Qualitative multiple criteria choice analysis

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THE DOMINANT REGIME METHOD

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THE DOMINANT REGIME METHOD

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1. Introduction to Qualitative Multiple Criteria Choice Analysis

Choice analysis (or evaluation) aims at rationalizing decision problems characterized by conflicting objectives regarding alternative options (or choice possibilities). Although theoretically the choice set may have an infinite number of options, in reality often a discrete set of distinct alternatives is taken into consideration. For instance, consumers make a choice out of a limited number of commodity bundles, or governments only consider a finite set of policy packages, plans or projects.

In the framework of discrete choice analysis with conflicting objectives several evaluation methods have been developed in the past decade, which served to make complex multidimensional choice problems more transparent. All these methods, usually coined multiple criteria choice methods, paid attention to the constituents of a choice problem, viz. the identification of relevant choice options, the definition of appropriate judgement criteria (emanating from conflicting objectives), the assessment of the numerical value of each judgement criterion for each choice option (e.g., by means of an impact analysis), the collection of prior information regarding each of the relevant decision criteria (e.g., by means of weights or interactive computer methods), the identification of relevant decision levels or of institutional decision procedures (in case of a multi-actor choice situation), and the specification of a suitable measurement system for available information (e.g., ratio, ordinal or fuzzy information).

Clearly, multi-attribute decision theory may under certain conditions prove a meaningful vehicle for analysing the abovementioned type of multi-dimensional choice problems (see Lancaster, 1971, and Keeney and Raiffa, 1976). However, in this case one usually needs a great deal of quantifiable information on measurable characteristics (or impacts) of a choice option as well as on trade-offs among different value criteria of the choice maker. In practice, such cardinal information (measured on a ratio scale) is often not available, so that conventional evaluation methods - either monetary-based methods (such as cost-benefit analysis, cost-effectiveness analysis or shadow project methods) or cardinal multiple criteria methods (such as goals-achievement analysis, concordance analysis or weighted summation analysis) - cannot be applied as methodologically sound tools for choice analysis. Usually choice problems are marked by the presence of qualitative, 'soft' or fuzzy i formation, and consequently the design of evaluation methods focusing on 'measuring the unmeasurable' (Nijkamp et al., 1985) is an important research direction in evaluation analysis. Such methods are based on ordinal, binary or nominal measurement scales and are called here 'qualitative multiple criteria choice methods'. These qualitative multidimensional judgement methods are essentially more in agreement with 'satisficing behaviour' (based on Simon's bounded rationality principle) than with conventional 'optimizing decision-making', as imprecise information precludes the attainment of an unambiguous maximum solution.

In the past years a wide variety of qualitative multiple criteria choice methods has been developed (see for a survey among others Rietveld, 1980, and Voogd, 1983). Examples of classes of such methods are:

- survey table methods (for instance, score card methods and computer-graphic methods)
- interactive computer methods (for instance, based on an interplay between expert and decision-maker)
- weighting methods (based on a set of weights reflecting the relative importance attached to the successive value criteria).

In the present paper, the attention will mainly be devoted to weighting methods. Weights can be assessed on the basis of different techniques, such as revealed preference methods (based on actual choice behaviour), stated preference methods (based on interview questionnaires, e.g.), or fictitious preference methods (based on simulation or dissimulation experiments, e.g.).

In the field of multiple criteria analysis a whole series of different qualitative evaluation methods has been developed, such as the eigenvalue (or prioritization) method (see Saaty, 1977, and Lootsma, 1980), the extreme expected value method (see Kmietowicz and Pearman, 1981, and Rietveld, 1980), the permutation method (see Mastenbroek and Paelinck, 1977), the frequency method (see Van Delft and Nijkamp, 1977), the multidimensional scaling method (see Nijkamp and Voogd, 1981) and the mixed data method (see Voogd, 1983). Unfortunately, various of these qualitative multiple criteria choice methods treat qualitative information as pseudo-cardinal information, so that their methodological basis is questionable. Despite the less correct treatment of categorical or ordinal information in several of these methods, they have become fairly popular analytical tools thanks to their simplicity. Some more complicated analytical techniques (such as geometric scaling methods) are scientifically more justified, but less accessible to decision-makers because of their statistical-mathematical contents.

