

Quadrature Mirror Filter Bank Design for Mitral Valve Doppler Signal Using Artificial Bee Colony Algorithm

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Abstract—In this paper, a new design method was developed for quadrature mirror filter (QMF) bank. Previous QMF bank design methods focused on frequency response properties such as pass band and stop band parameters with symmetrical filters. With the proposed method, QMF bank was designed using on time domain signals as mitral valve signal with unsymmetrical filter. The design problem was formulated as an objective function of difference between QMF bank's input and output signals. QMF bank signal disparity was compared with correlation and peak reconstruction error (PRE). QMF bank prototype filter parameters were optimized with Artificial Bee Colony (ABC) and Differential Evaluation (DE) Algorithms. Design outputs were compared with previous works.

Index Terms—Artificial Bee Colony (ABC) algorithm, Differential Evolution (DE) algorithm, quadrature mirror filter (QMF) bank.

I. INTRODUCTION

In recent years, the usage of the QMF banks has been applied in various subjects such as speech, image, video and data compression, coding, sub band coding in communication systems and signal processing [1]–[4].

Finite impulse response (FIR) and infinite impulse response (IIR) filters are two principal filters that are used with QMF banks. FIR filters have more stability, straightforwardness and linearity phase than IIR design. Therefore FIR filters are more widely used in the QMF bank design.

QMF bank was first introduced by Crosier *et al.* [5]. QMF design problem is solved by various design methods. These methods can be divided into conventional methods and metaheuristic methods including genetic algorithm (GA), differential evolution (DE), and artificial bee colony (ABC) algorithm [6]–[10].

QMF bank design methods usually consist on mean square errors in transition band, stopband and passband which are parameters of the filter frequency responses [7]–[9]. These methods focus on the frequency response

specifications rather than observing the input and output divergence or relationship of signals. Even in the best case conventional methods have phase delay and amplitude distortion for signal applications. In this work, difference between input and output is intended to be minimized.

There are several metaheuristic methods proposed to optimize digital filters. In this study, DE and ABC algorithms are used. DE algorithm, introduced by Price and Storn, is an effective and convenient algorithm in continuous optimization [11]. Most of the metaheuristic methods are used to design symmetrical filters. Filter symmetry is depicted with filter parameters flipping to right and thus the filters' lengths double. It means that the prototype filter designed with 8 filter length actually operates like the one with 16 filter length does. ABC is a new and robust optimization algorithm introduced by Karaboga and it is based on the intelligent foraging behaviour of honey bee swarms [12]. ABC and DE were used in digital filter bank design [13], [14].

In the next section of paper, QMF bank design methods are introduced. In section three, the algorithms used are explained and in section four, the formulation of the optimization problem is given. In the last section results are presented and evaluated.

II. DESIGN OF QMF BANK

QMF bank called as two channel filter bank is presented in Fig. 1. Input signal $x[n]$ is separated into two part signals to have equal bandwidth with two filters. The analysis filters H_0 and H_1 are low pass and high pass filters respectively. Analysis filter magnitude responses are images of each other $H_1(z) = H_0(-z)$ at sampling frequency $\pi/2$. The image condition of the frequency is the quarter of the sampling frequency 2π . The filtered signals are downsampled for signal compression or reducing complexity. The upsampled signals are sent to G_0 and G_1 synthesis filters. The upsampling operation equals the signal frequency as the former analysis filter frequency. Analysis and synthesis filters are used for eliminating aliasing and mirror effects in frequency domain [15]. Transfer function of the QMF bank

can be written as using z-transform

$$Y(z) = T(z)X(z) + A(z)X(-z), \quad (1)$$

where $T(z)$ is the distortion transfer function and it is

$$T(z) = \frac{1}{2}[H_0(z)G_0(z) + H_1(z)G_1(z)]. \quad (2)$$

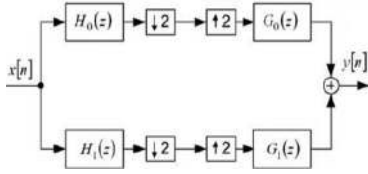


Fig. 1. Quadrature Mirror Filter Bank.

