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Quadrotors Flight Formation Control Using a Leader-Follower Approach*

D. A. Mercado[1], R. Castro[1] and R. Lozano[2]

Abstract— **In this paper it is presented a control strategy to solve the trajectory tracking and flight formation problem, in horizontal plane, of multiple unmanned aerial vehicles (UAVs) kind quadrotor, by means of a leader-follower scheme. Time scale separation of the translational and rotational quadrotor dynamics is used to achieve trajectory tracking. A sliding mode controller is proposed for the translational dynamic and provides the desired orientation for the UAV, which is controlled by a linear PD control. Finally, from the formation error dynamics, a sliding mode control is used by the follower to preserve the formation with respect to a leader. Experimental results, using a virtual leader and a follower in formation, are shown to evaluate the proposed control law.**

I. INTRODUCTION

Thanks to the huge potential in many civilian and military applications, especially for exploration, surveillance, search and rescue, UAVs have received a lot of attention in the last years. Particularly, quadrotors have become one of the most popular UAVs since their rotor's configuration produces cancellation of the reactive torques, simplifying considerably their analysis and control. Also, they are suitable for vertical take off and landing, as well as hovering, making them a good choice for maneuvering in small spaces.

Recent literature on quadrotors is quite vast, including modelling and control techniques for attitude stabilization, trajectory tracking and formation of multiples quadrotors. To cite some examples, in [1] it is presented an attitude stabilization control strategy for hover flight using nested saturations, while in [2] a sliding mode approach is used to accomplish position control of a quadrotor. In [3] a trajectory tracking control by means of a discrete time, feedback linearization control scheme is proposed. The formation and trajectory tracking control for multiple UAVs using consensus is discussed in [4], while in [5] a formation control strategy with a leader-follower approach is described and in [6] artificial potentials are used for the same purpose. Also, in [7] a trajectory tracking and formation control scheme for quadrotors by means of time scale separation and feedback linearization is presented.

In this paper, a control strategy for the problem of quadrotors flight formation in a leader follower scheme is presented. Time scale separation of the translational and rotational quadrotor dynamics is used together with a sliding mode controller and a PD controller. The contribution of the work consists in the design of a control strategy, simple and easy to implement which takes some ideas from previous works on mobile terrestrial robots [8]. Experimental results, where a virtual leader is used, are shown to evaluate the performance of the strategy.

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[1]D. A. Mercado and R. Castro are with Department of Electrical Engineering, CINVESTAV-IPN, Mexico. dmercado, rcastro@cinvestav.mx

[2]R. Lozano is with HEUDIASYC UMR 6599 CNRS-UTC, Compiègne, France. rlozano@hds.utc.fr

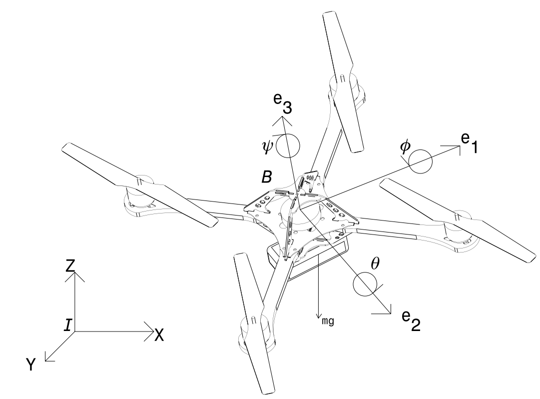

Fig. 1. Qadrotor in an inertial reference frame.