Consequently, there is apparently a conflict between simple but wrong methods on the one hand and complex but good methods on the other. In the search of a compromise between the requirements emanating from methodological soundness, accessibility and mathematical-statistical simplicity recently a new method has emerged, the so-called regime method (see Hinloopen et al., 1983). The present paper will present first some of the principles of this method (section 2), while next various new elements and extensions of this method will be developed (section 3). For pedagogical reasons our presentation will be based on some numerical examples. It will be concluded that the regime method is able to encapture a wide variety of qualitative multiple criteria choice problems based on both ordinal and mixed ordinal-cardinal data regarding both the characteristics (or impacts) of a choice option and the weights (or priorities) of a choice maker.

2. Principles of the Regime Method

The regime method for qualitative multiple criteria choice analysis is based on the following considerations;

- the technique should not use methodologically unpermitted operations (for instance, summation or multiplication of ordinal numbers)
- the technique should be as much accessible as possible to a choice maker
- the technique should be easily applicable on a computer
- the application of the regime method should in principle lead to an unambiguous solution, so that always a dominant choice option is identified.

In the sequel of this section, the essence and structure of the regime method will be further described.

Suppose we have a discrete choice problem with I choice options or alternatives i ($i=1,\ldots,I$), characterized by J judgement criteria j ($j=1,\ldots,J$). The basic information we have is composed of qualitative data regarding the ordinal value of all J judgement criteria for all I choice options. In particular we assume a partial ranking of all I choice options for each criterion j, so that the following effect matrix can be constructed:

$$E = \begin{bmatrix} e_{11} & \cdots & e_{1J} \\ \vdots & & \vdots \\ e_{I1} & \cdots & e_{IJ} \end{bmatrix} \qquad (2.1)$$

The entry e_{ij} ($i=1,\ldots,I$; $j=1,\ldots,J$) represents thus the rank order of alternative i according to judgement criterion j. Without loss of generality, we may assume a rank order characterized by the condition 'the higher, the better', in other words: if $e_{ij} > e_{i'j}$, then choice option i is preferable i' for judgement criterion j.

As there is usually not a single dominating alternative, we need additional information on the relative importance of (some of) the judgement criteria. In case of weighting methods this information is given by means of preference weights attached to the successive criteria. If we deal with ordinal information, the weights are represented by means of rank orders w_j $(j=1,\ldots,J)$ in a weight vector w:

$$w = (w_1,\ldots,w_J)^T \qquad (2.2)$$

Clearly, it is again assumed that $w_j > w_{j'}$ implies that criterion j is regarded as more important than j'.

Next, the regime method uses a pairwise comparison of all choice options, so that then the mutual comparison of two choice options is not influenced by the presence and effects of other alternatives. Of course, the eventual rank order of any two alternatives is co-determined by remaining alternatives (cf. the independence of irrelevant alternatives problem).

In order to explain the mechanism of the regime method, we will first define the concept of a regime. Consider two alternative choice options i and i'. If for criterion j a certain choice option i is better than i' (i.e. $s_{ii'j} = e_{ij} - e_{i'j} > 0$), it should be noted that in case of ordinal information, the order of magnitude of $s_{ii'j}$ is not relevant, but only its sign. Consequently, if $\sigma_{ii'j}$ = sign $s_{ii'j} = +1$, then alternative i is better than i' for criterion j. Otherwise, $\sigma_{ii'j} = -1$, or (in case of ties) $\sigma_{ii'j}=0$. By making such a pairwise comparison for any two alternatives i and i' for all criteria $j(j=1,\ldots,J)$, we may construct a $J\times1$ regime vector $r_{ii'}$, defined as:

$$r_{ii'} = (\sigma_{ii'1},\ldots,\sigma_{ii'J})^T, \qquad \forall\ i,i',\ i'\neq i \qquad (2.3)$$