Mirroring and aliasing free system transfer function $A(z)$ must be equal to zero. Hence, the analysis and synthesis filters can be defined as:

$$\begin{cases} G_0(z) = 2H_0(-z), \\ G_1(z) = -2H_1(-z). \end{cases} \quad (3)$$

Hence, overall transfer function is

$$Y(z) = T(z)X(z). \quad (4)$$

As seen in (3), other filters can be expressed with H_0 called as prototype filter. $T(z)$ is defined with prototype filter as

$$T(z) = \frac{1}{2}[H_0^2(z) - H_0^2(-z)]. \quad (5)$$

QMF bank has three type of distortion: amplitude distortion, phase distortion and aliasing distortion. Aliasing distortion is eliminated with prototype filter transformation with (3). If the prototype filter H_0 is a linear phase FIR filter, the transfer function in (5) has linear phase and its phase distortion is eliminated. Frequency response of H_0 is

$$H_0(e^{j\omega}) = A(\omega)e^{-j\omega(N-1)/2}, \quad (6)$$

where N is the filter length and $A(\omega)$ is the amplitude response of the filter. Here

$$A(\omega) = \sum_{n=0}^{N/2-1} 2h_0(n) \cos\left(\frac{N-1}{2}n - n\right)\omega, \quad (7)$$

where h_0 is the impulse response of the prototype filter $H_0(z)$. QMF bank's overall frequency response is

$$T(e^{j\omega}) = \frac{e^{-j\omega(N-1)}}{2} \left\{ |H_0 e^{j\omega}|^2 - (-1)^{N-1} |H_0 e^{j(\omega-\pi)}|^2 \right\}. \quad (8)$$

If N is odd, (8) results $T(e^{j\omega}) = 0$ at $\omega = \pi/2$ for amplitude distortion. In order to avoid the amplitude

distortion, N must be chosen even.

In the perfect reconstruction

$$\left| H_0(e^{j\omega}) \right|^2 + \left| H_0(e^{j(\omega-\pi)}) \right|^2 = 1, \quad (9)$$

must be satisfied and in the (9) frequency must be chosen as $\omega = \pi/2$. Hence,

$$\left| H_0(e^{j\pi/2}) \right| = 0.707. \quad (10)$$

H_0 prototype filter parameters' estimation by using optimization algorithms is the QMF design problem. In this paper, H_0 filter parameters are optimized with DE and ABC algorithms and results are compared. The objective function to be optimized by the optimization algorithms are explained in the section four.

III. ALGORITHMS

A. Differential Evolution Algorithm

DE optimizes the objective function by maintaining every candidate solution and creating new solutions by combining existing solutions. DE algorithm was particularly developed for real valued parameter optimization problems. This algorithm has simple structure and rapid convergence to optimum. The DE algorithm was applied for digital filter design and problems [11], [13].

The basic steps of the DE algorithm is as follows

<i>Initialization</i>
<i>Evaluation</i>
Repeat
<i>Mutation</i>
<i>Recombination</i>
<i>Selection</i>
Until (termination criteria)

The DE algorithm control parameters are NP (Number of population), CR (cross over rate) and F (Scaling factor).

The DE starts with generating a randomized population defined by NP. Population members are n-dimensional vectors and each parameter is between borders x_{ij}^{\min} and x_{ij}^{\max} .

In the mutation step, mutant vector is generated each target vector $x_{i,G}$ as below

$$v_{i,G+1} = x_{i,G} + K(x_{r_1,G} - x_{i,G}) + F(x_{r_2,G} - x_{r_3,G}), \quad (11)$$

where $i, r_1, r_2, r_3 \in \{1, 2, \dots, NP\}$ are randomly selected and not equal to each other. F is the scaling factor interacting with difference vector and K is the combination factor in (11).