II. QUADROTOR DYNAMIC MODEL

The quadrotor can be represented as a rigid body in space with mass m and inertia matrix J, subject to gravitational and aerodynamic forces. Let us consider an inertial coordinate frame $I = \{X \;\; Y \;\; Z\}$, fixed to ground and a body fixed coordinate frame, $B = \{e_1, e_2, e_3\}$ (see Fig. 1). Consider the vectors

$$\xi = [x \;\; y \;\; z]^T \tag{1}$$

$$\Phi = [\phi \;\; \theta \;\; \psi]^T \tag{2}$$

which stand for the position of the center of gravity, respect to the inertial frame I, and the Euler angles roll, pitch and yaw, respectively. The motion equations are given by the Newton-Euler equations in the inertial frame I (see e.g. [9])

$$m\ddot{\xi} = TRe_3 - mge_3 \tag{3}$$

$$J\dot{\Omega} = -\Omega_x J\Omega + \Gamma \tag{4}$$

where $T \in \Re^+$ is the total thrust from the motors, g is the gravity constant and $\Gamma \in \Re^3$ is the control torque defined in the body fixed frame B. $R \in SO(3) : B \to I$ is the rotational matrix from the body frame to the inertial one.

$$\Omega = [p \;\; q \;\; r]^T \tag{5}$$

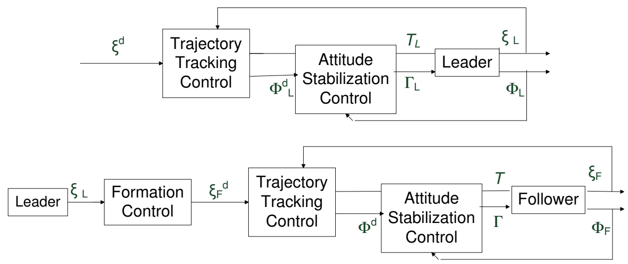

Fig. 2. Control estrategy block diagram.

represents the angular velocity in the body frame B. Ω_x stands for the skew symmetric matrix such that $\Omega_x \upsilon = \Omega x \upsilon$, this is the vectorial cross product. The kinematic relation between the generalized velocities $\dot{\Phi} = (\dot{\phi}, \dot{\theta}, \dot{\psi})$ and the angular velocity Ω is expressed by (see e.g. [10])

$$\Omega = Q\dot{\Phi} \tag{6}$$

with

$$Q = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta)\sin(\phi) \\ 0 & -\sin(\phi) & \cos(\theta)\cos(\phi) \end{bmatrix} \tag{7}$$

III. CONTROL STRATEGY

Assuming that the closed loop dynamics of rotation is much faster than the translational one, it is possible to separate the model in two independent subsystems [11]. The strategy, as shown in the block diagram of Fig. 2, is based on the designing of a controller for the translational dynamics such that it guarantees the trajectory tracking, providing as its output the desired orientation to be feed as input to the attitude stabilization controller. Such trajectory tracking control is used for both UAVs, the leader and the follower, so that they reach a desired position. A third control is designed for the follower quadrotor to solve the formation problem in the XY plane, with constant and equal height z for the leader and follower, so that the distance and orientation angle between them are kept to constant values λ and φ, respectively.

A. Trajectory Tracking Control

Defining the position error $\bar{\xi} = \xi - \xi_d$ and substituting into (3) leads to

$$m\ddot{\bar{\xi}} = (TRe_3)_d - mge_3 - m\ddot{\xi}_d \tag{8}$$

Let us consider the so called switching function [12]

$$\sigma_1 = k_1\bar{\xi} + k_2\int \bar{\xi}\,\mathrm{d}t + \dot{\bar{\xi}} \tag{9}$$

where k_1, k_2 are constant control parameters. It is desired that the system remains on the surface defined by $\sigma_1 = 0$, since on this surface one has the error dynamics

$$k_1\dot{\bar{\xi}} + k_2\bar{\xi} + \ddot{\bar{\xi}} = 0 \tag{10}$$

that assures to have an asymptotic convergence of $\bar{\xi} \to 0$ by choosing adequate values of k_1 and k_2. The so called equivalent control u_{eq} that guarantees the dynamics of the system to stay in the surface $\sigma_1 = 0$ is obtained from the condition $\dot{\sigma}_1 = 0$. Substituting (8) in this last equation leads to

$$k_1\dot{\bar{\xi}} + k_2\bar{\xi} + \frac{1}{m}(TRe_3)_d - ge_3 - \ddot{\xi}_d = 0 \tag{11}$$

