan & Thompson (1992) argued that magnetars – a class of neutron stars with super-strong (10^{14}–10^{15} G) magnetic fields – must exist. Treated at first with some skepticism, the magnetar paradigm has proved extremely successful in explaining the rich phenomenology of anomalous X-ray pulsars (AXPs) and soft gamma-ray repeaters (SGRs: e.g. Thompson & Duncan 1995, hereafter TD). In particular, one phenomenon which has an appealing explanation within the magnetar paradigm is that of giant flares from SGRs. They are thought to be powered by an impulsive release of magnetic energy stored in the neutron star. The release may be triggered by a fracture in the magnetically stressed crust (TD) or by a sudden reconnection in a twisted magnetosphere (Lyutikov 2003). So far, three giant flares have been observed: from SGR 0526−66 (Mazets et al. 1979), SGR 1900+14 (Hurley et al. 1999; Feroci et al. 1999) and SGR 1806−20 (Hurley et al. 2005; Palmer et al. 2005). In each of the flares' light-curves, there is intriguing evidence for QPOs with frequencies of tens of Hz. Firstly, the 43.5-Hz QPO was reported by Barat et al. (1983) in the 1979 flare from SGR 0526−66. Then recently Israel et al. (2005) have found several QPOs in the 2004 giant flare from SGR 1806−20, with frequencies from 18 to 90 Hz. These QPOs were detected with very high signal-to-noise ratios and lasted for ∼100 s; their presence was recently confirmed by Watts & Strohmayer (2006, hereafter WT) who used data from a different satellite. WT have also detected a relatively high-frequency QPO, of 625 Hz. Lastly, prompted by the Israel et al. observations, Strohmayer & Watts (2005) have found multiple QPOs in the light-curve of the 1998 giant flare from SGR 1900+14.

The QPOs have been widely associated with torsional modes of the neutron star crust (Barat et al. 1983 made the first suggestion, which was followed by the more detailed analyses of Duncan 1998, Israel et al. 2005; Strohmayer & Watts 2005 and Piro 2005). Indeed, it is attractive to associate a stable QPO with a mechanical mode of a neutron star, and the frequencies of crustal torsional modes are of the right order of magnitude.[1] However, we argue below that this

[1] The 625-Hz QPO found by WT is already somewhat problematic for the crustal mode picture. One can associate it with a crustal shear mode which has one radial node ($n = 1$; see e.g. Piro 2005). However, a large multitude of $l < 5$, $n = 1$ shear modes exist around that frequency, with spacings of several Hz. There is no physical reason why just one of them should be excited, and not many.

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interpretation is not viable within the magnetar paradigm. In the next section, we show that the crustal mode of a magnetar is not stable but decays within seconds by emitting Alfvén waves into the core. This in direct contradiction with the observations. Chris Thompson (private communication) has suggested that one may associate the QPOs with the magnetohydrodynamics (MHD) elastic modes of the *whole* neutron star. However, in Section 3 we argue that the frequency of such a global mode is likely to have an imaginary component, and the mode decays on a short time-scale. This is due to the continuous spectrum of MHD modes supported by the liquid core.

2 DECAY OF MAGNETAR CRUSTAL MODES

Magnetic fields mechanically connect the elastic crust with the liquid core. Both media can be considered as perfect conductors, and ideal MHD is applicable with magnetic field lines frozen into the media. The Alfvén wave speed in the core is given by

$$v_a = \sqrt{\frac{BB_{\rm cr}}{4\pi\rho\eta}} \simeq 3\times 10^7 \sqrt{\frac{B_{15}}{\rho_{14}\eta}}\ {\rm cm\,s^{-1}}. \quad (1)$$

Here $B = 10^{15}\ B_{15}$ G is the magnetic field inside the core, $\rho = 10^{14}\rho_{14}\ {\rm g\,cm^{-3}}$ is the density, $B_{\rm crit} \simeq 10^{15}$ G is the field inside superconducting flux tubes, and η is the fraction of the neutron star core involved in the Alfvén wave motion. If the neutrons are superfluid, then they decouple from the Alfvén wave and $\eta \sim 0.1$. If protons do not form a superconducting fluid, $B_{\rm crit}$ should be substituted by B in the above equation.

The time-scale of an Alfvén wave crossing the neutron star core is ~0.05 s. Therefore, even if the oscillation was initially localized in the crust, the rest of the core will get involved in less than a second. It is instructive to estimate the power W radiated in the Alfvén waves from the crust:

$$W \sim \rho\omega^2\zeta^2 v_a A, \quad (2)$$

where ω is the angular frequency of the oscillation, ζ is the amplitude of the crustal oscillation at the crust-core interface, and A is the inner surface area of the crust. The time-scale on which the energy is drained from the crustal mode is given by

$$\tau \sim \frac{M}{\rho v_a A} \sim 0.01\ {\rm s}. \quad (3)$$

Here M is the mass of the crust.