In the crossover step, parent (q) and mutated (v) vectors are used to create a trial vector $u_{ji,G+1}$. This is defined by

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } (rnd_j \leq CR) \text{ or } j = rn_i, \\ q_{ji,G+1} & \text{if } (rnd_j > CR) \text{ or } j \neq rn_i, \end{cases} \quad (12)$$

where CR is the crossover constant $\in [0,1]$, $j = 1, 2, \dots, D$; $r_j \in [0,1]$ is the array of random number and $m_{i_j} \in (1, 2, \dots, D)$ random index.

The child vector is evaluated with mutation and crossover operations. In the selection, the child and parent vectors are compared for their fitness value and better one is chosen.

B. Artificial Bee Colony Algorithm

ABC algorithm is a swarm algorithm based on bee's intelligent food search behaviour. There are three types of bees in this algorithm; onlooker, scout and employed bee. Each solution of the problem represents a food source and consists of an n-dimensional real valued vector. Employed bees exploit their food sources representing possible solutions. Onlooker bees wait in the hive and judge the quality of the food sources by interacting with employed bees. If an employed bee's food source is abandoned, the employed bee is converted to scout. Scout bees randomly search around for new food sources. ABC algorithm is widely and effectively used in digital filter design and optimization problems [14], [16], [17]. Basic steps of the ABC algorithm is

<i>Initialization</i>
<i>Evaluation</i>
Repeat
<i>The Employed</i>
<i>The Onlooker bee phase</i>
<i>The Scout bee phase</i>
<i>Memorize the best food source</i>
<i>Cycle=cycle+1</i>
Until

Step 1: Initialization of population is formed with randomly generated S possible solutions. Each solution is defined by $x_i = 1, 2, \dots, S$ and a fitness value is evaluated for a D dimensional solution. D is the number of optimization parameters.

Step 2: Produce new solution for each employed bee and evaluate it. Examine every new solution's fitness value.

Step 3: Calculate the probability value p_i for the solution i using (13)

$$p_i = \frac{F(x_i)}{\sum_{k=1}^S F(x_k)}, \quad (13)$$

where $F(x_i)$ is the fitness value of the solution. Complete the search for the onlooker bees just as in the employed bees.

Step 4: If a food source is abandoned, its associated employed bee becomes a scout bee. Send the scout bee for searching new sources randomly.

IV. PROBLEM FORMULATION

A new QMF bank design method proposed in this study is seen in Fig. 2. $x[n]$ is the input signal, $y[n]$ is the output of the QMF bank and e is the difference between input and output signals (error) which is objective criteria. DE and

ABC algorithms are used to design the prototype filter $H_0(z)$ by minimizing error.

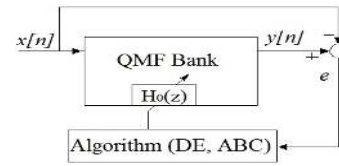


Fig. 2. QMF bank design schema.

In the proposed design method, the objective function $J(w)$, was optimized by employing the mean squared error (mse). Objective function optimized by ABC and DE algorithms is defined by

$$J(\omega) = \min \frac{1}{N} \sum_{n=1}^N (x(n) - y(n))^2. \quad (14)$$

Performance of the designed QMF banks is measured with peak reconstruction error (PRE) defined in (15) and Pearson's correlation given in (16). x_i and y_i are measured datasets, \bar{x} and \bar{y} are the means of the measured data sets in the correlation equation [18]:

$$PRE = \max_{\omega} \left\{ 20 \log_{10} \left(\left| H_0 e^{j\omega} \right|^2 + \left| H_0 e^{j\omega - \pi} \right|^2 \right) \right\}, \quad (15)$$

$$r = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^N (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^N (y_i - \bar{y})^2}}. \quad (16)$$

As input signal, the mitral valve signal, which is a real life biomedical signal, is used. A mitral valve signal has valve's opening and closing data called as cycle represented in Fig. 3. A mitral cycle data length is related to sampling rate, but it has nearly two thousands of data. In this work, we used 1400 samples of mitral valve data. The selected part is represented as inverse colored in Fig. 3. The detailed part of Mitral valve signal used for QMF bank design is shown in Fig. 4.