Thus, the regime vector contains only + and - signs (or in case of ties also 0 signs), and reflects a certain degree of (pairwise) dominance of choice option i with respect to i' for the unweighted effects for all J judgement criteria. Clearly, we have altogether I(I-1) pairwise comparisons, and hence also I(I-1) regime vectors. These regime vectors can be included in an $J\times I(I-1)$ regime matrix R:

$$R = \left[\begin{array}{cccccccc} \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ r_{14} & r_{13} & \cdots & r_{1I} & r_{21} & \cdots & r_{I1} & \cdots r_{I(I-1)} \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot & \cdot \end{array}\right] \qquad (2.4)$$
$$\underbrace{\qquad\qquad\qquad}_{I-1} \qquad \underbrace{\qquad\qquad\qquad}_{I-2}$$

It is evident that, if a certain regime vector $r_{ii'}$ would only contain + signs, alternative i would absolutely dominate i'. Usually however a regime vector contains both + and - signs, so that then additional information in the form of the weights vector (2.2) is required.

In order to treat ordinal information on weights, the assumption is now made here that the ordinal weights w_j $(j=1,\dots,J)$ are a rank order representation of an (unknown) underlying cardinal stochastic weight vector $\underline{w}^* = (\underline{w}_1^*,\dots,\underline{w}_J^*)^T$ with $\max_j\{\underline{w}_j^*\} = 1$, $\underline{w}_j^* \geq 0$, $\forall_j$. The ordinal ranking of the weights is thus supposed to be consistent with the quantitative information incorporated in an unknown cardinal vector w^*; in other words: $w_j > w_{j'} \rightarrow w_j^* > w_{j'}^*$. Next, we assume that the weighted dominance of choice option i with regard to i' can be represented by means of the following stochastic expression based on a weighted summation of cardinal entities (implying essentially a additive linear utility structure):

$$\underline{v}_{ii'} = \sum_{j=1}^{J} \sigma_{ii'j}\underline{w}_j^* \qquad (2.5)$$

If $\underline{v}_{ii'}$ is positive, choice option i is dominant with respect to i'. However, in our case we do not have information on the cardinal value of $\underline{w}_j^*$, but only on the ordinal value of w_j (which is assumed to be consistent with $\underline{w}_j^*$). Therefore, we introduce a certain probability $p_{ii'}$ for the dominance of i with respect to i':

$$p_{ii'} = \text{prob}\ (v_{ii'} > 0) \qquad (2.6)$$

and define as an aggregate probability measure:

$$p_i = \frac{1}{I-1} \sum_{i' \neq i} p_{ii'} \qquad (2.7)$$

Then it is easily seen that p_i is the average probability that alternative i is higher valued than any other alternative. Consequently, the eventual rank order of choice options is then determined by the rank order (or the order of magnitude) of the p_i's.

However, the crucial problem here is to assess $p_{ii'}$ and p_i. This implies that we have to make an assumption about the probability distribution function of both the $\underline{w}_j^*$'s and of the $s_{ii'j}$'s. In view of the ordinal nature of the w_j's, it is plausible to assume for the whole relevant area a uniform density function for the $\underline{w}_j^*$'s. The motive is that, if the ordinal weights vector w is interpreted as originating from a stochastic weight vector $\underline{w}^*$, there is without any prior information no reason to assume that a certain numerical value of $\underline{w}^*$ has a higher probability than any other value. In other words, the weights vector $\underline{w}^*$ can adopt with equal probability each value that is in agreement with the ordinal information implied by w. This argument is essentially based on the 'principle of insufficient reason', which also constitutes the foundation stone for the so-called Laplace criterion in case of decision-making under uncertainty (see Taha, 1976). However, if due to prior information in a specific case there is reason to assume a different probability distribution function (a normal distribution, e.g.), there is no reason to exclude this new information. Of course, this may influence the values of $p_{ii'}$ and hence the ranking of alternatives. The precise way in which in general rank order results will be derived from a probability distribution in case of qualitative information will be further discussed in section 3. But it may suffice to mention here that in principle the use of stochastic analysis, which is consistent with an originally ordinal data set, may help to overcome the methodological problem emanating from impermissible numerical operations on qualitative data.