In a completely rigorous treatment of the problem various corrections of the order of 1 would occur. Partial reflection of the Alfvén waves is possible, but it will not be a major correction since the wavelength and the relevant scaleheights (density, magnetic pressure) are comparable to the neutron star radius, and thus there is no impedance mismatch which is needed for significant reflection. We note that the situation here is qualitatively different from that considered by Blaes et al. (1989), who considered small-wavelength shear waves propagating from the depth of the crust into the neutron star magnetosphere, and found large reflection coefficients. Here we deal with global modes of the crust, which feature almost radius-independent horizontal displacements. The motion of the inner surface of the crust, together with frozen-in magnetic field-lines, should be considered as the boundary condition for launching the Alfvén waves into the core.

The geometry of the magnetic field is another obvious factor, but for a generic configuration we do not expect a qualitative change in radiated Alfvén-wave power. If, however, the magnetic field was confined to the crust or had a very small coherence length in its poloidal component, then the radiated Alfvén-wave power could decrease significantly. We do not know any compelling argument for either. Thus we conclude that even if the crustal mode was originally excited, magnetic stresses would significantly reduce the mode amplitude in a fraction of a second and redistribute the energy within a liquid core. Therefore, barring special magnetic-field geometry, a pure crustal mode cannot be associated with a stable QPO during the flare. One may hope to associate the *global* MHD–elastic mode with the QPO. This proposal (due to Chris Thompson) runs into potential difficulties, which we discuss in the next section.

3 GLOBAL MODES

Computing the global MHD modes of a star is clearly a difficult task. In recent tour-de-force study Reese, Rincon & Rieutord (2004, hereafter RRR) have found the eigenmodes of a shell of a conductive incompressible fluid with a dipole magnetic field (see also Rincon & Rieutord 2003, and Levin & D'Angelo 2004). However, we are not just limited by purely computational difficulties. In a magnetar core the magnetic pressure is subdominant, and there is virtually no conversion of the Alfvén waves into magnetosonic waves. The core responds to the wave as an incompressible fluid, and the wave propagates along the field lines. As different field lines have different lengths in the core, and different average Alfvén velocities along them, one might suspect that the core supports a continuum of the Alfvén modes. This is hard to prove generally, but below we, in the spirit of appendix B in RRR, present a toy model example where the Alfvén-mode spectrum can be computed explicitly.

Consider a perfectly conducting, incompressible fluid sandwiched in a box, with top and bottom plates also being perfect conductors (see Fig. 1).

The magnetic field $\boldsymbol{B}(y)$ is vertically directed and is a function of y only, threading both top and bottom plates. Gravity is zero, and magnetic pressure gradient is trivially compensated by the fluid pressure gradient. Consider now a z-independent Lagrangian displacement $\zeta(x, y)$ in the z direction. The equation of motion is

$$\frac{\partial^2\zeta}{\partial t^2} = c^2(y)\frac{\partial^2\zeta}{\partial x^2} - \gamma\frac{\partial\zeta}{\partial t}, \quad (4)$$

where $c(y) = \sqrt{T(y)/\rho(y)}$ and γ is a small damping constant added for generality; here $T = B^2/(4\pi)$ is the magnetic tension [in a superconducting fluid $T = BB_{\rm crit}/(4\pi)$] and we have allowed y-dependence of the fluid density ρ. Clearly, equation (4) separates

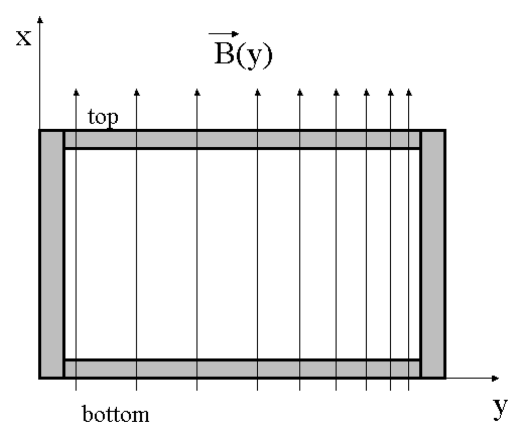

Figure 1. A toy model of the magnetar core. Perfectly conducting incompressible fluid is sandwiched between perfectly conducting top and bottom plates. The magnetic field - -shown by thin arrows – is vertically directed and is a function of y only. The z-axis is perpendicular to the page.

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in x and y, with eigenfunctions

$$\zeta_{ny_0}(x, y) = \sin(n\pi x/l_x)\delta(y - y_0)\exp(i\omega_{ny_0}t), \tag{5}$$

with eigenfrequencies

$$\omega_{ny_0} = \pm\sqrt{(n\pi/l_x)^2 T(y_0)/\rho(y_0) - \gamma^2/4} + i\gamma/2. \tag{6}$$

Here l_x is the box height, n is the integer number, and y_0 is the y-coordinate where the eigenfunction is localized. The spectrum is continuous!