Consider now $(TRe_3)_d$ to be the control input, then, the equivalent control u_{eq} is given by

$$u_{eq} = [(TRe_3)_d]_{eq} = m(ge_3 + \ddot{\xi}_d - k_1\dot{\bar{\xi}} - k_2\bar{\xi}) \tag{12}$$

It is interesting to observe that the equivalent control (12) coincides with the feedback linearization control presented in [7].
To attract the system dynamics to the surface $\sigma_1 = 0$ and keep it there, despite uncertainties and perturbations, a discontinuity is added. Let us consider the function $sgn(x)$ defined as

$$sgn(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases} \tag{13}$$

and the vector

$$Sgn(\sigma_1) = \begin{bmatrix} sgn(\sigma_{11}) \\ sgn(\sigma_{12}) \\ sgn(\sigma_{13}) \end{bmatrix} \tag{14}$$

where σ_{11}, σ_{12}, σ_{13} are the components of the vector σ_1. Then by making the assignment

$$\dot{\sigma}_1 = k_1\dot{\bar{\xi}} + k_2\bar{\xi} + \frac{1}{m}(TRe_3)_d - ge_3 - \ddot{\xi}_d = -L_\xi Sgn(\sigma_1) \tag{15}$$

with L_ξ a real, positive constant different from zero, it is possible to attract the system trajectories to the surface $\sigma_1 = 0$ in a finite time. The discontinuity control obtained from (15) is given by

$$(TRe_3)_d = u_{eq} - mL_\xi Sgn(\sigma_1) \tag{16}$$

In order to analyse the robustness of the control scheme, a model with bounded uncertainties $\Delta f(\xi)$ of the following form is considered:

$$m\ddot{\bar{\xi}} = (TRe_3)_d - mge_3 - m\ddot{\xi}_d + \Delta f(\xi) \tag{17}$$

where it is supposed that $||\Delta f(\xi)|| < \iota$ with ι a positive constant. Let us consider the Lyapunov's candidate function

$$V = \frac{1}{2}\sigma_1^T\sigma_1 \tag{18}$$

Differentiating (18) with respect to time leads to

$$\dot{V} = \sigma_1^T\dot{\sigma}_1 \tag{19}$$

or, equivalently,

$$\dot{V} = \sigma_1^T(k_1\dot{\bar{\xi}} + k_2\bar{\xi} + \ddot{\bar{\xi}}) \tag{20}$$

By substituting (17) in this last expression allows to write (20) as

$$\dot{V} = \sigma_1^T(k_1\dot{\bar{\xi}} + k_2\bar{\xi} + \frac{1}{m}(TRe_3)_d - ge_3 - \ddot{\xi}_d + \frac{1}{m}\Delta f(\xi)) \tag{21}$$

Using the control law (16)-(12), $\dot{V}$ takes the form

$$\dot{V} = \sigma_1^T(-L_\xi Sgn(\sigma_1) + \frac{1}{m}\Delta f(\xi)) \qquad (22)$$

from which

$$\dot{V} \leq ||\sigma_1||(-L_\xi + \frac{1}{m}\iota) \qquad (23)$$

and $\dot{V}$ will be negative defined when $L_\xi \geq \frac{1}{m}\iota$.
So the sliding mode control law (16)-(12) solves the trajectory tracking problem. It is important to notice that

$$R_d e_3 = \begin{bmatrix} R_{dx} \\ R_{dy} \\ R_{dz} \end{bmatrix} = \frac{(TRe_3)_d}{T_d} \qquad (24)$$

with $T_d = \|(TRe_3)_d\|$. Besides, if ψ_d is constant, it is possible to write ϕ_d and θ_d explicitly as

$$\phi_d = \arcsin\left(-\frac{R_{dy} - R_{dx}\tan(\psi_d)}{\sin(\psi_d)\tan(\psi_d) + \cos(\psi_d)}\right) \qquad (25)$$

$$\theta_d = \arcsin\left(\frac{R_{dx} - \sin(\phi_d)\sin(\psi_d)}{\cos(\phi_d)\cos(\psi_d)}\right) \qquad (26)$$

For the attitude stabilization control, a proportional-derivative controller is proposed, that acts on the orientation error defined by $\bar{\Phi} = \Phi - \Phi_d$, this is

$$\Gamma = -k_{do}\dot{\bar{\Phi}} - k_{po}\bar{\Phi} \qquad (27)$$

with $k_{do}, k_{po} \in \Re^+$.