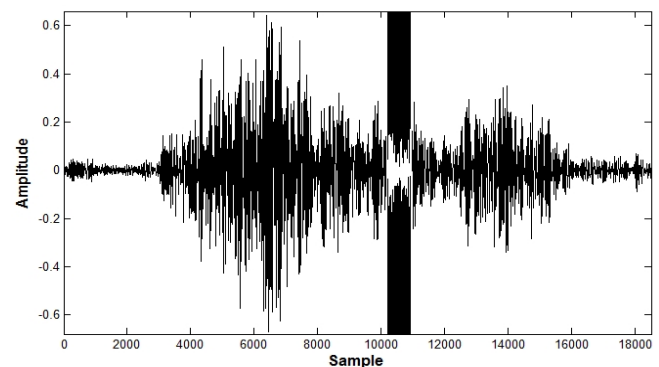


Fig. 3. Mitral valve cycle data.

QMF bank was designed for mitral valve signal by DE and ABC algorithms. ABC algorithm control parameters were evaluated for this problem. Firstly, different population sizes were applied from 20 to 120, with numbers increasing 10 by 10. After finding best population size, cycle was

studied between 500 and 5000. The last control parameter (limit) was searched between 100 and 500 for the best result. DE and ABC algorithm control parameter values used are given in Table I [19].

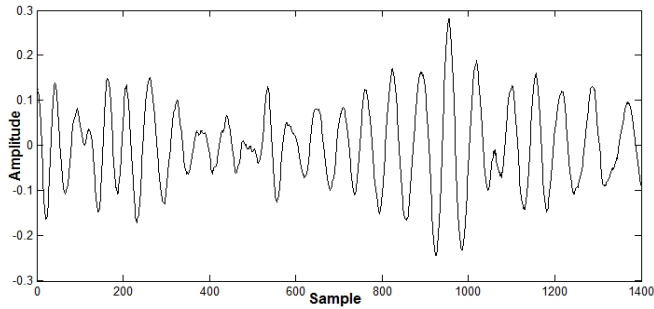


Fig. 4. Mitral valve signal used in QMF bank design.

TABLE I. CONTROL PARAMETER VALUES.

ABC Algorithm		DE Algorithm	
NP	20	NP	20
Limit	500	CR	0.9
Xmin-Xmax	[-1,1]	Xmin-Xmax	[-1,1]
Cycle	1000	Cycle	1000
		F	0.5

V. RESULTS AND DISCUSSIONS

The proposed QMF design method has been implemented in MATLAB. Unsymmetrical FIR filters were used for design in which number of parameters were altered between 8 and 100 degrees. Prototype filter parameters were designed with DE and ABC algorithms using proposed QMF design method presented in Fig 2. Overall filter parameters were obtained from the prototype filter parameters by using (3).

Evaluation results of the ABC and DE algorithm based design for different filters lengths are given in Table II. PRE and correlation results of the QMF bank are demonstrated for different filter lengths ranging from 8 to 100. In Table II, best results are given in bold. The best result of QMF bank found by ABC algorithms was obtained for 16 filter length. DE algorithm's best result was produced with 8 filter length.

TABLE II. REQUIREMENTS FOR THE EQUATIONS.