Another remark concerns the meaning embodied by $\sigma_{ii'j}$. Our approach implies that in case of a pairwise comparison of two non-numerically different alternatives the differences in effects are assumed to be negligible, i.e., $\sigma_{ii'j}=0$. This assumption corresponds essentially to that implied by Kendall's rank correlation coefficient. Thus, only if two distinct choice options can be distinguished in measurable terms (either cardinal or ordinal), we have a possibility to represent these differences somehow in numerical form. If these differences are measured in an ordinal sense, then these differences may again be interpreted as stochastic variables which stem from an underlying cardinal uniform probability distribution. This approach is then again based on the principle of insufficient reason and hence similar to that described above for the weights. The precise use of such a probabilistic approach will also be further explained in the next section.

So far we have neither paid much attention to a situation with ties, nor to that with mixed information (i.e., partly cardinal, partly ordinal data). Especially mixed information is a frequently occurring phenomenon in qualitative multiple criteria choice analysis. These various situations, viz. strictly ordinal data, ties and mixed information, are systematically included in the following scheme and will successively be dealt with in sections 3 and 4.

<table>
<tr><th colspan="2">criteria / effects</th><th colspan="2">ordinal</th><th colspan="2">mixed</th></tr>
<tr><th colspan="2"></th><th>no ties</th><th>ties</th><th>no ties</th><th>ties</th></tr>
<tr><td rowspan="2">ordinal</td><td>no ties</td><td>section 3</td><td>subsection 4.2</td><td colspan="2" rowspan="4">subsection 4.3</td></tr>
<tr><td>ties</td><td colspan="2">subsection 4.1</td></tr>
<tr><td rowspan="2">mixed</td><td>no ties</td><td colspan="2" rowspan="2"></td></tr>
<tr><td>ties</td></tr>
</table>

3. Structure of the Standard Regime Method

The regime method was originally developed for purely qualitative information on multidimensional choice problems, and aimed even at designing a choice evaluation method which did not need the use of a computer. However, it turned out that in case of many criteria (7 or more), of the presence of ties, or of the presence of mixed data, computer assistance was necessary. For simple examples (i.e., with a low number of criteria), however, numerical illustrations can without loss of generality directly be used to explain the basic steps of the regime algorithm.

In our case we assume the following choice problem. A decision-maker has to make a choice out of 3 alternative commodities (goods, plans, projects, etc.), which are characterized by 4 attributes (features, impacts, etc.), measured in a strictly ordinal sense (i.e., without ties). Then we may assume the following effect matrix E:

E = (3.1)

alternative \ criterion	1	2	3	4
1	3	2	1	1
2	2	1	2	3
3	1	3	3	2

Furthermore, we assume the following weight vector w:

$$\underline{w} = (\underline{w}_1 \quad \underline{w}_2 \quad \underline{w}_3 \quad \underline{w}_4)^T = (\ 4 \quad 3 \quad 2 \quad 1\ )^T \qquad (3.2)$$

which of course also implies the following consistency condition for the cardinal weights: $w_1^* > w_2^* > w_3^* > w_4^*$. A pairwise comparison of the information in E leads to the following regime matrix R:

$$R = \begin{array}{c} \begin{array}{cccccc} V_{12} & V_{13} & V_{21} & V_{23} & V_{31} & V_{32} \end{array} \\ \begin{bmatrix} + & + & - & + & - & - \\ + & - & - & - & + & + \\ - & - & + & - & + & + \\ - & - & + & + & + & - \end{bmatrix} \end{array} \qquad (3.3)$$