B. Formation Control

The translational dynamics of the leader UAV in the XY plane can be described as

$$\dot{x}_i = v_{ix}\cos(\psi_i) - v_{iy}\sin(\psi_i) \qquad (28)$$

$$\dot{y}_i = v_{ix}\sin(\psi_i) + v_{iy}\cos(\psi_i) \qquad (29)$$

$$\dot{\psi}_L = \omega_L \qquad (30)$$

where v_{ix}, v_{iy} are the velocity components in the x and y directions (body frame coordinates, B) ω_i is the angular velocity for the yaw angle and the subindex i defines either, the leader ($i = L$), or the follower ($i = F$).
It is desired to maintain the follower quadrotor to a distance λ and an angle φ from the leader (see Fig. 3). Let λ_x, λ_y be the x and y coordinates of the vector drawn from the mass center of the leader to the one of the follower, in the leader's body fixed frame (B_L). Then

$$\lambda_x = -(x_L - x_F)\cos(\psi_L) - (y_L - y_F)\sin(\psi_L) \qquad (31)$$

$$\lambda_y = (x_L - x_F)\sin(\psi_L) - (y_L - y_F)\cos(\psi_L) \qquad (32)$$

and also

$$\lambda_x = \lambda\cos(\varphi) \qquad (33)$$

$$\lambda_y = \lambda\sin(\varphi) \qquad (34)$$

Differentiating (31) with respect to time and using (28), (29), (30) and (32) one obtains

$$\dot{\lambda}_x = \lambda_y\omega_L + \dot{x}_F\cos(\psi_L) + \dot{y}_F\sin(\psi_L) - v_{Lx} \qquad (35)$$

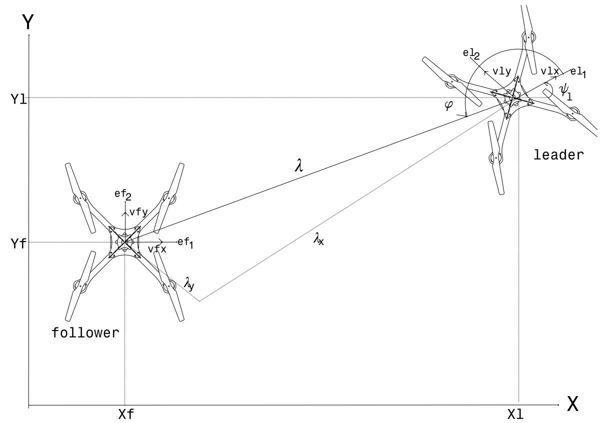

Fig. 3. Qadrotors formation in XY plane.

By defining the orientation error $e_\psi = \psi_F - \psi_L$ and employing the trigonometric identities for the sine and cosine of the difference of angles, as well as the equations (28) and (29), one gets

$$\dot{\lambda}_x = \lambda_y\omega_L + v_{Fx}\cos(e_\psi) - v_{Fy}\sin(e_\psi) - v_{Lx} \qquad (36)$$

Following a similar reasoning, one can write $\dot{\lambda}_y$ as

$$\dot{\lambda}_y = -\lambda_x\omega_L + v_{Fx}\sin(e_\psi) + v_{Fy}\cos(e_\psi) - v_{Ly} \qquad (37)$$

