

## QUADRATIC INTERPOLATION OF THE HEINZ MEANS

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*Abstract.* The main goal of this article is to present several quadratic refinements and reverses of the well known Heinz inequality, for numbers and matrices, where the refining term is a quadratic function in the mean parameters. The proposed idea introduces a new approach to these inequalities, where polynomial interpolation of the Heinz function plays a major role. As a consequence, we obtain a new proof of the celebrated Heron–Heinz inequality proved by Bhatia, then we study an optimization problem to find the best possible refinement. As applications, we present matrix versions including unitarily invariant norms, trace and determinant versions.

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### REFERENCES

- [1] M. BAKHERAD AND M. S. MOSLEHIAN, *Reverses and variations of Heinz inequality*, Linear Multilinear Algebra **63**, 10 (2015), 1972–1980.
- [2] R. BHATIA, *Interpolating the arithmetic-geometric mean inequality and its operator version*, Linear Algebra Appl. **413** (2006), 355–363.
- [3] J. BOURIN, *Some inequalities for norms on matrices and operators*, Linear Algebra Appl. **292** (1999), 139–154.
- [4] C. CONDE, *Young type inequalities for positive operators*, Ann. Funct. Anal. **4** (2013), no. 2, 144–152.
- [5] R. BHATIA AND C. DAVIS, *More matrix forms of the arithmetic-geometric mean inequality*, SIAM J. Matrix Anal. **14** (1993), 132–136.
- [6] R. KAUR, M. S. MOSLEHIAN, M. SINGH AND C. CONDE, *Further refinements of the Heinz inequality*, Linear Algebra Appl. **447** (2014), 26–37.
- [7] R. KAUR AND M. SINGH, *Complete interpolation of matrix versions of Heron and Heinz means*, Math. Inequal. Appl. **16** (2013), 93–99.
- [8] F. KITTANEH AND Y. MANASRAH, *Reverse Young and Heinz inequalities for matrices*, Linear Multilinear Algebra **59** (2011), 1031–1037.
- [9] F. KITTANEH AND Y. MANASRAH, *Improved Young and Heinz inequalities for matrices*, J. Math. Anal. Appl. **36** (2010), 262–269.
- [10] F. KITTANEH, *On the convexity of the Heinz mean*, Integral Equations Operator Theory **68** (2010), 519–527.
- [11] F. KITTANEH, *Norm inequalities for fractional powers of positive operators*, Lett. Math. Phys. **27** (1993), 279–285.
- [12] F. KITTANEH, M. KRNIĆ, *Refined Heinz operator inequalities*, Linear and Multilinear Algebra **61** (2013), 1148–1157.
- [13] M. KRNIĆ, *More accurate Young, Heinz, and Hölder inequalities*, Period. Math. Hung. **71** (2015), 78–91.
- [14] M. SINGH, *Inequalities involving eigenvalues for difference of Operator means*, Electron. J. Linear Algebra **27** (2014), 557–568.
- [15] M. SABABHEH, *Integral inequalities of the Heinz means as convex functions*, J. Math. Inequal. **10**, 2 (2016), 313–325.
- [16] M. SABABHEH, *Log and Harmonically log-convex functions related to matrix norms*, Oper. Matrices **10**, 2 (2016), 453–465.

- [17] M. SABABHEH, *Piecewise quadratic interpolation and applications to the Young inequality*, Results in Mathematics **72** (2017), 1315–1328. DOI 10.1007/s00025-016-0636-6.
- [18] M. SABABHEH AND M.S. MOSLEHIAN, *Advanced refinements of Young and Heinz inequalities*, J. Number Theory **172** (2017), 178–199.
- [19] L. ZOU, *Inequalities related to Heinz and Heron means*, J. Math. Inequal. **7**, 3 (2013), 389–397.