For instance, if we take regime r_{12}, it is easily seen that:

$$\underline{v}_{12} = \underline{w}_1^* + \underline{w}_2^* - \underline{w}_3^* - \underline{w}_4^* \qquad (3.4)$$

and:

$$p_{12} = \text{prob}\ (\underline{v}_{12} > 0) = \text{prob}\{(\underline{w}_1^* + \underline{w}_2^* - \underline{w}_3^* - \underline{w}_4^*) > 0\} \qquad (3.5)$$

The question is now whether we can make any valid statement regarding the value of p_{12}. In this case the previous question is easy to answer, given the information implied by (3.2), viz. $w_1 > w_2 > w_3 > w_4 > 0$. Thus it can directly be derived that $p_{12} = 1$.

If we next take the choice alternatives 1 and 3, we can easily derive $\underline{v}_{13}$ by means of (3.3), i.e.,

$$\underline{v}_{13}=\underline{w}_1{*}-\underline{w}_2{*}-\underline{w}_3{*}-\underline{w}_4{*} \qquad (3.6)$$

In this case, a priori no unambiguous statement regarding the value of p_{13} can be made, unless we use the probability approach outlined in the previous section. If we thus assume that all $\underline{w}_j{*}$'s are uniformly distributed, we have to identify the relative size of the four-dimensional hyperplane for which condition (3.6) holds. The relative size of the various hyperplanes which make up the envelopes of the information embodied in the weight vector may thus be regarded as a probability measure for the dominance of the alternative concerned. Of course, one has to take into account the standardization condition that $\max\{w_1{*}, w_2{*}, w_3{*}, w_4{*}\}=1$ and $w_1{*}\geq0$, $w_2{*}\geq0$, $w_3{*}\geq0$, $w_4{*}\geq0$ (see for further details Hinloopen, 1985, and Hinloopen and Smyth, 1985). Then by using conditions (3.2) and (3.6) in addition to the standardization condition, we can easily derive the value p_{13} in case of a uniform distribution:

$$p_{13} = 1/6 \qquad (3.7)$$

Finally, we will compare choice options 2 and 3. Then we have:

$$\underline{v}_{23}=\underline{w}_1{*}-\underline{w}_2{*}-\underline{w}_3{*}+\underline{w}_4{*} \qquad (3.8)$$

In this case, we can easily derive that:

$$p_{23} = 1/2 \qquad (3.9)$$

Now we can directly derive the total dominance of each choice option by means of (2.7), i.e.,

$$\left.\begin{array}{l} p_1 = 1/2(1 + 1/6)=7/12 \\ p_2 = 1/2(0 + 1/2)=1/4 \\ p_3 = 1/2(5/6+1/2)=2/3 \end{array}\right\} \qquad (3.10)$$

Thus, in our illustrative example the following final ranking of alternatives results:

alternative 3 > alternative 1 > alternative 2

It is evident that the foregoing example can easily be generalized in a formal notation, but as this notation is more cumbersome than illustrative we suffice to conclude here that the regime method provides a fairly direct and unambiguous solution to a strictly qualitative multiple criteria choice problem.

4. Ties and Mixed-Data in the Regime Analysis

In this section the additional problems caused by the presence of ties and mixed data for both the effects e_{ij} and the weights w_j will be dealt with.

4.1. Ties in the effect matrix

If the effect matrix contains ties (i.e., $e_{ij} = e_{i'j}$, in other words: equal rank orders of two alternatives i and i' for a specific criterion j), then the additional problems can easily be solved. This can easily be illustrated by including ties for the first criterion in the effect matrix (3.1):

	alternative \ criterion	1	2	3	4	
	1	2	2	1	1	
E =	2	2	1	2	3	(4.1)
	3	1	3	3	2	

In this case only the regime r_{12} will change, i.e.,

$$r_{12} = (0, +, -, -)^T \qquad (4.2)$$

so that:

$$\underline{v}_{12} = \underline{w}_2^* - \underline{w}_3^* - \underline{w}_4^* \qquad (4.3)$$