Considering the formation errors $e_x = \lambda_x^d - \lambda_x$, $e_y = \lambda_y^d - \lambda_y$, with λ^d and φ^d constant ($\dot{\lambda}_x^d = \dot{\lambda}_y^d = 0$) one obtains

$$\dot{e}_x = -(\lambda_y^d - e_y)\omega_L - v_{Fx}\cos(e_\psi) + v_{Fy}\sin(e_\psi) + v_{Lx} \qquad (38)$$

$$\dot{e}_y = (\lambda_x^d - e_x)\omega_L - v_{Fx}\sin(e_\psi) - v_{Fy}\cos(e_\psi) + v_{Ly} \qquad (39)$$

$$\dot{e}_\psi = \omega_F - \omega_L \qquad (40)$$

From the formation error dynamics (38), (39) y (40), a control law is now designed that allows to make the errors e_x, e_y and e_ψ to stay close to zero besides the presence of bounded uncertainties. For doing this, the follower velocities are considered as the formation control inputs. The dynamics of the formation error can be written as

$$\dot{\chi} = F(\chi) + G(\chi)\upsilon \qquad (41)$$

where

$$\chi = \begin{bmatrix} e_x \\ e_y \\ e_\psi \end{bmatrix} \qquad (42)$$

$$\upsilon = \begin{bmatrix} v_{Fx} \\ v_{Fy} \\ \omega_F \end{bmatrix} \qquad (43)$$

$$F(\chi) = \begin{bmatrix} e_y\omega_L + \gamma_1 \\ -e_x\omega_L + \gamma_2 \\ e_\psi \end{bmatrix} \qquad (44)$$

$$G(\chi) = \begin{bmatrix} -ce_\psi & se_\psi & 0 \\ -se_\psi & -ce_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (45)$$

with

$$\gamma_1 = v_{Lx} - \omega_L \lambda_y^d \quad (46)$$

$$\gamma_2 = v_{Ly} + \omega_L \lambda_x^d \quad (47)$$

It is easy to verify that the matrix $G(\chi)$ is always full rank. The sliding mode technique is proposed to be used for the development of a robust controller. Let the switching function be

$$\sigma_2 = \chi + k_f \int \chi dt \quad (48)$$

where k_f is a constant matrix to be chosen. It is desired that the error dynamics remains in the surface defined by $\sigma_2 = 0$ since on such a surface the error dynamics tends to zero by adequately choosing k_f. On this surface $\dot{\sigma}_2 = 0$, thus from (41), one gets

$$\dot{\sigma}_2 = \dot{\chi} + k_f \chi = F(\chi) + G(\chi)\upsilon_{eq} + k_f \chi = 0 \quad (49)$$

In the ideal case, this is, without perturbations or uncertainties, the following equivalent control υ_{eq} assures that the error dynamics remains on $\sigma_2 = 0$:

$$\upsilon_{eq} = G^{-1}(\chi)(-F(\chi) - k_f \chi) \quad (50)$$

To bring the system to the surface $\sigma = 0$ despite the presence of uncertainties and perturbations it is necessary to extend the region of attraction. More precisely, one makes the assignment

$$\dot{\sigma}_2 = \dot{\chi} + k_f \chi = F(\chi) + G(\chi)\upsilon + k_f \chi = -LSign(\sigma_2) \quad (51)$$

where L is a constant, positive control parameter. Then, the following discontinuous control is obtained

$$\upsilon = G^{-1}(\chi)(-F(\chi) - k_f \chi - LSgn(\sigma_2)) \quad (52)$$

In order to assure the attractiveness to the surface $\sigma_2 = 0$, even in the presence of bounded uncertainties and perturbations, a perturbed model for the error dynamics is considered, this is

$$\dot{\chi} = F(\chi) + \Delta F(\chi) + G(\chi)\upsilon \quad (53)$$

where $\Delta F(\chi)$ is an unknown uncertainty term satisfying $||\Delta F(\chi)|| < \iota_1$, being ι_1 a positive constant. Let us consider the Lyapunov's candidate function

$$V = \frac{1}{2}\sigma_2^T \sigma_2 \quad (54)$$

When differentiating V with respect to time one has

$$\dot{V} = \sigma_2^T \dot{\sigma}_2 = \sigma_2^T(\dot{\chi} + k_f \chi) \quad (55)$$

or, equivalently,

$$\dot{V} = \sigma_2^T(F(\chi) + \Delta F(\chi) + G(\chi)\upsilon + k_f \chi) \quad (56)$$