N	ABC		DE	
	Correlation	PRE	Correlation	PRE
8	0.9981	0.2332	0.9984	0.1852
12	0.9980	0.2348	0.9985	0.2929
16	0.9953	0.1587	0.9985	0.2094
20	0.9960	0.2858	0.9985	0.2290
24	0.9956	0.2379	0.9982	0.2984
28	0.9930	0.2818	0.9978	0.2446
32	0.9924	0.2923	0.9978	0.2739
40	0.9946	0.2857	0.9978	0.2739
48	0.9824	0.3007	0.9955	0.2764
64	0.9494	0.2869	0.9916	0.2110
80	0.9666	0.2889	0.9852	0.2326
100	0.9348	0.2461	0.4577	0.4577

The proposed method was compared with two recent papers [20]–[212] where predetermined prototype filter's parameters were optimized by using ABC and DE algorithms. We selected the example 1 in [20] (12 filter

coefficients and 24 filter lengths) and example 2 in [21] (16 filter coefficients and 32 filter lengths). Two examples of filters have stop band frequency at 0.6π and passband frequency at 0.4π . The selected part of mitral valve signal and entire mitral valve signal were processed with this QMF banks for the comparison. QMF bank input output signal correlation obtained by the proposed method was compared in Table III. Proposed method's correlation performance is better than the examples.

The best filter lengths found by of ABC and DE are 16 and 8 degrees, respectively. In Fig. 5, QMF bank input-output signals with 16 filter length designed by ABC algorithm are presented. QMF bank output and input signals designed by DE algorithm are given in Fig. 6.

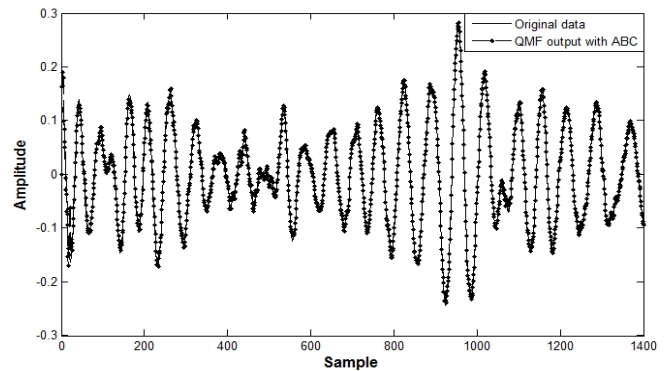


Fig. 5. QMF bank input-output for ABC algorithm.

TABLE III. COMPARISON OF CORRELATION.

	Filter Length	Selected Part	All Signal
ABC	16	0.9953	0.9920
DE	8	0.9984	0.9987
Example 1[20]	24	-0.7913	-0.2080
Example 2[21]	32	-0.6590	-0.6348

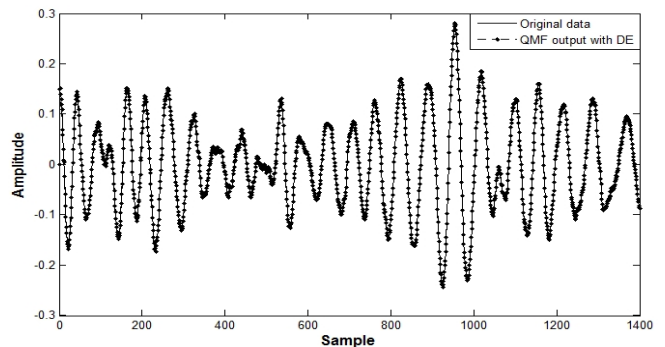


Fig. 6. QMF bank input-output for DE algorithm.

Samples are reduced for better visibility and output signal differences are depicted. Therefore, the difference between the outputs corresponding to input signal is presented for ABC and DE algorithms. The beginning part of the QMF bank output signals designed with ABC and DE is demonstrated in Fig. 7. Peak of the designed QMF bank output signals is compared on Fig. 8.

QMF input-outputs for the example 1 and example 2 are showed in Fig. 9. Examples' prototype filter parameters are given in Table IV. They are taken from the studies presented in [20]–[22]. The QMF bank prototype filter parameters found by using ABC and DE algorithms are given in Table IV. From Fig. 9 it is very clear that the proposed method produces much better correlation value.