On the basis of (4.3), we can easily derive - by assuming again a uniform probability distribution - that:

$$p_{12} = 1/2 \qquad (4.4)$$

The existence of ties has clearly consequences for the final rank order of plans, as in the present case we have:

$$\left.\begin{array}{l} p_1 = 1/2(1/2 + 1/6) = 1/3 \\ p_2 = 1/2(1/2 + 1/2) = 1/2 \\ p_3 = 1/2(5/6 + 1/2) = 2/3 \end{array}\right\} \qquad (4.5)$$

so that choice options 1 and 2 have changed position in the eventual rank order. Clearly, without any difficulty this procedure can be directly generalized for the existence of multiple ties.

4.2. Ties in the weights vector

Ties in the weight vector (i.e., $w_j = w_{j'}$) imply a different situation regarding the evaluation of values of the choice criteria. It is evident, that also the existence of ties in the weight vector will not affect the regime vector, but no doubt the probabilities $p_{ii'}$ will alter. This will be illustrated by using again the same effect matrix (3.1), whereas the weight vector is assumed to be equal to:

$$w = (\ 4 \quad 3 \quad 2 \quad 2\ )^T \qquad (4.6)$$

The treatment of ties will first be illustrated by comparing alternatives 1 and 2. In this case the regime matrix is still equal to (3.3), whilst v_{12} remains also unchanged, as can be seen from (3.4). It is clear that in this case the same result emanates, i.e.,

$$p_{12} = 1 \qquad (4.7)$$

If we next compare choice options 1 and 3, $\underline{v}_{13}$ and p_{13} do not change either in comparison with (3.6). However, in this case we may substitute the fact that $w_3^* = w_4^*$ into (3.6), so that the new condition becomes:

$$v_{13} = w_1^* - w_2^* - 2w_3^* \qquad (4.8)$$

By pursuing next the same stochastic analysis by means of a uniform distribution, we find the result:

$$p_{13} = 1/3 \qquad (4.9)$$

Finally, we will compare alternatives 2 and 3. In this case v_{23} becomes:

$$\begin{aligned} v_{23} &= w_1^* - w_2^* - w_3^* + w_4^* \\ &= w_1^* - w_2^* \end{aligned} \qquad (4.10)$$

Then it is evident that $p_{23}=1$, as condition (4.10) is always satisfied, given the initial condition (3.2).

Consequently, we may find the following final results:

$$\left.\begin{array}{l} p_1 = 1/2(1 + 1/3) = 2/3 \\ p_2 = 1/2(0 + 1) = 1/2 \\ p_3 = 1/2(2/3 + 0) = 1/3 \end{array}\right\} \qquad (4.11)$$

so that now the final ranking of alternatives becomes:

alternative 1 > alternative 2 > alternative 3

Thus, ties may exert a significant impact on the eventual rank order of choice options.

Finally, it is worth mentioning that also a situation of combined ties in both the effect matrix and the weight vector can be handled in the same way.

4.3. Mixed data

In case of mixed data in either the effect matrix or the weight vector, the regime method has to be significantly adjusted, irrespective of the presence of ties.

First we will consider a situation of mixed data in the weight vector, so that part of the weights is ordinal and another part cardinal in nature. Then we impose the condition that all stochastic weights $\underline{w}_j^*$ are standardized as follows:

$$\lambda_j = \frac{w_j^*}{\max_{j'=1,\ldots,J}\{w_{j'}^*\}} \qquad (4.12)$$

It is easily seen that in this case $\underline{\lambda}_j \leq 1$, while the highest value of $\underline{\lambda}_j$ is always equal to 1. The motive for this specific way of standardizing the weights is that (since $\underline{w}_j^*$ are uniformly distributed) also the vector $\underline{\lambda} = (\underline{\lambda}_1,\ldots,\underline{\lambda}_J)^T$ is uniformly distributed.