Substituting (52) in the previous expression one obtains

$$\dot{V} = \sigma_2^T(\Delta F(\chi) - LSgn(\sigma_2)) \quad (57)$$

from where

$$\dot{V} \leq ||\sigma_2||\iota_1 - L||\sigma_2|| \quad (58)$$

So, if L is chosen such that $L > \iota_1$, it is assured that $\dot{V}$ is negative definite. Therefore, the system will reach and remain in the surface $\sigma_2 = 0$ in a finite time. From the previous analysis, one can notice that the desired velocity is obtained to be feed as control input to the trajectory tracking control in the follower UAV. On the other hand, from (31), (32) one obtains the desired position

$$x_F = x_L + \lambda_x \cos(\psi_L) - \lambda_y \sin(\psi_L) \quad (59)$$

$$y_F = y_L + \lambda_x \sin(\psi_L) + \lambda_y \cos(\psi_L) \quad (60)$$

IV. EXPERIMENTAL RESULTS

In order to evaluate the proposed control strategy, an experimental platform was developed consisting of a quadrotor which has an embedded digital signal processor (DSP), an inertial measurement unit (IMU), four actuators (motors with drivers) and a wireless communication system to accomplish autonomous hover flight. In order to have the quadrotor position, a motion capture system integrated by twelve infra red cameras is employed. The capture system has a set of markers attached to the UAV, reflecting the infra red light emitted by the cameras. This system provides the quadrotor's position at a frequency of 100 Hz in a submillimetric accuracy, and sends it to the UAV using a wireless modem. The experiment described here consists of a virtual leader tracking a circular trajectory in the $z = 0.8m$ plane with a radius of 0.8 meters and a 10 seconds period, while a real follower UAV tries to stay in formation exactly on the position of the virtual leader; this is, a distance $\lambda = 0$m and an angle $\varphi = 0$rad with respect to the leader is selected. The results are shown in Fig. 4 through Fig. 12. First the x, y and z coordinates of the position are shown in Fig. 4, Fig. 5 and Fig. 6, respectively, while the position error is given in Fig. 7, where it can be appreciated that the performance of the control scheme proposed is quite satisfactory, specially in the x axis. Even if the errors do not converge to zero, they oscillate in a bounded neighbourhood of the origin. The orientation is shown in Fig.8, while velocities in x and y are given in Fig. 9 and Fig. 10. Finally, the top and space views can be observed in Fig.11 and Fig. 12, respectively. The observed errors could be due to problems with the experimental platform or bad tuned gains. It is desired to improve the experimental platform to get better results. Diferent helicopters and sensors are beeing studied.

V. CONCLUSIONS AND FUTURE WORK

In this work a control strategy to solve the trajectory tracking and flight formation problem, in a leader-follower scheme, for aerial robots (quadrotors) was presented. The strategy was successfully tested in real experiments.

Embedded positioning sensors, such as global positioning systems (GPS), have to be used instead of the motion capture system for outdoor applications. However, these sensors are much less precise, adding uncertainty to the system. Therefore, it is expected that the robustness property of the sliding mode technique can help to handle these uncertainties in future outdoor experiments.

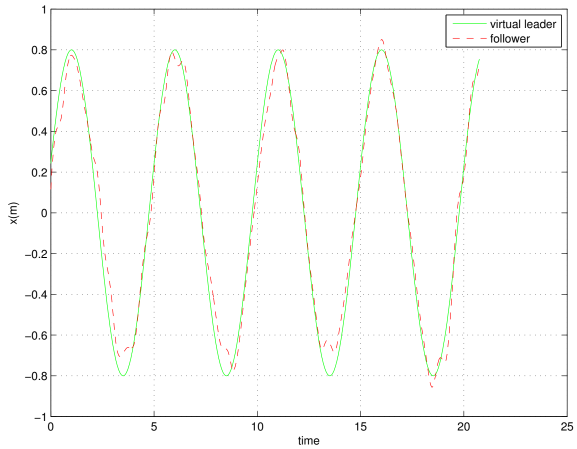

Fig. 4. X position.