It is notable that for a spherical shell numerical results of RRR indicate that the spectrum of toroidal modes is also continuous (and the eigenmodes are singular, as in equation 5 above). The continuity of part of the spectrum is a topological property, which should remain when external parameters (the shape of the box, density profile, etc.) change continuously. Because the basic MHD physics in our example, in the fluid shell of RRR and in the magnetar core are similar, we believe that the magnetar core supports a continuum of MHD modes.

We started this section by asking whether a global magnetar MHD–elastic mode can be responsible for an observed QPO. We can now see that such a mode of the whole magnetar will be coupled to a continuum of the MHD modes in the core. Experience from quantum mechanics tells us that a mode coupled to a continuum of other modes decays if its frequency lies within the continuum (cf. Fermi's Golden Rule). This is probably the situation here: the frequencies of the QPOs are tens of Hertz, above the base Alfvén-mode frequency (but obviously below that of an infinite number of higher-order Alfvén modes). Let us illustrate explicitly the decay of a global mode by returning to the simple example in Fig. 1. We now model the presence of a crust by assuming that the top plate is allowed to move in the z-direction but is also a harmonic oscillator (with restoring force provided by some external spring). The equation of motion of the top plate is given by

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}t^2} = -\omega_0^2 Z - F/M, \tag{7}$$

where Z is the displacement of the top plate, ω_0 is the proper frequency of the oscillator, F is the force due to magnetic stresses from the fluid below and M is the mass of the plate. When the top plate participates in a global mode of frequency ω, then $Z = Z_0 \exp(i\omega t)$. It is straightforward, by using equation (4), to solve for the fluid motion:

$$\zeta(x, y) = \frac{\sin[k(y)x]}{\sin[k(y)l_x]} Z, \tag{8}$$

where

$$k(y) = \sqrt{\omega^2 - i\omega\gamma}/c(y) \simeq \frac{\omega}{c(y)}\left(1 - i\frac{\gamma}{2\omega}\right). \tag{9}$$

The back-reaction force F can now be computed:

$$F = -\int T(y)\left(\frac{\partial\zeta}{\partial x}\right)_{x=l_x} \mathrm{d}y\mathrm{d}z$$
$$= -l_z\zeta\int T(y)k(y)\cot[k(y)l_x]\mathrm{d}y \tag{10}$$

where l_z is the z-dimension of the top plate. Let $\gamma = 0$. Then if ω is real and is in resonance with at least one of the continuum modes then $\cot kl_x$ will have a pole singularity at some $y = y_0$ in the range of integration. The singularity is lifted if γ is non-zero; in the limit of $\gamma \ll \omega$ the integration will produce an imaginary component of the amplitude of F:

$$\Im[F\exp(-i\omega t)] = -l_z\zeta_0\frac{T(y_0)k(y_0)\pi}{|l_x k'(y_0)|}. \tag{11}$$

Comparing the above equation with the equation (7), we see that our assumption that ω was real was not self-consistent. Within an order of magnitude, the imaginary part of ω is

$$\Im(\omega) \sim \frac{Tl_z l_y}{l_x M\omega}. \tag{12}$$

Substituting the parameters relevant for a magnetar ($B = 10^{15}$, $l_z = l_y = l_x = 10^6$ cm, $M = 3 \times 10^{31}$ g, $\omega \sim 300\,\mathrm{rad\,s^{-1}}$), we get

$$\Im(\omega) \sim 10\ \mathrm{s}^{-1}. \tag{13}$$

Even allowing for several corrections of the order of 1, we see that a global mode is expected to decay within a second. This is much shorter than the time-scale on which the QPO is observed, ~100 s.

One can ask the following question: is it possible that a global mode is not coupled to the MHD continuum in the core? After all, along with the MHD continuum the core can probably support discrete MHD modes as well (such modes are seen in RRR's numerical work). However, while some global mode may be decoupled from the continuum, the mode of our interest must involve the motion of the crust, in order to produce the observed QPO. Unless the magnetic field in the core has a high degree of symmetry (which seems unlikely, especially if one believes that it was generated by a turbulent dynamo), this crustal motion is bound to couple the QPO-generating global mode to the continuum of the core's MHD modes via the field-lines that thread the crust. Thus the decay of a global mode seems generic.

4 MAGNETAR: A TUNING FORK OR WET SPAGHETTI?

Observations argue for the former: QPO frequencies are stable over tens of seconds, something strongly indicative of a mechanical mode. However, we have shown that the theory argues for the latter. Strong MHD crust-core coupling destroys the stability of a purely crustal mode, and the presence of a continuum of MHD modes in the core makes a global crust-core mode decay within a second. So what is the origin of the magnetar QPOs? It can still be mechanical, but only if the magnetic field geometry is very different from what has been assumed – for example, if the magnetic field is largely confined to the crust or has an extremely incoherent poloidal component. Alternatively, the QPOs may have a magnetospheric origin; this was mentioned as a possibility in Barat et al. (1983), but has remained unexplored.

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