Next, we will consider the presence of mixed data in the effect matrix.

Also in this case the (stochastic qualitative) differences $\underline{s}^*_{ii'j}$ are assumed to be uniformly distributed. In order to be able to compare the differences across different criteria, $\underline{s}^*_{ii'j}$ is also standardized, i.e.,

$$\underline{d}_{ii'j} = \frac{\underline{s}^*_{ii'j}}{\max\{\underline{e}_{ij},\underline{e}_{i'j}\}} \qquad (4.13)$$

Note that $\underline{d}_{ii',j}$ is also uniformly distributed, either on the interval (0, 1) (if $\underline{e}_{ij} \geq \underline{e}_{i'j}$) or on the interval (-1, 0) (if $\underline{e}_{ij} < \underline{e}_{i'j}$).

In order to compare now 2 alternatives i and i', we define - instead of $\underline{v}_{ii'}$ from (2.5) - a new stochastic variable $\underline{z}_{ii'}$ as follows:

$$\underline{z}_{ii'} = \sum_{j=1}^{J} \underline{d}_{ii'j} \underline{\lambda}_j \qquad , \qquad (4.14)$$

while next $p_{ii'}$ is according to (2.6) defined as:

$$p_{ii'} = \text{prob } (\underline{z}_{ii'} > 0) \qquad (4.15)$$

The remaining part of the procedure is then simular to that described in section 2.

The various steps of the abovementioned exposition will now be illustrated by means of a simple numerical example. Assume the following effect matrix:

$$E = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad (4.16)$$

while w is assumed to be equal to:

$$w = (2 \quad 1)^T \qquad (4.17)$$

The resulting regime matrix R is:

$$R = \begin{bmatrix} + & - \\ - & + \end{bmatrix} \qquad (4.18)$$

If we want to compare choice option 1 with 2, we find:

$$\underline{v}_{12} = \underline{w}_1^* - \underline{w}_2^* \qquad (4.19)$$

Consequently, we may derive:

$$\begin{aligned} p_{12} &= \text{prob } (\underline{v}_{12} > 0) \\ &= \text{prob } \{(\underline{w}_1^* - \underline{w}_2^*) > 0\} \qquad (4.20) \\ &= 1 \end{aligned}$$

since we know from (4.17) that $w_1 > w_2$ and hence also $\underline{w}_1^* > \underline{w}_2^*$.

In addition, we know that $\underline{d}_{12,1}$ and $-\underline{d}_{12,2}$ are uniformly distributed on the interval (0,1). Note that $\underline{\lambda}_1 = 1$ and $\underline{\lambda}_2$ is uniformly distributed on (0,1). Therefore, we derive that:

$$\underline{z}_{12} = \underline{d}_{12,1} + \underline{d}_{12,2} \, \underline{\lambda}_2 \qquad (4.21)$$

By using next our standard procedure for uniform densities, we find ultimately:

$$p_{12} = \text{prob}\ (\underline{z}_{12} > 0) = 3/4 \qquad (4.22)$$

The proof of (4.22) runs as follows.

$$\text{prob}\ (\underline{z}_{12}>0) =$$
$$= \text{prob}\{(\underline{z}_{12}>0 | (\underline{d}_{12,1}>-\underline{d}_{12,2})\}.\text{prob}(\underline{d}_{12,1}>-\underline{d}_{12,2})$$
$$+\text{prob}\{(\underline{z}_{12}>0 | (\underline{d}_{12,1}<-\underline{d}_{12,2})\}.\text{prob}(\underline{d}_{12,1}<-\underline{d}_{12,2})$$
$$(4.23)$$

As both $\underline{d}_{12,1}$ and $-\underline{d}_{12,2}$ are uniformly distributed on the interval (0,1), we know that:

$$\text{prob}(\underline{d}_{12,1}>-\underline{d}_{12,2}) = \text{prob}(\underline{d}_{12,1}<-\underline{d}_{12,2}) = 1/2 \quad (4.23)$$

while also the following condition holds:

$$\text{prob}\{\underline{z}_{12} > 0 | (\underline{d}_{12,1} > -\underline{d}_{12,2})\} = 1 \qquad (4.25)$$