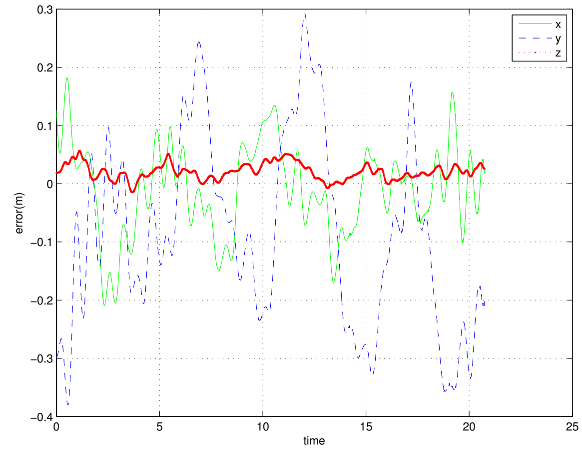

Fig. 7. Position error.

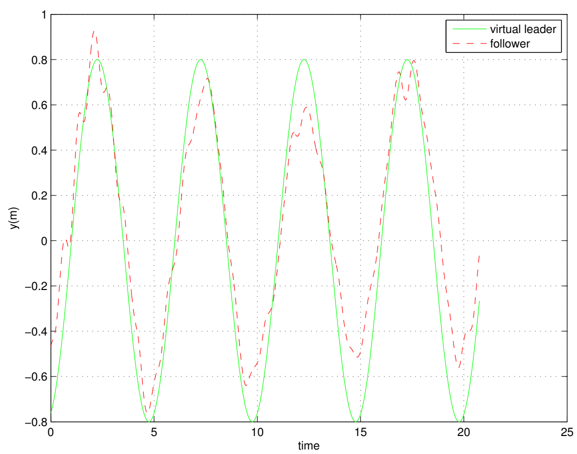

Fig. 5. Y position.

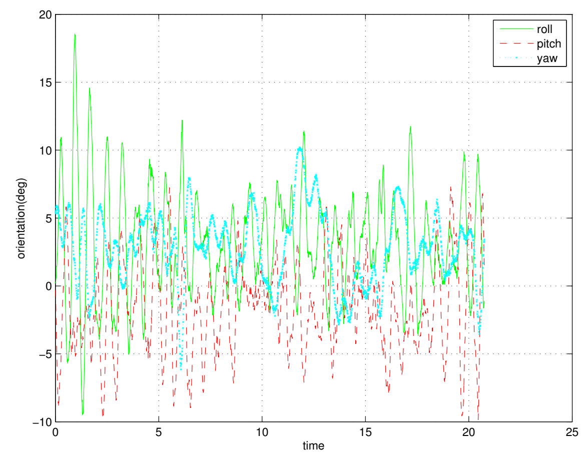

Fig. 8. Orientation.

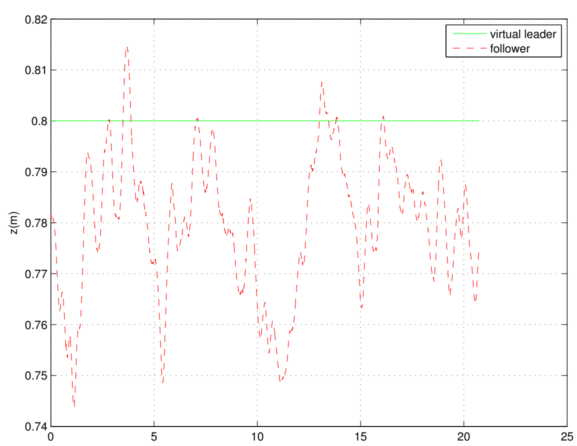

Fig. 6. Z position.