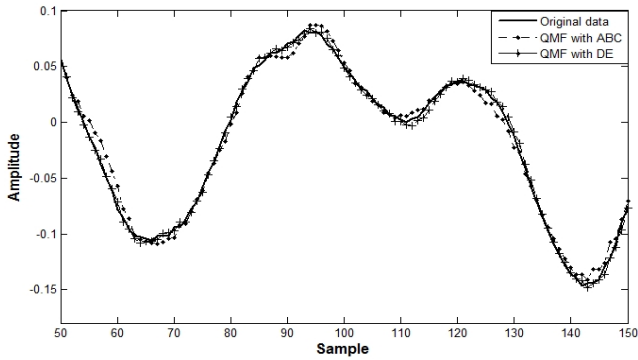


Fig. 7. QMF bank input-outputs for ABC and DE algorithms in the beginning part.

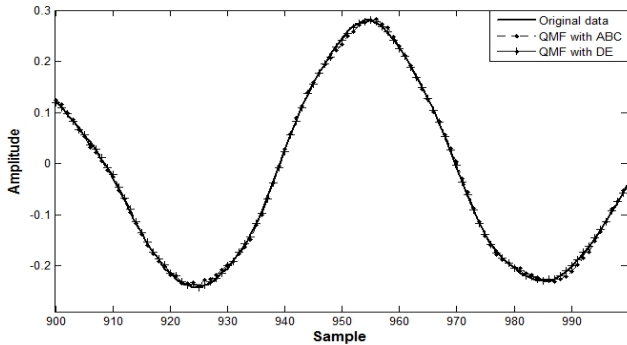


Fig. 8. QMF bank input-outputs for ABC and DE algorithms at the peaks of the signal.

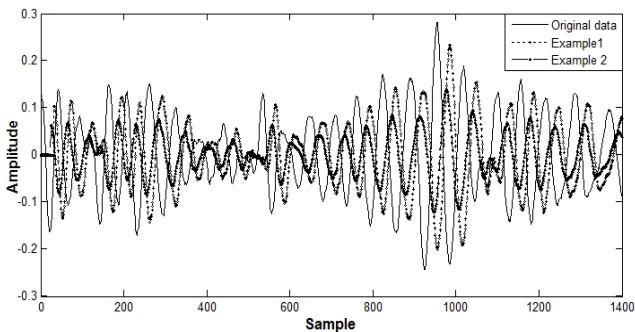


Fig. 9. QMF bank input-output for example 1 and example 2.

TABLE IV. PROTOTYPE FILTER PARAMETER VALUES OBTAINED FOR MINIMUM PRE VALUES AND COMPARED EXAMPLES.

ABC	DE	Example 1[20]	Example 2[21]				
b0	0.6613	b0	-1.0000	b0	0.0029	b0	0.0016
b1	0.9990	b1	-0.6057	b1	-0.0062	b1	-0.0028
b2	0.5163	b2	-0.0107	b2	-0.0025	b2	-0.0021
b3	-0.6233	b3	0.0529	b3	0.0150	b3	0.0073
b4	0.6190	b4	-0.2009	b4	-0.0012	b4	0.0016
b5	-0.9134	b5	0.1424	b5	-0.0289	b5	-0.0147
b6	0.4170	b6	0.3464	b6	0.0113	b6	0.0008
b7	0.6633	b7	-0.1431	b7	0.0519	b7	0.0258
b8	0.3230			b8	-0.0365	b8	-0.0074
b9	0.0909			b9	-0.0997	b9	-0.0424
b10	0.0271			b10	0.1261	b10	0.0215
b11	0.2336			b11	0.4678	b11	0.0693
b12	0.0700					b12	-0.0536
b13	-0.2173					b13	-0.1282
b14	-0.3521					b14	0.1680
b15	-0.0239					b15	0.5988