Next, in order to calculate $\text{prob}\{\underline{z}_{12}>0 | \underline{d}_{12}<-\underline{d}_{12,2})\}$, we define:

$$\underline{\partial}_{12,2} = -\underline{d}_{12,2}/\underline{d}_{12,2} \qquad (4.26)$$

It is also easily seen that:

$$\underline{d}_{12,1} < -\underline{d}_{12,2} \qquad (4.27)$$

Consequently, $\underline{\partial}_{12,1}$ is uniformly distributed on the interval (0,1).
Next we may calculate $\underline{z}_{12}$:

$$\underline{z}_{12} = -\underline{d}_{12,2} \cdot \underline{\partial}_{12,1} + \underline{d}_{12,2}\ \underline{\lambda}_2$$
$$= -\underline{d}_{12,2}(\underline{\partial}_{12,1} - \underline{\lambda}_2) \qquad (4.28)$$

This implies that:

$$\text{prob}\{\underline{z}_{12} > 0 | (\underline{d}_{12,1} < -\underline{d}_{12,2})\} =$$
$$= \text{prob}\{-\underline{d}_{12,2}(\underline{\partial}_{12,1} - \underline{\lambda}_2)>0\} = 1/2 \qquad (4.29)$$

Thus, in conclusion:

$$\text{prob}(\underline{z}_{12} > 0) = 1/2 + 1/2\ .\ 1/2 = 3/4 \qquad (4.30)$$

Q.E.D.

Next, we will assume that we have cardinal information on one of the criteria, say criterion 1. Let us assume then the following effect matrix:

$$E = \begin{bmatrix} 20 & 1 \\ 8 & 2 \end{bmatrix} \qquad (4.31)$$

Then $\underline{d}_{12,1} = \frac{20 - 8}{20} = 0.6,$ (4.32)

so that:

$$\text{prob}(\underline{z}_{12} > 0) = \text{prob}\ \{(0.6 + d_{12,2}\ \lambda_2) > 0\}$$
$$= 0.6 - 6 \ln 0.6 \approx 0.9 \qquad (4.33)$$

The proof of the latter calculation can easily be given.

It is already known that $\underline{\lambda}_2$ and $\underline{d}_{12,2}$ are independently uniformly distributed on (0,1). Then we have:

$\text{prob}\ (0.6 + \underline{d}_{12,2}\ \underline{\lambda}_2 > 0) = \text{prob}(\underline{d}_{12,2}\ \underline{\lambda}_2 > -0.6) =$
$1 - \text{prob}(-\underline{d}_{12,2}\ \underline{\lambda}_2 > 0.6)$

Since we know that $-\underline{d}_{12,2}$ and λ_2 are independent uniformly distributed on (0,1), we find:

$1 - \text{prob}(-\underline{d}_{12,2}\ \underline{\lambda}_2 > 0.6) =$

$$\int_{0.6} \int_{1/x} dy\ dx = 1 - 0.4 - 0.6 \ln 0.6 \approx 0.9$$

5. Concluding Remarks

The foregoing analysis has demonstrated that for discrete choice problems which are marked by complete (or partial) uncertainty in the form of ordinal (or mixed) information the dominant regime method may be an operational tool. It leads to a probability statement regarding the choice of alternatives, and in so doing it leads usually to a unique solution (which is a major advantage compared to other 'soft' multiple criteria choice methods - like the concordance method -, which often do not lead to an unambiguous solution). Also its ability to deal with both qualitative information (including ties) and mixed information, makes it a powerful vehicle for evaluation analysis, not only in the field of public choice theory but also in the field of consumer theory and marketing analysis. Various empirical applications (e.g., housing market, transportation and physical planning) have also demonstrated its usefulness in practical choice situations.

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