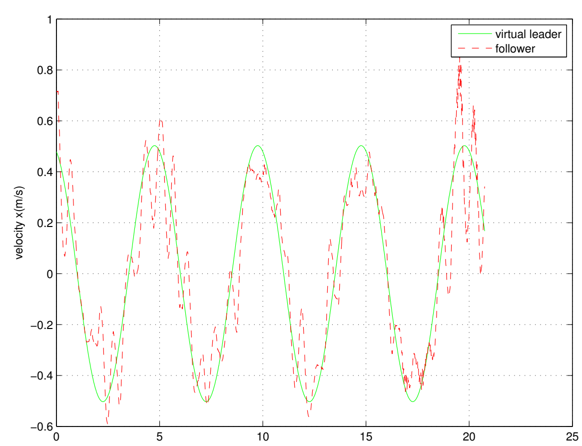

Fig. 9. X velocity.

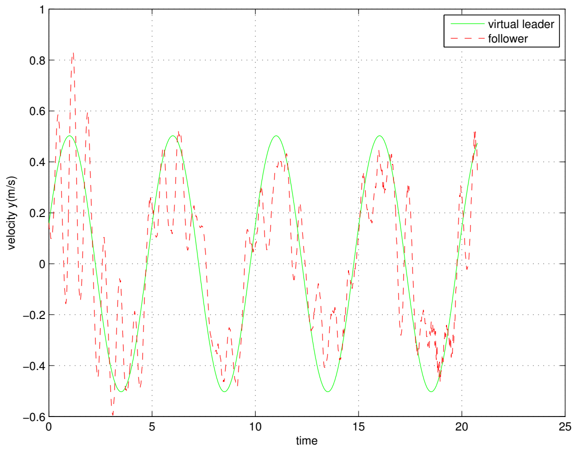

Fig. 10. Y velocity.

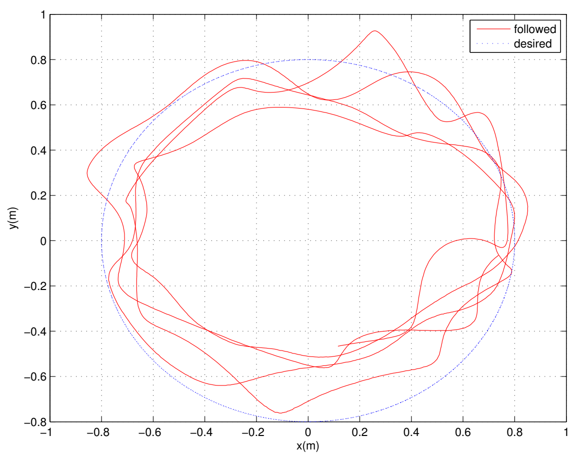

Fig. 11. Top view.

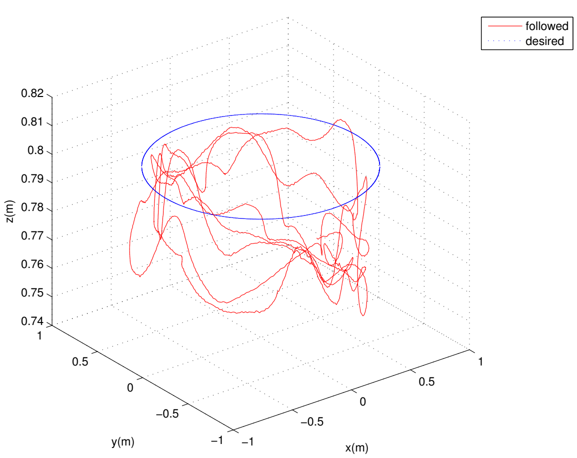

Fig. 12. Space view.

Serious issues were found when trying to implement the formation control with two UAVs in flight, because of the perturbation produced by the air stream generated during the flight. So future work includes solving this problem and implement the control strategy for two real quadrotors. Also, it is desired to extend the control strategy to more than one follower maneuvering in the space, as well as to explore and implement other control strategies for the trajectory tracking and the formation control.

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