VI. CONCLUSIONS

A new QMF design method was proposed. Previous QMF design methods examine frequency response parameters of analysis filter. These parameters such as passband,

transitionband and stopband are predetermined and used for error criteria. The proposed method offers QMF bank design using a time based signal. The mitral valve signal was used as real life signal. Analysis and synthesis filters were obtained by using prototype filter in the QMF bank design based filter. Filter parameters were obtained by ABC and DE algorithms. The best results were produced with 16 and 8 filter lengths. The filter performances and the input-output relations were evaluated with correlation and PRE values. The results are compared with two previous works and it is seen that the proposed method's correlation results are much better than others. It is concluded that the proposed method has minimum phase delay or amplitude distortion. The main advantage of proposed method is that it doesn't require any advanced error criteria or matrix based transformation.

REFERENCES

- [1] A. Kumar, G. K. Singh, R. S. Anand, "An improved method for the design of quadrature mirror filter banks using the Levenberg-Marquardt optimization", *Signal, Image Video Process*, vol. 7, no. 2, pp. 209–220, 2011. [Online]. Available: <http://dx.dio.org/10.1007/s11760-011-0209-9>
- [2] S. Chandran, "A novel scheme for a sub-band adaptive beam forming array implementation using quadrature mirror filter banks", *ElectronLett*, vol. 39, no. 12, pp. 891–892, 2003. [Online]. Available: <http://dx.dio.org/10.1049/el:20030579>
- [3] S. C. Chan, C. K. S. Pun, K. L. Ho, "New design and realization techniques for a class of perfect reconstruction two-channel FIR filter banks and wavelet bases", *IEEE Trans Signal Process*, vol. 52, no. 7, pp. 2135–2141, 2004. [Online]. Available: <http://dx.dio.org/10.1109/TSP.2004.828918>
- [4] P. Ghosh, S. Das, H. Zafar, "Adaptive differential evolution based design of two channel quadrature mirror filter banks for sub-band coding and data transmission", *IEEE Trans Syst, Man, Cybern*, vol. 42, no. 6, pp. 1613–1623, 2012. [Online]. Available: <http://dx.dio.org/10.1109/TSMCC.2012.2212007>
- [5] A. Crosier, M. Esteban, C. Galand, "Perfect channel splitting by use of interpolation/decimation/tree decomposition techniques", in *Proc. Int. Symposium On Information Circuits and Systems*, 1976.
- [6] V. K. Jain, R. E. Crochiere, "Quadrature mirror filter design in time domain", *IEEE Trans. Acoust. Speech Signal Process (ASSP)*, vol. 32, no. 4, pp. 353–361, 1984. [Online]. Available: <https://doi.org/10.1109/TASSP.1984.1164329>
- [7] O. P. Sahu, M. K. Soni, I. M. Talwar, "Marquardt optimization method to design two channel quadrature mirror filter banks", *Digital Signal Processing*, vol. 16, no. 6, pp. 870–879, 2006. [Online]. Available: <http://dx.dio.org/10.1016/j.dsp.2005.11.002>
- [8] S. M. Rafi, A. Kumar, G. K. Singh, "An improved particle swarm optimization method for multirate filter bank design", *Journal of the Franklin Institute*, vol. 350, pp. 757–759, 2013. [Online]. Available: <http://dx.dio.org/10.1016/j.jfranklin.2013.01.006>
- [9] J. Upendar, C. P. Gupta, G. K. Singh, "Designing of two channel quadrature mirror filter bank using particle swarm optimization", *Digital Signal Processing*, vol. 20, no. 10, pp. 304–313, 2010. [Online]. Available: <http://dx.dio.org/10.1016/j.dsp.2009.06.014>
- [10] Y. J. Yu, Y. C. Lim, "New natural selection process and chromosome encoding for the design of multiplierless lattice QMF using genetic algorithm", in *Proc. 8th IEEE Int. Conf. Electronics, Circuits and Systems*, 2001, vol. 3, pp. 1273–1276. [Online]. Available: <http://dx.dio.org/10.1109/ICECS.2001.957447>
- [11] R. Storn, K. V. Price, "Differential evolution a simple and efficient heuristic for global optimization over continuous spaces", *J. Global Optim.*, vol. 11, no. 4, pp. 341–359, 1997. [Online]. Available: <http://dx.dio.org/10.1023/A:1008202821328>
- [12] D. Karaboga, "An Idea based on honey bee swarm for numerical optimization", Erciyes University, Engineering Faculty, Computer Engineering Department, Tech. Rep. TR-06, 2005
- [13] N. Karaboga, "Digital IIR filter design using differential evolution algorithm", *Eurasip Journal On Applied Signal Processing*, no. 8, pp. 1269–1276, 2005. [Online]. Available: <http://dx.dio.org/10.1155/ASP.2005.1269>
- [14] T. Koza, N. Karaboga, S. Kockanat, "Aort valve doppler signal noise

- elimination using IIR filter designed with ABC algorithm”, *Int. Symposium on Innovations in Intelligent Systems and Applications (INISTA 2012)*, 2012, pp. 1–5. [Online]. Available: <http://dx.doi.org/10.1109/INISTA.2012.6247038>
- [15] P. P. Vaidyanathan, “Multirate digital filters, filter banks, polyphaser networks and applications: a tutorial”, in *Proc. IEEE*, 1990, vol. 78, no. 1, pp. 56–93. [Online]. Available: <http://dx.doi.org/10.1109/5.52200>
- [16] N. Karaboga, “A new design method based on artificial bee colony algorithm for digital IIR filters”, *Journal of the Franklin Institute-Engineering and Applied Mathematics*, vol. 346, pp. 328–348, 2009. [Online]. Available: <http://dx.doi.org/10.1016/j.jfranklin.2008.11.003>
- [17] S. K. Agrawal, O. P. Sahu, “Artificial bee colony algorithm to design two-channel quadrature mirror filter banks”, *Swarm and evolutionary computation*, vol. 21, pp. 24–31, 2015. [Online]. Available: <http://dx.doi.org/10.1016/j.swevo.2014.12.001>
- [18] J. M. Blackledge, *Digital Signal Processing: Mathematical and Computational Methods, Software Development and Applications. Horwood Publishing*, 2006, pp. 729–730. [Online]. Available: <https://doi.org/10.1533/9780857099457>
- [19] D. Karaboga, B. Akay, “A comparative study of artificial bee colony algorithm”, *Applied Mathematics and Computation*, vol. 214, pp. 108–132, 2009. [Online]. Available: <http://dx.doi.org/10.1016/j.amc.2009.03.090>
- [20] S. K. Agrawal, O. P. Sahu, “Artificial bee colony algorithm to design two-channel quadrature mirror filter banks”, *Swarm and Evolutionary Computation*, vol. 21, pp. 24–31, 2015. [Online]. Available: <http://dx.doi.org/10.1016/j.swevo.2014.12.001>
- [21] A. Kumar, G. K. Singh, R. S. Anand, “Design of quadrature mirror filter bank using particle swarm optimization (PSO)”, *ACEEE International Journal on Electrical and Power Engineering*, vol. 1, no. 1, pp. 41–45, 2009. [Online]. Available: <http://dx.doi.org/01.ijepe.01.0>
- [22] A. Dogan, T. Yalcinoz, M. Alci, “A Comparison of Heuristic Methods for Optimum Power Flow Considering Valve Point Effect”, *Elektronika ir Elektrotechnika*, vol. 22, no. 5, pp. 32–37, 2016. [Online]. Available: <http://dx.doi.org/10.5755/j01.eie.22.5.16340>